Tutorial 1

1. Determine whether or not each of the signal is periodic. In case a signal is periodic, specify its fundamental period.

a) $x_a(t) = 3\cos(5at + \pi/6)$

Soln.

Comparing it with $x(t) = A\cos(\Omega t + \phi)$

$\Omega = 5a$

$T = \frac{2\pi}{5a}$ which is irrational so non-periodic.

b) $x[n] = 3\cos[0.25n]$

Soln.

$w = 0.25$

$T = \frac{2\pi}{0.25}$ which is irrational so non-periodic

c) $x[n] = \cos[\pi n/2] - \sin[\pi n/8] + 3\cos(\pi n/4 + \pi/3)$

Soln.

$w_0 = \pi/2$

$T_0 = \frac{2\pi}{\pi/2} = 4$ which is rational.

$w_1 = \pi/8$

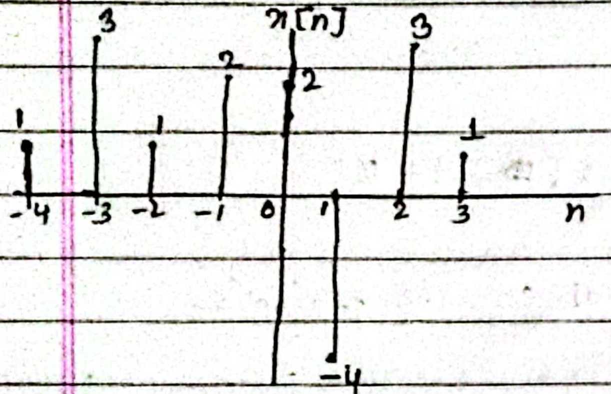
$T_1 = 16$

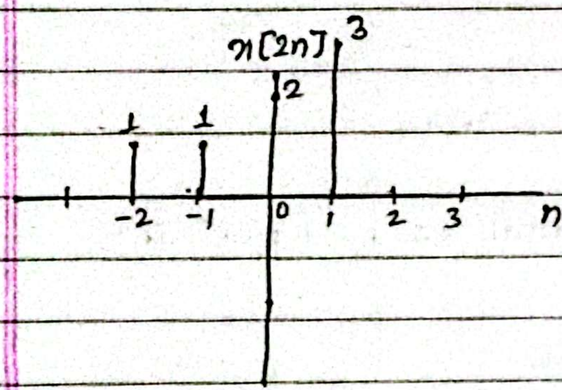
$w_2 = \pi/4$

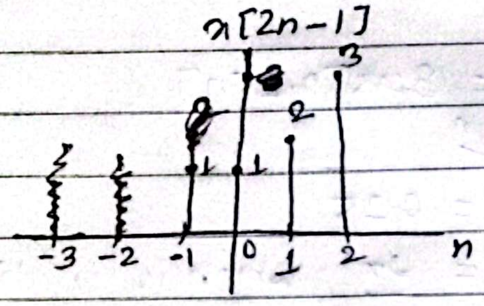
$T_2 = 8$

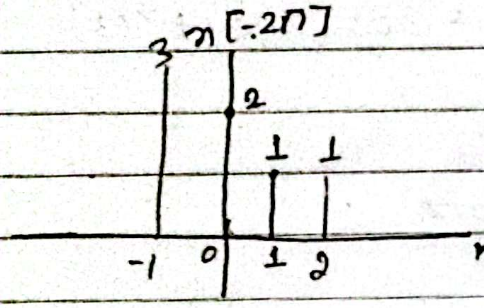
So the signal is periodic.

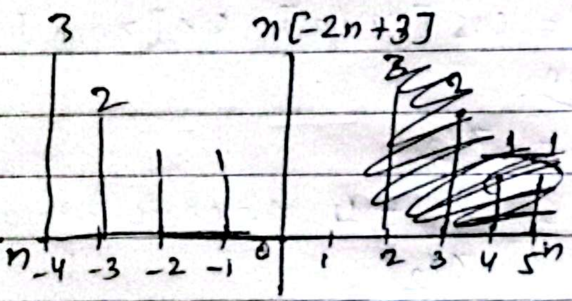
2. If $x[n] = \{1, 3, 1, 2, \underset{\uparrow}{2}, -4, 3, 1\}$. Plot $x[n]$, $x[2n-1]$, $x[-2n+3]$











3. An analog ECG signal contains useful frequencies upto 100 Hz.

a. What is the Nyquist rate?

Nyquist rate $(F_s) = 2 \times F_{max} = 2 \times 100\,Hz$

$= 200\,Hz$

6. Suppose that we sampled this signal at a rate of 150 samples/s. What is the highest frequency that can be represented uniquely?

Soln.

$F_s = 150$ samples/s

we know,

$F_s = 2F_{max}$

$F_{max} = \frac{150}{2}$ sample/s $= 75$ samples/s

8. An analog signal $x_a[t] = 2\sin(480\pi t) - 5\sin(920\pi t)$ is sampled 800 times/s

a. Determine Nyquist rate.
b. Determine folding frequency.
c. What are the frequencies in radian in discrete time signal $x[n]$?
d. What is the analog signal $y_a(t)$ you can reconstruct?

Soln.

$F_s = 800$ samples/s.

$F_{max} = 460$ Hz

Then,

$F_N = 2 \times 460$ Hz
$= 920$ Hz.

folding frequency $= \frac{F_s}{2} = \frac{800}{2} = 400$ samples/s.

$$x[n] = 2\sin\left(480\pi \times \frac{n}{800}\right) - 5\sin\left(920\pi \times \frac{n}{800}\right)$$

$$= 2\sin\left(2\pi \frac{24}{80}n\right) - 5\sin\left(2\pi \frac{46}{80}n\right)$$

$$= 2\sin\left[2\pi\left(\frac{3}{10}\right)n\right] - 5\sin\left(2\pi\left(\frac{17}{40}n\right)\right) = -2\sin\left(2\pi \frac{7}{10}n\right) - 5\left(2\pi \frac{17}{40}n\right)$$

$f_1 = 3/10 \quad f_2 = 17/40$

4. A digital communication link carries binary code words representing samples of an input signal $x_a(t) = 2\sin^2 300\pi t + 5\sin 500\pi t$. The link is operated at 10000 bits/sec and each input sample is quantized into 1024 different voltage levels.

Soln.

Link speed (V) = 10000 bits/s.

No. of quantization level (L) = 1024

Then, we know,

$L = 2^b$

or, $1024 = 2^b$

$b = 10$ bits

And,

a→ Sampling frequency $(F_s) = \frac{V}{b} = \frac{10000}{10} = 1000$ samples/s

folding frequency $(F_{max}) = \frac{F_s}{2} = \frac{1000}{2} = 500$ samples/s

b→ Nyquist rate $(F_N) = 2F_{max}$

$= 2\times 500 = 1000$ samples/s

c→ $x[b] = 2 \cdot \frac{1 - \cos 600\pi t}{2} + 5\sin 500\pi t$

$x[n] = 1 - \cos 2\pi\, 300 \times \frac{n}{1000} + 5\sin 2\pi\, 250 \times \frac{n}{1000}$

$= 1 - \cos\left(2\pi \frac{3}{10} n\right) + 5\sin\left(2\pi \frac{5}{20} n\right)$

$f_1 = \frac{3}{10}, \quad f_2 = \frac{5}{20}$

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d→ resolution (Δ) $= \frac{x_{max} - x_{min}}{L-1}$

$= \frac{6+6}{1000-1}$

$= 0.012\ V$

5. Determine the bit rate and the resolution in the sampling of a seismic signal with dynamic range of 0-1 volt if the sampling rate is $F_s = 20$ samples/s and we use 8 bit ADC. What is the maximum frequency?

Soln.

Dynamic range $= 0.1V \Rightarrow x_{max} - x_{min}$

no of bits (b) = 8

$F_s = 20$ samples/s.

Now, we haves,

$F_s = \frac{\text{bit rate}}{b}$

bit rate $= 20 \times 8 = 160$ bits/sample

And,

resolution $= \frac{\text{Dynamic range}}{2^b - 1}$

$= \frac{0.1}{2^8 - 1}$

$= 0.000392\ V \approx 0.392\ mV$

Maximum frequency $(F_{max}) = \frac{F_s}{2} = \frac{20}{2} = 10$ samples/s

6: Check for the memory, linearity, time invariance, causality and stability of the following system.

a) $y[n] = 4n\,x[n]$

Soln.

$y_1[n] = 4n\,x_1[n]$

$y_2[n] = 4n\,x_2[n]$

let $x[n] = a x_1[n] + b x_2[n]$

Here, $y[n] = 4n\,x[n]$

$= 4n\{a x_1[n] + b x_2[n]\}$

$a_1 y_1[n] + b y_2[n] = a\,4n\,x_1[n] + b\,4n\,x_2[n]$

$= 4n\{a x_1[n] + b x_2[n]\}$

so the system is linear.

For time invariance.

$y[n-k] = 4(n-k)\,x[n-k]$

$y(n,k] = 4n\,x[n-k]$

Since $y[n-k] \neq y(n,k)$ so time varying.

For causality.

$y[n] = 4n\,x[n]$

$y[1] = 4\,x[1]$

present value

Since o/p depend on present value so system is causal.

For stability.

Let the i/p be bounded.

$|x[n]| \leq M_x < \infty$

then, $|y[n]| = |4n\,x[n]|$

$\leq |4n\,M_x| \leq |4n|.M_x$

$\leq$ ~~M~~ so unstable.

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Sol^n.

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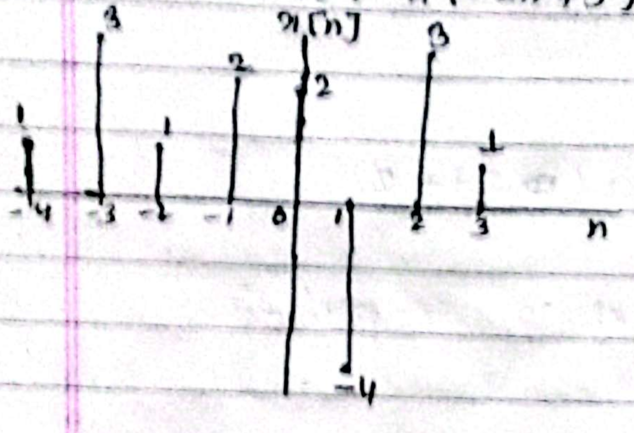

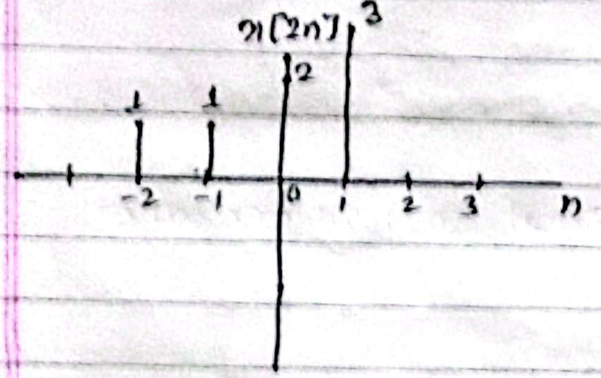

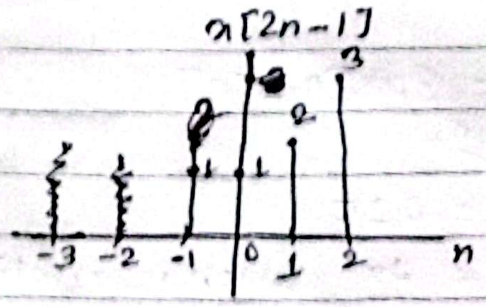

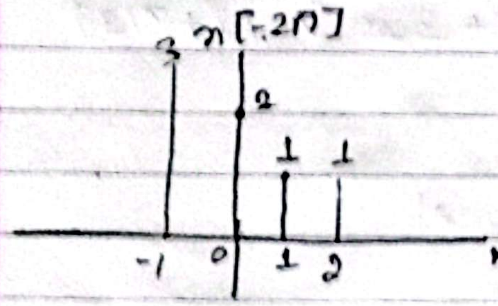

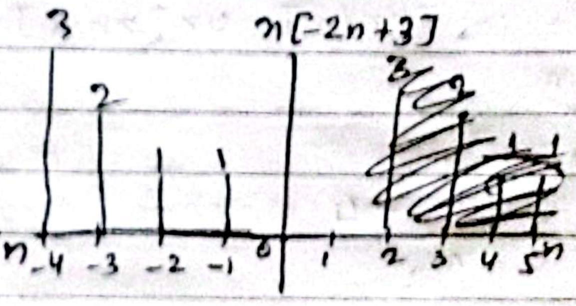

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$f_1 = \cancel{3}/10 \quad f_2 = \cancel{27}/40$

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$x[n] = 1 - \cos 2\pi \cdot \frac{300 \times n}{1000} + 5\sin 2\pi \frac{250 \times n}{1000}$

$= 1 - \cos\left(2\pi \frac{3}{10} n\right) + 5\sin\left(2\pi \frac{5}{20} n\right)$

$f_1 = \frac{3}{10}, \quad f_2 = \frac{5}{20}$

CamScanner

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8. Check for the memory, linearity, time invariance, causality and stability of the following system.

a) $y[n] = 4n\,x[n]$

Soln

$y_1[n] = 4n\,x_1[n]$

$y_2[n] = 4n\,x_2[n]$

Let $x[n] = a x_1[n] + b x_2[n]$

Here $y[n] = 4n\,x[n]$

$= 4n\{a x_1[n] + b x_2[n]\}$

$a y_1[n] + b y_2[n] = a\,4n\,x_1[n] + b\,4n\,x_2[n]$

$= 4n\{a x_1[n] + b x_2[n]\}$

So the system is linear.

For time invariance

$y[n-k] = 4(n-k)\,x[n-k]$

$y(n,k] = 4n\,x[n-k]$

Since $y[n-k] \neq y(n,k)$ so time varying

For causality.

$y[n] = 4n\,x[n]$

$y[1] = 4\,x[1]$

present value

Since o/p depend on present value so system is causal.

For stability.

Let the i/p be bounded.

$|x[n]| \le M_x < \infty$

then, $|y[n]| = |4n\,x[n]|$

$\le |4n M_x| \le |4n|.M_x$

so unstable.

3. $y[n] = 2x[n^2]$

Soln.

$y_1[n] = 2x_1[n^2]$ $\quad$ $y_2[n] = 2x_2[n^2]$

let $x[n] = ax_1[n] + bx_2[n]$

$x[n^2] = ax_1[n^2] + bx_2[n^2]$

we have, $y[n] = 2x[n^2]$

$= 2\{ax_1[n^2] + bx_2[n^2]\}$

$ay_1[n] + by_2[n] = 2a\,x_1[n^2] + 2bx_2[n^2]$

$= 2\{ax_1[n^2] + bx_2[n^2]\}$

Since $y[n] = ay_1[n] + by_2[n]$ so linear.

For time invariance.

O/p due to delayed i/p by k :

$y[n,k] = 2x[(n^2-k)]$

O/p due to delayed response by k

$y[n-k] = 2x[(n-k)^2]$

Here $y[n,k] \neq y[n-k]$ so time varying.

For causal

$y[n] = 2x[n^2]$

$y[1] = 2x[1]$

$y[2] = 2x[4]$

$y[-2] = 2x[4]$

Since o/p depends on present and future values so system is non causal.

For stability.

Let the i/p be bounded.

$|x[n]| \leq M_x < \infty$

$|x[n^2]| \leq M_x$.

$|y[n]| = |2x[n^2]|$

$\leq |2M_x|$

$\leq M_y < \infty$ so stable.

6. The accumulator.

$y[n] = y[n-1] + x[n]$

Soln

$y_1[n] = y_1[n-1] + x_1[n]$

$y_2[n] = y_2[n-1] + x_2[n]$

let $x[n] = a x_1[n] + b x_2[n]$

$y[n] = a y_1[n-1] + b y_2[n-1] + a x_1[n] + b x_2[n]$

And,

$a y_1[n] + b y_2[n] = a\{y_1[n-1] + x_1[n]\} + b\{y_2[n-1] + x_2[n]\}$

since $y[n] = a y_1[n] + b y_2[n]$ so linear.

For time invariance.

o/p due to delayed i/p.

$y[n,k] = y[n-1-k] + x[n-k]$

o/p due to delayed response

$y[n-k] = y[$.

For causal.

$y[n] = y[n-1] + x[n]$

O/p depends on past and present value so causal.

For stability.

let $x[n]$ be bounded.

$|x[n]| \leq M_x < \infty$

$|y[n]| = |y[n-1] + x[n]|$

$\leq |y[n-1] + M_x|$

time dependent so unstable.

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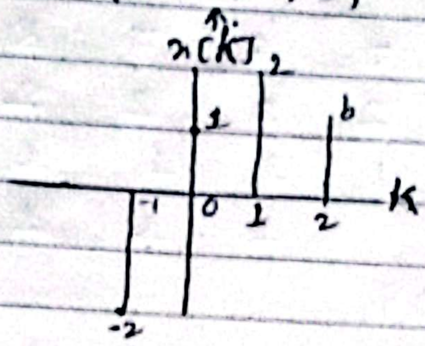
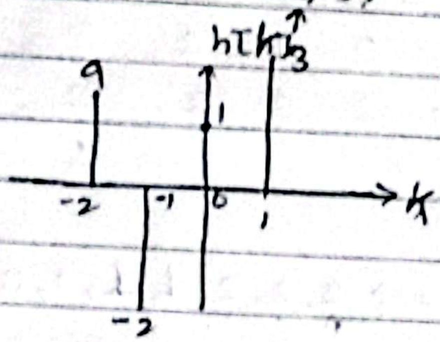
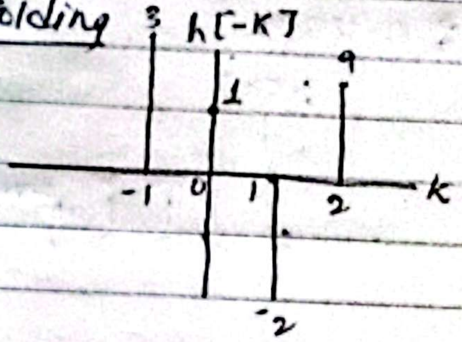
7. Determine $y[n]$.

a. $x[n] = \{-2, \underset{\uparrow}{1}, 2, b\}$ $\quad h[n] = \{a, -2, \underset{\uparrow}{1}, 3\}$

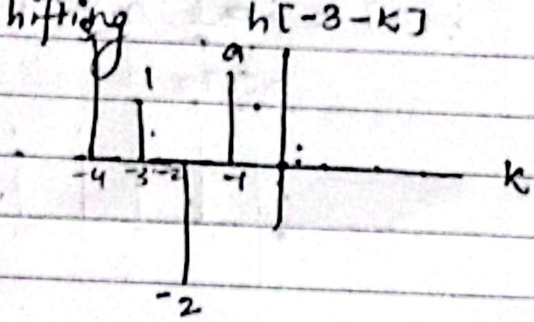




Folding



shifting



$y[-3] = -2a$
$y[-2] = 4 + a$
$y[-1] = -2 - 2 + 2a = -4 + 2a$
$y[0] = -6 + 1 - 4 + ab$
$= -9 + ab$
$y[1] = 3 + 2 - 2b$
$= 5 - 2b$
$y[2] = 6 + b$
$y[3] = 3b$

h[n] \ x[n]	-2	1	2	b
a	-2a	a	2a	ab
-2	4	-2	-4	-2b
1	-2	1	2	b
3	-6	3	6	3b

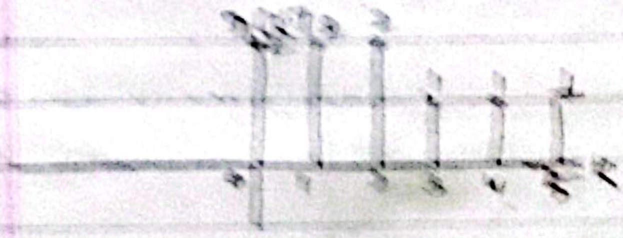
$x[n] = 2u[n] - u[n-3] - u[n-6]$; $h[n] = 0.8^n u[n]$

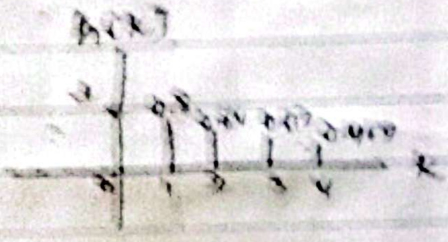
Sol:

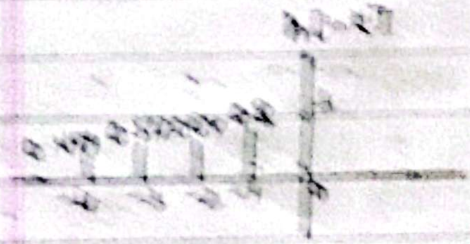
	0	1	2	3	4	5	6	7	8
$x[n]$	2	2	2	2	2	2	2	2	2
				-1	-1	-1	-1	-1	-1
							-1	-1	-1

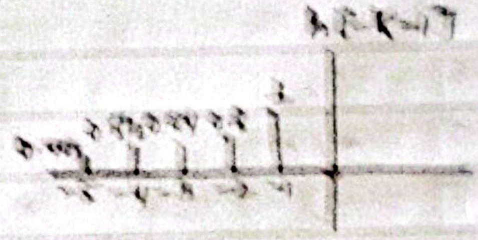
$x[n] = \{ \underset{\uparrow}{2}, 2, 2, 1, 1, 1, 0, 0, 0 \}$

$h[n] = \{ \underset{\uparrow}{1}, 0.8, 0.64, 0.512, 0.4096 \}$









$y[0] = 2$

$y[1] = 2 \times 0.8 + 2 \times 1 = 3.6$

$y[2] = 0.64 \times 2 + 0.8 \times 2 + 2 = 4.88$

$y[3] = 0.512 \times 2 + 0.64 \times 2 + 0.8 \times 2 + 1 = 4.904$

$y[4] = 0.4096 \times 2 + 0.512 \times 2 + 0.64 \times 2 + 0.8 + 1 = 4.923$

$y[5] = 0.4096 \times 2 + 0.512 \times 2 + 0.64 \times 1 + 0.8 + 1 = 4.282$

$y[6] = 0.4096 \times 2 + 0.512 + 0.64 + 0.8 = 2.77$

$y[7] = 0.4096 + 0.512 + 0.64 = 1.561$

$y[8] = 0.4096 + 0.512 = 0.921$

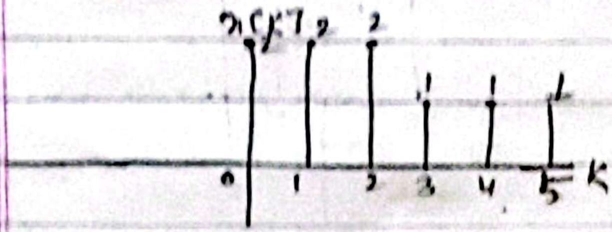
7b. $x[n] = 2u[n] - u[n-3] - u[n-6]$; $h[n] = 0.8^n u[n]$

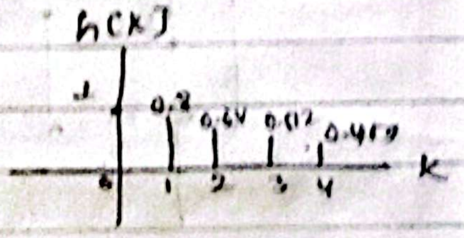
Soln :

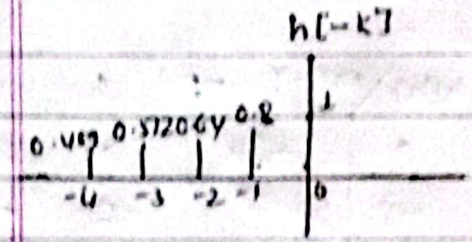
	0	1	2	3	4	5	6	7	8
$x[n]$	2	2	2	2	2	2	2	2	2
				1	1	1	1	1	1
							1	1	1

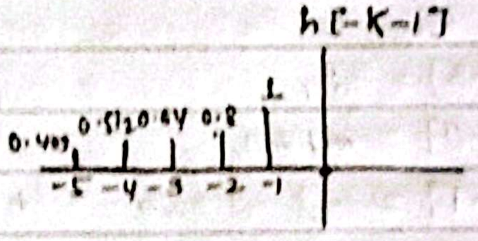
$x[n] = \{\underset{\uparrow}{2}, 2, 2, 1, 1, 1, 0, 0, 0\}$

$h[n] = \{\underset{\uparrow}{1}, 0.8, 0.64, 0.512, 0.4096\}$









$y[0] = 2$

$y[1] = 2\times0.8 + 2\times1 = 3.6$

$y[2] = 0.64\times2 + 0.8\times2 + 2 = 4.88$

$y[3] = 0.512\times2 + 0.64\times2 + 0.8\times2 + 1 = 4.904$

$y[4] = 0.409\times2 \; 0.512\times2 + 0.64\times2 + 0.8 + 1 = 4.922$

$y[5] = 0.409\times2 + 0.512\times2 + 0.64\times1 + 0.8 + 1 = 4.282$

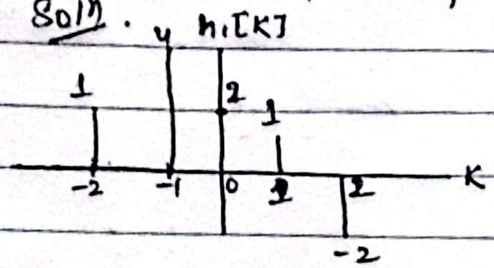
$y[6] = 0.409\times2 + 0.512 + 0.64 + 0.8 = 2.77$

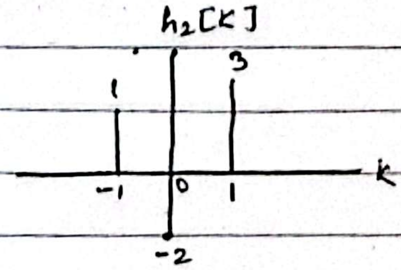
$y[7] = 0.409 + 0.512 + 0.64 = 1.561$

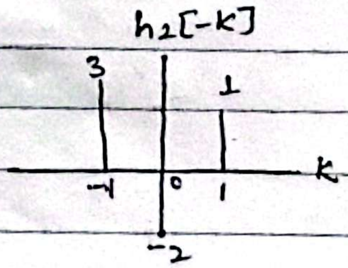
$y[8] = 0.409 + 0.512 = 0.921$

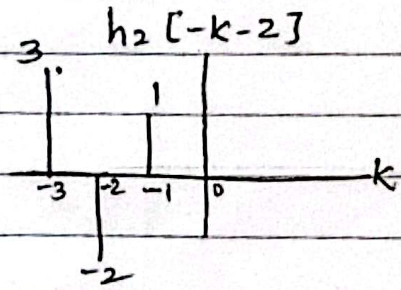
8. Given $h_1[n] = \{1, 4, \underset{\uparrow}{2}, 1, -2\}$ and $h_2[n] = \{1, \underset{\uparrow}{-2}, 3\}$. Find the overall response $h[n]$ if the two system are cascade interconnected. Also find the step response of this system.

Soln.









$h[-2] = -2 + 4 = 2$

$h[-1] = 3 - 8 + 2 = -3$

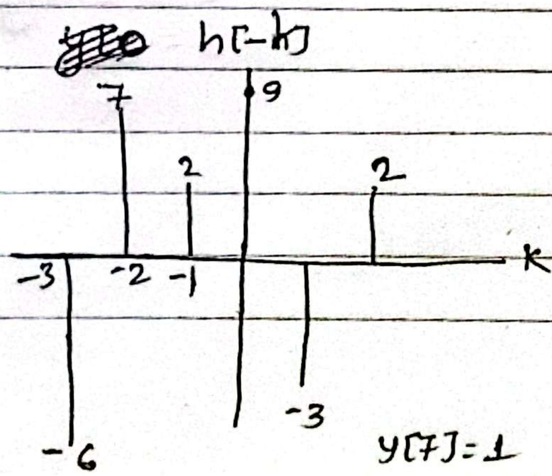
$h[0] = 12 - 4 + 1 = 9$

$h[1] = 6 - 2 - 2 = 2$

$h[2] = 3 + 4 = 7$

$h[3] = -6$

$h[n] = \{2, -3, \underset{\uparrow}{9}, 2, 7, -6\}$ $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1..\}$



$y[-2] = 2$

$y[-1] = -3 + 2 = -1$

$y[0] = 9 - 3 + 2 = 8$

$y[1] = 2 + 9 - 3 + 2 = 10$

$y[2] = 7 + 2 + 9 - 3 + 2 = 17$

$y[3] = 11$

$y[4] = -6 + 7 + 2 + 9 - 3 = 9$

$y[5] = -6 + 7 + 2 + 9 = 12$

$y[6] = -6 + 7 + 2 = 3$

$y[7] = 1$