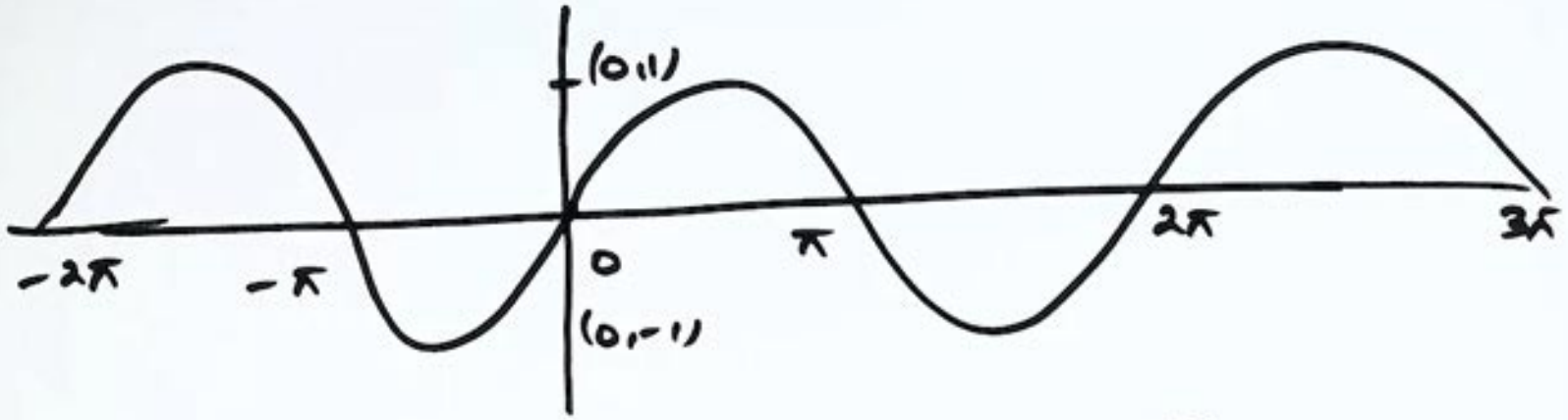
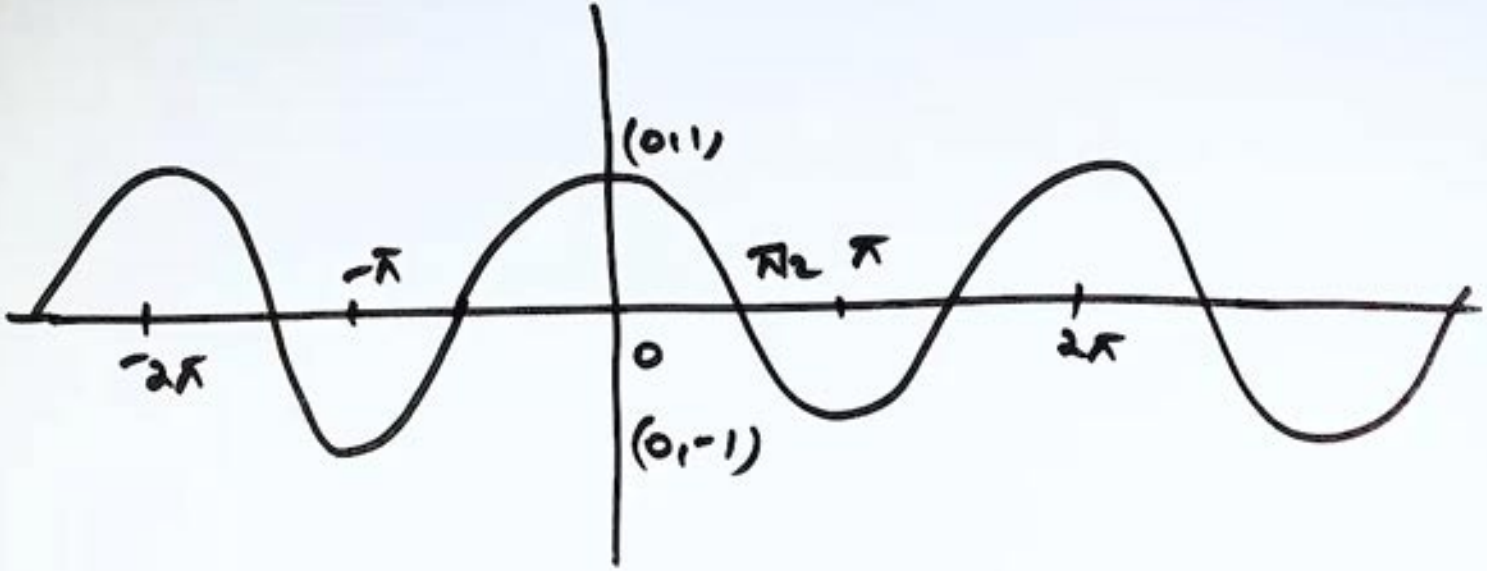
TRIGONOMETRIC INEQUALITIES



$y = \sin x$

$T = 2\pi$ Period.

Periodic



$y = \cos x$ $\quad T = 2\pi$

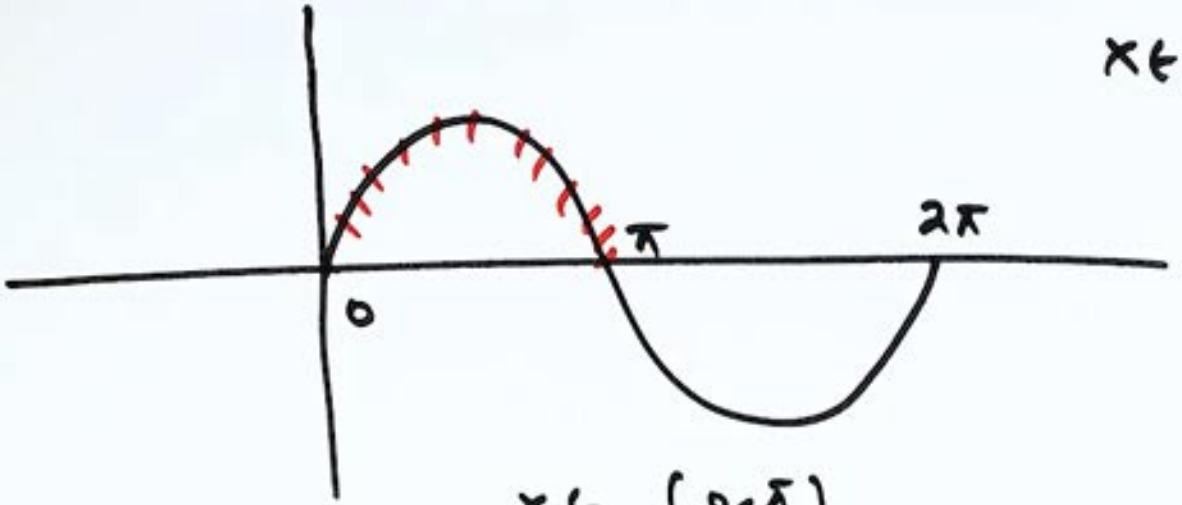
$\sin x / \cos x$ Inequality.

(1) Solve the inequality for $[0, 2\pi]$

(2) find the solution from $[0, 2\pi]$

(3) Add $2n\pi$ in the entire interval.

Q1 $\sin x \geq 0$

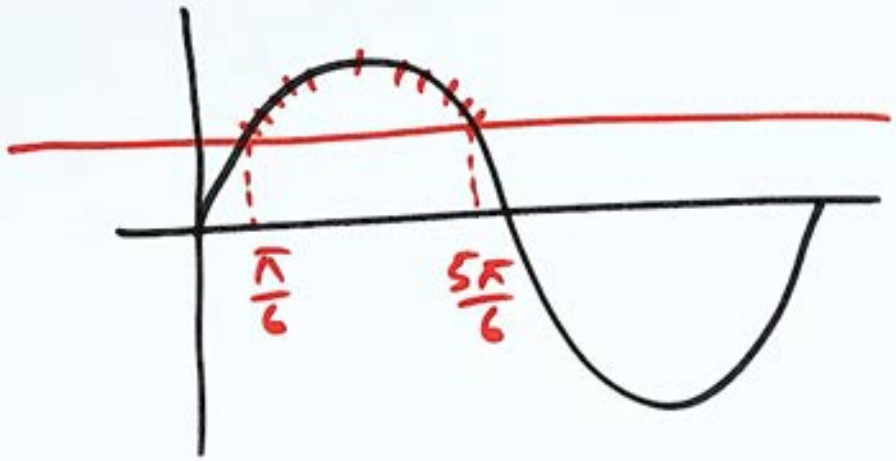


$x \in [0, \pi]$

$x \in [0, \pi]$

$x \in [2n\pi, 2n\pi + \pi]$ $n \in I$

Q $\sin x > \frac{1}{2}$



$y = \frac{1}{2}$

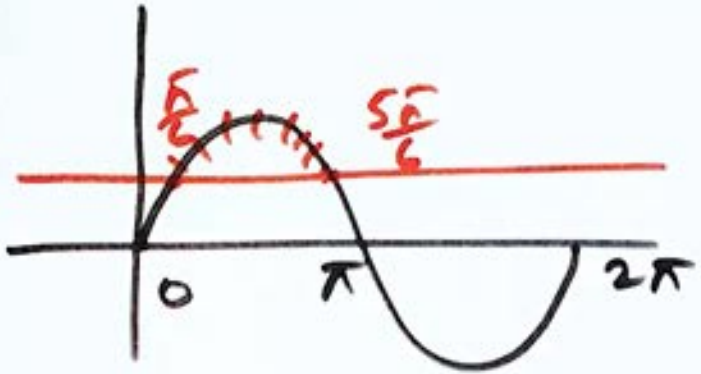
$\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

$x \in \left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$

Q $\sin 2x \geq \frac{1}{2}$

$2x = \theta$

$\sin\theta \geq \frac{1}{2}$



$\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

$\theta \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

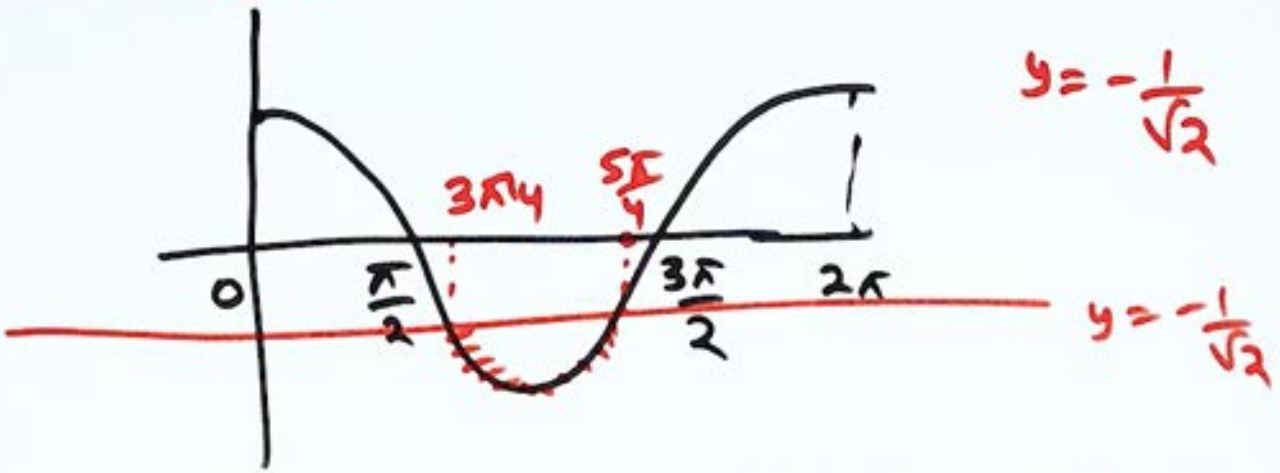
$2x \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

$x \in \left[n\pi + \frac{\pi}{12}, n\pi + \frac{5\pi}{12}\right]$

$n \in I$

Q $\cos x < -\frac{1}{\sqrt{2}}$

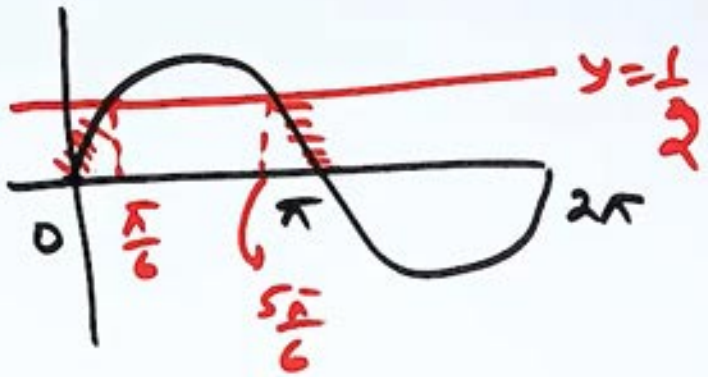
$\pi - \pi/4$ / $\pi + \pi/4$



$x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ $x \in \left(2n\pi + \frac{3\pi}{4}, 2n\pi + \frac{5\pi}{4}\right)$

$n \in I$

Q $\log_2(\sin x/2) < -1$



$0 < \sin x/2 < \frac{1}{2}$

$\frac{x}{2} = \theta$

$0 < \sin\theta < \frac{1}{2}$

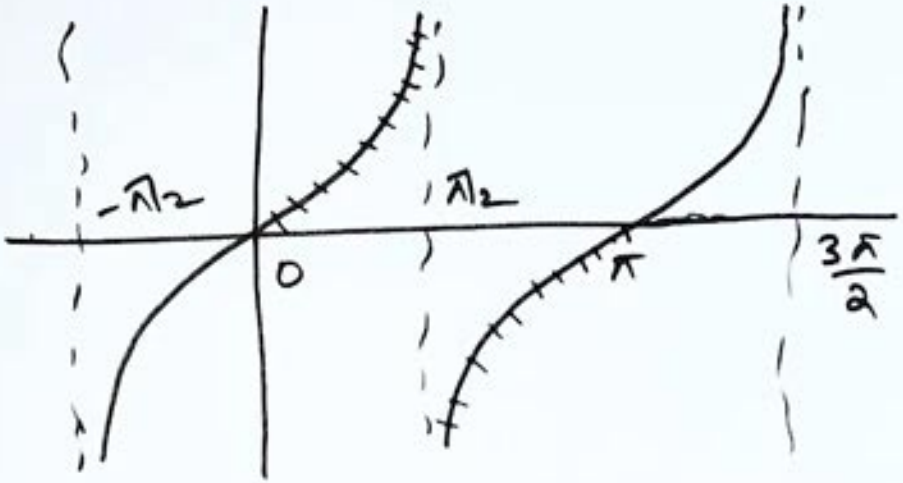
$\theta \in (0, \frac{\pi}{6}) \cup (5\frac{\pi}{6}, \pi)$

$\frac{x}{2} = \theta \in (2n\pi, 2n\pi + \frac{\pi}{6}) \cup (2n\pi + \frac{5\pi}{6}, 2n\pi + \pi)$

$x = (4n\pi, 4n\pi + \frac{\pi}{3}) \cup (4n\pi + \frac{5\pi}{3}, 4n\pi + 2\pi)$

$n \in I$

(3) Inequality of tanx.



$y = \tan x$

$T = \pi$

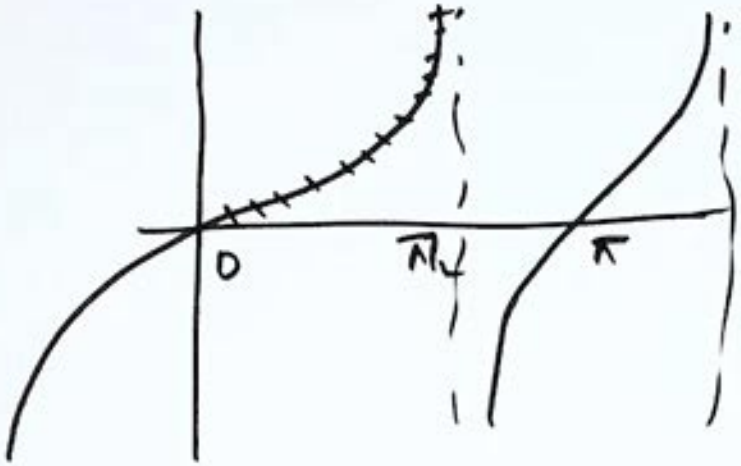
Period

(1) Solve the inequality for $[0,\pi] - \{\pi/2\}$

(2) find the interval from $[0,\pi] - \{\pi/2\}$

(3) Add $n\pi$ in the entire interval.

Q $\tan x > 0$



$x \in (0, \pi/2)$

Add $n\pi$

$x \in (n\pi, n\pi + \frac{\pi}{2})$

$n \in I$

Ans

Q $\tan^2 x - (\sqrt{3}+1)\tan x + \sqrt{3} < 0$

$\tan^2 x - \sqrt{3}\tan x - \tan x + \sqrt{3} < 0$

$\tan x(\tan x - \sqrt{3}) - 1(\tan x - \sqrt{3}) < 0$

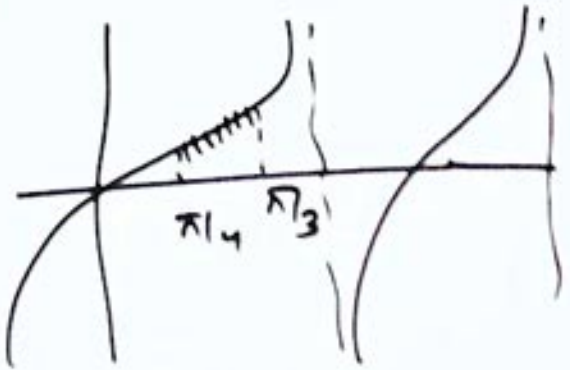
$(\tan x - 1)(\tan x - \sqrt{3}) < 0$

$1 < \tan x < \sqrt{3}$

$x \in (\pi/4, \pi/3)$

$x \in (n\pi + \frac{\pi}{4}, n\pi + \pi/3)$

$n \in I$



Q $\sin x \geq \cos 2x$

$\sin x \geq 1 - 2\sin^2 x$

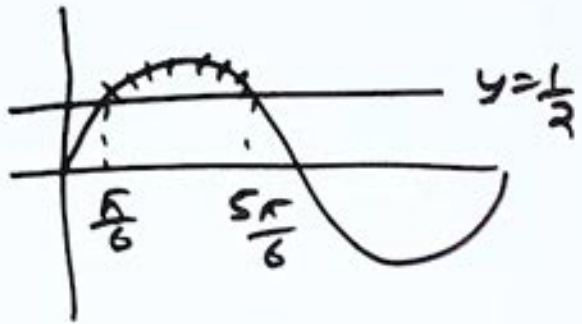
$2\sin^2 x + \sin x - 1 \geq 0$

$(2\sin x - 1)(\sin x + 1) \geq 0$

$\geq 0 \qquad \geq 0$

$2\sin x - 1 \geq 0$

$\sin x \geq \frac{1}{2}$



$\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

$\left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

$n \in I$

$\sin x + 1 \geq 0$

$\sin x \geq -1$

$\sin x = -1$

$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$ ✓

$x = 2n\pi - \frac{\pi}{2}$

$n \in I$

Q $\sin 2x > \sqrt{2}\sin^2 x + (2-\sqrt{2})\cos^2 x$

$2\sin x \cos x > \sqrt{2}\sin^2 x + (2-\sqrt{2})\cos^2 x$

Divide by $\cos^2 x$

$2\tan x > \sqrt{2}\tan^2 x + (2-\sqrt{2})$

$\tan x = t$

$2t > \sqrt{2}t^2 + (2-\sqrt{2})$

$\sqrt{2}t^2 - 2t + (2-\sqrt{2}) < 0$

$$\sqrt{2}t^2 - 2t + (2-\sqrt{2}) < 0$$

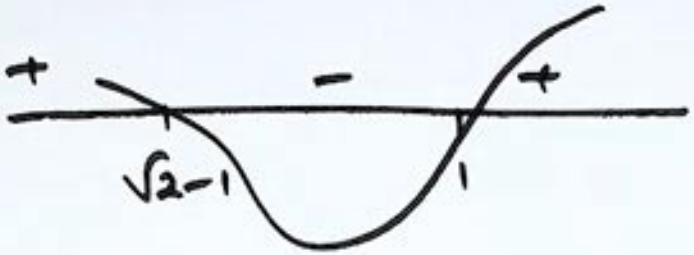
$$\sqrt{2}t^2 - (2-\sqrt{2}+\sqrt{2})t + (2-\sqrt{2}) < 0$$

$$\sqrt{2}t^2 - (2-\sqrt{2})t - \sqrt{2}t + 2-\sqrt{2} < 0$$

$$\sqrt{2}t(t-1) - (2-\sqrt{2})(t-1) < 0$$

$$(\sqrt{2}t - 2 + \sqrt{2})(t-1) < 0$$

$$\sqrt{2}(t - \sqrt{2} + 1)(t-1) < 0$$

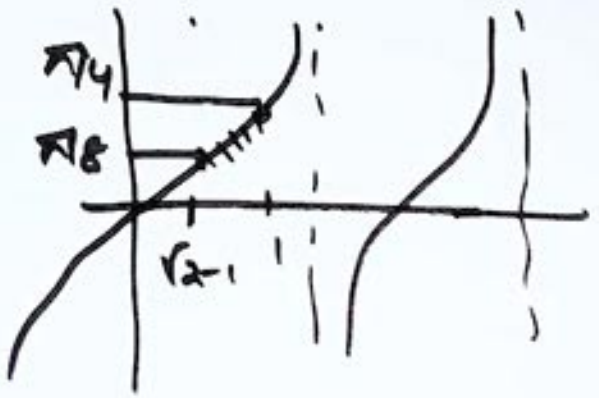


$t \in (\sqrt{2}-1, 1)$

$\tan x \in (\sqrt{2}-1, 1)$

$\frac{\pi}{8}, \frac{\pi}{4}$

$x \in (\pi/8, \pi/4)$



$x \in (n\pi + \frac{\pi}{8}, n\pi + \pi/4)$

$n \in I$