**Indian Institute of Technolgy Indore**
**Semester: Spring**
**Course: Numerical Methods (MA-204)**
**Tutorial-2**

---

1. The following data represents the function $f(x) = e^x$.

| $x$ | 1 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|
| $f(x)$ | 2.7183 | 4.4817 | 7.3891 | 12.1825 |

Estimate the value of $f(2.25)$ using the $(i)$ Newton's forward difference interpolation and $(ii)$ Newton's backward difference interpolation. Compare with the exact value. Obtain the bound on the truncation error.

2. Show that the truncation error on quadratic interpolation in an equidistant table is bounded by $\left(\frac{h^3}{9\sqrt{3}}\right) max|f'''(\zeta)|$.

3. In the following problems, find the maximum value of the uniform mesh size h that can be used to tabulate $f(x)$ on $[a, b]$ ,using quadratic interpolation such that $|\ Error\ | \leq \varepsilon$.

   $(i)$ $f(x) = (2 + x)^4, \quad [a, b] = [1, 2], \quad \varepsilon = 10^{-4}.$
   $(ii)$ $f(x) = x^2 e^x, \quad [a, b] = [0, 1], \quad \varepsilon = 5 \times 10^{-6}.$

4. Determine the step size that can be used in the tabulation of $f(x) = \sin x$ in the interval $[0, \frac{\pi}{4}]$ at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than $5 \times 10^{-8}$.

5. In the following problems, find the maximum value of the uniform mesh size h that can be used to tabulate $f(x)$ on $[a, b]$ ,using cubic interpolation such that $|\ Error\ | \leq \varepsilon$.

   $(i)$ $f(x) = e^x, \quad [a, b] = [1, 2.5], \quad \varepsilon = 10^{-4}.$
   $(ii)$ $f(x) = \cos 2x, \quad [a, b] = [0, \frac{\pi}{4}], \quad \varepsilon = 10^{-6}.$

6. A function $f(x)$ is approximated by the interpolating polynomial

$$P(x) = c_0 + c_1(x - 1) + c_2(x - 1)^2 + c_3(x - 1)^3, 1 \leq x \leq 2.$$

Determine the parameters $c_0, c_1, c_2$ and $c_3$ such that $P(1) = f(1), P(2) = f(2)$, $p'(1) = f'(1)$ and $p'(2) = f'(2)$.

7. Construct the Hermite interpolation polynomial that fits the data.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 4 | -6 | -22 |
| $f'(x)$ | -5 | -14 | -17 |

8. (a) Construct the Hermite interpolation polynomial that fits the data

| $x$ | 2 | 3 |
|---|---|---|
| $f(x)$ | 29 | 105 |
| $f'(x)$ | 50 | 105 |

Interpolate $f(x)$ at $x = 2.5$.

(b) Fit the cubic polynomial $P(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$ to the data given in part $(a)$. Are these polynomials same?

9. Obtain the quadratic splines representing the function defined by the following data.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ | 1 | 3 | 11 | 31 |

Assume $f''(0) = M(0) = 0$. Interpolate at $x = 1.5$ and 2.5.

10. Obtain the cubic spline approximation valid in $[3, 4]$, for the function given in the tabular form

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 3 | 10 | 29 | 65 |

under the natural spline conditions $f''(1) = M(1) = 0$ and $f''(4) = M(4) = 0$.

11. Find whether the following functions are splines or not.

(i)

$$f(x) = \begin{cases} x^2 - x + 1, \; 1 \le x \le 2 \\ 3x - 3, \; 2 \le x \le 3. \end{cases}$$

(ii)

$$f(x) = \begin{cases} -x^2 - 2x^3, \; -1 \le x \le 0 \\ -x^2 + 2x^3, \; 0 \le x \le 1. \end{cases}$$

(iii)

$$f(x) = \begin{cases} -x^2 - 2x^3, \; -1 \le x \le 0 \\ x^2 + 2x^3, \; 0 \le x \le 1. \end{cases}$$

12. Find the values of $\alpha$ and $\beta$ such that the function

$$f(x) = \begin{cases} x^2 - \alpha x + 1, \; 1 \le x \le 2 \\ 3x - \beta, \; 2 \le x \le 3. \end{cases}$$

is a quadratic spline.