SOLUTION COUNTING

Q find the number of solution of

$\sin x = \frac{1}{2}$ in $[0, 2\pi]$.

2 solution

(11) $[0, 4\pi] \rightarrow [0, 2\pi] \cup [2\pi, 4\pi] \rightarrow 4$

(2) (2)

$[0, 2\pi]$ → 1 cycle

$[0, 4\pi]$ → 2 cycles

1 cycles → 2 solution

2 cycles → 4 solution

(iii) $[0, 100\pi]$ → 50 cycles

50 cycles → 100 solution

Q find number of solution of

$\cos x = -\frac{\sqrt{3}}{2}$ in $[0, 50\pi]$

$[0, 2\pi] \rightarrow$ 2 solutions

25 cycles $\rightarrow$ $50 = 25 \times 2$ solutions

Q Number of solutions of

$\sin x = 1$ in $[0, 10\pi]$

$x = \frac{\pi}{2}$ in $(0, 2\pi)$ → 5 cycles

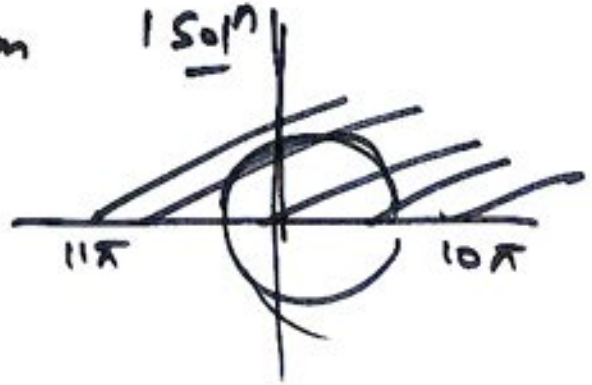
1 cycle → 1 solution

5 cycles → $5 \times 1 = 5$ solution.

(11) $[0, 11\pi]$

(11) $[0, 11\pi] = [0, 10\pi] \cup [10\pi, 11\pi]$

$\downarrow$ 5 Solution $\quad \downarrow$ 1 Soln



Total = 6

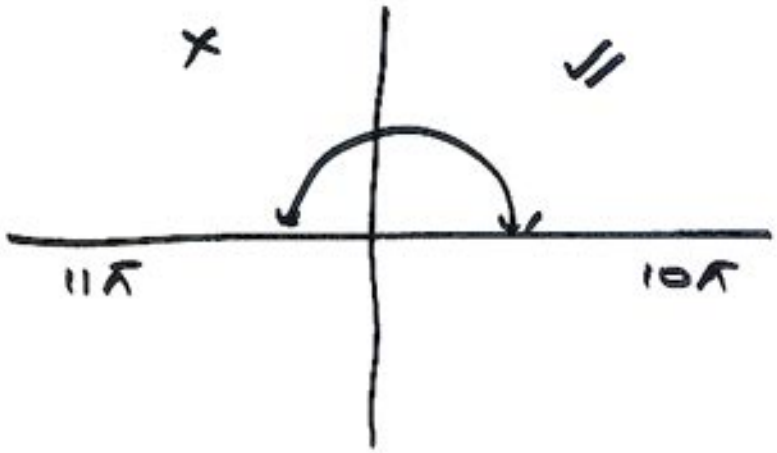
Q Number of solutions of $\cos x = \frac{1}{2}$ in $[0, 11\pi]$

$[0, 11\pi] = [0, 10\pi] \cup [10\pi, 11\pi]$

$[0, 10\pi]$ → 5 cycles

$[0, 2\pi]$ → 2 solutions

5 cycles → 10 solutions



$\cos x = \frac{1}{2}$

$[10\pi, 11\pi) \rightarrow$ 1 solution

$10 + 1 =$ 11 solution

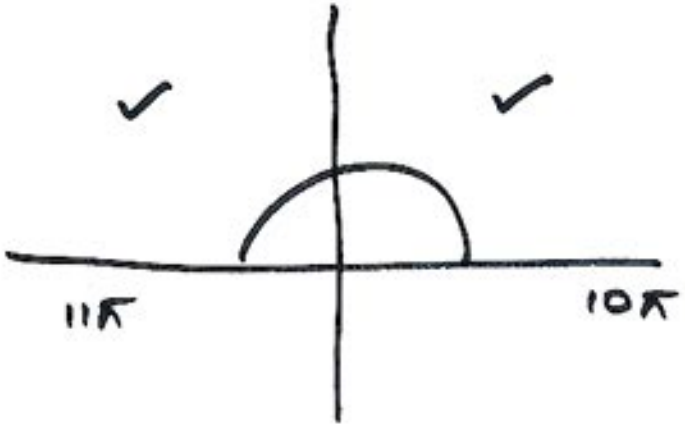
Q Number of solutions of

$\sin x = \frac{1}{2}$ in $[0, 11\pi]$

$[0, 11\pi] = [0, 10\pi] \cup [10\pi, 11\pi]$

$[0, 2\pi] \rightarrow$ 2 Solution

5 cycles $\rightarrow$ $5 \times 2 = 10$ Solution



$\sin x = \frac{1}{2}$

2 Solutions

$10 + 2 = 12$ Solutions

TYPE III FACTOR METHOD

Q1 find general solution for x

$(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

Also find number of solutions from $[0, 8\pi]$

Soln $(2\sin x - \cos x)(1+\cos x) = 1-\cos^2 x$

$(2\sin x - \cos x)(1+\cos x) = (1+\cos x)(1-\cos x)$

$1+\cos x = 0$

$\cos x = -1$ (R)

$x = (2n-1)\pi$ / $x = 2n\pi \pm \pi$

$n \in I$

(11) $2\sin x - \cos x = 1 - \cos x$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = n\pi + (-1)^n \frac{\pi}{6}$ $n \in I$

$\sin x = \frac{1}{2}$

$[0, 2\pi] \rightarrow$ 2 + 1

(3)

$\cos x = -1$

(π)

$[0, 8\pi]$

↓

4 cycles

4 cycles $\rightarrow$ $4 \times 3 = 12$ Solutions

Q2 find general value of θ

$$\cot^2\theta + 3\operatorname{cosec}\theta + 3 = 0$$

Also find number of solutions from $[0, 100\pi]$

$$\operatorname{cosec}^2\theta - 1 + 3\operatorname{cosec}\theta + 3 = 0$$

$$\operatorname{cosec}^2\theta + 3\operatorname{cosec}\theta + 2 = 0$$

$$(\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta + 2) = 0$$

$\operatorname{cosec}\theta = -1$ $\qquad$ $\operatorname{cosec}\theta = -2$

$\sin\theta = -1$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$$\begin{cases} \theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right) \\ \theta = 2n\pi - \frac{\pi}{2} \end{cases}$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$[0, 100\pi]$ → 50 cycles

$\sin\theta = -1$ → $\frac{3\pi}{2}$ ①

$\sin\theta = -\frac{1}{2}$ → 3rd, 4th ②

$[0, 2\pi]$

3 Solutions

50 cycles ⟶ $50 \times 3 = 150$ Solutions

Q

$$\cos 2\theta - (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$$

$$(2\cos^2\theta - 1) - (\sqrt{2}+1)\left(\frac{\sqrt{2}\cos\theta - 1}{\sqrt{2}}\right) = 0$$

$$(\sqrt{2}\cos\theta + 1)(\sqrt{2}\cos\theta - 1) - \left(1 + \frac{1}{\sqrt{2}}\right)(\sqrt{2}\cos\theta - 1) = 0$$

$$(\sqrt{2}\cos\theta - 1)\left(\sqrt{2}\cos\theta + 1 - 1 - \frac{1}{\sqrt{2}}\right) = 0$$

$\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 2n\pi \pm \frac{\pi}{4}$

$n \in I$

$\sqrt{2}\cos\theta - \frac{1}{\sqrt{2}} = 0$

$\cos\theta = \frac{1}{2}$

$\theta = 2n\pi \pm \frac{\pi}{3}$

$n \in I$

Q $\cot x - \cos x = 1 - \cot x \cos x$

$1 - \cot x \cos x - \cot x + \cos x = 0$

$(1 + \cos x) - \cot x (1 + \cos x) = 0$

$(1 - \cot x)(1 + \cos x) = 0$

$\cos x = -1 \qquad \cot x = 1$

$\cot x = 1$

$\tan x = 1$

$x = n\pi + \frac{\pi}{4}$

$\cos x = -1$

$x = 2n\pi \pm \pi$ or $(2n-1)\pi$

Rejected

$\cot x$ = not defined

$x = n\pi$

Q $2\cos x \cdot \cos 2x = \cos 2x$

$\cos 2x = 0$

$2x = (2n-1)\frac{\pi}{2}$

$x = (2n-1)\frac{\pi}{4}$ ✓

or

$2x = 2n\pi \pm \pi/2$

$x = n\pi \pm \pi/4$ ✓

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = 2n\pi \pm \frac{\pi}{3}$

$n \in ?$