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CS 726 : HW1

1 (a) Prove that all trails not of the form $x \rightarrow \dots \leftarrow y$ are blocked by z.

(a) We can prove the above state using d-seperation rule. The other possible trails are:

$x \rightarrow \dots z \longrightarrow y$

$x \leftarrow z \dots \longleftarrow y$

$x \leftarrow \dots z \dots \rightarrow y$

In all these 3 cases, X & Y are blocked by Z because of d-seperation rule.

(b) Prove that all trails of the form $x \rightarrow w_1 \dots \leftarrow y$ with exactly one V-node is blocked.

(b) Using d-seperation principle

$x \rightarrow w_1 \leftarrow Y$ x & Y will be blocked if w_1 is not observed.

If the trail is like:

$x \rightarrow w_1 \dots \rightarrow w \leftarrow \dots y$ → Exactly one V-node

Z lies in this trail ⇒ There will be loop in the BN

~~⇒ There this will be~~ ⇒ It is not a V-node

Hence, the trail of these forms with exactly one V-node is blocked (We say this using d-seperation rule as shown in part (a))

(c) Prove that all trails of the form $x \rightarrow w_1 \dots \leftarrow y$ with more than one V-node is also blocked.

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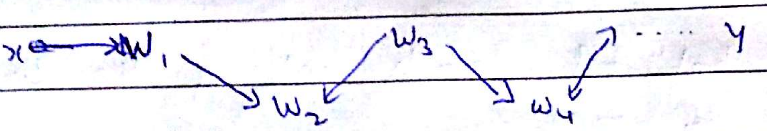
(c) Consider a general trail in a BN having multiple v-node



Suppose for any v-node, z is the part in that v-node. This will result in the formation of loop ($\because$ z = Pa(x) ∪ Pa(y)) But formation of loop is not allowed in the BN.

Hence, we cannot observe "z" for a valid BN and using d-seperation principle we claim that ~~the~~ trails of the above type would also be blocked.

3 (a) (i) $A \perp\!\!\!\perp G \mid F$

Correct

Reason:

node B & C are v-node

~~We are not observing B & C & hence this path A to G is~~

Consider the following path $A \to E \to F \to G$

Using d-seperation rule, we claim that

$A \perp\!\!\!\perp G \mid F$

(ii) $A \perp\!\!\!\perp D$

Correct

Reason:

~~B~~ We see that all the trails b/n A & D have a v-node. If we do not observe this v-node Then A & D are independent.

(iii) $B \perp\!\!\!\perp G \mid C, F$

Wrong

Consider the trail ~~BA~~ B — A — C — D — G

So using d-seperation rule B & G cannot be independent if you "observe" C (since it is a v-node)

(iv) F ⊥⊥ D | C

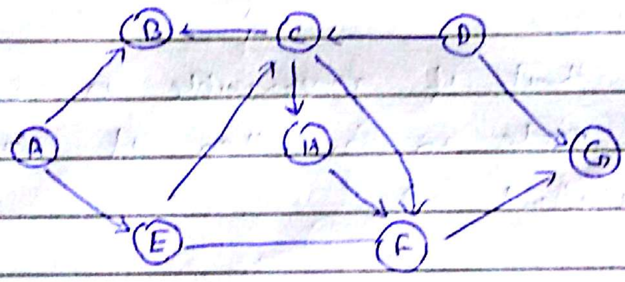
Wrong

Consider the trail F - E - C - D

Here, C is a v-node & you are observing C.

Hence using d-seperation rule F & D become dependent given C.

(b)



We can go to H from G using the two path

Path 1

G → D → C → H

D block the trail b/n H & G (using D-seperation) Hence D ∈ S

Path 2

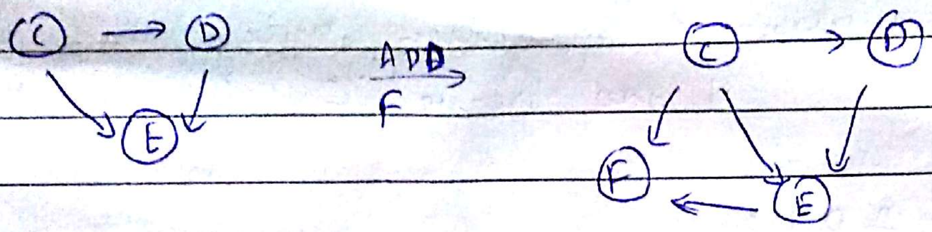
G → F → H

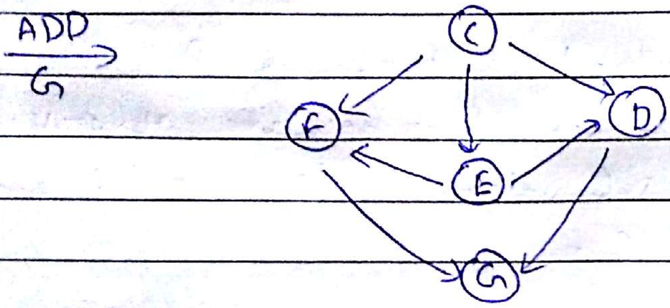
F also blocks the trail b/n G & H. Moreover there is ~~not~~ no v-node in the path.

F ∈ S

Hence set S = {D, F}

(c) Start adding node in topological order & connect the edges.



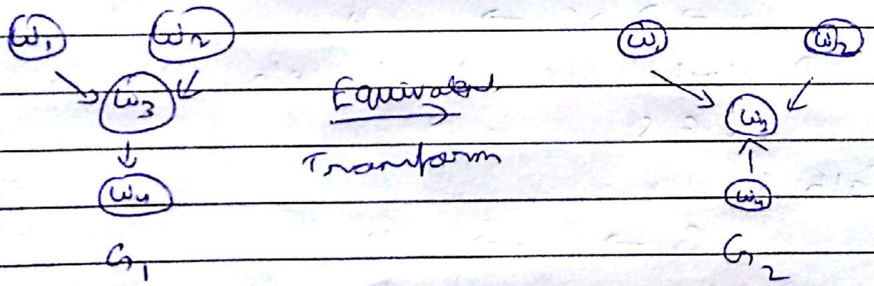


4. (a) Prove that if a variable is not a descendent of a V-node in G_1, then it cannot be a descendent in equivalent G2.

Consider a V-node

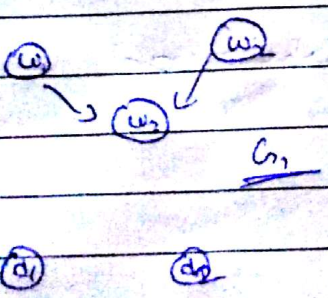
Case 1:

G_1: Assume V-node has a descendent



In G_2, the transformation is possible but w_4 has become parent. <u>V-node is destroyed</u>

Case 2: node



w_3 has ~~not~~ no descendent d_1 & d_2 are non connected

If we transform this graph to equivalent graph G_2. G_2

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cannot have a descendent because of the property of equivalent graph that they should have same skeleton.

(b) In equivalent graph G_1 & G_2 V-nodes are preserved.

Hence if in G_1, ~~there~~ ~~is~~ a node is not a V-node, then after transformation, that node cannot become V-node.

Consider the following 3 type of trails ~~ab~~

X .. ← Z ← .. Y
X .. → Z → .. Y
X .. ← Z → .. Y

~~The~~ X & Y are d-seperate

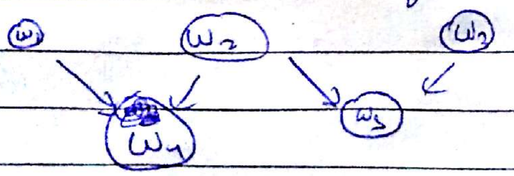
If we transform (equivalent transform) these type of trail we get ~~similar~~ the following 3 types of trails are possible.

X .. ← Z ← Y
X .. → Z → .. Y
X ← Z → Y

The resultant transform will also ~~have~~ satisfy d-seperate rule.

We already know that V-nodes are preserved.

2. Consider a BN having V-node for ex



This type of graph will have O(n) edges

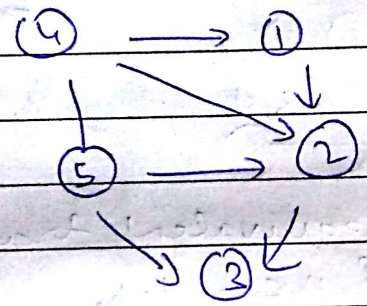
Do the transformation in the following way

(1) Pick the V-node one by one in topological order

(2) After all the V-node gets exhausted pick the other node topological.

This transformation will cause $O(n^2)$ edge with adversial ordering of vertices

For ex → After transformation the previous graph would be



This holds for a general graph having V-nodes.