Indian Institute of Technolgy Indore
Semester: Spring
Course: Numerical Methods (MA-204)
Tutorial-5

In all the examples you can solve till 5 iterations.(if the number of iterations are not mentioned.)

1. Can you use the Bisection method to find a zero of $f(x) = x^3 - 3x + 1$ in the interval $[0, 2]$.
2. Use the Bisection method to find a zero of $f(x) = x^3 - 3x + 1$ in the interval $[0, 1]$.
3. $a = 6$, $b = 7$, $\epsilon = 0.0005$. How many iterations are needed such that $|x - r| \leq \epsilon$ (Bisection Method).
4. Use the Bisection method to find the root of $x = \cos x$ with an absolute error < 0.02 in the interval [0.5,0.9].
5. Solve $2x^3 - 2.5x - 5 = 0$ for the root in the interval [1,2] by Regula-Falsi method.
6. Find a root of $3x + \sin x - e^x = 0$ using Regula-Falsi method.
7. Find the roots between $[2, 3]$ of $x^3 - 2x - 5 = 0$, by using Regula-Falsi method.
8. Use the Secant method to find the root of $f(x) = x^6 - x - 1$ with two initial points $x_0 = 1$ and $x_1 = 1.5$.
9. Use the Secant method to find the root of $f(x) = \cos x + 2 \sin x + x^2$ with two initial points $x_0 = 0$ and $x_1 = -0.1$.
10. Find the root of $x^4 - x - 10 = 0$ using fixed point iteration scheme.

    (a) Consider $g_1(x) = \frac{10}{x^3 - 1}$ with initial guess 2.

    (b) Consider $g_2(x) = (x + 10)^{\frac{1}{4}}$ with initial guess 1,2 and 4.

    (c) Consider $g_3(x) = \frac{(x+10)^{\frac{1}{2}}}{x}$ with initial guess 1.8.

    What can you say about the convergence of $g_1(x), g_2(x)$ and $g_3(x)$.

11. In the diagram shown the floating ball has a specific gravity of 0.6 and has a radius of 5.5 cm. You are asked to find the depth to which the ball is submerged when floating in water.

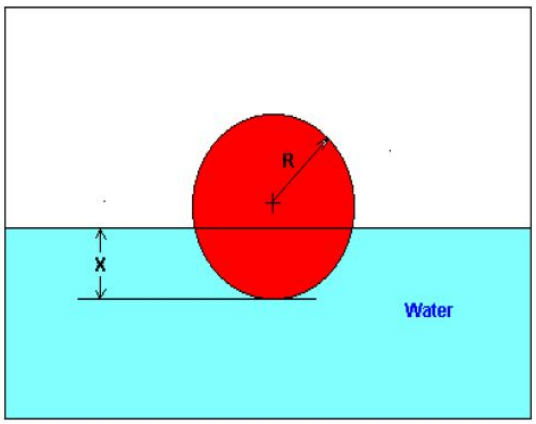

Figure 1: Diagram of the floating ball

The equation that gives the depth x to which the ball is submerged under water is given by $x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$.

(i) (a) Use the bisection method of finding roots of equations to find the depth x to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

(b) Find the absolute relative approximate error at the end of each iteration, and the number of significant digits at least correct at the end of each iteration.

(ii) (a) Use the Secant method of finding roots of equations to find the depth x to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

(b) Find the absolute relative approximate error and the number of significant digits at least correct at the end of each iteration.