CS 726: Homework 3 (Due Sep 1 , 2016)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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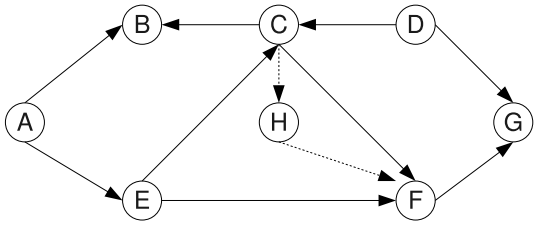
1. In a Bayesian network the potentials are over a variable and its immediate parents. For the graph below (ignore variable H and its dotted lines)



   (a) List all the potentials.

..1

   (b) Draw the initial induced graph at the start of a VE algorithm run.

..1

   (c) Show the induced graph after running the VE algorithm on variables A, B, E, C in that order.

2. In class we presented the variable elimination algorithm for computing marginal probability $\Pr(x_i)$ for one variable x_i. Now, suppose you have to compute $\Pr(x_i)$ and $\Pr(x_j)$ for two different x_i and x_j.

   (a) Present a sketch of an efficient algorithm for this. The standard VE algorithm for computing $\Pr(x_i)$ is given below.

   1: $\mathcal{F}^1$=set of factors in the graphical model.
   2: G^1=Initial graph after triangulation.
   3: **for** $t = 1 \ldots n-1$ **do**
   4: $\quad \pi_t = v =$ a simplicial vertex in G^t other than x_i
   5: $\quad \mathcal{F}_v = \{F \in \mathcal{F}^t s.t. v \in F\}$
   6: $\quad m_v = \sum_v \prod_{F \in \mathcal{F}_v} F()$
   7: $\quad \mathcal{F}^{t+1} = \mathcal{F}^t - \mathcal{F}_v + \{m_v\}$
   8: $\quad G^{t+1} = G^t - \{v\}$
   9: **end for**
   10: $\Pr(x_i) =$ product of factors in $\mathcal{F}^n$ suitably normalized.

   Your algorithm should use the same notations and may be stated in terms of modifications to the numbered steps above.

(b) Provide a graphical model and an x_i, x_j where you cannot reuse any work between computing $\Pr(x_i)$ and $\Pr(x_j)$

..2

(c) Can you provide a general characterization of cases such as above?

..2

(d) For what cases, do you get maximum reuse of work across the two?

..2

3. Let G be a chordal graph with a junction tree T. We add an edge between two non-adjacent vertices x_i, x_j in G to get a new graph G'.

(a) Show how you will modify T to get the junction tree T' for G' under the following cases. Justify your answer.

i. x_i, x_j are simplicial and their neighbors in G are the same.

..2

ii. There is no path between x_i and x_j in G.

..2

(b) Show a case where T and T' have no cliques in common and T has at least three nodes.

..3

Total: 21