Theoretical Part A

$$P(\text{price} \mid L, S) = \frac{P(L \mid \text{Price}) \cdot P(S \mid \text{price}) \cdot P(\text{Price})}{P(L) \cdot P(S)}$$

From the dataset

Total = 10

Exp = 4

Aff = 3

Cheap = 3

$\text{Probability} = \frac{\text{Count}}{\text{total}}$

$P(\text{exp}) = 4/10 = 0.4$

$P(\text{Aff}) = 3/10 = 0.3$

$P(\text{cheap}) = 3/10 = 0.3$

Conditional Probability

→ For Expensive

$P(L = \text{Urban} \mid \text{price} = \text{exp}) = \frac{2}{4} = 0.5$

$P(S = \text{med} \mid \text{price} = \text{exp}) = 0/4 = 0$

→ For Affordable

$P(L = \text{Urban} \mid \text{price} = \text{Aff}) = \frac{1}{3} = 0.333$

$P(S = \text{medium} \mid \text{Price} = \text{Aff}) = \frac{1}{3} = 0.333$

→ PTO

→ For cheap

$P(L = \text{Urban} \mid \text{Price} = \text{cheap}) = \frac{1}{3} = 0.33$

$P(L = \text{medium} \mid \text{Price} = \text{cheap}) = \frac{0}{3} = 0$

→ Now, using Naive Bayes formula

$P(\text{Price} \mid L = \text{urban}, S = \text{medium})$

$\propto \big(P(L = \text{urban} \mid \text{Price}) \times P(S = \text{medium} \mid \text{Price}) \times P(\text{Price})$

$P(\text{Exp} \mid L = \text{Urban}, S = \text{medium})$

$= 0.5 \times 0.0 \times 0.4$

$= 0$

$P(\text{Aff} \mid L = \text{Urban}, S = \text{medium})$

$= 0.33 \times 0.33 \times 0.3$

$= 0.0333$

$P(\text{cheap} \mid L = \text{Urban}, S = \text{medium})$

$= 0.333 \times 0.0 \times 0.3$

$= 0$

→ PTO

As, P(Exp) = 0 & P(cheap) = 0

Prediction of medium size house in Urban locality $\boxed{\text{Affordable}}$

Theoretical Part B

$$4x_1 - 3x_2 + x_3 = -10,$$
$$2x_1 + x_2 + 3x_3 = 0,$$
$$-x_1 + 2x_2 - 5x_3 = 17,$$

Matrix form

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & -10 \\ 2 & 1 & 3 & 0 \\ -1 & 2 & -5 & 17 \end{array}\right]$$

→ Apply Gaussian elimination

$$R_1 \to \frac{1}{4} R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -3/4 & 1/4 & -10/4 \\ 2 & 1 & 3 & 0 \\ -1 & 2 & -5 & 17 \end{array}\right]_1$$

$$R_2 \to R_2 - 2R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -3/4 & 1/4 & -10/4 \\ 1 & 1-(-3/2) & 3-\frac{1}{2} & 0-\left(\frac{-10}{2}\right) \\ -1 & 2 & -5 & 17 \end{array}\right]$$

$$R_3 \longrightarrow R_3 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -3/4 & 1/4 & -5/2 \\ 0 & 11/2 & 5/2 & 5 \\ 0 & 5/4 & -19/4 & -59/2 \end{array}\right]$$

$$R_2 = \frac{2}{11} R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -3/4 & 1/4 & -5/2 \\ 0 & 1 & 5/11 & 10/11 \\ 0 & 0 & -6 & -16.8 \end{array}\right]$$

$$R_3 \longrightarrow \frac{-1}{6} R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3/4 & 1/4 & -5/2 \\ 0 & 1 & 5/11 & 10/11 \\ 0 & 0 & 1 & 2.8 \end{array}\right]$$

$\longrightarrow$ Substitute in equations

$R_3 \longrightarrow \boxed{x_3 = 2.8}$

$R_2 \rightarrow x_2 + \frac{5}{11} x_3 = \frac{10}{11}$

$x_2 + \frac{5}{11}(2.8) = \frac{10}{11}$

$x_2 = \frac{10}{11} - \frac{14}{11}$

$\boxed{x_2 = -\frac{4}{11}}$

$R_1 \longrightarrow x_1 - \frac{3}{4} x_2 + \frac{1}{4} x_3 = -\frac{5}{2}$

$x_1 - \left(\frac{3}{4} \cdot -\frac{4}{11}\right) + \frac{1}{4}(2.8) = -\frac{5}{2}$

$x_1 + \frac{3}{11} + \frac{7}{10} = -\frac{5}{2}$

$x_1 = -\frac{5}{2} - \frac{7}{10} - \frac{3}{11}$

$\boxed{x_1 = -3.5}$