Quantitative Management Modelling-Assignment 2

Topic: Mathematical formulation for LP problem

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By: Khushboo Yadav

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1. (Computer Center Staffing) You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.
   Time of day Minimum number of consultants required to be on duty 8 am–noon 4 Noon–4 pm 8 4 am–8 pm 10 8 am–midnight 6

   Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consultants are paid $14 per hour. Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

Solution:

Full Time:

Let, x1= number of full-time consultants working morning shift (8am-noon)

x2= number of full-time consultants working afternoon shift (12pm-8pm)

x3= number of full-time consultants working evening shift (4pm- midnight)

Part Time:

$y1$= no. of part-time consultants working first shift(8am-12pm)

$y2$= no. of part-time consultants working second shift (12pm-4pm)

$y3$= no. of part-time consultants working third shift(4pm-8pm)

$y4$= no. part-time consultants working fourth shift (8pm- midnight)

Objective function is:

Minimize = (\$14/hour)(8 hours)($x1+x2+x3$) + (\$12/hour)(4hours)($y1+y2+y3+y4$)

$= 112(x1+x2+x3) +48(y1+y2+y3)$

Constraints:

Subject to, $x1+y1>=4$

$x1+x2+y2>=8$

$x2+x3+y3>=10$

$x3+y4>=6$

$x1>=2y1$

$x1+x2>=2y2$

$x2+x3>=2y3$

$x3>=2y4$

and

$x1>=0, x2>=0, x3>=0, y1>=0, y2>=0, y3>=0, y4>=0$

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a) Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?

Solution:

Minimum no. of consultant required is 28 and the time slots are divided into 4 hours shift. . Thus, the total no. of employees for full time is 28/4=7.

Total no. of employees for part time work = 14.

As mentioned above, that one full time consultant should always be there for a part time consultant. This can lead to below constraints:

$x1>=y1$ (shift 1),
$x1+x2>=y2$ (shift 2),
$x2+x3>=y3$ (shift 3),
$x3>=y4$ (shift 4)

Since, part-time workers cost less per hour in comparison to full-time consultants, we should try to maximize their number in total.
Therefore, the numbers of part-time workers are $y4=3$, $y3=5$, $y2=4$ and $y1=2$
Similarly, based on part time workers, full time workers will be $x1=2$, $x3=3$, $x2=2$

Therefore, minimum-cost daily staffing plan = total full-time cost + total part-time cost

Total cost which must minimized= $112*(x1+x2+x3) + 48*(y1+y2+y3+y4)$
Therefore, the minimum cost of staffing is $= 112*(2+3+2) +48*(3+5+4+2)$
$= 784 + 672$
$=1456$

Answer: The minimum cost will be $1456 with 7 full time consultants and 14 part time consultants.

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b. After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. Full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Solution:
Full-time workers are entitled to 1hr lunch break during their 8-hr. shift. Assuming that if 1 hr. lunch break is considered in their 8-hr. shift but not for billing.
As we know that, the full-time consultant is paid $14/hr. rate and they work 7hrs (8hrs-1hrs)
=($14/hr.) *(7)
=$98

Therefore, salary per head for full-time consultant is $98 a day and for the 7 members in full-time will be $98*7= $686

Since, part-time consultants are not getting lunch break, cost for them will remain same i.e., $672.

Therefore, minimum cost daily staffing plan are:
= total full-time cost + total part-time cost
=686+672
= 1358

Answer:
The minimum cost daily staffing plan is $ 1358. The minimize cost for full-time consultant is $98.

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2. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a longterm contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

Solution:

The decision variables are:

X1 = number of collegiates
X2 = number of mini backpacks

Objective function is: Maximize Z=32X1+24X2

Constraints: Subject to, 3X1+2X2≤5000
X1≤1000
X2≤1200
45X1+40X2≤84000

and X1, X2≥0

Graphical solution:

Since, the problem has only 2 decision variables, we can use graphical method to solve it. Below are the coordinates of constraints which will be used to plot on the graph:

C1: 3x1+2x2 <= 5000

X1	1666.6	0
X2	0	2500

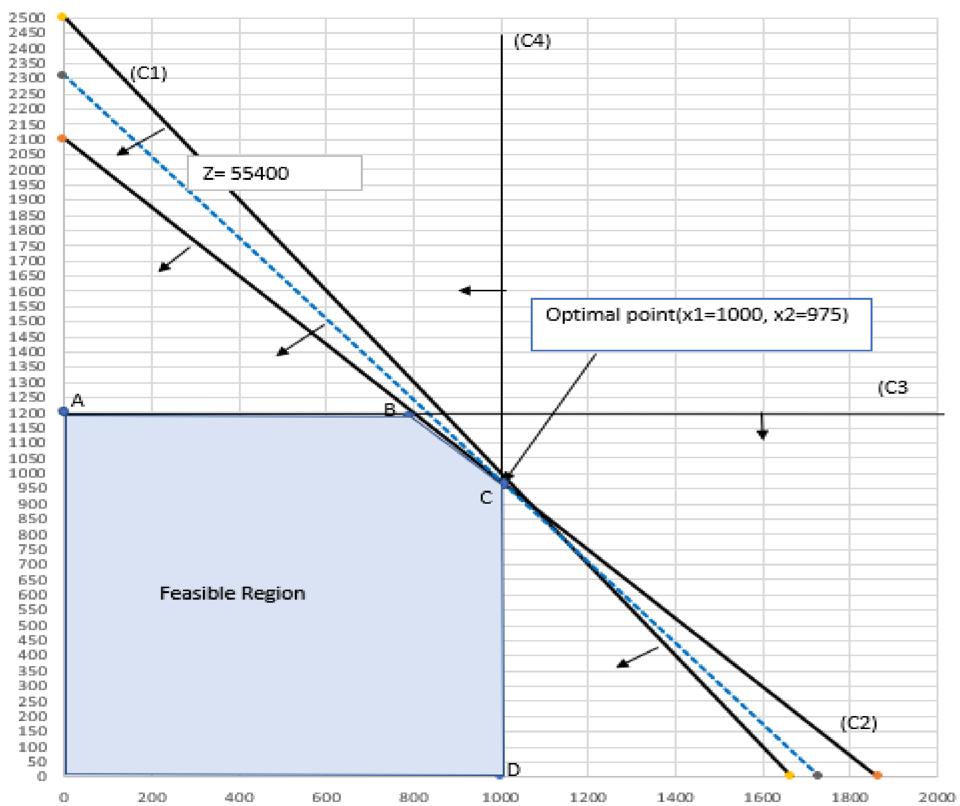
C2: 45x1+40x2<=84000

X1	1866.6	0
X2	0	2100

C3: x1<=1000
C4: x2<=1200

Based on the plotted results, feasible region with intersection points A, B, C, D is found.
A= (0,1200),
B= (800,1200),
C= (1000,975),
D= (1000,0)

Applying objective function into the graph we can determine the optimal points.



Answer:
 The optimal point is (1000,975). Therefore, applying that to the objective function:
Z = 32(x1)+24(x2)
   = 32(1000) + 24(975)
   = 55400.
Hence, the number of collegiate is 1000 , mini backpacks is 975 producing maximum profit of $55400.

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3. (Weigelt Production) The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs, if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.
a. Define the decision variables
b. Formulate a linear programming model for this problem.
c. Solve the problem using lpsolve, or any other equivalent library in R.

Solution:
Considering that the plant 1 produces X1, Y1 & Z1 units of size large, medium and small respectively. Plant 2 produces X2, Y2 & Z2 units of size large, medium and small respectively. Similarly, Plant 3 produces X3, Y3 & Z3 units of size large, medium and small respectively.

the Total Profit can be calculated as:
$Z = 420X1 + 360Y1 + 300Z1 + 420X2 + 360Y2 + 300Z2 + 420\ X3 + 360\ Y3 + 300\ Z3$

As mentioned above, plant 1 can produce 750 units per day. And plant 2 & 3 can produce 900 and 450 units respectively. Therefore,
$X1 + Y1 + Z1 \leq 750,$
$X2 + Y2 + Z2 \leq 900,$
$X3 + Y3 + Z3 <= 450$

Also, Plant 1, 2 & 3 have 13000, 1200 & 5000 square feet of space available respectively for in-process storage of a day's production. And each unit of large, medium and small size products require 20, 15 & 12 square feet respectively. Therefore,

$20X1 + 15Y1 + 12Z1 \leq 13000$
$20X2+15Y2 + 12Z2 \leq 12000$
$20X3 + 15Y3 + 12Z3 \leq 5000$

The sales forecast indicates that 900, 1200 & 750 units of large, medium and small sizes respectively are sold per day. Therefore,
$X1 + X2 + X3 \leq 900$
$Y1 + Y2 + Y3 \leq 1200$
$Z1+ Z2 + Z3 <= 750$

Also, to avoid layoffs, the plants should use the same percentage of their excess capacity to produce the new products. Therefore,

$1/750(X1 + Y1 + Z1)– 1/900(X2 + Y2 +Z2) = 0$
i.e., 900 X1 +900 Y1 +900 Z1-750 X2 -750 Y2 -750 Z2= 0

$1/900(X2 + Y2 + Z2)– 1/450(X3 + Y3 + Z3) = 0$
i.e., 450 X2 +450 Y2 +450 Z2 -900 X3 -900Y3 -900 Z3 = 0

Decision variables:

X1 = number of large units produced per day at Plant 1
Y1 = number of medium units produced per day at Plant 1
Z1 = number of small units produced per day at Plant 1
X2 = number of large units produced per day at Plant 2
Y2 = number of medium units produced per day at Plant 2
Z2 = number of small units produced per day at Plant 2
X3 = number of large units produced per day at Plant 3
Y3 = number of medium units produced per day at Plant 3
Z3 = number of small units produced per day at Plant 3

Objective:

Maximize $Z= 420\ X1 + 360\ Y1 + 300\ Z1 + 420\ X2 + 360\ Y2 + 300\ Z2+420X3+360Y3+300Z3$

Constraint:

subject to
$X1 + Y1 + Z1\leq 750$
$X2 + Y2 + Z2\leq900$
$X3 + Y3 + Z3\leq450$
$20X1 + 15Y1 + 12Z1\leq13000$
$20X2 + 15Y2 + 12Z2\leq12000$
$20X3 + 15Y2 + 12Z3\leq5000$
$X1 + X2 + X3\leq900$
$Y1 + Y2 + Y3\leq1200$
$Z1 + Z2 + Z3\leq750$

$900 X1 + 900 Y1 + 900 Z1 - 750 X2 - 750 Y2 - 750 Z2 = 0$
$450 X2 + 450 Y2 + 450 Z2 - 900 X3 - 900 Y3 - 900 Z3 = 0$
And
$X1, X2, X3, Y1, Y2, Y2, Z1, Z2, Z3 >= 0$