TYPE#6 USING BOUNDNESS OF FUNCTIONS

Q1 $2^{\frac{1}{\sin^2 x}}\left(\sqrt{y^2-2y+2}\right) \leq 2$

$2^{\operatorname{cosec}^2 x}\sqrt{(y-1)^2+1}$

$\operatorname{cosec}^2 x \geq 1$

$2^{\operatorname{cosec}^2 x} \geq 2$

$\sqrt{y^2-2y+2}$

$\sqrt{y^2-2y+1+1}$

$\sqrt{(y^2-2y+1)+1}$

$\sqrt{(y-1)^2+1}$

$$\sqrt{\underbrace{\underbrace{(y-1)^2}_{\geq 0}+1}_{\geq}} \geq 1$$

$$2^{\operatorname{cosec}^2 x} \geq 2$$

mulipl

$$2^{\operatorname{cosec}^2 x} \cdot \sqrt{y^2-2y+2} \geq 2$$

≤ 2 (given in question)

$$\left(\underbrace{2^{\operatorname{cosec}^2 x}}_{2}\underbrace{\sqrt{y^2-2y+2}}_{1}\right)_{min} = 2$$

$2^{\operatorname{cosec}^2 x} = 2$

$\operatorname{cosec}^2 x = 1$

$\sin^2 x = 1$

$x = n\pi \pm \frac{\pi}{2}$ ✓ $n \in I$

$\sqrt{y^2 - 2y + 2} = 1$

$y^2 - 2y + 2 = 1$

$(y-1)^2 = 0$

$y = 1$ ✓

Q $1-2x-x^2 = \tan^2(x+y) + \cot^2(x+y)$

$x^2+2x-1+\tan^2(x+y)+\cot^2(x+y)=0$

$\underbrace{x^2+2x+1}-2+\tan^2(x+y)+\cot^2(x+y)=0$

$(x+1)^2+\underbrace{\tan^2(x+y)+\cot^2(x+y)-2}=0$

$(x+1)^2+\underbrace{\tan^2(x+y)+\cot^2(x+y)-2\tan(x+y)\cot(x+y)}=0$

$$\underline{(x+1)^2} + \underline{(\tan(x+y) - \cot(x+y))^2} = 0$$

if $a^2+b^2=0 \Rightarrow a=0 \ \& \ b=0$

$x+1=0$ $\qquad$ $\tan(x+y) - \cot(x+y) = 0$

$x=-1$ $\qquad$ $\tan(x+y) = \cot(x+y) = \dfrac{1}{\tan(x+y)}$

$$\tan^2(x+y) = 1$$

$$x+y = n\pi \pm \frac{\pi}{4}$$

$$\boxed{y = n\pi \pm \pi/4 + 1} \quad n \in \underline{I}$$

Q $\cos x + \cos 2x + \cos 3x = 3$

$(\cos\theta)_{max} = 1$

$\cos x = 1$ & $\cos 2x = 1$ & $\cos 3x = 1$

$x = 2n\pi \quad \cap \quad 2x = 2p\pi \quad \cap \quad 3x = 2q\pi$

$x = p\pi \qquad x = \frac{2q\pi}{3}$

$[0, 2\pi]$ $\quad 0, 2\pi \qquad 0, \pi, 2\pi \qquad 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

$x = 2n\pi \quad n \in I$

Q

$$2\sin x = 3x^2 + 2x + 3$$

$$= 3\left(x^2 + \frac{2x}{3} + 1\right)$$

$$= 3\left(\left(x+\frac{1}{3}\right)^2 - \frac{1}{9} + 1\right)$$

$$= \underbrace{3\left(\underbrace{\left(x+\frac{1}{3}\right)^2}_{\geq 0} + \frac{8}{9}\right)}_{\geq 8/9} \geq \frac{8}{3}$$

Not possible

no solution

$$\text{RHS} \geq \frac{8}{3} \qquad \text{LHS} = 2\sin x \leq 2$$

Q Good

$$\sin x\left(\cos\frac{x}{4}-2\sin x\right)+\left(1+\sin\frac{x}{4}-2\cos x\right)\cdot\cos x=0$$

$$\sin x\cos\frac{x}{4}-2\sin^2 x+\cos x+\cos x\sin\frac{x}{4}-2\cos^2 x=0$$

$$\left(\sin x\cos\frac{x}{4}+\cos x\sin x/4\right)+\cos x-2\left(\sin^2 x+\cos^2 x\right)=0$$

$$\sin\left(x+\frac{x}{4}\right)+\cos x-2=0$$

$$\sin\frac{5x}{4}+\cos x=2$$

$$\underbrace{\sin\frac{5x}{4}}_{\le 1} + \underbrace{\cos x}_{\le 1} = \underset{=}{2}$$

$\sin\frac{5x}{4} = 1$ & $\underline{\cos x = 1}$

$$\frac{5x}{4} = 2q\pi + \frac{\pi}{2}$$

$$\frac{5x}{4} = (4q+1)\frac{\pi}{2}$$

$$✓\ x = \frac{2\pi}{5}(4q+1)$$

$q \in I$

✓

$\cos x = 1$

$x = 2p\pi$

$p \in I$

<u>Intersection</u>

$$x = (4q+1)\frac{2\pi}{5} \qquad x = 2p\pi$$

$$(4q+1)\cancel{2}\frac{\cancel{\pi}}{5} = \cancel{2}p\cancel{\pi}$$

$$p = \frac{4q+1}{5} \qquad p, q \in I$$

$$q \neq 0$$

q	1	6	11
p	1	5	9

1, 5, 9, 13, 17 - - .

$p = 1, 5, 9, 13, \ldots$

$p = T_n = 1 + (n-1)4 = \underline{\underline{4n-3}}$ ✓

$\underline{n \in I}$

$x = 2p\pi$

$x = 2(4n-3)\pi$

$x = (8n-6)\pi$

$\underline{\underline{n \in I}}$

$q \quad 1 \quad 6 \quad 11$

$q = 1 + (n-1) \times 5$

$= 5n - 4$

$x = \frac{2\pi}{5}(4q+1)$

$$x = \frac{2\pi}{5}(4q+1) \quad \text{where } q = 5n-4$$

$$x = \frac{2\pi}{5}(20n - 16 + 1)$$

$$= \frac{2\pi}{5}(20n - 15) = 2\pi(4n-3)$$

$$= (8n-6)\pi$$

$$n \in I$$

Q $\sin 3x + \cos 2x = -2$

mI $\sin 3x = -1$ $\cos 2x = -1$ (Self practice)

intersection

mII $3\sin x - 4\sin^3 x + 1 - 2\sin^2 x = -2$

$4\sin^3 x + 2\sin^2 x - 3\sin x - 3 = 0$

$\sin x = 1$

$(\sin x - 1)(4\sin^2 x + 6\sin x + 3) = 0$

$\underline{\sin x = 1}$

$x = 2n\pi + \frac{\pi}{2}$

or

$n\pi + (-1)^n \frac{\pi}{2}$

$n \in I$

$$\underbrace{4\sin^2 x + 6\sin x + 3 = 0}$$

$\underline{\underline{D < 0}}$

No real roots.

Q Good

Number of solutions of the equation in $[0, \pi]$

$$\sin x + 2\sin 2x = 3 + \sin 3x$$

$$\sin x + 4\sin x\cos x = 3 + 3\sin x - 4\sin^3 x$$

$$4\sin^3 x + 4\sin x\cos x - 2\sin x - 3 = 0$$

$$2\sin x(2\sin^2 x + 2\cos x - 1) - 3 = 0$$

$$\sin x\,(4(1-\cos^2 x) + 4\cos x - 2) - 3 = 0$$

$$\sin x\,(4(1-\cos^2 x)+4\cos x-2)-3=0$$

$$\sin x\,(4-4\cos^2 x+4\cos x-2)-3=0$$

$$\sin x\,(-4\cos^2 x+4\cos x+2)-3=0$$

$$\sin x\,(\underline{4\cos^2 x-4\cos x}-2)+3=0$$

$$\sin x\,((2\cos x-1)^2-3)+3=0$$

$$\sin x\,(2\cos x-1)^2-3\sin x+3=0$$

$$\underbrace{\sin x}_{\geq 0}\,\underbrace{(2\cos x-1)^2}_{\geq 0}+3\underbrace{(1-\sin x)}_{\geq 0}=0$$

No solution