Data Science, 2022.

Tutorial 5:
Evaluation and Measurement -
Hypothesis testing.

Ex1 $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

Level of significance $= \alpha = 0.10$.

Test statistic: Binomial variable X with $p = 0.7$ and $n = 15$.

$x = 8$ and $np_0 = 15 \times 0.7$
$= 10.5$

$$p = 2P(X \leq 8 \text{ when } p = 0.7)$$
$$= 2\sum_{x=0}^{8} b(x; 15, 0.7)$$
$$= 2 \times 0.1311 \text{ (From binomial Probability Table)}$$
$$= 0.2622$$

$\therefore p > 0.10$ i.e $p > \alpha$

$\therefore$ Do not reject H_0. Conclusion:- There is insufficient reason to doubt the builder's claim.

Ex 2. $H_0 : p = 0.6$.

$H_1 : p > 0.6$.

level of significance : $\alpha = 0.05$

Given that $x = 70$, $n = 100$

$p_0 = 0.6$.

$\therefore \quad Z = \dfrac{x - np_0}{\sqrt{np_0q_0}}$.

$\therefore \quad Z = \dfrac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}}$.

$\therefore Z = 2.04$.

$P = P(Z > 2.04)$.

$P = 0.0207$ (from table).

As $P < \alpha$, we reject H_0 and conclude that new drug is superior.

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Ex 3

Let P_1 be the proportion of Mumbai voters & P_2 be the proportion of surrounding area residents.

$$\hat{P}_1 = \frac{120}{200} = 0.6$$

$$\hat{P}_2 = \frac{240}{500} = 0.48$$

$$\hat{P}_P = \frac{120+240}{700} = 0.514$$

$\therefore \alpha = 5\% = 0.05$

Hypothesis :-

$H_0 : \quad P_1 \le P_2$

$H_1 : \quad P_1 > P_2$

$$\cancel{z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_1)(1-\hat{P}_2)}}}$$

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$= \frac{0.6 - 0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200}+\frac{1}{300}\right)}}$$

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$\therefore z = 2.869$

$\therefore p = P(\cancel{z} > 2.869)$

$\therefore p = 0.0044$

As $p < \alpha$, we reject H_0 & conclude that the proportion of Mumbai voters favouring the proposal is higher than the proportion of surrounding area voters.

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Ex Q4)

a) Null Hypothesis.

$H_0 : p = 0.2$

Alternative hypothesis.

$H_1 : p > 0.2$

The critical region is in right tail.

b) Null Hypothesis :-

$H_0 : \mu = 3.$

Alternative hypothesis :-

$H_1 : \mu \neq 3.$

The critical region is in both tails

c) Null hypothesis :-

$H_0 : p = 0.15.$

Alternative hypothesis :-

$H_1 : p < 0.15$

The critical region is in left tail.

d) Null Hypothesis :-
$H_0 : \mu = 500$.

Alternative hypothesis :-
$H_1 : \mu > 500$.

The critical region is in the right tail

e) Null Hypothesis :-
$H_0 : \mu = 15$

Alternative Hypotheses :-
$H_1 : \mu \neq 15$.

The critical region is in both tails.

Ex 5 Let μ_1 and μ_2 be the population mean "robustness" of laptops supplied by company A & B respectively.

$H_0 : \mu_1 = \mu_2$.

$H_1 : \mu_1 \neq \mu_2$

Significance level $= \alpha = 0.05$.

$$\overline{x_1} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$= \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8.0 + 6.5 + 9.2 + 7.0}{10}$$

$$\therefore \overline{x_1} = 7.95$$

$$\overline{x_2} = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i}$$

$$= \frac{11.0 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11 + 11.1 + 10.2 + 9.6}{10}$$

$$\overline{x_2} = 10.26$$

$$S_1^2 = \frac{1}{n_1 - 1}\left[\sum_{i=1}^{n_1} x_{1i}^2 - n_2 \overline{x_1}^2\right]$$

$$S_1^2 = \frac{10.865}{9} = 1.207.$$

$$S_2^2 = \frac{1}{n_2 - 1}\left[\sum_{i=1}^{n_2} x_{2i}^2 - n_2 \bar{x}_2^2\right]$$

$$S_2^2 = \frac{2.924}{9} = 0.325$$

Since sample variances are quite different, we cannot assume that population variances are equal, so we will use the unpooled t-test.

The degrees of freedom for this test are calculated as:-

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{9}\left(\frac{1.207}{10}\right)^2 + \frac{1}{9}\left(\frac{0.325}{10}\right)^2}$$

$$= 10.3 \approx 10$$

The test statistics used to test these hypothesis is

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

which under the null hypothesis follows approximately t-distribution with $v = 10$ degrees of freedom.

Also, under the null hypothesis we have $\mu_1 - \mu_2 = 0$, so the value of test statistic is

$$T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.9$$

Since the test is two sided, then the value of test is the doubled area under the density curve of t-distribution with 10 degrees of freedom, right of the absolute value of test statistic.

$|t| = |-5.9| = 5.9$ i.e.

the pvalue $= 2P(T \geq |t|)$

$= 2P(T \geq 5.9)$

$t_{0.0005}(10) = 4.587$ and since $|t| = 5.9$ is even greater than $P(T \geq 5.9) < 0.0005$

So pvalue < 0.001

As $p < \alpha$, we reject the H_0 in favor of alternative hypothesis and conclude that the mean robustness of laptops is not the same for the two companies.