CS 726: Homework 2 (Due August 25 , 2016)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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1. Consider a distribution over four variables $V = \{x_1, x_2, x_3, x_4\}$ such that $\forall\ i, j,\ x_i \perp\!\!\!\perp x_j$ but no other CI holds. Indicate what happens when you attempt to draw a network via each of the three methods below

   (a) BN using the fixed order algorithm.

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   (b) MRF using the Markov blanket method.

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   (c) MRF using the pairwise conditional independence method

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(d) Give a numerical example of such a distribution over three variables.

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2. Let G be a BN over variables $V = \{x_1, \ldots, x_n\}$. Let $MB_G(x_i) = Pa(x_i) \cup Ch(x_i) \cup Sp(x_i)$. Let $Y_i = V - MB_G(x_i)$ Show that $x_i \perp\!\!\!\perp Y_i | MB_G(x_i)$. [Hint: use d-separation and consider different types of trails]

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3. Let $P(x_1, \ldots, x_n)$ be a positive distribution and H be a graph over nodes $x_1, \ldots, x_n$ such that in P, $x_i \perp\!\!\!\perp x_j | V - \{x_i, x_j\}$ for all (x_i, x_j) that are not adjacent in H. That is, P satisfies the pairwise CIs over H. Prove that $x_i \perp\!\!\!\perp V - \text{nbr}_H(x_i) | \text{nbr}_H(x_i)$ where $\text{nbr}_H(x_i)$ denotes the set of neighbors of x_i in H. That is, P satisfies the local CIs over H. [Hint: For positive distributions If $A \perp\!\!\!\perp B | C, D$ and $A \perp\!\!\!\perp C | B, D$ then $A \perp\!\!\!\perp (B, C) | D$]

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4. Start with a grid graph with three vertices on each side — that is a total of 9 vertices and 12 edges. Treat this as an undirected graphical model.

   (a) Triangulate this graph so that it is now chordal. [Hint: you will need more than four edges to triangulate the graph.]

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   (b) Use the simplicial vertices based algorithm to convert the above chordal graph into a Bayesian network.

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$\boxed{\textbf{Total: 20}}$