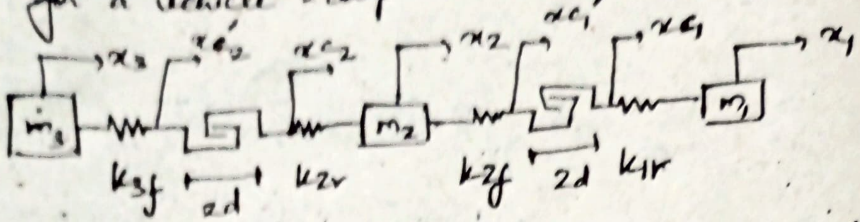
Q Longitudinal natural frequencies & mode shapes for a vehicle - coupler model.



Let variables be as shown in the above figure. We know,

$m_1 = m_2 = m_3 = m$

And,

$k_{1r} = k_{2f} = k_{2r} = k_{3f} = k$

There are 2 couplers,

Case 1: Both couplers are engaged.

$$\Rightarrow x_{c_1} - x_{c_1}' = 2d \longrightarrow (1)$$

And,

$$x_{c_2} - x_{c_2}' = 2d \longrightarrow (2)$$

Equations of motion:

$$\left\{\begin{array}{l} m\ddot{x}_1 + k(x_1 - x_{c_1}) = F_{r_1} + F_{g_1} + F_{db} \\ m\ddot{x}_2 + k(x_2 - x_{c_1}') + k(x_2 - x_{c_2}) = F_{r_2} + F_{g_2} \\ m\ddot{x}_3 + k(x_3 - x_{c_2}') = F_{r_3} + F_{g_3} \end{array}\right.$$

AND

$$k(x_1 - x_{c_1}) = k(x_{c_1}' - x_2)$$

$$\Rightarrow (x_1 + x_2) = x_{c_1}' + x_{c_1}$$

$$\Rightarrow x_1 + x_2 = 2x_{c_1} - 2d \quad \text{From (1)}$$

$$\Rightarrow \frac{x_1 + x_2}{2} + d = x_{c_1}$$

$$\frac{x_1 + x_2}{2} - d = x_{c_1}'$$

Similarly,

$$\frac{x_2 + x_3}{2} + d = x_{c_2}$$

$$\frac{x_2 + x_3}{2} - d = x_{c_2}'$$

Substituting in #

$$\#\begin{cases} m\ddot{x}_1 + k\left(\frac{x_1 - x_2}{2} - d\right) = Fr_1 + Fg_1 + F_y db \\ m\ddot{x}_2 + k\left(\frac{x_2 - x_1}{2} + d\right) + k\left(\frac{x_2 - x_3}{2} - d\right) = Fr_2 + Fg_2 \\ m\ddot{x}_3 + k\left(\frac{x_3 - x_2}{2} + d\right) = Fr_3 + Fg_3 \end{cases}$$

Hence, Mass Matrix $M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$

Stiffness Matrix

$$K = \begin{bmatrix} k/2 & -k/2 & 0 \\ -k/2 & k & -k/2 \\ 0 & -k/2 & k/2 \end{bmatrix}$$

Putting into MATLAB: $m = 60\times10^3$ kg, $k = 10^{\circ}$1e6 N/m

$\omega_1 = 0.0000$, mode shape $= [-0.0024 \;\; -0.0024 \;\; -0.0024]$
$= [1, 1, 1]$

$\omega_2 = 9.1287$, mode shape $= [-0.0029 \;\; 0.0000 \;\; 0.0029]$
$= [1, 0, -1]$

$\omega_3 = 15.8114$, mode shape $= [0.0017, -0.0033, 0.0017]$
$= [-0.5, +1, -0.5]$

Case 2: One coupler is not engaged

If say second coupler is not engaged then k_{2r}, k_{2f} are not stretched.

\# Now becomes,

$$m\ddot{x}_1 + k\left(\frac{x_1 - x_2}{2} - d\right) = Fr_1 + Fg_1 + F_y db$$
$$m\ddot{x}_2 + k\left(\frac{x_2 - x_1}{2} + d\right) = Fr_2 + Fg_2$$
$$m\ddot{x}_3 = Fr_3 + Fg_3$$

Hence, $M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$ $K = \begin{bmatrix} k/2 & -k/2 & 0 \\ -k/2 & k/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Putting into MATLAB:

$\omega_1 = 0$, mode shape $= [0, 0, 0.0041]$
$= [0, 0, 1]$

$\omega_2 = 0$, mode shape $= [-0.0029, -0.0029, 0]$
$= [1, 1, 0]$

$\omega_3 = 12.9099$, mode shape $= [-0.0029, 0.0029, 0]$
$= [1, -1, 0]$

<u>Case 3</u>: Both couplers disengaged

$\Rightarrow$

$$m\ddot{x}_1 = F_{r_1} + F_{g_1} + F_{gdb}$$
$$m\ddot{x}_2 = F_{r_2} + F_{g_2}$$
$$m\ddot{x}_3 = F_{r_3} + F_{g_3}$$

$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$, $K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Putting into MATLAB:

$\omega_1 = 0$ mode shape $= [0.0041, 0, 0]$
$= [1, 0, 0]$

$\omega_2 = 0$ mode shape $= [0, 0.0041, 0]$
$= [0, 1, 0]$

$\omega_3 = 0$ mode shape $= [0, 0, 0.0041]$
$= [0, 0, 1]$