TRIGONOMETRIC EQUATIONS

TYPES OF SOLUTION

(1) PRINCIPAL SOLUTION

(2) GENERAL SOLUTION

(1) **PRINCIPAL SOLUTION:-**

They are the solution of trigonometric equation for $[0, 2\pi)$

Q1 $\sin\theta = \frac{1}{2}$ find principal solution

$\frac{\pi}{6} / 30^\circ$ $150^\circ, \frac{5\pi}{6}$

$\frac{\pi}{6} \mid \frac{5\pi}{6}$ $30^\circ / 150^\circ$.

Q2 $\cos x = \frac{1}{\sqrt{2}}$

1^{st}, 4^{th}.

$\frac{\pi}{4}$, $2\pi - \frac{\pi}{4}$

$\frac{\pi}{4}$, $\frac{7\pi}{4}$

$\pi - \theta$	θ
$\pi + \theta$	$2\pi - \theta$

Q3 $\tan x = \sqrt{3}$

1^{st}, 3^{rd}

θ, $\pi + \theta$.

$\frac{\pi}{3}$, $\pi + \frac{\pi}{3}$

$\frac{\pi}{3}$, $\frac{4\pi}{3}$.

Ans

Q4 $\sin x = -\frac{\sqrt{3}}{2}$

$3^{rd}, 4^{th}.$

$\sin x = \frac{\sqrt{3}}{2}$

$\frac{\pi}{3}$ ✓

$\pi + \theta, \quad 2\pi - \theta$

$\pi + \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3}$

$\frac{4\pi}{3}, \frac{5\pi}{3}$

Q5 $\tan x = -\frac{1}{\sqrt{3}}$

$2^{nd}, 4^{th}.$

$\tan x = \frac{1}{\sqrt{3}} \quad x = \frac{\pi}{6}$

$\pi - \theta, \quad 2\pi - \theta$

$\pi - \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$

$\frac{5\pi}{6}, \quad \frac{11\pi}{6}$

Q6 $\sin x = \frac{1}{3}$

$x = \underline{\sin^{-1}\left(\frac{1}{3}\right)} \rightarrow \theta$

$\underline{1^{st}}$

$1^{st}, \ 2^{nd}$

$\theta, \ \pi - \theta$

$\sin^{-1}\frac{1}{3}, \ \pi - \sin^{-1}\frac{1}{3}$

Q7 $\tan x = 3$

$x = \underbrace{\tan^{-1}3}_{\theta}$

$1^{st}, \ 3^{rd}$

$\theta, \ \pi + \theta$

$\tan^{-1}3, \ \pi + \tan^{-1}3$

Q8 $\tan x = -3$

If $\tan x = 3$

$x = \underbrace{\tan^{-1} 3}_{\theta}$

2^{nd}, 4^{th}

$\pi - \theta$, $2\pi - \theta$

$\pi - \tan^{-1} 3$, $2\pi - \tan^{-1} 3$

Q9 $\sin x = -\frac{1}{5}$

If $\sin x = \frac{1}{5}$

$x = \underbrace{\sin^{-1} \frac{1}{5}}_{\theta}$

3^{rd}, 4^{th}

$\pi + \theta$, $2\pi - \theta$

$\pi + \sin^{-1} \frac{1}{5}$, $2\pi - \sin^{-1} \frac{1}{5}$

concept

1) $\sin x = 0$

$0, \pi, 2\pi \ldots$

$x = n\pi$

2) $\cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$x = (2n-1)\frac{\pi}{2}$

3) $\cos x = 1$

$0, 2\pi, 4\pi, 6\pi \ldots$

$x = 2n\pi$

(4) $\cos x = -1$

$\pi, 3\pi, 5\pi, \ldots$

$x = (2n-1)\pi$

(5) $\sin x = 1$

$x = \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi + \frac{\pi}{2} \ldots$

$\sin(2\pi + \theta) = \sin\theta$

$x = 2n\pi + \frac{\pi}{2}$

(6) $\sin x = -1$

$x = -\frac{\pi}{2}, \frac{3\pi}{2}, 2\pi + \frac{3\pi}{2} \ldots$

$x = 2n\pi - \frac{\pi}{2}$

(7) $\tan x = 0$

$0, \pi, 2\pi - -$

$x = n\pi$

(8) $\tan x$ = not defined

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} - -$

$x = (2n-1)\frac{\pi}{2}$ Then $\tan x$ is not defined

GENERAL SOLUTIONS

Eg // $\sin x = \frac{1}{2}$

Periodic function

$T = 2\pi$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$2\pi + \frac{\pi}{6}, \quad 2\pi + \frac{5\pi}{6}$

$\sin(2\pi + \theta) = \sin\theta$

$4\pi + \frac{\pi}{6}, \quad 4\pi + \frac{5\pi}{6}$

$\vdots$

infinite solutions

* It is a general formula in $(n \in I)$ which can represent infinite solutions of the equation.

Eg $\sin x = \frac{1}{2}$

(1) $\sin\alpha = \sin\beta$

$\sin\alpha - \sin\beta = 0$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0$

$\cos\left(\frac{\alpha+\beta}{2}\right) = 0$ or $\sin\left(\frac{\alpha-\beta}{2}\right) = 0$

$\frac{\alpha+\beta}{2} = (2p-1)\frac{\pi}{2}$ $\qquad$ $\frac{\alpha-\beta}{2} = q\pi$

$\alpha+\beta = (2p-1)\pi$ $\qquad$ $\alpha-\beta = 2q\pi$

$\alpha = (2p-1)\pi - \beta$ $\qquad\qquad$ $\alpha = 2q\pi + \beta$

$p, q \in I$

$$\boxed{\alpha = n\pi + (-1)^n \beta}$$

$n \in I$

$n = 2q$

$n = 2p-1$

$$\boxed{\alpha = n\pi + (-1)^n \beta} \quad n \in I$$

$\beta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin x \geq 0 \quad \left[0, \frac{\pi}{2}\right]$

$\sin x < 0 \quad \left[-\frac{\pi}{2}, 0\right)$

Q1 $\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$x = n\pi + (-1)^n \frac{\pi}{6}$ $\quad$ $x = n\pi + (-1)^n \frac{5\pi}{6}$ ✓

$n \in I$

Q $\sin x = -\frac{\sqrt{3}}{2}$

$x = -\frac{\pi}{3}$ / $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ / $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x = n\pi + (-1)^n\left(-\frac{\pi}{3}\right)$ / $n\pi + (-1)^n\left(\frac{4\pi}{3}\right)$

✓✓ ✓

N.C.E.R.T

Q1 $\sin x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}$

$x = n\pi + (-1)^n \frac{\pi}{4}$

$n \in I$

Q2 $\sin x = \frac{1}{3}$

$x = \sin^{-1}\frac{1}{3}$

// $x = n\pi + (-1)^n \left(\sin^{-1}\frac{1}{3}\right)$

or $x = n\pi + (-1)^n(\alpha)$

where $\alpha = \sin^{-1}\frac{1}{3}$

Q1 $\sin x = 1$

$x = 2n\pi + \frac{\pi}{2} \longrightarrow$ $n=0$ $\frac{\pi}{2}$

$n=1$ $2\pi + \frac{\pi}{2}$

$x = n\pi + (-1)^n \frac{\pi}{2}$

$n=0$ $\pi/2$

$n=1$ $\pi/2$

$n=2$ $2\pi + \frac{\pi}{2}$

Q2 $\sin x = -1$

Q2 $x = 2n\pi - \frac{\pi}{2}$

$x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

$n \in I$