Data Science, 2022

Tutorial 7

Machine learning 2

Bayesian Network.

Travel $\longrightarrow$ Foreign Purchase
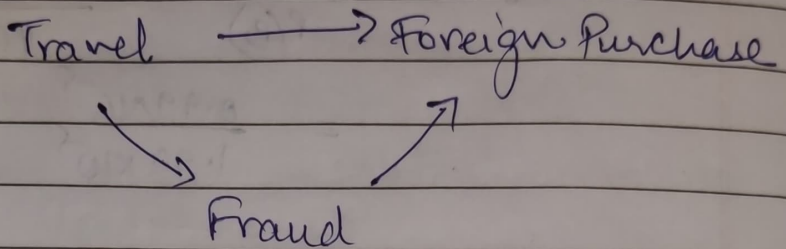
Fraud

- Increased probability of travel makes fraud more likely. Travel can cause Fraud.
- Increased probability of foreign purchase makes fraud more likely. Foreign purchase is evidence for fraud.
- Travel and fraud can each cause foreign purchase. Travel explains foreign purchase and so is evidence against fraud.

True	False
0.05	0.95

Travel $\longrightarrow$ Foreign Purchase
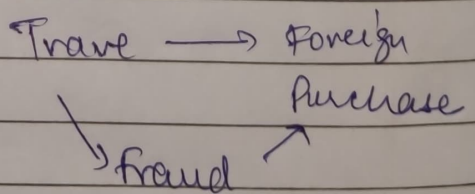
Fraud

Travel	True	False
True	0.01	0.99
False	0.002	0.998

Travel	Fraud	True	False
True	True	0.9	0.1
False	True	0.1	0.9
True	False	0.9	0.1
False	False	0.01	0.99

Q) Foreign purchase = true
   travel = ?
   fraud = ?

$P($Fraud=true | Foreign purchase = True$)$

$= \alpha \times P($Fraud$= T \frac{1}{2}$ travel $= T) *$
     $P($Foreign purchase $= T,$ travel$=T,$
        fraud $= T) \times P($Travel $=$ true$)$
   $+ P($Fraud$=T$ | travel $= F) *$
     $P($Foreign purchase $= T,$ travel $=$ False,
        Fraud $=T) * P($Travel $=$ False$)]$

$= \alpha \left[0.01 * 0.9 * 0.05 + 0.002 * 0.1 * \right.$
                                      $\left. 0.95 \right]$

$= \alpha \left[0.00045 + 0.00019 \right]$

$= 0.00064 \alpha$

$P(\text{Fraud} = \text{False} \mid \text{Foreign-purchase} = T)$

$$= \alpha \left[\begin{array}{l} P(\text{Fraud} = F, \text{travel} = T) * \\ P(\text{Foreign-purchase} = T \mid \text{True} = \text{true}, \\ \quad \text{Fraud} = \text{False}) * \\ P(\text{travel} = T) \\ \\ + \\ \\ P(\text{Fraud} = F \mid \text{travel} = F) \cdot * \\ P(\text{Foreign-purchase} = T, \text{travel} = F, \\ \quad \text{Fraud} = F) * \\ P(\text{travel} = F) \end{array} \right]$$

$$= \alpha \left[\begin{array}{l} 0.99 * 0.9 * 0.05 + \\ 0.998 \times 0.01 \times 0.95 \end{array} \right]$$

$$= \alpha \left[0.04455 + 0.009481 \right]$$

$$= 0.054031 \, \alpha$$

$$\alpha = \frac{1}{0.00064 + 0.054031} = 18.291$$

$P(\text{Fraud} = \text{true} \mid \text{Foreign purchase} = T)$

$$= 0.00064 \, \alpha$$

$$= 0.00064 \times 18.291$$

$$= 0.01170$$

$$= 1.17\% //$$

2) $P(\text{fraud} = T \mid \text{foreign purchase} = T, \text{travel} = T)$.

$$= \alpha \times 0.00045.$$

$P(\text{fraud} = F \mid \text{foreign-purchase} = T, \text{travel} = T)$

$$= 0.04455 \times \alpha$$

$$\alpha = \frac{1}{0.00045 + 0.04455} = 22.222$$

$P(\text{fraud} = T \mid \text{foreign-purchase} = T, \text{travel} = T)$

$$= \alpha \times 0.00045$$
$$= 22.222 \times 0.00045$$
$$= 0.01$$

$\therefore P(\text{fraud} = T \mid \text{foreign-purchase} = T,$
$$\text{travel} = T)$$
$$= 1 \, \%$$