Q $\underline{\sin\theta + \sin 5\theta} = \sin 3\theta$

$\underline{\theta \in [0, \pi]}$

$2\cancel{\sin 3\theta}\cos 2\theta = \cancel{\sin 3\theta}$

$\sin 3\theta = 0 \qquad \cos 2\theta = \frac{1}{2}$

$\underline{3}\theta = n\pi \qquad 2\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{n\pi}{3} \qquad \theta = n\pi \pm \frac{\pi}{6}$

$\underline{n \in I}$

$\theta = \frac{n\pi}{3}$

$0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

$\theta = n\pi \pm \frac{\pi}{6}$

$n=0$

$\theta = \pm \pi/6$ $\quad \pi/6$

$n=1$

$\theta = \pi \pm \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$n=2$

$\theta = 2\pi \pm \pi/6 > \pi$ ✗

$$\boxed{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi}$$

6 solutions

Q $\sin 3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$

find number of solutions in $[0, \pi]$

$3\sin\theta - 4\sin^3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$

$\cancel{\sin\theta}(3 - 4\sin^2\theta) = 4\cancel{\sin\theta} \sin 2\theta \sin 4\theta$

$\sin\theta = 0$

$0, \pi$

2 solutions

$$3-4\sin^2\theta = 4\sin 2\theta \sin 4\theta$$

$$= 2(2\sin 2\theta \sin 4\theta)$$

$$= 2(\cos 2\theta - \cos 6\theta)$$

$$3-4\sin^2\theta = 2(1-2\sin^2\theta - \cos 6\theta)$$

$$3-\cancel{4\sin^2\theta} = 2-\cancel{4\sin^2\theta} - 2\cos 6\theta$$

$$-2\cos 6\theta = 1$$

$$\cos 6\theta = -\frac{1}{2}$$

$\theta \in [0, \pi]$

$6\theta \in [0, 6\pi]$

3 cycles

1 cycle → 2 solⁿ

3 cycles → 6 solⁿ

Ans

$6+2$

$= 8$

Q $\dfrac{\sin 6x}{\sin x} = 8\cos x \cdot \cos 2x \cos 4x$ $\longrightarrow$ $\sin x \neq 0$

$$\sin 6x = 8\cos x \sin x \cos 2x \cos 4x$$

$$= 4\,(\underbrace{2\sin x \cos x})\cos 2x \cos 4x$$

$$= 4\sin 2x \cos 2x \cos 4x$$

$$= 2\,(\underbrace{2\sin 2x \cos 2x})\cos 4x$$

$$= \underbrace{2\sin 4x \cos 4x} = \sin 8x$$

$$\sin 6x = \sin 8x$$

$$\sin 6x = \sin 8x$$

$$\sin 8x - \sin 6x = 0$$

$$2\cos 7x \sin x = 0$$

$\sin x = 0$ & $\cos 7x = 0$

$\times$

$$\cos 7x = 0$$

$$7x = (2n-1)\frac{\pi}{2}$$

$$x = (2n-1)\frac{\pi}{14} \quad n \in I$$

Q $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$ $\theta \in [0, \pi]$

$$4\cos\theta \cos 2\theta \cos 3\theta = 1$$

$$2(2\cos\theta\cos 3\theta)\cos 2\theta = 1$$

$$2[\cos 4\theta + \cos 2\theta]\cos 2\theta = 1$$

$$2[2\cos^2 2\theta - 1 + \cos 2\theta]\cos 2\theta = 1$$

$$\cos 2\theta = t$$

$$4t^3 + 2t^2 - 2t - 1 = 0$$

$$4t^3 + 2t^2 - 2t - 1 = 0$$

$$t = -\frac{1}{2} \qquad 4\left(-\cancel{\tfrac{1}{8}}\right) + \cancel{2}\left(\cancel{\tfrac{1}{4}}\right) + \cancel{1} - \cancel{1} = 0$$

$$(2t+1)(2t^2-1) = 0$$

$$t = -\frac{1}{2} \qquad t^2 = \frac{1}{2}$$

$$\cos 2\theta = -\frac{1}{2} \qquad \cos^2 2\theta = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$2\theta = 2n\pi \pm \frac{2\pi}{3} \qquad 2\theta = n\pi \pm \pi/4$$

$$\theta = n\pi \pm \pi/3 \qquad \theta = \frac{n\pi}{2} \pm \pi/8$$

$\underline{n \in I}$

$n\pi \pm \frac{\pi}{3}$

$n=0 \quad \pi/3$

$n= \quad \pi - \frac{\pi}{3}$

$\frac{\pi}{3}, \frac{2\pi}{3}$ ✓

$\frac{n\pi}{2} \pm \frac{\pi}{8}$

$n=0 \quad \pi/8$

$n=1 \quad \frac{\pi}{2} \pm \pi/8$

$n=2 \quad \pi \pm \pi/8$

$\pi - \pi/8$ ✓

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$

6 solutions

Q $\underbrace{\cos^2 x} + \cos^2 2x + \cos^2 3x + \cos^2 4x = 2$

$$\frac{1+\cos 2x}{2} + \frac{1+\cos 4x}{2} + \frac{1+\cos 6x}{2} + \frac{1+\cos 8x}{2} = 2$$

$$\cancel{1}+\cos 2x + \cancel{1}+\cos 4x + \cancel{1}+\cos 6x + \cancel{1}+\cos 8x = \cancel{4}$$

$$\cos 2x + \cos 4x + \cos 6x + \cos 8x = 0$$

$$\boxed{\text{Sum} = \frac{\sin\frac{nd}{2}}{\sin d/2}\cos\left(\frac{T_1+T_n}{2}\right)}$$

$n=4$ $d=2x$

$T_1 = 2x$ $T_4 = 8x$

$$\frac{\sin 4x}{\sin x}\cos\left(\frac{2x+8x}{2}\right)=0$$

$$\frac{\sin 4x\cos 5x}{\sin x}=0 \qquad \sin x\neq 0$$

$\sin 4x=0$ or $\cos 5x=0$

$4x=n\pi$

$x=\frac{n\pi}{4}$ $n\in I$

$n\neq 4m$

$\cos 5x=0$

$5x=(2n-1)\frac{\pi}{2}$

$x=(2n-1)\frac{\pi}{10}$

$n\in I$

Q $\underbrace{\tan\theta \tan(\pi/3+\theta)\tan(\pi/3-\theta)} = \frac{1}{\sqrt{3}}$

$$\tan\theta \tan\left(\frac{\pi}{3}+\theta\right)\tan\left(\frac{\pi}{3}-\theta\right) = \tan 3\theta$$

$$\tan 3\theta = \frac{1}{\sqrt{3}}$$

$$3\theta = n\pi + \frac{\pi}{6}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{18} \quad n \in I$$

Q $\operatorname{cosec}x - \operatorname{cosec}2x = \operatorname{cosec}4x$

$x \neq n\pi \qquad 2x \neq n\pi \qquad 4x \neq n\pi$

$\qquad\qquad x \neq \frac{n\pi}{2} \qquad x \neq \frac{n\pi}{4}$

$$\frac{1}{\sin x} - \frac{1}{\sin 2x} = \frac{1}{\sin 4x}$$

$$\frac{\sin 2x - \sin x}{\sin x \cancel{\sin 2x}} = \frac{1}{2\cancel{\sin 2x}\cos 2x}$$

$$\frac{\sin 2x - \sin x}{\sin x} = \frac{1}{2\cos 2x}$$

$$2\cos 2x \sin 2x - 2\cos 2x \sin x = \sin x$$

$$\sin 4x - [\sin 3x - \sin x] = \sin x$$

$$\sin 4x - \sin 3x + \cancel{\sin x} = \cancel{\sin x}$$

$$2\cos\frac{7x}{2}\sin\frac{x}{2} = 0$$

$$\sin\frac{x}{2} = 0$$

$$x = 2n\pi$$

$$\cos\frac{7x}{2} = 0$$

$$\frac{7x}{2} = (2n-1)\frac{\pi}{2}$$

$$x = (2n-1)\frac{\pi}{7}$$

$x = (2n-1)\frac{\pi}{7}$

$\frac{2n-1}{7}$ = integer

Problem

= odd integer

$\frac{2n-1}{7} = 2p-1$

$2n-1 = 14p-7$

$2n = 14p-6$

$n = 7p-3$

$p=1 \quad n=4$

$p=2 \quad n=\underline{\underline{11}}$

$x = (2n-1)\frac{\pi}{7}$

$n \in I \quad n \neq 7p-3$

✓ $\quad p \in I$