Indian Institute of Technolgy Indore
Semester: Spring
Course: Numerical Methods (MA-204)
Tutorial-4

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1. Prove that if A is positive definite, then $det(A) > 0$. (Hint: use the theorem on Cholesky decomposition)

2. Let $A = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$. $a)$ Prove that A is positive definite. (b) Calculate the Cholesky factor of A. (c) Find the other upper triangular matrices R such that $A = R^T R$. (d) Let A be $n \times n$ positive definite matrix. How many upper triangular matrices R such that $A = R^T R$ are there?

3. Let
$$A = \begin{bmatrix} 4 & -2 & 4 & 2 \\ -2 & 10 & -2 & -7 \\ 4 & -2 & 8 & 4 \\ 2 & -7 & 4 & 7 \end{bmatrix} \text{ and } b = \begin{bmatrix} 8 \\ 2 \\ 16 \\ 6 \end{bmatrix}.$$
(a)Use the inner product formulation of Cholesky's method to show that A is positive definite and compute Cholesky factor. (b) Use forward and back substitution to solve the system $Ax = b$.

4. Determine whether or not each of the following is positive definite.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 10 \\ 3 & 10 & 16 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 3 & 3 \\ 3 & 10 & 7 \\ 3 & 5 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 4 & 8 \\ 4 & -4 & 1 \\ 8 & 1 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

5. Use the Gauss-Seidel iteration method with initial approximation $(x_1, x_2, x_3) = (0, 0, 0)$ to approximate the solution to the system of equations.
$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3. \end{aligned}$$
Continue the iterations until two successive approximations are identical when rounded to three significant digits.

6. Which of the following systems of linear equations has a strictly diagonally dominant coefficient matrix?
(a) $\begin{aligned} 3x_1 - x_2 &= -4 \\ 2x_1 + 5x_2 &= 2 \end{aligned}$

(b) $\begin{aligned} 4x_1 + 2x_2 - x_3 &= -1 \\ x_1 \qquad + 2x_3 &= -4 \\ 3x_1 - 5x_2 + x_3 &= 3. \end{aligned}$

7. Apply the Gauss-Seidel method to the system
$x_1 - 5x_2 = -4$
$7x_1 - x_2 = 6,$
using the initial approximation $(x_1, x_2) = (0, 0)$, and show that the method diverges.

8. Interchange the rows of the above system to obtain one with a strictly diagonally dominant coefficient matrix. Then apply the Gauss-Seidel method to approximate the solution to four significant digits.

9. Using the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

10. $S_3(x)$ is the piecewise cubic Hermite interpolating approximate of $f(x) = \sin x \cos x$ in the abscissas 0, 1, 1.5, 2, 3. Estimate the error $\max_{0 \le x \le 3} |f(x) - S_3(x)|.,$

11. Obtain the Chebyshev linear polynomial approximation of first and second degree to the function $f(x) = x^3$ on $[0, 1]$.