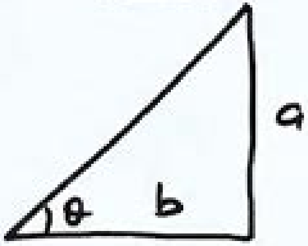
TYPE # 2 Equation of the form

$$\boxed{a\cos x + b\sin x = c}$$

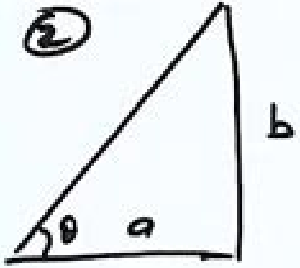
Divide by $\sqrt{a^2+b^2}$

$$\frac{a}{\sqrt{a^2+b^2}}\cos x + \frac{b}{\sqrt{a^2+b^2}}\sin x = \frac{c}{\sqrt{a^2+b^2}}$$

①



②



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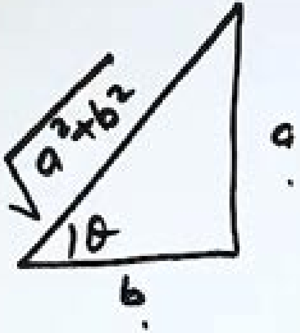


$$\sin\theta = \frac{a}{\sqrt{a^2+b^2}} \quad \cos\theta = \frac{b}{\sqrt{a^2+b^2}}$$

$$\sin\theta \cdot \cos x + \cos\theta \sin x = \frac{c}{\sqrt{a^2+b^2}}$$

$$\sin(x+\theta) = \frac{c}{\sqrt{a^2+b^2}}$$

$$\sin x = \sin\beta$$

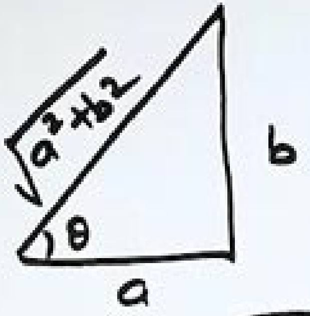
$$\beta = \sin^{-1}\frac{c}{\sqrt{a^2+b^2}}$$

$$x+\theta = n\pi + (-1)^n\beta \qquad n \in I$$

where $\beta = \sin^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

$$\boxed{x = n\pi + (-1)^n\beta - \theta}$$

$n \in I$



$$\sin\theta = \frac{b}{\sqrt{a^2+b^2}} \qquad \cos\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{a}{\sqrt{a^2+b^2}}\cos x + \frac{b}{\sqrt{a^2+b^2}}\sin x = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos\theta\cos x + \sin\theta\sin x = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos(x-\theta) = \frac{c}{\sqrt{a^2+b^2}}$$

$$\beta = \cos^{-1}\frac{c}{\sqrt{a^2+b^2}}$$

$$x - \theta = 2n\pi \pm \beta$$

$$\boxed{x = 2n\pi \pm \beta + \theta}$$

where $\beta = \cos^{-1}\frac{c}{\sqrt{a^2+b^2}}$

$n \in \mathbb{Z}$

Q $\sqrt{3}\sin x + \cos x = \sqrt{2}$

$\sqrt{a^2+b^2}$

$a = \sqrt{3} \quad b = 1 \quad \sqrt{a^2+b^2} = 2$

$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

① $\sin \pi/3 \qquad \cos \pi/3$

$\cos x \cos\frac{\pi}{3} + \sin x \sin \pi/3 = \frac{1}{\sqrt{2}}$

$\cos(x - \pi/3) = \frac{1}{\sqrt{2}}$

$$x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{4}$$

$$\boxed{x = 2n\pi \pm \pi/4 + \frac{\pi}{3}} \quad n \in I$$

$$x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{3} \quad \Big| \quad x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{3}$$

$$x = 2n\pi + \frac{7\pi}{12} \quad \Big| \quad x = 2n\pi + \frac{\pi}{12} \quad n \in I$$

Ans

m 11

$$\underbrace{\frac{\sqrt{3}}{2}}_{\cos\frac{\pi}{6}}\sin x + \underbrace{\frac{1}{2}}_{\sin\frac{\pi}{6}}\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x+\frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

✓

$$x+\frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{4}$$

$$x = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{6} \qquad n \in I$$

Q $\sin x + \cos x = \sqrt{2}$

$a = 1 \quad b = 1 \quad \sqrt{a^2+b^2} = \sqrt{2}$

$\underbrace{\frac{1}{\sqrt{2}}}_{\sin \pi/4} \sin x + \underbrace{\frac{1}{\sqrt{2}}}_{\cos \pi/4} \cos x = 1$

$\underline{\cos x \cos \frac{\pi}{4}} + \underline{\sin x \sin \frac{\pi}{4}} = 1$

$\cos(x - \pi/4) = 1$

$x - \pi/4 = 2n\pi$

$x = 2n\pi + \pi/4$

$n \in I$

Q $3\cos x + 4\sin x = 5$

$a=3 \quad b=4 \quad \sqrt{a^2+b^2} = 5$

$\underbrace{\frac{3}{5}}_{\cos\theta}\cos x + \underbrace{\frac{4}{5}}_{\sin\theta}\sin x = 1$

$\cos x\cos\theta + \sin x\sin\theta = 1$

$\cos(x-\theta) = 1$

$x-\theta = 2n\pi$

$\cos\theta = 3/5$

$x = 2n\pi + \theta$

where $\theta = \cos^{-1}3/5$

$n \in I$

Q

$$\sin x + \cos x = \sqrt{3}$$

$a=1 \quad b=1 \quad \sqrt{a^2+b^2} = \sqrt{2}$

$$\underbrace{\frac{1}{\sqrt{2}}}_{\sin \pi/4} \sin x + \underbrace{\frac{1}{\sqrt{2}}}_{\cos \pi/4} \cos x = \sqrt{3/2}$$

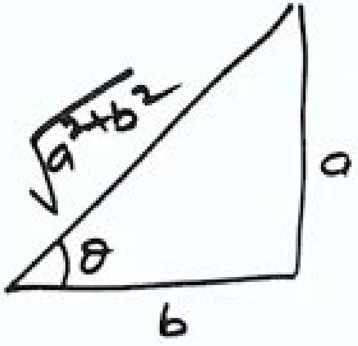
$$\cos\left(x - \frac{\pi}{4}\right) = \sqrt{\frac{3}{2}}$$

>1

No Solution

$$a\sin x + b\cos x = c$$

$$\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\sin\theta}\sin x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\cos\theta}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$



$$\cos(x-\theta) = \frac{c}{\sqrt{a^2+b^2}} \checkmark$$

$$-1 \le \cos(x-\theta) \le 1$$

$$-1 \le \frac{c}{\underbrace{\sqrt{a^2+b^2}}} \le 1$$

$$-\sqrt{a^2+b^2} \le c \le \sqrt{a^2+b^2}$$

$$\boxed{c \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]}$$

Equation will posses Solution

$$\Leftarrow \quad |c| \le \sqrt{a^2+b^2}$$

$$|c| > \sqrt{a^2+b^2}$$

no solution

Q $\sin x + \cos x = \sqrt{3}$

$\sqrt{a^2+b^2} = \sqrt{2}$

$c = \sqrt{3}$

$c > \sqrt{a^2+b^2}$

no solution

Q $\sin x + \cos x = 1.5$

$\sqrt{a^2+b^2} = \sqrt{2}$

$= \underline{\underline{1.414}}$

$c = 1.5$

$c > \sqrt{a^2+b^2}$

no solution

Q If $k\cos x - 3\sin x = k+1$.

find the value of 'k' for which given equation has real solutions

$$|c| \le \sqrt{a^2+b^2}$$

$a = k \qquad b = -3 \qquad c = k+1$

$$\sqrt{a^2+b^2} = \sqrt{k^2+9}$$

$|c| \leq \sqrt{a^2+b^2}$

$|k+1| \leq \sqrt{k^2+9}$ (Both sides are +ve)

(Squaring)

$|x|^2 = x^2$

$|k+1|^2 \leq \sqrt{k^2+9}$

$\cancel{k^2}+1+2k \leq \cancel{k^2}+9$

$2k \leq 8$

$k \leq 4$

$(-\infty, 4]$ ms

Q $1+\sin^3 x+\cos^3 x=\frac{3}{2}\sin 2x$

$\sin 2x = 2\sin x\cos x$

$1+\sin^3 x+\cos^3 x = 3\sin x\cos x$

$a=1 \quad b=\sin x \quad c=\cos x$

$a^3+b^3+c^3 = 3abc$

(1) $a=b=c$ (2) $a+b+c=0$ ✓

$\sin x=\cos x=1$ ✗

$a+b+c=0$

$1+\sin x+\cos x=0$

$\sin x+\cos x=-1$

$a=1 \quad b=1 \quad \sqrt{a^2+b^2}=\sqrt{2}$

$\underbrace{\frac{1}{\sqrt{2}}}_{\sin \pi/4}\sin x+\underbrace{\frac{1}{\sqrt{2}}}_{\cos \pi/4}\cos x=-\frac{1}{\sqrt{2}}$

$\cos(x-\pi/4)=-\frac{1}{\sqrt{2}} \rightarrow \frac{3\pi}{4}$

$$\cos(x - \pi/4) = -\frac{1}{\sqrt{2}}$$

$$x - \pi/4 = 2n\pi \pm \frac{3\pi}{4}$$

$$x = 2n\pi \pm \frac{3\pi}{4} + \frac{\pi}{4}$$

$$x = 2n\pi + \frac{3\pi}{4} + \frac{\pi}{4}$$

$$\underline{x = 2n\pi + \pi} \checkmark$$

$$x = 2n\pi - \frac{3\pi}{4} + \pi/4$$

$$\underline{x = 2n\pi - \frac{\pi}{2}}$$

$$n \in I$$

Q

$$3\sqrt{3}\sin^3x + \cos^3x + 3\sqrt{3}\sin x\cos x = 1$$

$$(\sqrt{3}\sin x)^3 + (\cos x)^3 - 1 = -3\sqrt{3}\sin x\cos x$$

$$\underline{(\sqrt{3}\sin x)}^3 + \underline{(\cos x)}^3 + \underline{(-1)}^3 = 3\,\underline{(-1)}\,\underline{(\sqrt{3}\sin x)}\,\underline{(\cos x)}$$

(1) $a = b = c$

$$\underline{\sqrt{3}\sin x} = \underline{\cos x} = \underline{-1}$$

$\times$

(2) $a + b + c = 0$

$$\sqrt{3}\sin x + \cos x - 1 = 0$$

$$\sqrt{3}\sin x + \cos x = 1$$

$$a = \sqrt{3} \quad b = 1 \quad \sqrt{a^2+b^2} = 2$$

$$\underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{\pi}{3}}\sin x + \underbrace{\frac{1}{2}}_{\cos\frac{\pi}{3}}\cos x = \frac{1}{2}$$

$$\cos\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$$

$$x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}$$

① $x = 2n\pi + \frac{\pi}{3} + \pi/3$

$x = 2n\pi + \frac{2\pi}{3}$

② $x = 2n\pi \cancel{-\frac{\pi}{3}} \cancel{+\pi/3}$

$x = 2n\pi$

$n \in I$