• $h^*(n)$ we do not know

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PART = A

Q1 Define admissible heuristic. Give one example of an admissible heuristic and one example of a non-admissible heuristic.

Defination: A heuristic function $h(n)$ is called admissible, if for every node n, it never over estimate the true cost of reaching the goal from n

$$h(n) \leq h^*(n).$$

Where $h^*(n)$ is the optimal cost from the n to goal

* Example of an admissible Heristic

Suppose we have four direction up, down, left, right, where the cost of moving at each step is 1

our source is (2,3)
our goal is (5,5)

heuristic = Manhattan distance $= |x_{goal} - x_n| + |y_{goal} - y_n|$

$$h(2,3) = |5-2| + |5-3| = 5$$

The true cost from (2,3) to (5,5) is 5 mean that at least we have to take 5 move

→ Here $h(n)$ never overestimate, it is admisible

Example of Non-admisible

Now, suppose we define

$$h(n) = 2\times \text{(manhattan distance)}$$

At node (2,3)

$$h(2,3) = 2\times 5$$

But the true cost was only 5 and here it is 10.

So, since $h(n) = 10 \geq h^*(n) = 5$

This heuristic overestimate the cost

Q2. Define consistent heuristic. Explain how it is related to triangle inequality

A cosistent heuristic is heuristic function $h(n)$ that satisfies the condition

$$h(n) \leq c(n, n') + h(n')$$

for every node n, every successor n' of n, and where ~~actual~~ $c(n, n')$ is the actual cost of reaching n' from n

- $h(n)$ - heuristic estimate from node n to goal
- $c(n, n') \rightarrow$ actual step cost from node n to n'
- $h(n') \rightarrow$ heuristic from neighbour n' to goal.

* Relation Between Consistency & Triangle Inequality

→ We know the condition of consistent heuristic
i.e = $h(n) \leq c(n, n') + h(n')$

→ It is same as Triangle inequality
i.e

$$Dist(A, C) \leq Dist(A, B) + Dis(B, C)$$

- $h(n) \Rightarrow$ " estimate distance from n to goal $:= Dis(A, C)$
- $c(n, n')$ distance from n to n' to goal $(A-B)$
- $h(n')$ estimate distance from n' to goal $(B-C)$

3. Can a heuristic be admisible but not consistent Provide reasoning

Yes, a heuristic can be admissible but not consistent
Admisible mean it never overestimate the true cost to goal :
Cosistency means it also satisfies the triangle inequality - like condition across every edges

$$h(n) \leq c(n, n') + h(n')$$

Let's take an example:
Take Three nodes $S - A - G$ with cost

- $c(S, A) = 1$
- $c(A, G) = 4$

So, True cost will be:

- $h^*(G) = 0$
- $h^*(A) = 4$
- $h^*(S) = 5$

Now, we will look heuristic, let these be

- $h(G) = 0$
- $h(A) = 3$ $(3 \leq 4 \rightarrow$ admisible$)$
- $h(S) = 5$ $(5 \leq 5 \rightarrow$ admisible$)$

→ Check Consistency:
Let's check consistency from $S \rightarrow A$

$$h(S) \leq c(S, A) + h(A)$$

$$5 \leq 1 + 3 = 4 \qquad 5 \leq 4 \quad X$$

This is not True. $(5 > 4)$
So, the heuristic is non-consistent, even though each value individually is admisible

Q4. Why is admissibility necessary for A* to be optimal? Why is consistency necessary?

- Admissibility is necessary because if the heuristic never overestimate the true cost, A* is guaranteed to find the optimal solution

- Consistency is necessary because it ensures $f(n) = g(n) + h(n)$ is non-decreasing along the path, so A* does not need to reopen nodes - making search efficient.

→ This allow A* to safely use a closed list (expand node do not need to be revisited)

PART-B

Worked Example (Pen/Pencil)

1. Consider the following graph:

$A \xrightarrow[2]{} B \xrightarrow[2]{} C$ (goal)

$A \xrightarrow[5]{} C$

with $h(A)=3$, $h(B)=1$, $h(C)=0$ Show that A* find the optimal path

* Find the sortest path from A to C using A* Search

A* uses formula = $f(n) = g(n) + h(n)$
$g(n)$: cost from start to n node
$h(n)$: heuristic estimate from n to goal
$f(n)$: estimate total cost of path through n to goal

Step 0: Start At A.
$g(A) = 0$
$h(A) = 3$
$f(A) = g(A) + h(A) = 0+3 = 3$
open = [(A, f=3)]
closed = [ ]

Step 1 Expand A

- B : cost = 2 → $g(B)=2$, $h(B)=1$
$f(B) = g(B) + h(B) = 2+1 = 3$

- C : cost = 5 → $g(C)=5$ $h(C)=0$
$f(C) = g(C) + h(C) = 5+0 = 5$

open = [(B, f=3), (C, f=5)]
closed = [A]

Step 2 : Expand B (lowest $f = 3$)

- C : cost : $g(C) = 4$, $h(C) = 0$

$f(C) = g(C) + h(C)$
$= 4 + 0 = 4$

compare : Existing C in open $f = 5$ (via $A \to C$ directly)
new path C $f = 4$ (via ($A \to B - C$)
Replace old entry of C with $f = 4$

open = [(C, $f = 4$)]
closed = [A, B]

Step 3 : Expand C
C is the goal node
- Total cost = 4 (via A - B - C)

$\because$ A* finds the path $A \to B \to C$
Other path ($A \to C$ directly) cost 5 which is more

A* has found the optimal path

= With $h(A) = 5$, $h(B) = 5$, $h(C) = 0$, Show that Step by step how A* fails to find the optimal path

$f(n) = g(n) + h(n)$

Step 0: Start at A

$g(A) = 0$
$h(A) = 0$
$f(A) = 0 + 5 = 5$
open = [(A, f=5)] , closed = [ ]

Step 1: expand A
From A we have two neighbour

- B: $g(B) = 2$
  $h(B) = 5$
  $f(B) = g(B) + h(B)$
  $2 + 5 = 7$
- C: $g(C) = 5$
  $h(C) = 0$
  $f(C) = g(C) + h(C)$
  $= 5 + 0$
  $= 5$

open = [(C, f=5), (B, f=7)]
closed = [A]

Step 2: expand C (lowest f=5)
C is the goal via A → C, cost = 5
Stop A* terminate here, since it found the goal.

A* search return path A → C cost = 5
But the optimal path is A → B → C cost = 4

here heuristic is not admisible $\left\{\begin{matrix} h(A) = 5 \\ h B = 5 \end{matrix}\right\}$ overestimate
∴ A* fails & terminating prematurely

2: For the same graph, test consistency:
case:1 $h(A) = 3$, $h(B) = 1$, $h(C) = 0$
case: $h(A) = 4$, $h(B) = 5$, $h(C) = 0$
Identify which violet's the consistency condition:

Consistency condition
$$h(n) \leq c(n, n') + h(n')$$

case 1: check each edge
$A \to B$ (cost) $= 2$
$h(A) \leq c(A,B) + h(B)$
$3 \leq 2+1$ $\therefore 3 \leq 3$ ✓

$B \to C$ (cost $= 2$)
$h(B) \leq c'(B,C) + (C)$
$1 \leq 2+0$ $1 \leq 2$ ✓

$A \to C$ (cost $= 5$)
$h(A) \leq c(A,C) + h(C)$
$3 \leq 5+0$ $3 \leq 5$ ✓

Case 2: check each edge:

$A \to B$ (cost $= 2$)
$h(A) \leq c(A,B) + h(B)$
$4 \leq 2+5 = 4 \leq 5$ ✓

$\cdot B \to C$ (cost $= 2$)
$h(B) \leq c(B,C) + h(C)$
$5 \leq 2+0 = 5 \leq 2$ ✗

$A \to C$ (cost $= 5$)
$h(A) \leq c(A,B) + h(C)$
$4 \leq 5+0$ $4 \leq 5$ ✓

$\therefore$ Case 1 is ~~to~~ consistent case 2 is not consistent
bec = $B \to C$ voilate the consistency

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PART → D

1. Why A* is guaranteed optimal if both admissibility and consistency hold

A* is guaranteed to be optimal if the heuristic is both admissible and consistent because:

1. Admissibility ensures the heuristic never overestimate the ~~true~~ true cost to reach the goal $h(n) \leq h^*(n)$.
   This means that A* will never ignore the true shortest path thinking its more expensive than it really is

2. Consistency ensures that $f(n) = g(n) + h(n)$ is non decreasing along path, so, once a ~~Path~~ node is expand, the cost found is guaranteed to be the lowest Possible

in conclusion:
A* always expands node in order ~~to~~ of the lowest estimated total cost, never missis the optimal path, and find the shortest path efficiently.

Q2. Construct your own maze example where admissibility fails and show A* produces a non optimal result.

A heuristic h(n) is admissible if it never overestimate the true cost to the goal if il overestimate, A* may choose a "Seemingly" path that actually suboptimal

S	o	o	o	o
x	x	o	x	o
o	o	o	x	o
o	x	x	o	o
o	o	o	o	G

S - start (0,0)
G - Goal (4,4)
x → wall / obstacle

optimal path avoiding wall →
(0,0) → (0,1) → (0,2) → (0,3) → (0,4) → (1,4) → (2,4) → (3,4) → (4,4)
~~Total~~ COST = 8 steps

Let we define admisibility fails heuristic
$h(n)$ = Manhattan distance + 2

- For (0,0) → $h = |4-0| + |4-0| + 2 = 10$
- For (0,1) → $h = 9$

And so on like this

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Here I will show how It can Pick the wrong Path

step 1: Start (0,0)
- $g(s) = 0$
- $h(s) = |4-0| + |4-0| + 2 = 10$
- $f(s) = g+h = 0+10 = 10$
- Add to open list [(0,0)]

step 2: Expand (0,0)
- Neighbours (0,1) open, (1,0) Block
- $g(0,1) = 1$
- $h(0,1) = |4-0| + |4-1| + 2 = 9$
- $f(0,1) = g+h = 1+9 = 10$
- add open list [(0,1)]

step 3: Expand (0,1)
- Neighbour (0,2), open, (1,1) block, (0,0) -> already visited
- $g(0,2) = 2$
- $h(0,2) = |4-0| + |4-2| + 2 = 8$
- $f(0,2) = g+h = 2+8 = 10$ open list add = [(0,2)]

step 4: Expand (0,2)
- Neigh: (0,3) open, (1,2) open, (0,1) - already visited

1. $g(0,3)$
   - $g = 3$
   - $h = |4-0| + |4-3| + 2 = 7$
   - $f = 3+7 = 10$

   open list = [(0,3), (1,2)]

2. $g(1,2)$
   - $g = 3$
   - $h = |4-1| + |4-2| + 2 = 7$
   - $f = 3+7 = 10$

   open list = [(0,3), (1,2)]

-> here A* may pick (1,2) and lead to long Path.

problem:

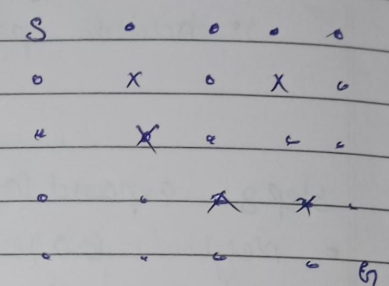
- A* may now explore (2,2) instead of (0,3)
- Due to overestimating heuristic, A* thinks alternative (1,2) -> (2,2) - - - is ~~better~~ equally good, better
- In reality optimal path is (0,0) -> (0,1) -> (0,2) -> (0,3) - - - - -

Q3: Construct your own maze example where consistency fails and show how A* may expand nodes incorrectly:

inconsistent

We define heuristic h(n) as manhattan distance

Except for node (2,2)
where we desperately set
$h(2,2) = 10$ (



* Normally, true manhattan at 2,2, would be 4, but we assume 10 to create inconsistency

→ Why This Violet Consistency

Take neighbour (2,3):

- $h(2,3) = |4-2| + |4-3| = 3$
- Edge cost $c((2,2),(2,3)) \to 1$

Consistency condition
$h(2,2) \leq 1 + h(2,3) = 10 \leq 4$ ✗

So consistency is violted

→ How it will create a problem or misbehaves

- Start (0,0) → expand toward (1,2) then (2,2)
- At (2,2), $g = 4$ but $h(2,2) = 10$, so $f = 14$ → looks very bad
- A* skips (2,2) and explore loger path

Q4. Construct, one example which is admissible but inconsistent:

Example for admissible but inconsistent

Graph =

(start)

$S \longrightarrow A \longrightarrow G$ (goal)

Edge cost:

- cost $(S, A) = 2$
- cost $(A, G) = 2$

So the true shortest path cost from S to G is $2+2=4$

Heuristic Value

- $h(G) = 0$
- $h(A) = 1$
- $h(S) = 4$

Check admissibility:

A heuristic is admissible if for every node n, $\rightarrow h(n) \leq h^*(n)$

$\cancel{h(n) \leq c(n, n') + h(n')}$

- $h(G) = 0 \leq 0$
- $h(A) = 1 \leq$ truecost $(A \rightarrow G) = 2$
- $h(S) = 4 \leq$ true_cost $(S \rightarrow G) = 4$

So, the heuristic is admissible.