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SE367 - Introduction to Cognitive Science
Project

Analysing Mathematical Abilities in Infants

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1 Introduction

Inferring causal relations from a given set of instances, application of Bayes Rule as a mathematical phenomenon is introduced to us quite late in life. Often one restricts this just to the mere mathematical formulae or statistical data. However, we must realize that the ability to skilfully use data for inferring resides in us even if we are not aware of its existence. Every human being, irrespective of educational background, scientific ability, age or social involvement utilises the concept of learning from instances presented in our day to day lives. Based on the distribution observed and inferences drawn, we base our plans and actions. Surprisingly, even young infants of the age of 8 months have adequate ability to observe and learn from their surroundings. We must also take into account the finding that infants only attribute goals to actions when unambiguous information about means selection can be obtained. In this study, we aim to analyse the ability of infants around the age of 8-12 months to predict the optimum action, once they are provided varying sample actions. Specifically, we aim to test whether babies have the innate capability of inferring the unitary method rule.

2 Literature Review

Several experiments have been conducted to rate the quality of abilities possessed by the infants and have given quite a high accuracy of children predicting the likely outcomes or being surprised by the occurrence of a less likely or unfavourable outcome. Most of these experiments include binary outcomes, as in the case of two different coloured toys [1]. The authors, Hyowon Gweon and Laura Schulz, from the Early Childhood Cognition Lab at MIT show through two experiments that infants have the ability to track the statistical dependence between agents, objects and outcomes and can use minimal data to draw inferences that support rational action. In their particular experiment, they show how an infant infers whether the agent or the object is faulty through its actions in the experimental settings. These experiments expose the baby to very less data points and yet the authors observe remarkable inference abilities through their behaviour. In [2] the authors build upon their work in [1] and provide a Bayesian inference model hypothesis of children's inductive inference abilities. They study and propose different methods of knowledge acquisition in children and argue that the acquisition method is as important as the representation for performing various tasks. Other studies analyse the ability of infants to infer the mean optimum action from a set of given instances, [3] if they are given enough information. In other words, it has been suggested that infants can, in most instances, derive and infer the actions from a set of continuous actions, leading to a goal. [4] present an interesting paper titled 'Occam's Rattle', where the authors explore and show that children prefer simpler probabilistic explanations (simple quantified by the number of causes in an explanation) over other competing explanations during inference. This is very similar to the Occam's Razor Principle which is very widely used in Information Theory and Data Science.

3 Proposed methodology

In order to study the variation in behaviour of infants as young as a year, we need to devise an experiment that could actually be convincing enough to engage them. The responses from adults can be easily collected and analysed. However, getting infants to do the required task and inferring their responses is another challenge. Mood swings and unpredictable interests of the infants need to be taken into consideration while devising the methodology suitable for them. Since study show that infants consider both the sample and the sampling process in inductive generalization [5], we need to work out an effective procedure for the experiment. Keeping in mind the aforementioned constraints, we devise an experiment that could hopefully lend us reliable data for analysing the ability of infants to apply the unitary method (or proportional mean calculation).

Ability to apply the unitary method for calculating the suitable value for a given x, when several instances of pairs (x, y) are given, can also be considered as learning a function $f(x)$ and then calculating its value at a specific x. Given some instances spread out in the dataset, one can look at it as inferring the mean value optimum for the given condition, once the relevant values are known for some other instances. Unitary method implies this function we need to learn is a linear one, and in fact most instances of everyday life have a simple linear aspect to them. Thus our model also tends to explore the ability of linear function estimation in our participants. This approach could have 2 models - one that simply tries to extrapolate information, ie given some x_i, y_i pairs, our test point lies on either ends of $x_1, x_2 ... x_n$. In this case the participant simply needs to predict a value greater than the max y_i or smaller than the min y_i. However in order to test the presence of unitary method analysis, this method would prove inefficient. Therefore, our model proposes the testing of points which lie in between the initial samples shown. This would allow us to aptly justify whether the choice is merely a guess according to raised (or decreased) quantity or if it is an attempt to approximate the unitary calculation.

3.1 Participants

The prospective participants for our experiment are infants of the age of 12-13 months. We choose the infants of this age since we aim to test the youngest possible babies for these skills in order to aptly test and wonder at the unacknowledged talent and beauty of human mind. Also, babies at this age can be expected to participate in the experiment a bit more willingly than the younger ones, and tend to offer less instances of unaccountable impulsive responses, which could otherwise hamper the analysis (act as noise in the data). We'd prefer to keep the noise to as low as possible since the requirement of data for proper inference increases with the increase in noise.

3.2 Experimental Design

3.2.1 Step 1

For achieving the required goal, we propose a setup wherein we hope to grab the attention of the babies and arouse the appropriate interest in them to carefully view and infer the variation of the function.
We shall depict our x_i values by number of fishes, while y_i shall be the number of food granules given to them. For a given number of fishes (x_i), we shall depict that only a specific quantity of food is optimal (giving positive feedback), while anything more

or less than that represents an unwanted condition, making all the fishes sad (negative feedback). Initially, we show x_i fishes and an output of y_i food granules as mentioned above. Next, we bring in another x_j, comparatively much higher or much lower than x_i and vary the number of granules as earlier. This time we proportionally change the value of optimum food granules. Now a number between x_i and x_j is chosen as the test case x. We ask the baby to tell us what quantity of granules should be provided. We give the babies options of potential candidates of food quantities to select from. Our hypothesis is that if the value guessed by the babies lies within y_i and y_j, we can say that the babies have a basic sense of proportion and that this sense is not just a rough categorical model but applies approximately to the actual proportion model. This basic experiment would allow us to proceed further into testing the accuracy of their ability. Since the babies cannot count exactly, we definitely can't expect them to guess the exact value. This is why our initial experiment shall be implemented using widely varied data points, with a large scope of positive values between them.

3.2.2 Step 2

Now, we vary the number of fish (x_j) with a comparatively smaller step size (maximum difference between x_j's (as compared to step 1, while proportionally varying the optimal food granules. The performance of babies is recorded for a specific step size for various x_i, y_i values. We also vary the closeness of the potential values of y to the optimum one. This trend shall be repeated for a sufficient number of times with increasingly small step sizes. The performance variation observed with the varying parameters would assist in providing a quantitative experimental measure of the accuracy of the mathematical ability possessed by a young brain.

3.2.3 Variations in Design

We propose various possible ways to depict the optimality of the particular y. This could be done by the production of a soothing cheerful music at the positive instance while the absence of any such event for the negative instances. Alternatively, the baby's parent can be asked to participate with her. Each time the value is the one intended, the parent would be elated and hug the baby, while the parent would be sad in the other instances. Since babies tend to imitate what they see and also learn to relate to the feelings of their parents at an early age, this would instill similar emotions in the baby. Thus, the babies would surely try to make their parent (or indirectly themselves) happy by choosing the exact optimum value of food granules.

The way the overall experiment is presented can also be varied. We could either have actual fishes in bowls and feed them, or else we could use cartoon representations of fishes and artificially shake them sending out beautiful bubbles through the water or depicting a shift to dullness to signify despair. Another possible variation is the conduction of this entire setup on a digital apparatus. Here, we could add different visual variations, amazing colours and celebration scene for the correct number, while a depressing decaying scene for the incorrect ones.

To record the response, we could use separate eye tracking devices, by installing the setups for the different y_is at a considerable distance, or in the case of the digital setup, we could make use of the web camera's eye tracking abilities. In the latter case, we may provoke the baby to touch the respective block of y that he considers optimum.

4 Procedure

4.1 Step 1

4.1.1 Train Phase

1. Show an image of x_i number of fish
2. Let the experimenter choose the correct option from three output options and depict positive feedback. The three output options are, the correct output y_i, a number of granules greater than y_i and a number of granules less than y_i
3. Show the same image and options again. This time, when the experimenter chooses the wrong option, negative feedback is given to the baby (as discussed above)
4. Repeat the above three steps for new x_j's for some k iterations which is very different from the previous value of x_i

4.1.2 Test Phase

5. Show an image of some x_k, where $x_k \in [min(x_i, x_j), max(x_i, x_j)]$
6. Place the screen close to the baby and ask the baby to indicate from its answer from three options, where option2 is the true value $option1 < min(y_i, y_j)$ and $option3 > max(y_i, y_j)$.
(Note: The order in which the options appear should not be kept constant, lest the baby learn the position of the correct answer)

4.2 Step 2

We primarily repeat the same steps as above, with some added variations as described in the Methodology section.

1. Repeat steps 1, 2 and 3 as above for various values of x with a minimum difference of k between any two x_i values.
2. Record the results, as in the test phase above with the 2 false candidates sufficiently apart from the actual value of y.
3. Repeat steps 1 and 2 with successively smaller values of k.
4. Repeat steps 1 and 3, but this time, increase the difficulty by bringing the false candidates presented closer in value to the optimum candidate. Record the observations.

5 Expected Results

For the experiment defined above, we present our estimated results. Since the experiment for the proposed behavioral aspect i.e. unitary method (or equivalently proportional understanding) in babies has not been conducted before, we can only report our expected results. What happens on the conduction of an actual experiment would hopefully match the patterns defined, but there always lies the possibility of completely unexpected observations.

5.1 Step 1

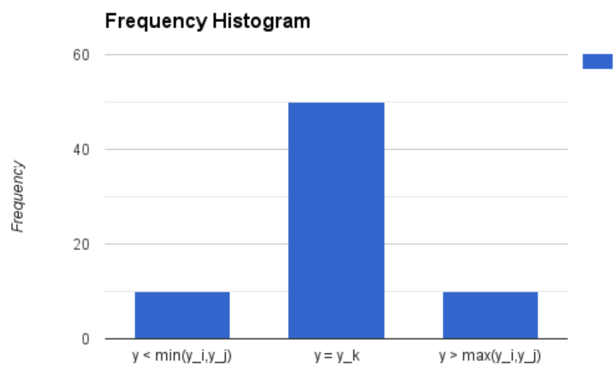

Figure 1: Expected Results on Step 1

Figure 1 shows the variation in responses for the first case, where in the presented values are the least confusing and the easiest to guess. But still we allot a finite and considerable probability to the wrong decisions, since what appears easy to us, may not be so for an infant. Since no particular inclination can be expected in general towards a lower or higher guess, we allot equal frequency to both.

5.2 Step 2

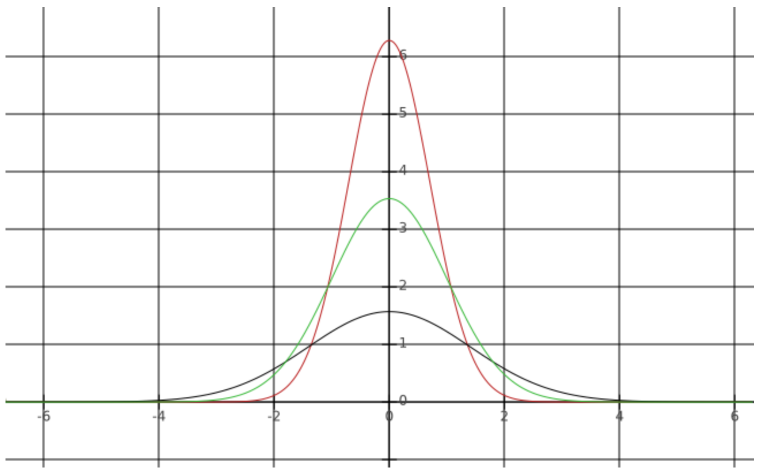

Figure 2: Expected Results on Step 2

The three Gaussians (Figure2) depict 3 different cases of the varying step sizes. The one with highest variance represents an instance of the smallest step size, while the low variance, high peak graph depicts largest step size. The one in the middle carries a medium value of the step size. The expected characteristics are such because as the step

size decreases, the variance of the values available for learning or guessing the function also decrease, thus increasing the accuracy on the task. But this does not imply that the function has been learnt better (since only the mean value might be learnt instead of the function), yet the guess would surely be closer to the expected value.
These Gaussians are drawn assuming an extension of the above experiment with continuously varying values of y. Each Gaussian gives the frequency of examinees that would select the respective x for a particular value of $y_i, y_j andy_k$.

References

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