

MA 204
Tutorial Sheet 3

1. Solve the following system of equations by Gauss- Jordan elimination method.

$$\begin{aligned}(a) \quad x + 2y + z &= 0 \\ 2x + 2y + 3z &= 3 \\ x + 3y &= -2 \\ (b) \quad 2x + 3y + z &= 9 \\ x + 2y + 3z &= 6 \\ 3x + y + 2z &= 8\end{aligned}$$

2. Solve the following linear systems by Gauss- Jordan method, with partial pivoting if necessary (but without scaling).

$$\begin{aligned}(a) \quad 4x + y + z &= 4 \\ x + 4y - 2z &= 4 \\ 3x + 2y - 4z &= 6 \\ (b) \quad 2x + 3y + z &= 9 \\ x + 2y + 3z &= 6 \\ 3x + y + 2z &= 8\end{aligned}$$

3. Find the inverse of the following matrices by the Gauss- Jordan method:

$$(a) \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} \quad (b) \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad (c) \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

4. Solve the following systems of equations by (Gauss-Jordan) matrix inversion method, Doolittle's and Crout's methods. Are the answers you are getting in each method the same?

$$\begin{aligned}(a) \quad x + y &= 0 \\ y + z &= 1 \\ x + z &= 3 \\ (b) \quad x + y + z &= 2 \\ 2x + 2y + 3z &= 7 \\ 5x - y + 13z &= 0\end{aligned}$$

5. Solve the following systems of equations by Doolittle's and Crout's methods:

$$\begin{aligned}(a) \quad 10x + y + z &= 12 \\ x + 10y + z &= 12 \\ x + y + 10z &= 12 \\ (b) \quad 3x + 5y + 2z &= 8 \\ 8y + 2z &= -7 \\ 6x + 2y + 8z &= 26\end{aligned}$$

6. Verify whether the following matrices are positive definite.

$$(a) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix} \quad (b) \begin{pmatrix} 15 & 4 & -2 & 9 & 0 \\ 4 & 7 & 1 & 1 & 1 \\ -2 & 1 & 18 & 6 & 6 \\ 9 & 1 & 6 & 19 & 3 \\ 0 & 1 & 6 & 3 & 11 \end{pmatrix}$$

7. Solve the following systems of equations by Cholesky's method, if the method is applicable. If it is not applicable, give the reason.

$$(a) \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ 8 \end{pmatrix} \quad (b) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14 \\ 20 \\ 14 \end{pmatrix}$$

8. Solve the following system of equations by Gauss- Jacobi method with initial approximation other than $(1, 1, 1)$:

$$\begin{aligned} 8x + 2y - 2z &= 8 \\ x - 8y + 3z &= -4 \\ 2x + y + 9z &= 12 \end{aligned}$$

9. Solve the following linear system by Jacobi method with

$$X^{(0)} = (0 \ 0 \ 1)^T$$

Find out three iterations:

$$\begin{aligned} 10x_1 + 3x_2 + x_3 &= 14 \\ 2x_1 - 10x_2 + 3x_3 &= -5 \\ x_1 + 3x_2 + 10x_3 &= 14 \end{aligned}$$