Assignment: Parameter Estimation

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Q1) random sample

$(X_1, X_2 \ldots X_n)$

size $= n$

normal population

mean $= \theta_1$

variance $= \theta_2$

} 2 parameters

find MLE of these 2 parameters

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

ans. $\mu = \theta_1$

$\sigma^2 = \theta_2$

density func

$$f(x_1) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x-\theta_1)^2}{2\theta_2}}$$

joint density func

$$L(x_1, x_2 \ldots x_n) = \frac{1}{(2\pi\theta_2)^{n/2}} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

taking natural log

$$\ln L = \ln\left(\frac{1}{(2\pi\theta_2)^{n/2}} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}\right)$$

$$= \ln\left(\frac{1}{(2\pi\theta_2)^{n/2}}\right) + \ln\left(e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_1-\theta_1)^2}\right)$$

$$= \ln\left(\frac{1}{(2\pi\theta_2)^{n/2}}\right) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2 \quad - 1$$

now diff 1 wrt θ_1 and equate to 0

$$\frac{d}{d\theta_1}(\ln L) = \frac{d}{d\theta_1}\left(\frac{1}{(2\pi\theta_2)^{n/2}}\right) - \frac{d}{d\theta_1}\left(\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2\right)$$

$$0 = 0 - \frac{1}{2\theta_2}\sum_{i=1}^{n}\frac{d}{d\theta_1}(x_i-\theta_1)^2$$

$$0 = \sum_{i=1}^{n} 2(x_i - \theta_1)\cdot(-1)$$

$$0 = -2 \sum_{i=1}^{n} (x_i - \theta_1)$$

$$0 = \sum_{i=1}^{n} x_i - n\theta_1$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}$$

θ_1 = sample mean = $\bar{X}$ — (2)

ans

for θ_2

$$\ln L = \ln\left(\frac{1}{(2\pi\theta_2)^{n/2}}\right) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

$$= -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

differentiate wrt θ_2 and equate to 0

$$0 = -\frac{n}{2}\frac{d}{d\theta_2}\ln(2\pi\theta_2) - \frac{1}{2}\sum_{i=1}^{n}(x_1 - \theta_1)^2 \frac{d}{d\theta_2}\theta_2^{-1}$$

$$0 = -\frac{n}{2}\frac{d}{d\theta_2}\ln(2\pi\theta_2) - \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta_1)^2\frac{d}{d\theta_2}\theta_2^{-1}$$

$$0 = -\frac{n}{2}\cdot\frac{1}{\cancel{2\pi}\theta_2}\cdot\cancel{2\pi} - \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta_1)^2\cdot\left(\frac{-1}{\theta_2^2}\right)$$

$$\frac{n}{2\theta_2} = \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta_1)^2\cdot\left(\frac{1}{\theta_2^2}\right)$$

$$n = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

from 2 $\theta_1 = \bar{x}$

$$\therefore \theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = S^2 = \text{sample variance}$$

ans

Q2) random sample

$x_1, x_2 \cdots x_n$

distribution

$B(m, \theta)$

$\theta \rightarrow$ unknown

$m \rightarrow$ known +ve integer

ans given distribution is a Binomial distribution

$$\text{pmf} = f(x) = {}^{m}C_x \ \theta^{x} (1-\theta)^{x}$$

joint density function

$$L(X_1, X_2 \cdots X_n) = \prod_{i=1}^{n} {}^{m}C_{x_i} \ \theta^{x_1} (1-\theta)^{(m-x_i)}$$

$$= \prod_{i=1}^{n} {}^{m}C_{x_i} \ \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{\sum_{i=1}^{n} (m-x_i)}$$

taking natural log

$$\ln L = \ln\left(\prod_{i=1}^{n} {}^{m}C_{x_1}\right) + \ln \theta^{\sum_{i=1}^{n} x_1}$$

$$+ \ln (1-\theta)^{\sum_{i=1}^{n} (m-x_i)}$$

$$\ln L = \ln\left(\prod_{i=1}^{n} {}^{m}C_{x_1}\right) + \sum_{i=1}^{n} \ln \theta$$

$$+ \sum^{n} (m - x_i) \ln (1-\theta)$$

$$+ \sum_{i=1}^{n} (m - x_i) \ln(1-\theta)$$

differentiating wrt to θ and equate to zero

$$0 = \frac{d}{d\theta}\left(\ln\left(\prod_{i=1}^{n} {}^{m}C_{x_i} \right)\right)$$

$$+ \sum_{i=1}^{n} x_i \frac{d}{d\theta} \ln \theta$$

$$+ \sum_{i=1}^{n} (m - x_i) \frac{d}{d\theta} \ln(1-\theta)$$

$$0 = 0 + \sum_{i=1}^{n} x_i \frac{1}{\theta} + \sum_{i=1}^{n} (m - x_i) \frac{-1}{(1-\theta)}$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \left(nm - \sum_{i=1}^{n} x_i \right)\left(\frac{1}{1-\theta} \right)$$

$$\frac{1-\theta}{\theta} = \frac{nm - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i}$$

$$\frac{1}{\theta} - \cancel{1} = \frac{nm}{n\ldots} - \cancel{1}$$

$$\frac{1}{\theta} - \cancel{1} = \frac{n\,m}{\sum_{i=1}^{n} x_i} - \cancel{1}$$

$$\theta = \left(\frac{\sum_{j=1}^{n} x_j}{n} \right) \frac{1}{m}$$

$$\theta = \frac{\bar{X}}{m}$$ ans