# Parametric Estimation Assignment

**Ques.1**

$(x_1, x_2, \ldots x_n) \rightarrow$ random sample

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu \rightarrow$ mean
$\theta \rightarrow$ variance

MLE of 2 parameters

$$L(x_1 \ldots x_n) = L(x_1) \times L(x_2) \ldots L(x_n)$$

$$= \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_1-\mu)^2}{2\theta_2}} \times \ldots \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_n-\mu)^2}{2\theta_2}}$$

$$L = \frac{1}{\sqrt{2\pi\theta^2}} \prod_{i=1}^{n} e^{-\frac{(x_i-\mu)^2}{\theta_2}}$$

Taking log on both sides

$$\ln\left(\frac{1}{\sqrt{2\pi\theta^2}} e^{-\frac{(x_i-\mu)^2}{2\theta_2}}\right) = \ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right) + \ln\left(e^{-\frac{(x-\mu)^2}{2\sigma_2^2}}\right) \quad \ldots(1)$$

$$\ln\left(\frac{1}{\sqrt{2\pi\theta}}\right) = \ln(2\pi\theta_2)^{-\frac{1}{2}} \quad \ldots(2)$$

(2) in (1), and $\theta_2 = \sigma^2$

$$-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x_1-\mu)^2}{2\theta} \ln(e)$$

$\Rightarrow -\frac{1}{2}(\ln 2\pi + \ln \theta) - \frac{(x_1 - \mu)^2}{2\theta} \ln(e)$

$\Rightarrow -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\theta - \frac{(x-\mu)^2}{2\theta}$

↳ similar for n-1 terms

taking $\frac{\delta}{\delta\mu}$ on both sides.

$\frac{\delta(L(x_1 \ldots x_n))}{\delta\mu} = 0 - 0 + \frac{x_1 - \mu}{\theta} + \ldots \frac{x_n - \mu}{\theta}$

$= \frac{1}{\theta}(x_1 - \mu \ldots x_n - \mu)$

$= \frac{1}{\theta}[(x_1 + x_2 \ldots x_n) - n\mu]$ ... (2)

Now $\frac{\delta}{\delta\theta}$

$\frac{\delta L(x_1 \ldots x_n)}{\delta\theta} = 0 - \frac{n}{2} \times \frac{1}{\theta} + \frac{(x_1 - \mu)^2}{2\theta^2} + \ldots \frac{(x_n - \mu)^2}{2\theta^2}$

$= -\frac{n}{2\theta} + \frac{1}{2\theta^2}((x_1 - \mu)^2 + \ldots$

MLE over $\mu$ =

(2) = 0

$$\theta = \frac{1}{\theta}\left[(x_1 - \mu) + \dots + (x_n - \mu)\right]$$

$$x_1 + x_2 + \dots + x_n = n\mu$$

$$\mu = \frac{x_1 + x_2 + \dots + x_n}{n} \rightarrow \text{mean}$$

MLE over $\theta = 0$

$$L_\theta = -\frac{n}{2\theta} + \frac{1}{2\theta^2}\left((x_1-\mu)^2 + \dots + (x_n-\mu)^2\right)$$

(divide by $2\theta^2$) both sides

$$n\theta = (x_1-\mu)^2 + \dots + (x_n-\mu)^2$$

$$\theta = \frac{(x_1-\mu)^2 + \dots + (x_n-\mu)^2}{n} \rightarrow \text{variance}$$

Que 2 Binomial distribution →

$x_1 \dots x_n \in B(m, \theta)$

$\theta \in (0, 1)$

m is the

Bino find $\theta$

$p(x, \theta) = {}^{m}C_{x}\, \theta^{x} (1-\theta)^{m-x}$

$L(x_1, \dots x_n) = \prod_{i=1}^{n} p(x_i, \theta)$

taking log on both sides.

$\ln(L(x_1 \dots x_n)) = \sum_{i=1}^{n} \ln({}^{m}C_{x_i}) + x_i \ln\theta + (m-x_i)\log(1-\theta)$

NOW $\frac{\delta}{\delta\theta}$ on both sides

$\frac{\delta \ln L(x_1 \dots x_n)}{\delta\theta} = 0 + \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}$

$= \sum_{i=1}^{n} \frac{x_i(1-\theta) - (m-x_i)\theta}{\theta(1-\theta)}$

$= \sum_{i=1}^{n} \frac{x_i - m\theta}{\theta(1-\theta)} \quad \dots (1)$

MLE over $\frac{\delta}{\delta\theta} = 0$

$(1) = 0$

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Page No.
Date : / /20

$$\sum_{i=1}^{m} x_i - m\theta = 0 \qquad \left(\sum_{i=1}^{n} \frac{x_i}{n} = \bar{x}\right)$$

$\hookrightarrow$ mean

$$\theta = \frac{\sum_{i=1}^{mn} x_i}{mn} = \frac{\bar{x}}{m}$$

$$\therefore \theta = \frac{\bar{x}}{m}$$

$\cancel{\bar{x} = m\theta}$