Quantitative Management Modelling-Assignment 3

Topic: Conduct post-optimality and sensitivity analysis for an LP problem.

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The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

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1. **Solve the problem using lpsolve, or any other equivalent library in R.**

rmd and lp file attached.

2. Identify the shadow prices, dual solution, and reduced costs

Answer:

The shadow price of each constraint variable tells how much objective function value will change for one unit increase in that constraint. It is an estimated price for something that is not normally priced or sold in the market.

R code for shadow price provides a numeric vector of length m+n (where m is the number of constraints in m and n is the number of decision variables in lprec)

Shadow Price for the below mentioned constraints and decision variable:

Constraints:	
Plant1_Production	0.0 (non-Binding)
Plant2_Production	0.0 (non-Binding)
Plant3_Production	0.0 (non-Binding)
Plant1_Storage_Space	12
Plant2_Storage_Space	20
Plant3_Storage_Space	60
Plant1_Sales_Forecast	0.0 (non-Binding)
Plant2_Sales_Forecast	0.0 (non-Binding)
Plant3_Sales_Forecast	0.0 (non-Binding)
Capacity1	0.20
Capacity2	0.466
Decision Variable:	
Plant1_Large	0.0 (non-Binding)
Plant1_Medium	0.0 (non-Binding)
Plant1_Small	-24.0
Plant2_Large	-40
Plant2_Medium	0.0 (non-Binding)
Plant2_Small	0.0 (non-Binding)
Plant3_Large	-360
Plant3_Medium	-120
Plant3_Small	0.0 (non-Binding)

We can see several constraints with zero shadow price, it indicates that an additional unit does not increase profits or objective function. These are non- Binding constraint since a change in its RHS does not affect the optimal solution or objective function at all. The shadow price of a constraint is defined for a "one unit" change in the constraint.

Reduced Cost: -

It is the amount by which an objective function coefficient would have to improve (increase for maximization, decrease in minimization)

R code for reduced cost provides a list of size n (where n is the number of decision variables in lprec) containing the values of the lower limits and upper limits of the objective function.

	3.60e+02	<=	Plant1_Large	<=	4.60e+02
	3.45e+02	<=	Plant1_Medium	<=	4.20e+02
	-1.00e+30	<=	Plant1_Small	<=	3.24e+02
	-1.00e+30	<=	Plant2_Large	<=	4.60e+02
	3.45e+02	<=	Plant2_Medium	<=	4.20e+02
	2.52e+02	<=	Plant2_Small	<=	3.24e+02
	-1.00e+30	<=	Plant3_Large	<=	7.80e+02
	-1.00e+30	<=	Plant3_Medium	<=	4.80e+02
	2.04e+02	<=	Plant3_Small	<=	1.00e+30

3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.

Range for Shadow Prices:

-1.00E+30	<=	Plant1_Production	<=	1.00E+30
-1.00E+30	<=	Plant2_Production	<=	1.00E+30
-1.00E+30	<=	Plant3_Production	<=	1.00E+30
1.04E+04	<=	Plant1_Storage_Space	<=	1.39E+04
1.00E+04	<=	Plant2_Storage_Space	<=	1.25E+04
4.80E+03	<=	Plant3_Storage_Space	<=	5.40E+03
-1.00E+30	<=	Plant1_Sales_Forecast	<=	1.00E+30
-1.00E+30	<=	Plant2_Sales_Forecast	<=	1.00E+30
-1.00E+30	<=	Plant3_Sales_Forecast	<=	1.00E+30
-4.00E+04	<=	Capacity1	<=	5.00E+04
-1.50E+04	<=	Capacity2	<=	3.00E+04
-1.00E+30	<=	Plant1_Large	<=	1.00E+30
-1.00E+30	<=	Plant1_Medium	<=	1.00E+30
-8.61E+02	<=	Plant1_Small	<=	1.11E+02
-1.00E+02	<=	Plant2_Large	<=	2.50E+02
-1.00E+30	<=	Plant2_Medium	<=	1.00E+30
-1.00E+30	<=	Plant2_Small	<=	1.00E+30
-5.00E+01	<=	Plant3_Large	<=	2.50E+01
-1.33E+02	<=	Plant3_Medium	<=	6.67E+01
-1.00E+30	<=	Plant3_Small	<=	1.00E+30

Range for Reduced Price:

	3.60e+02 <= Plant1_Large <= 4.60e+02
	3.45e+02 <= Plant1_Medium <= 4.20e+02
	-1.00e+30 <= Plant1_Small <= 3.24e+02
	-1.00e+30 <= Plant2_Large <= 4.60e+02
	3.45e+02 <= Plant2_Medium <= 4.20e+02
	2.52e+02 <= Plant2_Small <= 3.24e+02
	-1.00e+30 <= Plant3_Large <= 7.80e+02
	-1.00e+30 <= Plant3_Medium <= 4.80e+02
	2.04e+02 <= Plant3_Small <= 1.00e+30

3. **Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?**

We have 11 constraints in the primal solution, so we will have 11 decision variables each constraint, also, we have 9 decision variables in above problem, so we will have 9 constraints in its dual relates to each decision variable.

The solution to the dual problem provides a lower bound to the solution of the primal (minimization) problem.

Since, we have 11 decision variables in its dual. Let's say v1, v2, v3, v4, v5, v6,v7,v8,v9,v10,v11 are variables which are also non-negative. Variable v10 and v11 are unrestricted in sign because its constraints have equality sign. Since these variables are unrestricted therefore, they can have a positive sign as well as negative.

Minimize: 750v1 +900v2+450v3 +13000v4 + 12000v5 +5000v6 +900v7 +1200v8 +750v9 +0v10 +0v11

Subject To: v1+ 20 v4 + v7 + 900 v10 + 450 v11 >=420

v1+15 v4 + v8 +900 v10 +450 v11>=360

v1 +12 v4 + v9 + 900 v10+ 450 v11>=300

v2 + 20 v5 + v7 - 750 v10 >=420

v2 +15 v5 + v8 - 750 v10 >=360

v2 + 12 v5 + v9 - 750 v10 >=300

v3 + 20 v6+ v7 - 750 v11 >=420

v3 + 15 v6 + v8 - 750 v11 >=360

v3 + 12 v6+ v9 - 750 v11 >=300

here, v1, v2, v3,v4,v5,v6,v7,v8,v9 >= 0 and v10,v11 are unrestricted in sign.

Answer:

The objective function value for the dual problem using this solution is the same as the objective function value for the primal problem with the corresponding solution. The **Z value came out to be exact same for both primal and dual solutions.**

Therefore, the dual solution corresponding to the optimal primal solution is both optimal and feasible. Hence, the solution agrees with what we observed for the primal problem.