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Data Science, 2022

Tutorial 6:

Machine Learning I

Q) 1) $P(H) = \lambda$

$P(T) = 1-\lambda$

$P(H \text{ at } (k+1)^{th} \text{ toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } k+1^{th} \text{ toss})$

$= (1-\lambda)^k \lambda.$

b) Let M be no. of tosses required to get the first head and let $S = E[M]$

As tosses are independent & expectation is additive

$$S = \lambda + (1-\lambda)(S+1)$$

$$S = \lambda + S + 1 - \lambda S - \lambda$$

$$S\lambda = 1$$

$$\boxed{S = \frac{1}{\lambda}}$$

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Q2) $X \rightarrow$ random variable.

a) Variance of X:

$$var(X) = E[(X - E[X])^2]$$

To prove $var(X) = E[X^2] - E[X]^2$.

Given that :-

$$var(X) = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - 2E[XE[X]] + E[X]^2$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2 \quad -(1)$$

b) $E[X] = 0$ and $E[X^2] = 1$.

To find 1) variance of X.

2) If $Y = a + bX$, $var(Y) = ?$

(1) $var(X) = E[X^2] - E[X]^2$ from (1)

$$= 1 - 0^2$$

$$var(X) = 1$$

(2) $Y = a + bX$.

$$E[Y^2] = E[(a+bX)^2]$$

$$= E[a^2 + 2abX + b^2X^2]$$

$$= a^2 + 2ab\,E[X] + b^2E[X^2]$$

$$= a^2 + 2ab(0) + b^2(1)$$

$$E[Y^2] = a^2 + b^2$$

$$E[Y] = E[a+bX]$$
$$= a + bE[X]$$
$$= a + b(0)$$
$$E[Y] = a$$

$$var(Y) = E[Y^2] - E[Y]^2$$
$$= a^2 + b^2 - a^2$$
$$var(Y) = b^2$$

Q3) Let A be the event that "Aku predicts that a given horse is a winning horse".
Let ~A be the eventt that "Aku predicts that the given horse is not a winning horse".

Similarly, let B be the event that the given horse wins and ~B be the event that given horse doesnot win.

a) Given a horse, the probability that it wins is:

$$P(B) = P(B, A) + P(B, \sim A)$$
$$= P(B/A)\,P(A) + P(B/\sim A)\,P(\sim A)$$
$$= 0.99 \times 10^{-5} + (1 - 0.9999) \times (1 - 10^{-5})$$
$$P(B) = 1.99 \times 10^{-5} \quad \text{——} \quad (1)$$

b) Probability that Anu predicts that the black beauty is winning

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(A|B)P(A)}{P(B)}$$

$$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$ — from ①.

$$P(A/B) = 0.497$$