Data Science, 2022.
Tutorial 4 :-
Independent Component Analysis.

Ex 1 Mixing statistically independent sources.

$$var(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i\right)^2 \right\rangle - \left\langle \sum_i w_i s_i \right\rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i\right)^2 \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right)^2$$

$$= \left\langle \left(\sum_i w_i s_i\right)\left(\sum_j w_j s_j\right) \right\rangle$$

$$- \left(\sum_i w_i \langle s_i \rangle\right)\left(\sum_j w_j \langle s_j \rangle\right)$$

$$= \left\langle \sum_{i,j} w_i w_j s_i s_j \right\rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_{i,j} w_i w_j \langle s_i, s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$+ \sum_{i,j;\, i \neq j} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2 \right) \quad +$$

$$\sum_{i,j;\, j\neq i} w_i w_j \left(\langle s_i\rangle\langle s_j\rangle - \langle s_i\rangle\langle s_j\rangle\right)$$

s_i & s_j are statistically independent for $i \neq j \Rightarrow$

$$\langle s_i\rangle\langle s_j\rangle - \langle s_i\rangle\langle s_j\rangle = 0.$$

Also $var(s_i) = 1$

$$\therefore \; Var(x) = \sum_i w_i^2$$

To guarantee that the mixture has unit variance,

$$var(x) = 1$$

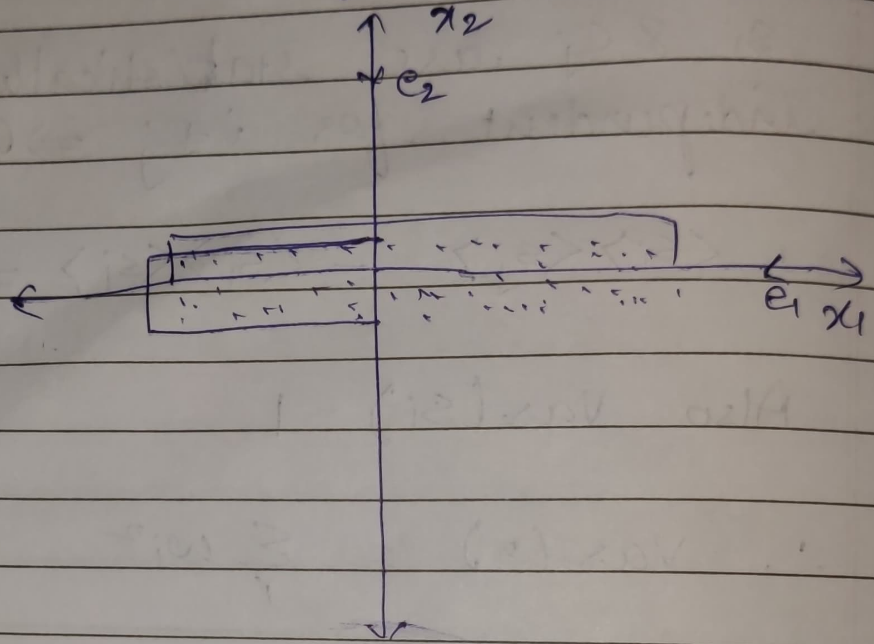
$$\therefore \; \sum_i w_i^2 = 1$$

$\therefore$ The following constraint has to be imposed on the weights w_i for the mixture to have unit variance.

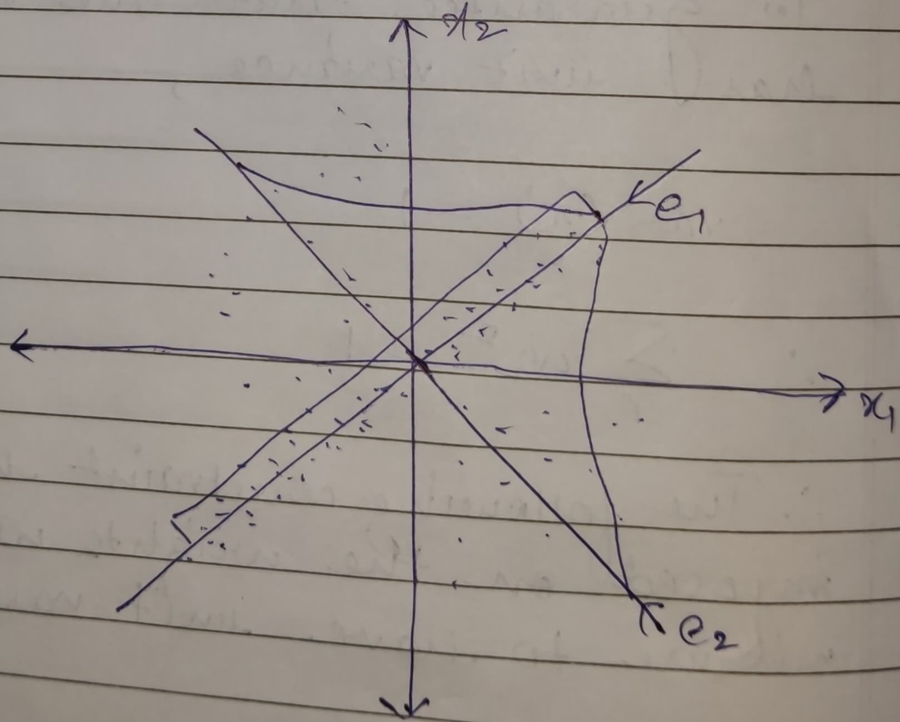
$$\sum_i w_i^2 = 1$$

Ex 2 Guess independent components and distribution from data.

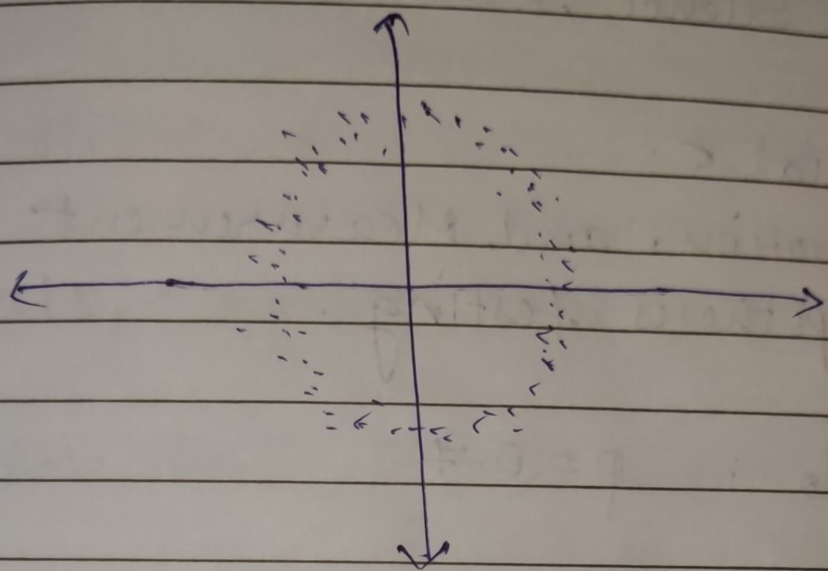
a)



b)



c)



This cannot be separated into independent components.