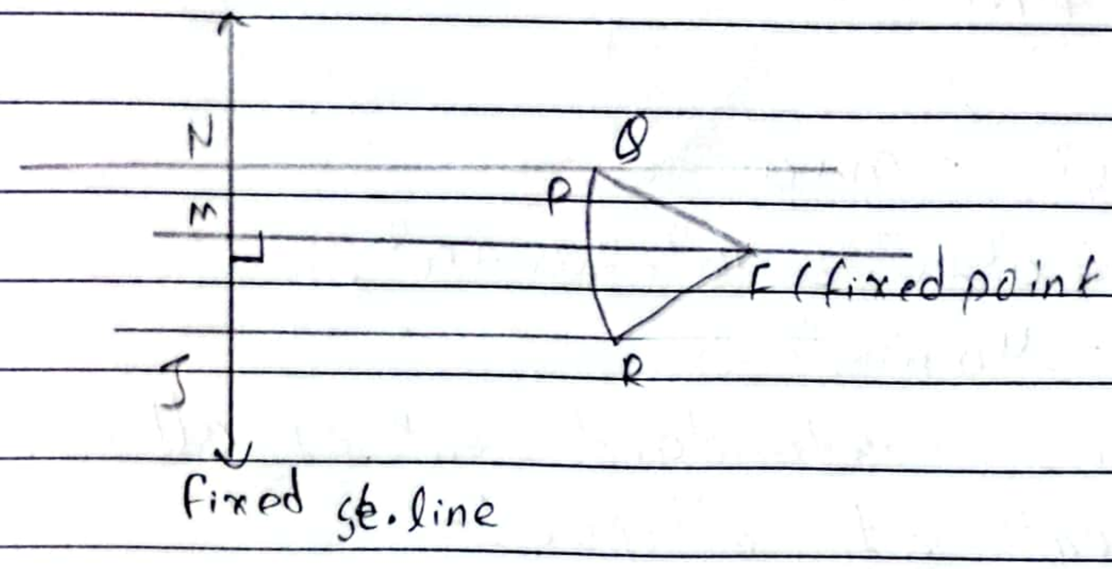
Tutorial – II

(1) Define conic section and derive the standard equation of parabola, ellipse and Hyperbola.

:- A conic section is a locus of a point 'p' which moves in a plane such that the ratio of its distance from fixed point to it's distance from fixed st. line is always constant



$$\frac{PF}{PM} = \frac{RF}{RJ} = \frac{QF}{QN}$$

Standard eqn of Parabola :-

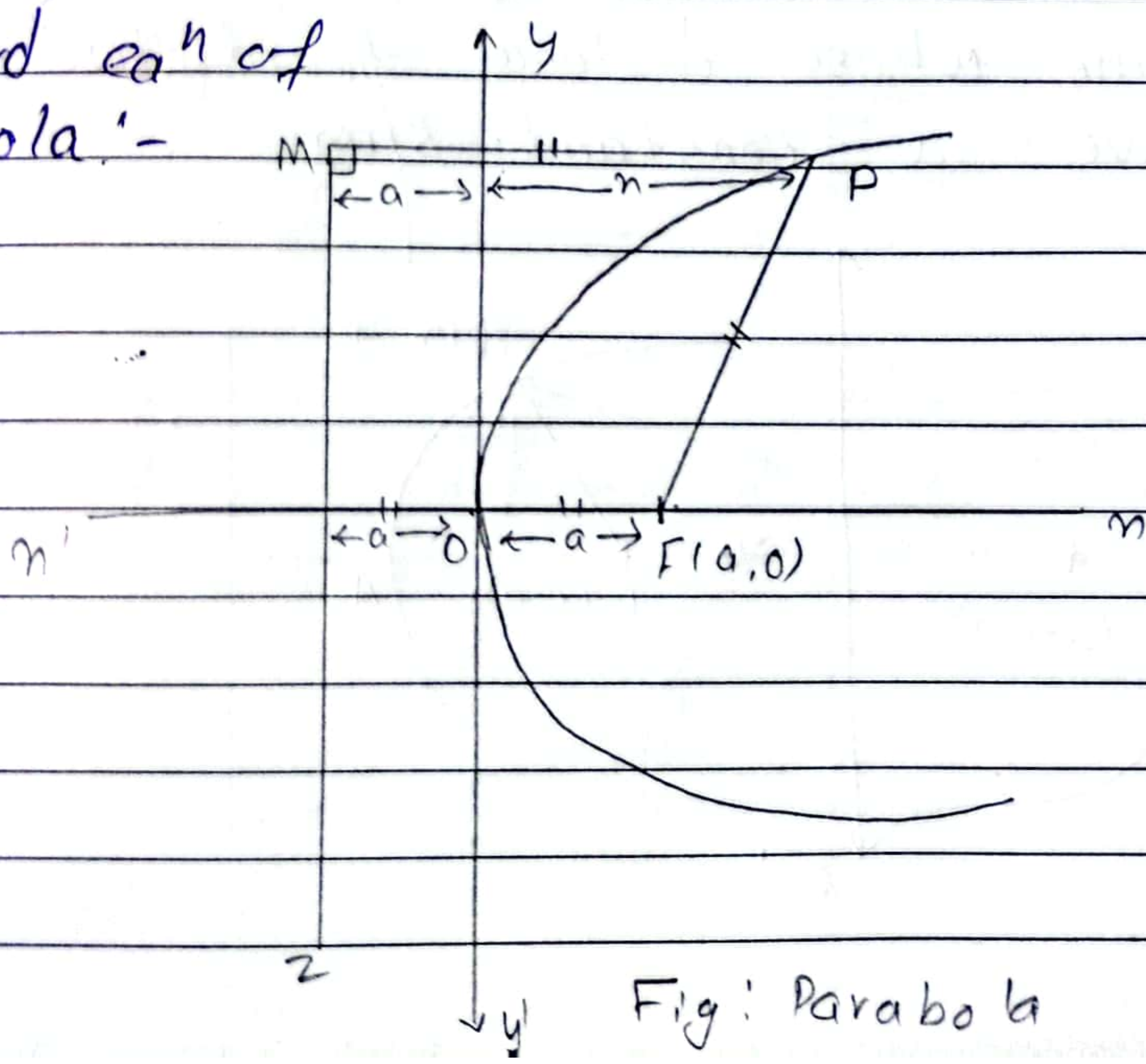


Fig: Parabola

let $O(0,0)$ be vertex of parabola, $F(a,0)$ be focus of parabola, ZM be directrix of parabola.
$OF = OF' = a$

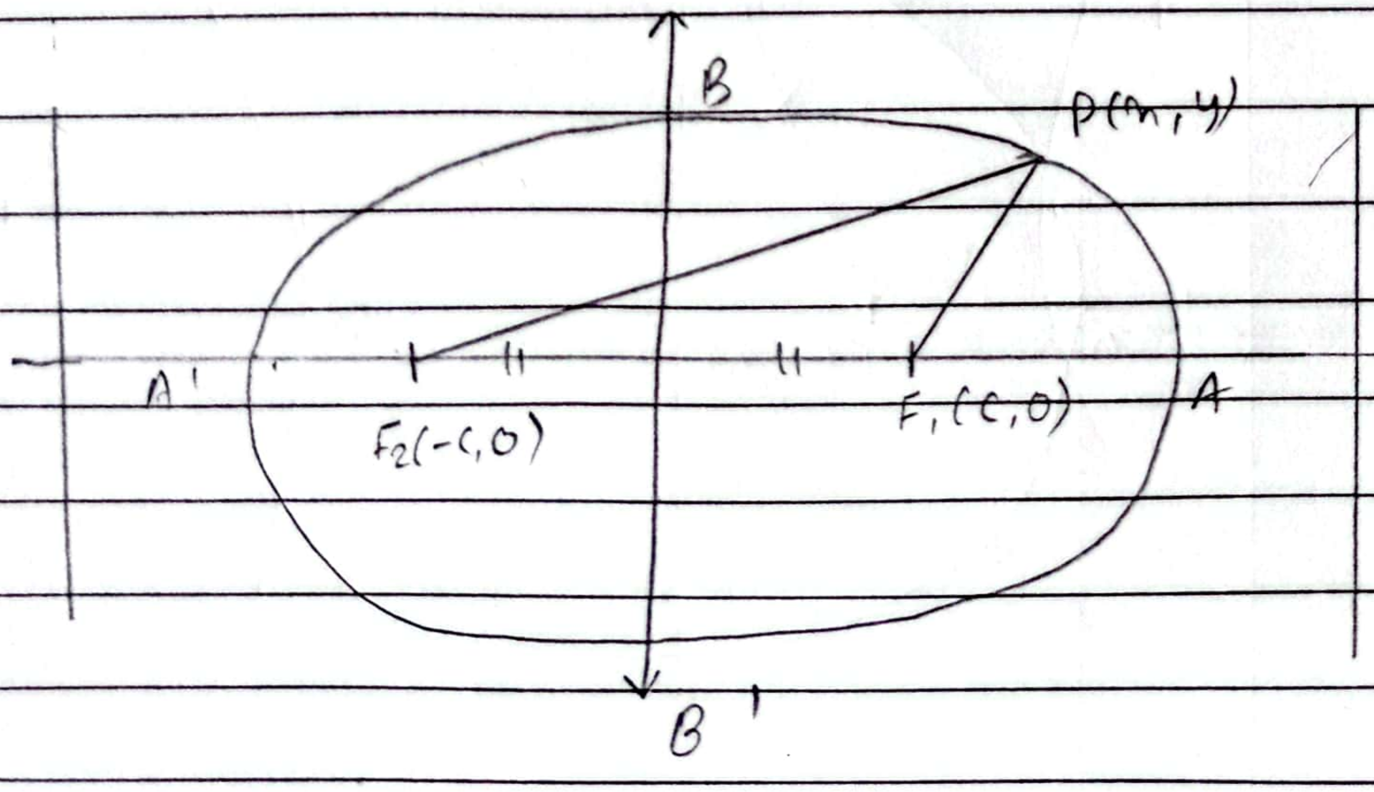
$P(x,y)$ be any point on the parabola, join P and F

Draw $PM \perp ZM$
By def^n of parabola,
$PF = PM$

$$\sqrt{(x-a)^2 + y^2} = x + a$$
$$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$
$$y^2 = 4ax$$
which is standard eq^n of ellipse.

Standard eq^n of ellipse :-

An ellipse is the set of points (or locus) in the plane whose distance from two fixed points have a constant sum.



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$PF_1 + PF_2 = \text{constant}$ (major axis)
$= 2a$

let $O(0,0)$ be centre of ellipse

- $AA' = 2a$ be major axis, $BB' = 2b$ be minor axis of ellipse. $F_1(c,0)$ and $F_2(-c,0)$ be focii of the ellipse. Let $P(x,y)$ be any point on the ellipse join PF_1 and PF_2.

Now, by defn of ellipse

$PF_1 + PF_2 = \text{constant } (2a)$

using distance formula,

$$\sqrt{(x-c)^2+y^2} + \sqrt{(x+c)^2+y^2} = 2a$$

$$\sqrt{(x-c)^2+y^2} = 2a - \sqrt{(x+c)^2+y^2}$$

S.O.B.S

$$x^2 - 2xc + c^2 + y^2 = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + x^2 + 2xc + c^2 -$$

$$-4xc - 4a^2 = -4a\sqrt{(x+c)^2+y^2}$$

$$xc + a^2 = a\sqrt{(x+c)^2+y^2}$$

SOBS

$$x^2c^2 + 2xca^2 + a^4 = a^2(x^2 + 2xc + c^2 + y^2)$$

$$x^2c^2 + 2x\cancel{c}a^2 + a^4 = a^2x^2 + 2x\cancel{c}a^2 + a^2c^2 + a^2y^2$$

$$a^2x^2 + a^2y^2 - x^2c^2 + a^2c^2 - a^4 = 0$$

$$x^2(a^2-c^2) + a^2y^2 = a^2(a^2-c^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{(a^2-c^2)} = 1$$

From fig:- $PF_1 + PF_2 > F_1F_2$

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major axis > c+c

$2a > 2c$

$a > c$

put $a^2 - c^2 = b^2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which is standard eqn of ellipse.

standard eqn of hyperbola.

Hyperbola is the set of points (or 'locus) in the plane whose distance from two fixed points have a constant difference.

$PF_2 - PF_1 = \text{constant}$

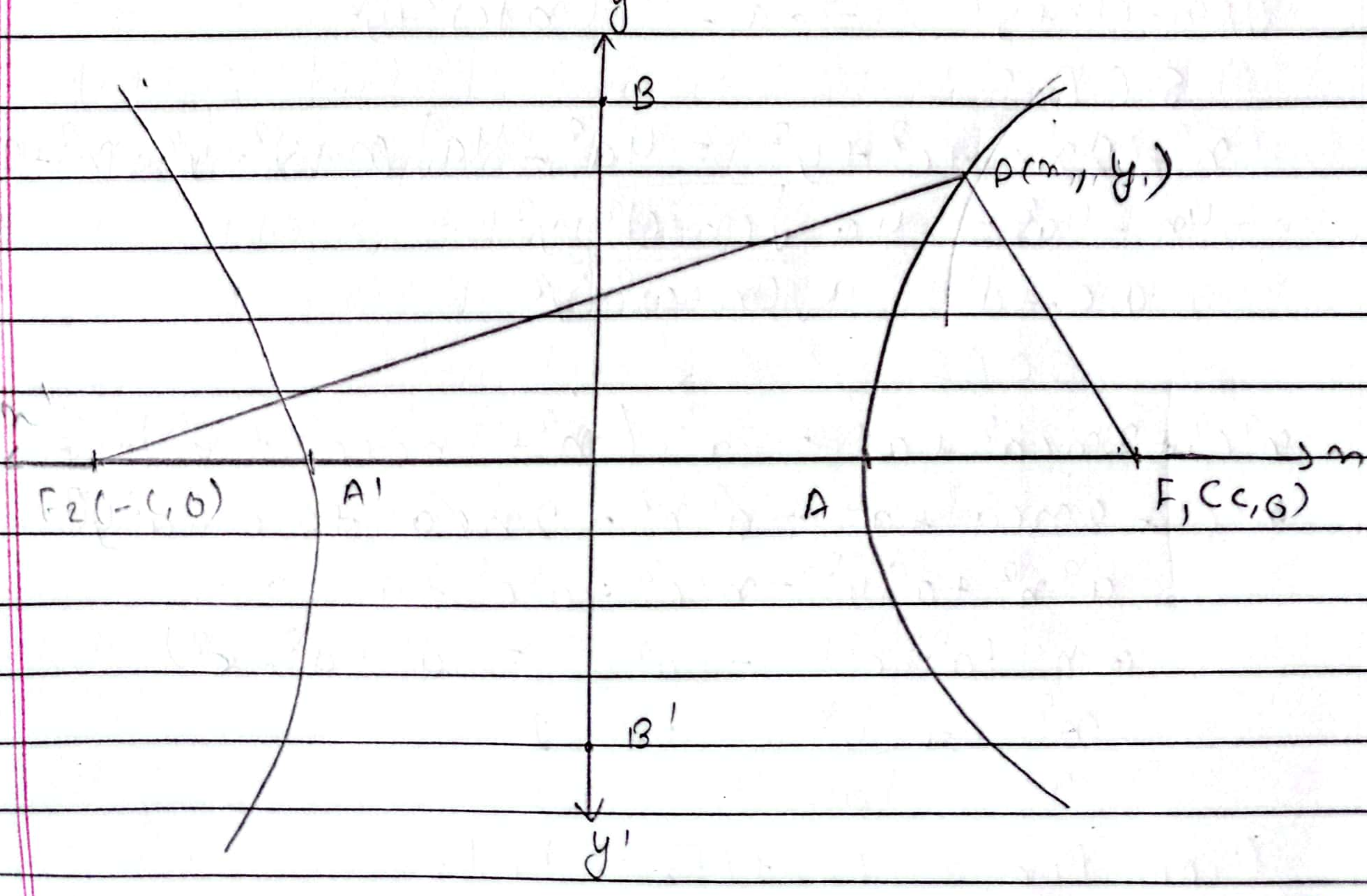


Fig: hyperbola.

Let O(0, 0) be centre of hyperbola $F_1(c, 0)$ and $F_2(-c, 0)$ be foci of the hyperbola $AA' = 2a$ be transverse axis and $BB' = 2b$ be conjugate axis. Let $P(x, y)$ be any point on the hyperbola join (P and F_1) join (P and F_1)

Now,

by ~~fe~~ defn of hyperbola,

$PF_2 - PF_1 = 2a$ (constant)

by distance formula,

$$\sqrt{(x + c^2) + y^2} = 2a + \sqrt{(x-c)^2 + y^2}$$

S.O.B.S

$$x^2 + 2x + c^2 + y^2 = 4a^2 + 4a\sqrt{(x-c)^2+y^2} + x^2 - 2xc + c^2 + y^2$$

$$-4xc - 4a^2 = 4a\sqrt{(x-c)^2+y^2}$$

$$xc - a^2 = a\sqrt{(x-c)^2+y^2}$$

S.O.B.S

$$x^2c^2 - 2a^2xc + a^4 = a^2x^2 - 2xca^2 + a^2c^2 + a^2y^2$$

$$x^2(c^2-a^2) - a^2y^2 = a^2c^2 - a^4$$

$$x^2(c^2-a^2) - a^2y^2 = a^2(c^2-a^2)$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = a^2$$

From defn :-

$PF_2 - PF_1 < F_1F_2$

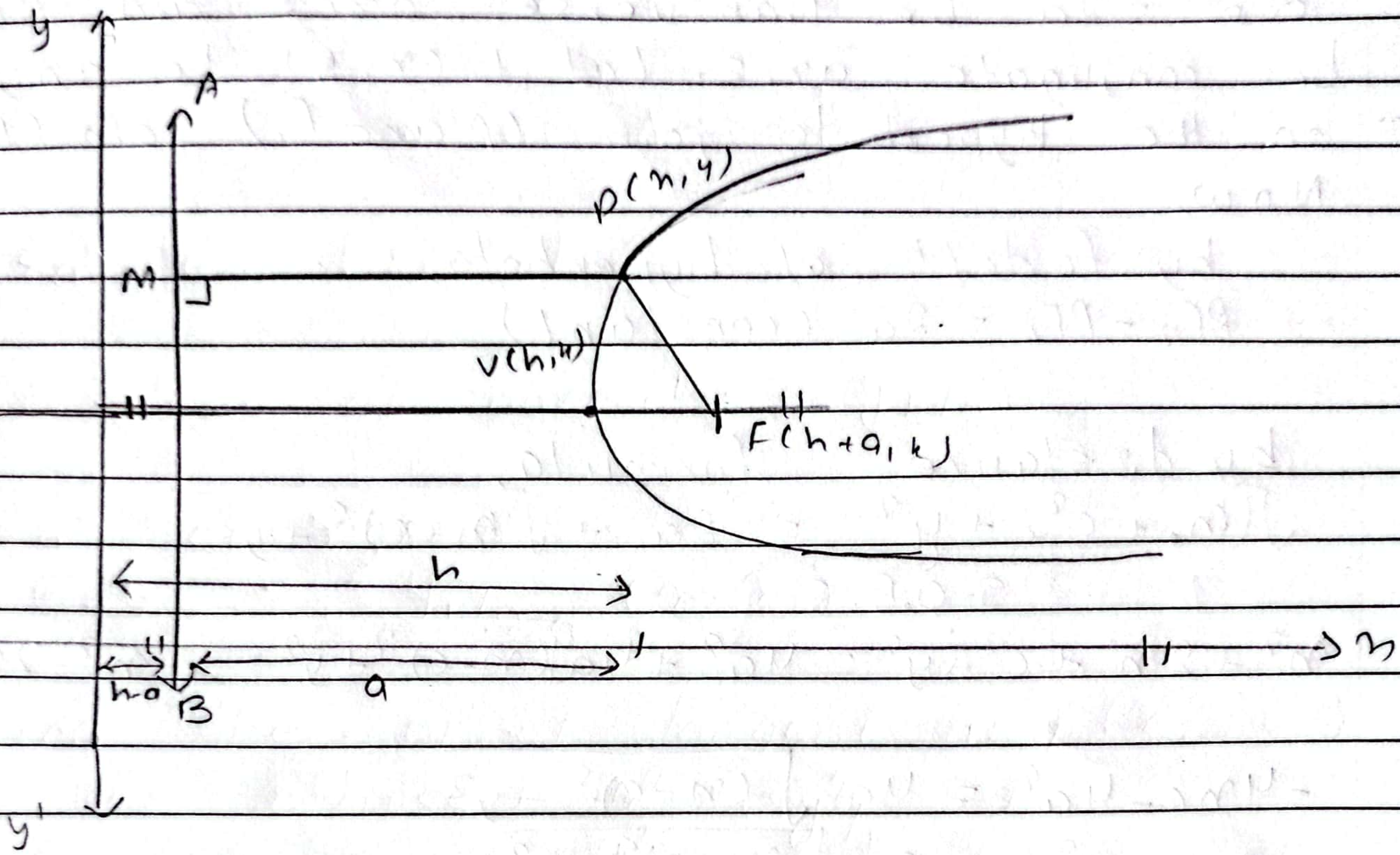
$2a < 2c$

$a < c$

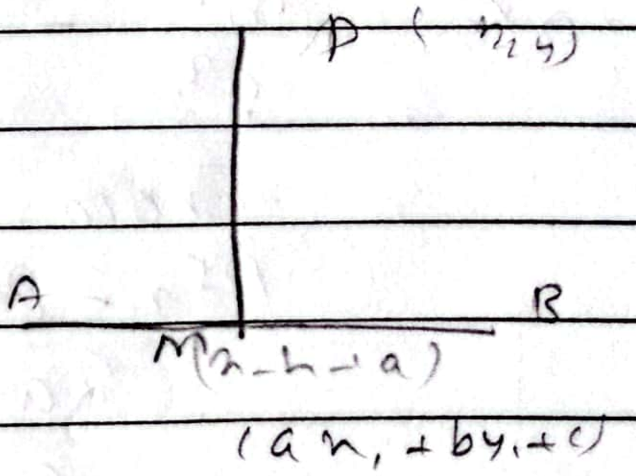
$b^2 = c^2 - a^2$

$\frac{x^2}{a^2} - y^2/b^2 = 1$ which is required eqn of hyperbola.

Q. Derive the eqn of parabola with vertex (h,k) and focus (h+a, k).



let V(h,k) be vertex of parabola, F(x+a,k) be focus of parabola. AB be directrix.
let P(x,y) be any point on the parabola. join P and F draw PM ⊥ AB. Now the eqn of directix is:-



$x = h - a$, $\quad x - h + a = 0$.

Now, by defn of parabola

$PF = PM$

$$\sqrt{(x-h-a)^2 + (y-k)^2} = \frac{x-h+a}{\sqrt{1^2+0^2}}$$

PM is the perpendicular distance calculated as, $PM = \dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}$

$$(x-h-a)^2 + (y-k)^2 = (x-h+a)^2$$
$$(x-h)^2 - 2(x-h)a + a^2 + (y-k)^2 = (x-h)^2 + 2(x-h)a + a^2$$

$$(y-k)^2 = 4a(x-h)$$

(3) Find the equation of tangent and normal at $p(x_1, y_1)$ on the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow$ Eqn of parabola is $y^2 = 4ax$

$$y^2 = 4ax$$
diff. wrt x.
$$2y \frac{dy}{dx} = 4a$$

or, $\frac{dy}{dx} = \frac{2a}{y}$ at (x_1, y_1),

$$\frac{dy}{dx} = \frac{2a}{y_1}$$

$$\text{Slope}\left(\frac{dy}{dx}\right) = \frac{2a}{y_1}$$

Eqn of tangent at $P(x_1, y_1)$ to the ~~the~~ parabola $y^2 = 4ax$, is :-

$$y - y_1 = m(x - x_1)$$
$$y - y_1 = \frac{2a}{y_1}(x - x_1)$$

$$yy_1 - y_1^2 = 2ax - 2ax_1 \quad (y_1^2 = 4ax_1)$$
$$yy_1 = 2ax - 2ax_1 + 4ax_1$$
$$yy_1 = 2a(x - x_1 + 2x_1)$$
$$yy_1 = 2a(x + x_1)$$

which is required eqn of tangent.

Eqn of Normal:-

Slope of normal $= -y_1/2a$

Eqn of normal is:-

$$y - y_1 = -y_1/2a\,(x - x_1)$$

Find the eqn of tangent at $p(x_1, y_1)$ to the ellipse $\frac{x^2}{a^2} + y^2/b^2 = 1$

$\Rightarrow$ Eqn of ellipse is:-

$$b^2x^2 + a^2y^2 = a^2b^2$$

diff wrt x. we get,

$$2b^2x + 2a^2y\frac{dy}{dx} = 0$$

or, $$\frac{-b^2x}{a^2y} = \frac{dy}{dx}$$

slope $(dy/dx) = \frac{-b^2x}{a^2y}$ at (x_1, y_1)

$$= \frac{-b^2x_1}{a^2y_1}$$

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Eqⁿ of tangent at $P(x_1, y_1)$.

Slope $\frac{-b^2x_1}{a^2y_1}$ is:

$$y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$$

$$a^2yy_1 - a^2y_1^2 = -b^2xx_1 + b^2x_1^2$$
$$a^2yy_1 + b^2x_1x_1 = a^2y_1^2 + b^2x_1^2$$

$$(\because b^2x_1^2 + a^2y_1^2 = a^2b^2)$$

$$a^2yy_1 + b^2xx_1 = a^2b^2$$
dividing by a^2b^2 on both sides,
$$\frac{yy_1}{b^2} + \frac{xx_1}{a^2} = 1 \quad H$$
which is required eqⁿ of tangent of ellipse.

Eqⁿ of hyperbola is:-
$$b^2x^2 - a^2y^2 = a^2b^2$$
diff. w.r.t x,
we get,
$$2b^2x - 2a^2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{b^2x}{a^2y}, \text{ at } (x_1, y_1)$$

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slope $\frac{dy}{dx} = \frac{b^2x_1}{a^2y_1}$

Eqn of tangent at $P(x_1, y_1)$ with slope $\frac{b^2x}{a^2y_1}$

$$y - y_1 = \frac{b^2x_1}{a^2y_1}(x - x_1)$$

or, $a^2yy_1 - b^2xx_1 = a^2y_1^2 - b^2x_1^2$
or, $b^2x_1^2 - a^2y_1^2 = b^2xx_1 - a^2yy_1$

at (x_1, y_1), $b^2x_1^2 - a^2y_1^2 = a^2b^2$

$$b^2xx_1 - a^2yy_1 = a^2b^2$$

dividing by a^2b^2 on both sides

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

which is required eqn of tangent

) Find the condition that the line $lx+my+n=0$ is tangent to the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Also find point of contact.

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⇒ we have,

Eqn of parabola is:-

$y^2 = 4ax$ --- (i)

Eqn of line, $y = mx + c$ --- (ii)

from (i) & (ii),

$(mx+c)^2 = 4ax$

$m^2x^2 + 2mxc + c^2 = 4ax$

$m^2x^2 + (2mc - 4a)x + c^2 = 0$ --- (iii)

which is quadratic in x.

So x gives two values but the line --(ii) is tangent to the parabola --(i) for this the discriminant of eqn (iii) must be zero

i.e. $b^2 - 4ac = 0$

$(2mc - 4a)^2 - 4m^2c^2 = 0$

$4m^2c^2 - 16amc + 16a^2 - 4m^2c^2 = 0$

$a^2 - mc = 0$

$\frac{a}{m} = c$, $c = a/m$

which is required condition

we have,

Eqn of ellipse : $b^2x^2 + a^2y^2 = a^2b^2$ --(i)

Eqn of line :- $y = mx + c$ --(ii)

from (i)

$y^2 = \frac{a^2b^2 - b^2x^2}{a^2} = b^2 - \frac{b^2x^2}{a^2}$

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$$(mx+c)^2 = \frac{a^2b^2 - b^2x^2}{a^2}$$

$$(m^2x^2 + 2mxc + c^2)a^2 = a^2b^2 - b^2x^2$$
$$a^2m^2x^2 + 2a^2mxc + a^2c^2 = a^2b^2 - b^2x^2$$
$$(a^2m^2+b^2)x^2 + (2a^2mc)x + a^2c^2 - a^2b^2 = 0 \text{ --- (iii)}$$

which is quadratic in x,
So x gives two values but the line --- (iii) is tangent to the ellipse --- (i). for this the discriminant of eqn (iii) must be zero.

$$(2a^2mc)^2 - 4(a^2m^2+b^2)(a^2c^2 - a^2b^2) = 0$$
$$4a^4m^2c^2 - 4a^4m^2c^2 + 4a^4b^2m^2 - 4a^2b^2c^2 + 4a^2b^4 = 0$$
$$4a^2b^2(a^2m^2 - c^2 + b^2) = 0$$
$$c^2 = a^2m^2 + b^2.$$
which is required condition.

Hyperbola,
$\Rightarrow$ Eqn of hyperbola, $b^2x^2 - a^2y^2 = a^2b^2$ --- (i)
Eqn of line, $y = mx + c$, --- (ii)

from (i) & (ii),
$$b^2x^2 - a^2(mx+c)^2 = a^2b^2$$
$$b^2x^2 - a^2m^2x^2 - 2a^2mxc - a^2c^2 = a^2b^2$$
$$x^2(b^2 - a^2m^2) - (2a^2mc)x - a^2c^2 - a^2b^2 = 0 \text{ --- (ii)}$$
$a = b^2 - a^2m^2$, $b = -2a^2mc$, $c = -(a^2c^2 + a^2b^2)$

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which is quadratic in x, so x gives two values, but the line ...(ii) is tangent to the hyperbola ...(i), for this the discriminant of eqn (iii) must be zero.

$$b^2 - 4ac = 0$$

$$4a^4m^2c^2 + 4(b^2 - a^2m^2)(a^2c^2 + a^2b^2) = 0$$
$$4a^4m^2c^2 + 4b^2a^2c^2 - 4a^4m^2c^2 + 4a^2b^4 - 4a^4m^2b^2 = 0$$

$$4b^2a^2(c^2 + b^2 - m^2a^2) = 0$$
$$c^2 = m^2a^2 - b^2$$
which is required condition.

(5) Obtain the vertex, focus, directrix, line of symmetry and length of latus rectum of the following parabola and sketch.
(i) $x^2 - 4y + 2x - 3 = 0$ (ii) $4y^2 + 3x - 8y - 2 = 0$

=>

(i) $x^2 - 4y + 2x - 3 = 0$
$x^2 + 2x = 4y + 3$
~~$(x)^2 + 2.x.1 + (1)^2 = (4y + 3/2^{-1})$~~
$x^2 + 2.x.1 + 1^2 = 4y + 3 + 1$
$(x + 1)^2 = 4(y + 1)$
$(x+1)^2 = 4.1(y+1)$
comparing with, $(x - h)^2 = 4a(y - k)$
$h = -1$, $a = 1$, $k = -1$

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vertex $(h,k) = (-1,-1)$
Focus $(h+a, k) = (0,-1)$
eqn of directrix $y = k-a = -2$
length of latus rectum $= 4a = 4$ #

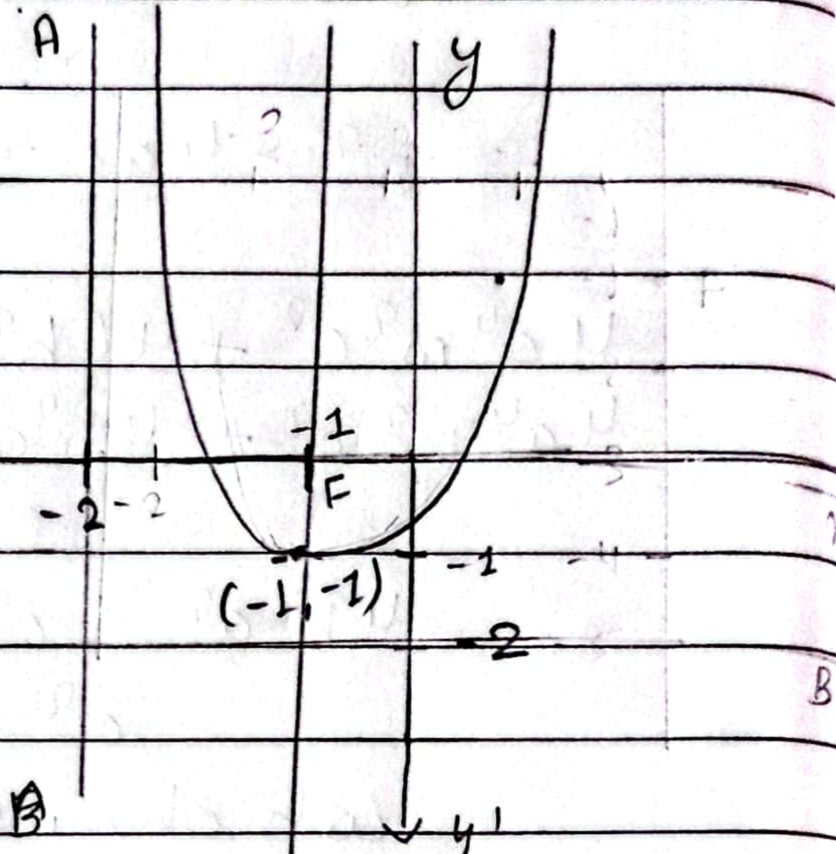

Fig: parabola Q.6a

Q no. 6b $4y^2+3x-8y-2=0$

$4y^2-8y = -3x+2$
or, $4(y^2-2y) = -3x+2$
or, $4(y^2-2.y.1+1^2) = -3x+2+4$
or, $4(y-1)^2 = -3x+6$
or, $(y-1)^2 = -\frac{3}{4}(x-2)$

or, $(y-1)^2 = -4 \cdot \frac{3}{16}(x-2)$

Comparing with $(y-k)^2 = -4a(x-h)$
$k=1$, $a=3/16$, $h=2$
vertex $(h,k) = (2,1)$
Focus $(h-a, k) = \cancel{(2, 13/16)}$ $(29/16, 2)$
eqn of directrix $x = h+a = 35/16$
length of latus rectum $= 4a = 3/4$

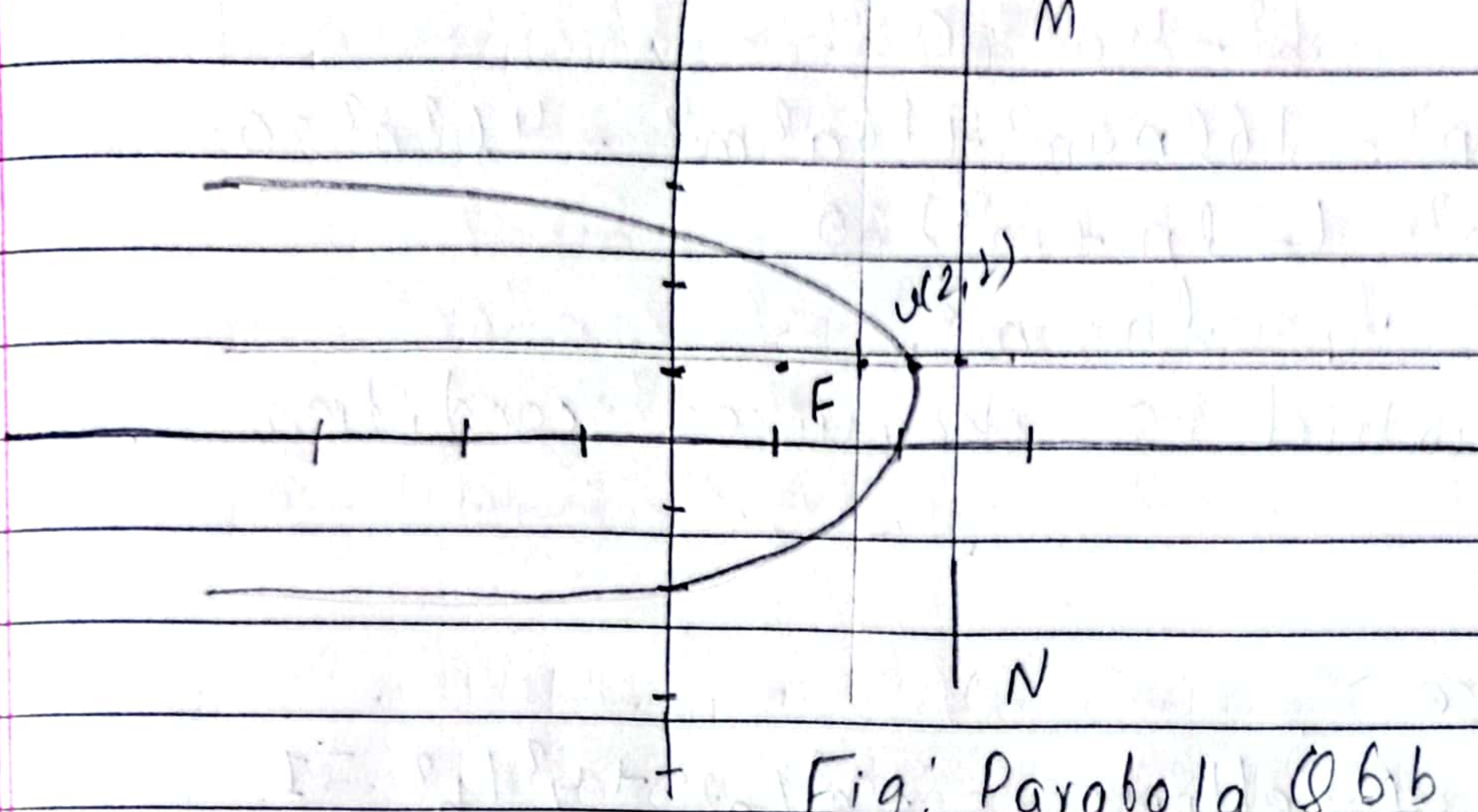

Fig: Parabola Q 6.b

(5) Find the condition that the line $lx + my + n = 0$ is tangent to the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + y^2/b^2 = 1$ and hyperbola $\frac{x^2}{a^2} - y^2/b^2 = 1$

Also find point of contact.

$\Rightarrow$ Eqn of parabola is:-

$$y^2 = 4ax \quad \text{---(i)}$$

and line is:-

$$lx + my + n = 0 \quad \text{---(ii)}$$

$$y = \frac{-lx - n}{m}$$

Solving (i) & (ii) we get,

$$l^2x^2 + 2lnx + n^2 = 4am^2x$$

$$l^2x^2 + (2ln - 4am^2)x + n^2 = 0, \quad \text{---(iii)}$$

which is quadratic in x,

So x gives two values but the line (ii) is tangent to the parabola ---(i) for this discriminant of eqn (iii) must be zero.

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i.e. $b^2 - 4ac = 0$

or, $4l^2n^2 - 16lnam^2 + 16a^2m^4 - 4l^2n^2 = 0$

or, $16am^2(-ln + m^2) = 0$

$\therefore ln = m^2$

which is required condition.

(ii) ellipse

Eqn of ellipse is: $x^2/a^2 + y^2/b^2 = 1$

$b^2x^2 + a^2y^2 = a^2b^2$ --(i) and line is:-

$lx + my + n = 0$

$$y = \frac{lx - n}{m} \quad \text{--(ii)}$$

Solving (i) & (ii), we get,

$m^2b^2x^2 + a^2(l^2x^2 + 2lxn + n^2) = a^2b^2m^2$

or, $x^2(m^2b^2 + a^2l^2) + (2a^2ln)x + a^2n^2 - a^2b^2m^2 = 0$ -(iii)

which is quadratic in x,

So x gives two values, but the line--(ii) is tangent to the ellipse --(i). The discriminant of eqn (iii) must be zero.

i.e. $b^2 - 4ac = 0$.

or, $4a^4l^2n^2 - 4(m^2b^2 + a^2l^2)(a^2n^2 - a^2b^2m^2) = 0$

or, $4a^4l^2n^2 - 4a^2b^2m^2n^2 + 4a^2b^4m^4 - 4a^4l^2n^2 + 4a^4l^2b^2m^2 = 0$

or, $4a^2b^2m^2(-n^2 + m^2b^2 + a^2l^2) = 0$

$\therefore a^2l^2 + m^2b^2 = n^2$

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which is required condition.

E_q^n of hyperbola is:-

$$x^2/a^2 - y^2/b^2 = 1$$

$$b^2x^2 - a^2y^2 = a^2b^2 \quad \text{--- (i)}$$

and line is:-

$$lx + my + n = 0; \quad y = \frac{lx - n}{m} \quad \text{--- (ii)}$$

Solving (i) & (ii)

$$b^2x^2m^2 - a^2l^2x^2 - 2a^2lnx - n^2a^2 = a^2b^2$$

$$(b^2m^2 - a^2l^2)x^2 - (2a^2ln)x - n^2a^2 - a^2b^2m^2 = 0 \quad \text{--- (iii)}$$

which is quadratic in x, so x gives two values.

But the line -(ii) is tangen to the hyperbola --(i). For this the discriminant of -(iii) must be zero.

i.e. $b^2 - 4ac = 0$

$$4a^4l^2n^2 + 4(b^2m^2 - a^2l^2)(a^2n^2 + a^2b^2m^2) = 0$$

$$4a^4l^2n^2 + 4(b^2m^2 - a^2l^2)(a^2n^2 + a^2b^2m^2 = 0$$

$$4a^4l^2n^2 + 4a^2b^2n^2m^2 + 4a^2b^4m^4 - 4a^4l^2n^2 - 4a^4b^2m^2l^2 = 0$$

$$4a^2b^2m^2(n^2 + b^2m^2 - a^2l^2) = 0$$

$$a^2l^2 - b^2m^2 = n^2$$

which is required condition

Point of contact :-

1) Ellipse

To find point of contact

we have the eqn of tangent at (x_1, y_1) to the ellipse $x^2/a^2 + y^2/b^2 = 1$ is:-

$$\frac{xx_1}{a^2} + \frac{yy_1}{a^2} = 1$$

Also, $lx + my + n = 0$ is tangent to the ellipse, $x^2/a^2 + y^2/b^2 = 1$

$$\therefore \frac{x_1/a^2}{l} = \frac{y_1/a^2}{m} = -1/n$$

$$x_1 = \frac{-a^2 l}{n}, \quad y_1 = \frac{-ma^2}{n}$$

$\therefore$ point of contact of (x_1, y_1) is $\left(\frac{-a^2 l}{n}, \frac{-a^2 m}{n}\right)$

2) Hyperbola

To find point of contact.

We have the eqn of tangent at (x_1, y_1) to the ~~ellipse~~ hyperbola $x^2/a^2 - y^2/b^2 = 1$,

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Also the line,

$lx + my + n = 0$ is tangent to the
eqn of hyperbola, $x^2/a^2 - y^2/b^2 = 1$

$$\frac{x_1/a^2}{l} = \frac{-y_1/b^2}{m} = -1/n$$

$$x_1 = \frac{-la^2}{n}, \quad y_1 = \frac{mb^2}{n}$$

∴ point of contact of (x_1, y_1) is :-
$\left(-\frac{la^2}{n}, \frac{mb^2}{n}\right)$

(iii) Parabola,
To find point of contact,
we have eqn of tangent at (x_1, y_1) to
the parabola, $y^2 = 4ax$,

$$yy_1 = 2a(x + x_1)$$

Also, $lx + my + n = 0$ is tangent to the
eqn of parabola, $y^2 = 4ax$.

6) ~~Obtain the vertex, focus, directrix, l~~

7) Obtain the centre, vertices, coordinates of foci, directrix, eccentricity and length of latus rectum of the following ellipse $9x^2 + 5y^2 - 30y = 0$

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Q. $9x^2 + 5y^2 - 30y = 0$

$5(y^2 - 6y) + 9x^2 = 0$

$9(x^2) + 5(y^2 - 6y) = 0$

$9(x - 0)^2 + 5(y^2 - 2.3y + 3^2) = 45$

$\frac{(x-0)^2}{5} + \frac{(y-3)^2}{9} = 1$

comparing with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$h = 0, \quad k = 1, \quad a = \sqrt{5}, \quad b = 3$

$e^2 = a^2 + b^2 = \cancel{5 + 9, c = \sqrt{14}}$

$a < b$

$c^2 = b^2 - a^2 = 3^2 - 5 = 4$

$c = \pm 2$

centre$(h, k) = (0, 1)$

foci $(h, \cancel{\pm c}, k \pm c) = \cancel{(0 \pm 2, 1)}$ $(0, 1 \pm 2)$

vertices $(h, \cancel{a}\, k \pm b) = (\cancel{0 \pm \sqrt{5}}, 1)$ $(0, 1 \pm 3)$

major axis $= 2b = \cancel{2\sqrt{5}}\ 18$

minor axis $= 2a = 2\sqrt{5}$

eccentricity $e = c/b = 2/3$

eqn of directrix ; $y = k \pm b/e$

$= 1 \pm 9/2$

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length of latus rectum $= \frac{2a^2}{b} = \frac{2\times5}{3} = \frac{10}{3}$

Obtain the centre, vertices, coordinates of foci, directrix, eccentricity and length of latus rectum of the following hyperbola
$4(y+3)^2 - 9(x-2)^2 = 1$

$4(y+3)^2 - 9(x-2)^2 = 1$

$\frac{(y+3)^2}{9} - \frac{(x-2)^2}{4} = \cancel{36}$

$\frac{(y+3)^2}{1/4} - \frac{(x-2)^2}{1/9} = 1$

comparing with $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

we get,

$k = -3$, $b = \pm\frac{1}{2}$ $a = \pm\frac{1}{3}$ $h = 2$

$c^2 = a^2 + b^2 = 13/36$, $c = \pm\frac{\sqrt{13}}{6}$

vertices $(h, k \pm b) = (2, -3 \pm 1/2)$
coordinates of foci, $(h, k \pm c) = (2, -3 \pm \sqrt{13}/6)$
Eqn of directrix $y = k \pm b/e = -3 \pm \frac{3\sqrt{13}}{26}$

eccentricity $(e) = c/b = \frac{\sqrt{13}}{12}$

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$9x^2 + 18x - 16y^2 + 32y = 151$

$9(x^2 + 2x) - 16(y^2 - 2y) = 151$

$9(x^2 + 2.x.1 + 1^2) - 16(y^2 - 2.y.1 + 1^2) = 151 + 9 - 16$

$9(x+1)^2 - 16(y-1)^2 = 144$

$\frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1$ ...(i)

$c^2 = a^2 + b^2 = 16 + 9 = 25$

$c = \pm 5$

Comparing (i) with $\frac{(x-h)^2}{a^2} - \frac{(y-k^2)}{b^2} = 1$

$h = -1, \quad k = 1, \quad a = 4, \quad b = 3$

Now,

~~centre $(h \pm c, k) = (-1 \pm 5, 1)$~~

~~vertices $(h \pm a, k) = (-1 \pm 4, 1)$~~

~~coordinates of foci~~

centre $(h, k) = (-1, 1)$

vertices $(h \pm c, k) = (-1 \pm 5, 1)$

coordinates of foci $(h \pm a, k) = (-1 \pm 4, 1)$

Eqn of directrix, $\qquad$ eccentricity(e) $= c/a$

$y = h \pm a/e$ $\qquad = 5/4$

$= -1 \pm \frac{4}{5/4} = \frac{-1 \pm 16}{5}$

length of latus rectum $= \frac{2b^2}{a} = \frac{18}{4} = 9/2$

Gurukul

Find the equation of parabola having focus and directrix.
Focus$(1,2)$ directrix $x+2y+4=0$.

Given, the focus of the parabola as $F:(1,2)$
and directrix, M $x+2y+4=0$

let $P(x,y)$ be the point on the parabola,
such that,

$$FP^2 = PM^2$$

$$(x-1)^2 + (y-2)^2 = \frac{(x+2y+4)^2}{1^2+2^2}$$

on simplifying,

$$5(x^2-2x+1+y^2-4y+4) = x^2 + 2x(2y+4) + (2y+4)^2$$

$$5x^2 - 10x + 5y^2 - 20y + 25 = x^2 + 4xy + 8x + 4y^2 + 16y + 16$$

$$4x^2 - 4xy + y^2 - 18x - 36y + 9 = 0$$

$$(2x-y)^2 - 18x - 36y + 9 = 0$$

which is required eqn of parabola.

Find the equation of ellipse having focus, directrix and eccentricity $F(-1,1)$, $4x-3y=0$ and $e=5/6$

Let F be the given focus of ellipse and $P(h,k)$ be a point on the ellipse.

$\therefore$ distance $FP = \sqrt{(h+1)^2 + (k-1)^2}$

Also, shortest distance of P from directrix,

$$= PD = \left|\frac{4h-3k}{\sqrt{16+9}}\right| = \left|\frac{4h-3k}{5}\right|$$

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Now, from definition of eccentricity,

$e = \frac{FP}{DP}$

$\therefore \frac{5}{6} = \frac{\sqrt{(h+1)^2 + (k-1)^2}}{\left|\frac{4h-3k}{5}\right|}$

$\therefore \frac{5}{6}\left|\frac{4h-3k}{5}\right| = \sqrt{(h+1)^2 + (k-1)^2}$

S.O.B.S

$\frac{25}{36}\left(\frac{4h-3k}{5}\right)^2 = (h+1)^2 + (k-1)^2$

$(4h-3k)^2 = 36(h+1)^2 + 36(k-1)^2$

$16h^2 \mp 24hk + 9k^2 = 36h^2 + 36h + 36 + 36k^2 - 36k + 36$

$20h^2 + 27k^2 + 36h - 36k + 24hk + 72 = 0$

Hence, the required eqn of ellipse is:-

$\cancel{8}20x^2 + 27\cancel{b}y^2 + 36x - 36y + 24xy + 72 = 0$

) Find the equation of hyperbola having focus, directrix and eccentricity are $(6,0)$, $4x-3y=6$, $e = 5/4$

=) Using definition of hyperbola,

$PS^2 = e^2 \cdot PM^2$,

~~where, PS~~

let, eqn of directrix is $4x - 3y - 6 = 0$

let S be the corresponding focus, then $S = (6,0)$

Gurukul

Given, $e = 5/4$

Let, $P(x, y)$ be any point on the required hyperbola.
Let PM be the length of the perpendicular from P to the directrix.

Then, $\frac{PS}{PM} = e = 5/4$

$$PS^2 = \frac{25}{16} PM^2$$

or, $(x-6)^2 + (y-0)^2 = \frac{25}{16} \left| \frac{4x - 3y - 6}{5} \right|^2$

or, $16(x-6)^2 + 16y^2 = (4x - (3y+6))^2$

or, $16(x^2 - 12x + 36) + 16y^2 = 16x^2 - 4x(3y+6) + (3y+6)^2$

or, $16x^2 - 192x + 576 + 16y^2 = 16x^2 - 12xy - 24x + 9y^2 + 36y + 36$

or, $\cancel{9y^2 - 12xy + 168x - 7y^2 - 540}$

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12) A tangent to $y^2 = 16x$ makes an angle $60°$ with x-axis. Find the equation of tangent and it's point of contact.

=) Given parabola $y^2 = 16x$

Inclination of the tangent is
$\theta = 60°$, $m = \tan 60° = \sqrt{3}$

Therefore eqn of tangent is a,

$$y = mx + \frac{a}{m}$$

$$y = \sqrt{3}x + \frac{4}{\sqrt{3}}$$

$$\sqrt{3}y = 3x + 4$$

Point of contact, $= \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$$= \left(\frac{4}{3}, \frac{8}{\sqrt{3}}\right)$$