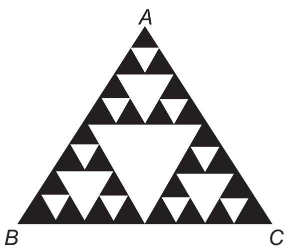
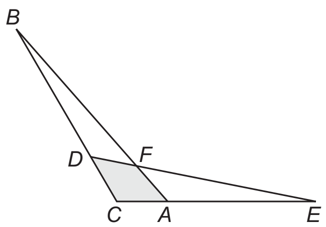
2 Mock Test Paper—2

1. In the diagram, all triangles are equilateral. If AB = 16, then the total area of all the black triangles is



(1) $25\sqrt{3}$ (2) $27\sqrt{3}$
(3) $35\sqrt{3}$ (4) $37\sqrt{3}$

2. In the figure below $DC = AC = 1$ and $CB = CE = 4$. If the area of triangle ABC is equal to S then the area of the quadrilateral $AFDC$ is equal to:



(1) $\frac{S}{2}$ (2) $\frac{S}{4}$
(3) $\frac{S}{5}$ (4) $\frac{2S}{5}$

3. Find the number of zeroes in
$100^1 * 99^2 * 98^3 * 97^4 * 96^5 * \_\_\_\_\_\_ * 1^{100}$
(1) 970 (2) 1070
(3) 1120 (4) 1124

4. The crew of an 8-member rowing team is to be chosen from 12 men ($M_1, M_2 \ldots, M_{12}$) and 8 women ($W_1, W_2 \ldots, W_8$). There have to be 4 people on each side with at least one woman on each side. Further it is also known that on the right side of the boat (while going forward) W_1 and M_7 must be selected while on the left side of the boat M_2, M_3 and M_{10} must be selected. What is the number of ways in which the rowing team can be arranged?
(1) $1368 \times 4! \times 4!$ (2) $1200 \times 4! \times 4!$
(3) $1120 \times 4! \times 4!$ (4) $728 \times 4! \times 4!$

5. In a community of 20 families, every family is expected to have the number of offsprings in an Arithmetic Progression with a common difference of 2, starting with 2 offsprings in the first family. Further for every child in the family every family is expected to deposit the same number of gold coins as there are children in the family towards a community contribution) for each child the family has. What will be the total number of gold coins collected in the community?
(1) 11480 (2) 11280
(3) 10880 (4) 10280

6. A boy plays a mathematical game wherein he tries to write the number 1998 into the sum of 2 or more consecutive positive even numbers (e.g. 1998 = 998 + 1000). In how many different ways can he do so?
(1) 5 (2) 6
(3) 7 (4) 8

7. Find the remainder when 2^{650} is divided by 224.
(1) 124 (2) 66
(3) 32 (4) 28

8. How many numbers are there between 100 and 1000, which have exactly one of their digits as 8?
(1) 300 (2) 1444
(3) 225 (4) 729

9. $f(n)$ is a function which is defined in three different domains as

$$f(n) = \begin{cases} 2n+4 \text{ if } n < -1 \\ 3n+5 \text{ if } -1 \le x < 1 \\ 2n-5 \text{ if } x \ge 1 \end{cases}$$

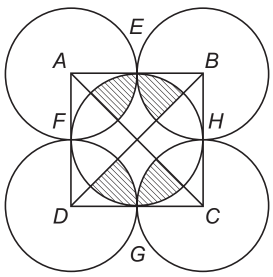
Find the value of (–5)?

(1) –20 (2) –12
(3) –32 (4) –36

10. Aman while getting bored in his school starts to draw lines on a fixed pattern. First, he draws a line 10 mm long. Then he constructs a square with this line as its diagonal. He then draws another square with a side of the first square as its diagonal. This process is repeated 'x' times, until the perimeter of the x^{th} square is less than $\frac{1}{12.5}$ mm. What is the least value of 'n'?
(1) 16 (2) 18
(3) 20 (4) 22

11. In the world championship of table tennis, players play best of 5 game matches. In every game the first player who scores 21 points wins. Service changes alternately between the 2 players after every five points. A player can score points both during his service and his opponent's service. Raman beat Ching Mai by 21-16 in a game. 24 of the 37 points played were won by the player serving. Who served first?
(1) Raman (2) Ching Mai
(3) Either could have served first
(4) Indeterminate

12. A square of side 12 m is drawn and a circle is inscribed in it. Now with



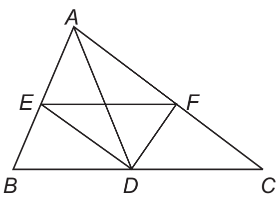
Now with each vertex as center, a circle is drawn passing through the midpoints of the two sides of the square, which meet at the vertex. Find the area of the shaded portion. (in m^2)
(1) $18\pi - 36$ (2) $36\pi - 72$
(3) $9\pi - 72$ (4) $18\pi - 72$

13. In a bombing of the Nathula pass, the Indian troops have to destroy a bridge on the pass. The bridge is such that it is destroyed when exactly 2 bombs hit it. A MIG-27 is dispatched in order to do the bombing. Flt Lt. Rakesh Sharma needs to ensure that there is at least 97% probability for the bridge to be destroyed. He knows that when he drops a bomb on the bridge the probability of the bomb hitting the bridge is 90%. Weather conditions and visibility being poor he is unable to see the bridge from his plane. How many bombs does he need to drop to be 95% sure that the bridge will be destroyed?
(1) 3 (2) 4
(3) 5 (4) 6

14. In a bombing of the Nathula pass, the Indian troops have to destroy a bridge on the pass. The bridge is such that it is destroyed when exactly 3 bombs hit it.
Flt Lt. Rakesh Sharma for his extraordinary skills is given the responsibility to drop the bombs again. If he drops a maximum of 5 bombs and he knows that when he drops a bomb on the bridge the probability of the bomb hitting the bridge is 90%. What will be the probability that the bridge would have been destroyed? (assume the bridge is visible to Flt Lt Sharma this time).
(1) 0.98348 (2) 0.98724
(3) 0.99144 (4) 0.99348

15. In a peculiar chessboard containing 12 × 12 squares, what is the greatest number of 5 × 5 squares that can be traced?
(1) 8 (2) 16
(3) 64 (4) 32

16. In the following figure (not drawn to scale) $\angle DEF$ = 42°. Find the other two angles of ΔDEF if DE and DF are the angle bisectors of $\angle ADB$ and $\angle ADC$ respectively.



(1) 28° and 11°0 (2) 65° and 73°
(3) 67° and 71° (4) 48° and 90°

17. The sides of a triangle have 4, 5 and 6 interior points marked on them respectively. The total number of triangles that can be formed using any of these points
(1) 371 (2) 415
(3) 286 (4) 421

18. The sides of a triangle have 4, 5 and 6 interior points marked on them respectively. The total number of triangles that can be formed using any of these points and/or the vertices would be?
(1) 704 (2) 415
(3) 684 (4) 421

19. The number of digits lying between 2000 and 7000 (including 2000 and 7000) which have at least two digits equal but at most 3 digits equal is:
(1) 2476 (2) 2474
(3) 2475 (4) 2473

20. The number of ways in which three distinct numbers in AP can be selected from 1, 2, 3, …, 24 is
(1) 144 (2) 276
(3) 572 (4) 132

ANSWER KEY

1. (~~4~~) 2	2. (4)	3. (4)	4. (1)
5. (1)	6. (3)	7. (3)	8. (3)
9. (4)	10. (2)	11. (1)	12. (2)
13. (1)	14. (3)	15. (3)	16. (4)
17. (4)	18. (1)	19. (1)	20. (4)

Solutions and Shortcuts

Solution 1: Level of Difficulty (1)

In the diagram, there are 27 black triangles. If the entire diagram was divided into the smallest size equilateral triangles, there would be $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$ equilateral triangles). Thus, 27/64 of D *ABC* is coloured black, so 37/64 is unshaded.

Area of triangle *ABC* = $\left\{\sqrt{(3)/4}\right\} \times 16 \times 16 = 64\sqrt{3}$. Hence, the area of the unshaded portion is $37\sqrt{3}$. Drop a perpendicular from *A*, meeting *BC* at *D*. Since *D ABC* is equilateral and $AB = 16$, then $BD = DC = 8$. Hence, Option 4 is correct.

Solution 2: Level of Difficulty (3)

The best way to solve problems like these is to assume specific case: Assume *ACB* and *ECB* to be right angled. Also assume that the point *C* is at the origin (0, 0). Then by the information we can conclude that *E* would be (4, 0), *B*(0, 4), *A*(1, 0) and *D*(0, 1).

Then the equations of the lines *AB* and *ED* would be $4x + y = 4$ and $x + 4y = 4$ respectively.

The intersection point would be got by equating these lines: $F = (4/5, 4/5)$.

Area of *CFA* is 2/5. Hence, area of *CAFD* = 2*2/5 = 4/5. But area of $ABC = S = 2$. Hence, choice (4) is the right answer.

Solution 3: Level of Difficulty (2)

$$= (1 + 6 + 11 + 16 + 26 + 31\_\_\_ + 96) + (1 + 26 + 51 + 76)$$
$$= 20*48.5 + 4*38.5$$
$$= 970 + 154 = 1124$$

Solution 4: Level of Difficulty (3)

In this question you will first have to complete the selection of 4 people for either side and then arrange the rowers on each side (which would be done by using 4!)

The solution would depend on the following structure—the structure would vary based on whether you select 2 more men for the right side or you select 1 man and 1 woman for the right side or you select 2 women for the right side.

The solution would be given by:

$$ {}^{12}C_2 \times 4! \times {}^{8}C_1 \times 4! + {}^{12}C_1 \times {}^{8}C_1 \times 4! \times {}^{7}C_1 \times 4! + {}^{8}C_2 \times 4! \times {}^{6}C_1 \times 4! = 1368 \times 4! \times 4!$$

Hence, Option 1 is the correct answer.

Solution 5: Level of Difficulty (3)

The number of children in the family would be: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32 ,34, 36, 38, 40.

Hence, the number of gold coins selected would be:
$2 \times 2 + 4 \times 4 + 6 \times 6 + \ldots 40 \times 40 = 4 + 16 + 36 + 64 + 100 + 144 + 196 + 256 + 324 + 400 + 484 + 576 + 676 + 784 + 900 + 1024 + 1156 + 1296 + 1444 + 1600 = 11480$

Solution 6: Level of Difficulty (3)

For answering this question we need to plan the use of the factors of 1998.

$1998 = 2 \times 3^3 \times 37 \rightarrow 16$ factors viz. 1×1998, 2×999, 3×666, 6×333, 9×222, 18×111, 27×74, 54×37.

Thus we could form 7 APs as follows:

(1) An AP with 2 terms and average 999
(2) An AP with 3 terms and average 666 and so on $\rightarrow$ 7 ways.

Solution 7: Level of Difficulty (1)

$$2^{650} = 25 \cdot (2^3)^{215} = 32 \times 8^{215}/224 = 8^{215}/7$$

gives us a remainder of 1. Since we have cut the numerator and denominator by 32 in the process, the actual remainder must be 32.

Solution 8: Level of Difficulty (2)

There are exactly 72 numbers in which 8 comes at the units place. Given by $8 \times 9 \times 1$

Similarly at the tens place there are $8 \times 1 \times 9$ ways

Also at the hundred's place there are $1 \times 9 \times 9 = 81$ ways

So, there are $72 + 72 + 81 = 225$ numbers between 100 and 1000 which have exactly one of their digits as 8.

Solution 9: Level of Difficulty (1)

$$\begin{aligned} f(-3) &= 2(-5) + 4 = -6 \\ f(-6) &= 2(-6) + 4 = -8 \\ f(-8) &= 2(-8) + 4 = -12 \\ f(-12) &= -20 \\ f(-20) &= -36 \end{aligned}$$

Solution 10: Level of Difficulty (2)

(3) Diagonal of first square = 10 mm.

Side of first square = $10/\sqrt{2}$

Side of second square = $10 \times 1/(\sqrt{2})^2 \ldots$

Side of x^{th} square = $10 \times 1/(\sqrt{2})^x$

$\Rightarrow 4 \times 10 \times 1/(\sqrt{2})^x < 0.08 \Rightarrow 1/(\sqrt{2})^x < 0.002$

$\Rightarrow (1/2)^{x/2} < 2/1000 \Rightarrow x$ should be at least 18.

Solution 11: Level of Difficulty (3)

In all 37 services are made. Two cases are possible.

(a) Raman has served 20 times and Ching Mai has served 17 times.
(b) Ching Mai has served 20 times and Raman has served 17 times.

We would need to see what occurs in each case to understand which of these situations holds true.

Case (a): In this case Raman starts serving. Let *X* be the score obtained by Raman in his services and *Y* be the score obtained by Raman when Ching Mai serves. The following table would emerge:

Player who serves	*No. of balls serves*	*Scores of Raman*	*Scores of Ching Mai*
R	C		
R	20	X	$20 - X$
C	17	Y	$17 - Y$

Now, $X + Y = 21$, $X + 17 - Y = 24$.
$\Rightarrow X = 14 \Rightarrow$ we get an acceptable solution.

Case (b): In this case Ching Mai starts serving.

Player who serves	*No. of balls serves*	*Scores of Raman*	*Scores of Ching Mai*
R	C		
C	20	X	$20 - X$
R	17	Y	$17 - Y$

$\Rightarrow \quad X + Y = 21$
$\Rightarrow \quad 20 - X + Y = 24$
$\Rightarrow \quad Y = 12.5$ (inadmissible)

Hence Raman started first.

Solution 12: Level of Difficulty (2)

In order to solve the question you need to find the difference between the area of the quarter circle (inside the square) and the right angled triangle at the center of the inner circle having base and height equal to 6 m each. The required area would be 4 times this value.

Hence, the required answer would be given by:

$$4x\,[36\pi/4 - 18] = 36\pi - 72$$

Solution 13: Level of Difficulty (3)

The probability if he drops 3 bombs will be given by: Hit and Hit OR Miss and Hit and Hit OR Hit and Miss and Miss $= 0.9 \times 0.9 + 0.1 \times 0.9 \times 0.9 + 0.9 \times 0.1 \times 0.9 = 0.81 + 0.081 + 0.081 = 0.972 > 0.97$.

Hence, 3 bombs would give him a probability of higher than 97% for the bridge to be destroyed.

Solution 14: Level of Difficulty (3)

The bridge would be destroyed under the following conditions: In 3 bombs OR In 4 Bombs Or In 5 Bombs Hit, Hit and Hit OR One miss and 3 hits (3 ways) OR 2 misses and 3 hits ($4C_2 = 6$ ways)

$$= (0.9)^3 + 3 \times (0.9)^3 \times (0.1)^1 + 6 \times (0.9)^3 \times (0.1)^2$$
$$= 0.729 + 3 \times 0.0729 + 6 \times 0.00729 = 0.99144.$$

Solution 15: Level of Difficulty (2)

The required answer would be given by $8 \times 8 = 64$ (Because if you take the top side of the square on the top side of the chess board, you would be able to trace out 8 squares. Similarly you would get exactly 8 squares in each row when you move to the next row of the chess board. This would continue till the 8th row of the chess board is taken as the top row of the 5×5 square. Hence, $8 \times 8 = 64$).

Solution 16: Level of Difficulty (2)

The measure of the angle *EDF* has to be 90° since it should be half of the 180° angle. Hence, the required answer has to be option 4.

Solution 17: Level of Difficulty (3)

You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

This can be done in: $4 \times 5 \times 6 + {}^4C_2 \times {}^{11}C_1 + {}^5C_2 \times {}^{10}C_1 + {}^6C_2 \times {}^9C_1 = 120 + 66 + 100 + 135 = 421$

Solution 18: Level of Difficulty (3)

In order to solve this question, you will first need to solve assuming that neither of the three vertices (say A, B or C) are used. For this we can solve the following way: You can form triangles by taking 1 point from each side, or by taking 2 points from any 1 side and the third point from either of the other two sides.

This can be done in: $4 \times 5 \times 6 + {}^4C_2 \times {}^{11}C_1 + {}^5C_2 \times {}^{10}C_1 + {}^6C_2 \times {}^9C_1 = 120 + 66 + 100 + 135 = 421$

Then, you need to consider situations where you are either taking one of the vertices of the triangle *ABC* or taking two vertices from the triangle. Thus assuming 6 points on *AB*, 4 on *AC* and 5 on *BC* the following possibilities emerge:
Taking $A - 5C_2 + 6 \times 5 + 6 \times 4 + 5 \times 4 = 10 + 30 + 24 + 20 = 84$
Taking $B - 4C_2 + 4 \times 5 + 4 \times 6 + 5 \times 6 = 80$
Taking $C - 6C_2 + 6 \times 4 + 6 \times 5 + 5 \times 4 = 89$
Taking $AB - 9$
Taking $AC - 11$
Taking $BC - 10$
Thus the total number of ways is: $421 + 84 + 80 + 89 + 9 + 11 + 10 = 704$

Solution 19: Level of Difficulty (3)

Numbers having 3 digits equal—Starting with 2: Numbers with 3 twos-222_ $\rightarrow$ 9 numbers (and 3 such cases so a total of 27 such numbers) Numbers with 3 digits other than twos $\rightarrow$ like 2000, 2111, 2333 … 2999 = 9 numbers

Total of 36 numbers

Similarly, numbers starting with 3 and having 3 digits common would be 36

numbers starting with 4 and having 3 digits common would be 36

numbers starting with 5 and having 3 digits common would be 36

numbers starting with 6 and having 3 digits common would be 36

Total numbers having 3 digits equal = 180.

Numbers having 2 digits equal—Again we first count number of numbers that are starting with 2 and having 2 digits equal and then multiply the same by 5 to get the answer: In order to find the number of numbers starting with

2 and having 2 digits equal we need to divide the same into:

(a) Numbers starting with 2 and having 2 twos and two more digits equal $3C_1 \times 9 = 27$

OR

(b) Numbers starting with 2 and having 2 twos and two more digits which are not equal to one another-

$$3C_1 \times 9C_1 \times 8C_1 = 216$$

OR

(c) Number of numbers starting with 2 and having two digits which are same and a fourth digit which is other than 2 as well as different from the other digit which has been used for 2 digits same.

$$9C_1 \times 3C_1 \times 8C_1 = 216$$

Thus, the total number of such numbers is 459.

Totally, $459 \times 5 + 180 + 1$ (for 7000) = 2476

Solution 20: Level of Difficulty (2)

If we take common difference = 1, then (1, 2, 3), (2, 3, 4) … (22, 23, 24). There will be 22 combinations.

If we take common difference = 2, then total 20 combinations are possible i.e. (1, 3, 5), (2, 4, 6), (3, 5, 7), …, (20 , 22, 24).

If we take c.d. = 3, 18 combinations are possible

If we take c.d. = 4, 16 combinations are possible

(There will be two combinations in last i.e. (1, 12, 23) & (2, 13, 24) with c.d. = 11

So total no. of combinations

$$= 22 + 20 + 18 + 16 + \ldots + 4 + 2$$
$$= 2 \times (1 + 2 + 3 + \ldots + 11)$$
$$= 2 \times \frac{11 \times 12}{2} = 132$$