Kusal lamsal
Roll no: 15

1 a.

→ Sol^n.

Total = 5 + 4 + 3 + 2

= 14

The total no. of selection is

$$\binom{14}{4} = \frac{14!}{(14-4)!\,4!}$$

$$= 1001$$

(i) one of each kind

→

Engineers	Economist	doctor	nurse
1	1	1	1

then

$$\binom{5}{1}\cdot\binom{4}{1}\binom{3}{1}\binom{2}{1}$$

$$= 5.4.3.2 = 120$$

Probability that each profession is represented in committee

$$P(A) = \frac{120}{1001}$$

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ii) no economist

probability that there is no economist

$= 5+3+2$

$= 10$

Then,

$$\binom{10}{4} = \frac{10!}{(10-4)!\,4!}$$

$$= 210$$

probability that there is no economist

$$P(\overline{B}) = \frac{210}{1001} = \frac{30}{143}$$

iii) at least one nurse

~~Probar~~ probability for no nurse

total $= 5+4+3 = 12$

$$\binom{12}{4} = \frac{12!}{(12-4)!\,4!} = 495$$

$$P(\overline{N}) = \frac{495}{1001} = \frac{45}{91}$$

Probability that at least one nurse

$$P(N) = 1 - P(\overline{N}) = 1 - \frac{45}{91} = \frac{46}{91}$$

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1b

Ans. Here

A : Chip is actually good.

$\bar{A}$: chip is actually defective.

B : Tested ~~def~~ device is indicated to be defective.

then

from given,

$$P(\bar{A}) = 2\% = 0.02$$

$$P(B|A) = 1 - 0.95$$
$$= 0.05$$

$$P(B|\bar{A}) = 0.94$$

Total probability

$$P(B) = P(A)\,P(B|A) + P(\bar{A})\,P(B|\bar{A})$$
$$= (0.98 \times 0.05) + (0.02 \times 0.94)$$
$$= 0.0678$$

then By baye's theorem.

probability for actual defective $P(\bar{A}|B)$

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$$P(\bar{A}|B) = \frac{P(\bar{A})P(B|\bar{A})}{P(B)}$$

$$= \frac{0.02 \times 0.94}{0.0678}$$

$$= 0.2773$$ #

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2a.

x:	0	1	2	3	4	5	6	7	8
$P(x)$	K	3K	5K	7K	9K	11K	13K	15K	17K

i) Determine K

We have

$$\sum_{i=1}^{n} P(x_i) = 1$$

or, $P(0) + P(1) + \dots + P(8) = 1$

or, $K + 3K + 5K + 7K + 9K + 11K + 13K + 15K + 17K = 1$

or $81K = 1$

or, $K = \frac{1}{81}$

ii) find $P(x<3)$

$= P(0) + P(1) + P(2)$

$= K + 3K + 5K$

$= \frac{1}{81} + \frac{3}{81} + \frac{5}{81}$

$= \frac{1}{9}$

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$P(x \geq 3)$

$\therefore P(3) + P(4) + \cdots + P(8)$

$= 7K + 9K + 11K + 13K + 15K + 17K$

$= \frac{7}{81} + \frac{9}{81} + \frac{11}{81} + \frac{13}{81} + \frac{15}{81} + \frac{17}{81}$

$= \frac{8}{9}$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= 3K + 5K + 7K + 9K$

$= \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81}$

$= \frac{8}{27}$

iii) Find variance of function x.

We have

Variance $\{V(x)\} = E[x^2] - [E(x)]^2$

So

$E(x) = \sum_{x} x P(x)$

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$= 0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + \dots + 8 \cdot P(8)$

$= 0 + 3K + 2 \times 5K + 3 \times 7K + 4 \times 9K + 5 \times 11K + 6 \times 13K + 7 \times 15K + 8 \times 17K$

$= \frac{3}{81} + \frac{10}{81} + \frac{21}{81} + \frac{36}{81} + \frac{55}{81} + \frac{78}{81} + \frac{105}{81} + \frac{136}{81}$

$= \frac{148}{27} = 5.3$

and.

$E(x^2) = \sum_x x^2 P(x)$

$= 0^2 \times P(0) + 1^2 \cdot P(1) + \dots 8^2 \cdot P(8)$

$= 3K + 4 \cdot 5K + 9 \cdot 7K + 16 \cdot 9K + 25 \cdot 11K + 36 \cdot 13K + 49 \cdot 15K + 64 \cdot 17K$

$= \frac{3}{81} + \frac{20}{81} + \frac{63}{81} + \frac{144}{81} + \frac{275}{81} + \frac{468}{81} + \frac{735}{81} + \frac{1088}{81}$

$= \frac{932}{27} = \cancel{33.5}\ \cancel{34.518}\ 33.552$

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Also $[E(x)]^2 = (5.3)^2 = 28.09$

$\therefore V(x) = E(x)^2 - [E(x)]^2$

$= 33.552 - 28.09$

$= 5.462$

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2b.

Ans. Sol^n.

let x = no. of engine in working condition

Possible value of x : 0, 1, 2, 3, 4.

~~$P(x \geq 2)$~~

$$P(x \geq 2) = \binom{4}{2} 0.1^2 . 0.90^2 + \binom{4}{3} . 0.1^1 . 0.9^3$$

$$+ \binom{4}{4} . 0 . 1^0 . 0.9^4$$

$$= 0.996$$

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3a.

Ans. Sol^n.

Total no of accident per year = 4.

then

probability for less than 4 accident per year

$$P[x<4] = F(3) = \sum_{x=0}^{3} \frac{e^{-4} 4^x}{x!}$$

$$= \sum_{x=0}^{3} \frac{e^{-4} \cdot 4^x}{x!}$$

$$= 0.4334 \text{ Ans.}$$

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3b.

Ans. given

$$f(x,y) = 8xy \quad \text{for} \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

i) Marginal density of x

$$= f(x) \cdot \int_y f(x,y)\, dy$$

$$= \int_0^1 8xy\, dy$$

$$= 8x \left[\frac{y^2}{2} \right]_0^1$$

$$= 4x \cdot [1 - 0]$$

$$= 4x.$$

ii) Conditional density function of x given y

$$f(x|y) = \frac{f(x,y)}{f(y)} \quad ; \quad f(y) \neq 0$$

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So. $f(y) = \int_x f(x,y)\,dx$

$= \int_0^1 8xy\,dx$

$\frac{8y}{2}\left[x^2\right]_0^1$

$= 4y$

$\therefore f(x|y) = \frac{8xy}{4y} = 2x.$

iii) Determine whether variable are independent

$\Rightarrow$ $f(x).f(y)$

$= 4x.4y$

$= 16xy \neq f(x,y)$

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4a.

Ans. Mean $(\mu) = 4.76$ sec.

S.D $(\sigma) = 0.04$ sec.

then.

i) $P(x < 4.6)$

$= P\left(\frac{x-\mu}{0.04} < \frac{4.6-4.76}{0.04}\right)$

$= \cancel{Prob(Z < -4)}$

$= \cancel{Prop(Z > 4)}$

$= P(Z < -4)$

$= F(-4)$

ii) between 4.7 to 4.9 sec.

$= P(4.7 < x < 4.9)$

$= P\left(\frac{4.7-\mu}{0.04} < \frac{x-\mu}{0.04} < \frac{4.9-\mu}{0.04}\right)$

$= P\left(\frac{4.7-4.76}{0.04} < \frac{x-0.76}{0.04} < \frac{4.9-4.76}{0.04}\right)$

$= P(-1.5 < z < 3.5)$

$= P(-1.5 < z < 0) + P(0 < z < 3.5)$

$= P(0 < z < 1.5) + P(0 < z < 3.5)$

$= (0.5 - 0.4332) + (0.5 - 0.4998)$

$= 0.067$

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4 b.

Ans. let x = lifetime hrs of a battery.

$$\lambda = \frac{1}{50} \text{ hrs.}$$

We have,

$$F(x) = \lambda e^{-\lambda x}$$

$$= \frac{1}{50} e^{-1/50\, x}$$

probability that semiconductor will last for 100 hrs

$$P(x \geq 100)$$

$$= \int_{100}^{\infty} F(x)\, dx$$

$$= \int_{100}^{\infty} \frac{1}{50} e^{-1/50\, x}\, dx$$

$$= \frac{1}{50} \left[\frac{e^{-1/50\, x}}{-1/50} \right]_{100}^{\infty}$$

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$$= -1\left[e^{-\infty} - e^{-100/50}\right]$$

$$= -1\left[0 - e^{-2}\right]$$

$$= e^{-2}$$

$$= \frac{1}{e^{2}}$$

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3a.

Ans. ~~Ze~~ We can represent Markov chain by a matrix entering the probabilities of event for one state to another state which is called transition probability matrix.

Transition Matrix is said to be regular if some power of P has all positive entries (i.e. strictly greater than 0) otherwise chain is irregular.

Here,

$$P = \begin{bmatrix} 0.2 & 0.8 \\ 1 & 0 \end{bmatrix}$$

Now,

$$P^2 = P \cdot P \begin{bmatrix} 0.2 & 0.8 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0.2 & 0.8 \\ 1 & 0 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 0.01 + 0.8 & 0.16 + 0 \\ 0.2 + 0 & 0.8 + 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 0.84 & 0.16 \\ 0.2 & 0.8 \end{bmatrix}$$

$$P^3 = P^2 \cdot P = \begin{bmatrix} 0.84 & 0.16 \\ 0.2 & 0.8 \end{bmatrix} \times \begin{bmatrix} 0.2 & 0.8 \\ 1 & 0 \end{bmatrix}$$

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$$= \begin{bmatrix} 0.328 & 0.672 \\ 1.2 & 0.16 \end{bmatrix}$$

Here all the entries are greater than 0 in any power of P, so given matrix is regular.

5 b.

Ans. The given Transition probability matrix is

$$P = \begin{bmatrix} 1/6 & 5/6 \\ 1/12 & 11/12 \end{bmatrix}$$

Two step trancition probability

$$P^2 = P \cdot P = \begin{bmatrix} 1/6 & 5/6 \\ 1/12 & 11/12 \end{bmatrix} \begin{bmatrix} 1/6 & 5/6 \\ 1/12 & 11/12 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 131/144 \end{bmatrix} = \begin{bmatrix} 0.0973 & 0.9027 \\ 0.0902 & 0.9097 \end{bmatrix}}$$

~~Three step transition~~

$$= \begin{bmatrix} 7/72 & 65/72 \\ 13/144 & 131/144 \end{bmatrix}$$

Three step transition probability matrix

$$P^3 = P^2 \cdot P = \begin{bmatrix} 7/72 & 65/72 \\ 13/144 & 131/144 \end{bmatrix} \cdot \begin{bmatrix} 1/6 & 5/6 \\ 1/12 & 11/12 \end{bmatrix}$$

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$$= \begin{bmatrix} 79/864 & \frac{785}{864} \\ \frac{157}{1728} & \frac{1571}{1728} \end{bmatrix}$$

Then n step T.P.M is

$$P^n = \frac{1}{\alpha+\beta}\left\{\begin{bmatrix} \beta & \alpha \\ \beta & \alpha \end{bmatrix} + (1-\alpha-\beta)^n \begin{bmatrix} \alpha & -\alpha \\ -\beta & \beta \end{bmatrix}\right\}$$

$$\alpha = 1 - 0.17 = 0.83$$
$$\beta = 1 - 0.92 = 0.08$$

$$P^n = \frac{1}{0.83+0.08}\left\{\begin{bmatrix} 0.08 & 0.83 \\ 0.08 & 0.83 \end{bmatrix} + (1-0.83-0.08)^n \begin{bmatrix} 0.83 & -0.83 \\ -0.08 & 0.08 \end{bmatrix}\right\}$$

$$= \frac{1}{0.91}\left\{\begin{bmatrix} 0.08 & 0.83 \\ 0.08 & 0.83 \end{bmatrix} + (0.09)^n \begin{bmatrix} 0.83 & -0.83 \\ -0.08 & 0.08 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 0.087 & 0.913 \\ 0.087 & 0.913 \end{bmatrix} + (0.09)^n \begin{bmatrix} 0.913 & -0.913 \\ -0.087 & 0.087 \end{bmatrix}\Big\}$$

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As $n \to \infty$

$$\varphi = \lim_{n \to \infty} P^n = \begin{bmatrix} 0.087 & 0.913 \\ 0.087 & 0.913 \end{bmatrix} \begin{bmatrix} \pi \\ \pi \end{bmatrix}$$

$$\therefore \varphi = \pi \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{bmatrix} 0.087 & 0.913 \end{bmatrix}$$

$$= \begin{bmatrix} 0.087 & 0.913 \end{bmatrix}$$

8a.

Ans. Given,

Rate of arrival of customer $= \lambda = 15$ per hour

$= \frac{15}{60} = \frac{1}{4}$ min

Mean $(\mu) = 2$ minutes $= \frac{1}{2}$ per min

Traffic intensity $(\rho) = \lambda/\mu = \frac{1/4}{1/2} = 1/2$

i) Average no of customer in system $= \frac{\rho}{1-\rho}$

$= \frac{1/2}{1-1/2} = 1$

$\therefore L_q = \rho^2/1-\rho = (1/2)^2/1-1/2 = 1/2$

ii) Expected no. of customer in system

$L_s = \frac{\rho}{1-\rho}$

$= \frac{1/2}{1-1/2} = 1.$

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6 b

Ans. Soln.

$\lambda = 15$ per hour $= 15/60 = 1/4$ per min

$\mu = 6$ per hour $= 6/60 = 1/10$ per min

$S = 3.$

$$\rho = \frac{\lambda}{\mu S} = \frac{1/4}{\left(\frac{1}{10}\right)\cdot 3} = \frac{1}{4} \times \frac{10}{3} = 0.83$$

$$S \cdot \rho = 3 \times 0.83$$
$$= 2.5$$

$$P_0 = \sum_{n=0}^{2} \left\{ \frac{(2.5)^n}{n!} + \frac{(2.5)^3}{3!(1-0.83)} \right\}^{-1}$$

$$= \left\{ 1 + \frac{2.5}{1!} + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!(0.17)} \right\}^{-1}$$

$$= \{1 + 2.5 + 3.125 + 15.319\}^{-1}$$

$$P_0 = 0.045$$

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(I) What fraction of time all the typist will be busy

$$P(N \geq S) = \frac{(S\rho)^S}{S!\,(1-\rho)} \times P_0$$

$$= \frac{(2.5)^3}{3!\,(1-0.83)} \times 0.045$$

$$= 0.69$$

(II) What is average no. of letters waiting to be typed?

→

$$L_q = \frac{1}{S \cdot S!} \frac{(S\rho)^{S+1}}{(1-\rho)^2} \times P_0$$

$$= \cancel{\frac{(2.5)^4}{3 \cdot 3!\,(1-0.8}} = \frac{(2.5)^4}{3 \cdot 3!\,(1-0.83)^2} \times 0.045$$

$$= \frac{39.0625}{0.5202} \times 0.045$$

$$= 3.379$$

#.

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7. Write short notes on:

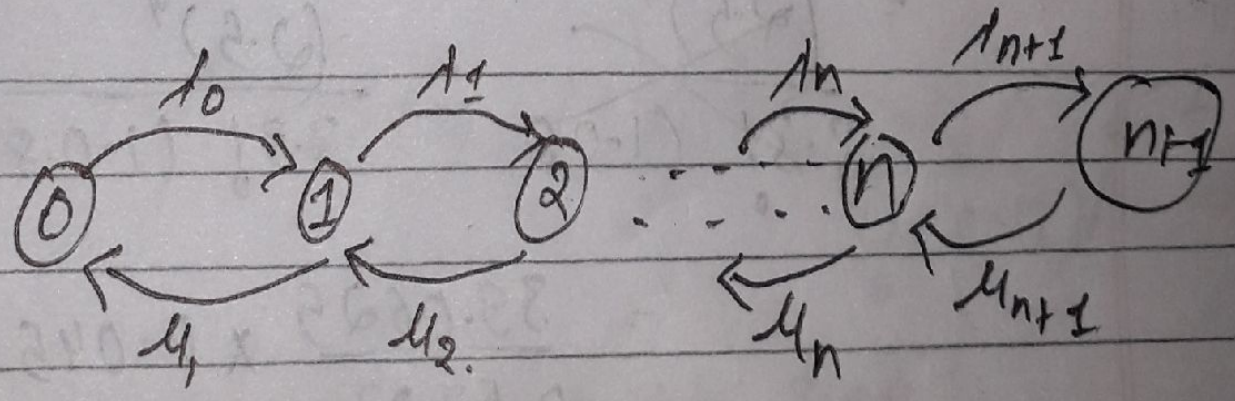
b) Birth and death process

A continuous M.C. $X(t)$ with the state space $S = \{0, 1, 2, \dots\}$ is called birth and death process if the following condition is satisfied.

$$P[X(t+h) - X(t) = K \mid X(t) = n]$$

$$= \begin{cases} \lambda_n h + o(h), & \text{if } K = 1 \ \& \ n \geq 0 \\ \mu_n h + o(h), & \text{if } K = -1 \ \& \ n \geq 0 \\ 1 - (\lambda_n + \mu_n) h + o(h), & \text{if } K = 0, \ n \geq 0 \end{cases}$$

where $\lambda_0, \lambda_1, \lambda_2, \dots$ are positive constants called Birth rates and $\mu_1, \mu_2, \mu_3 \dots$ are the constants called death rates.



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(c) Conditional probability

It is the probability of an event occurring given that another event has already occurred. The concept is one of the quintessential concepts in probability theory.

Formula for conditional probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(A|B)$ is conditional probability; the probability of event occurring given that event B has already occurred.

$P(A \cap B)$ is joint probability of events A & B the probability that both events A & B occur.

$P(B)$: is probability of event B.

It is applied to calculate the conditional probability of events that are neither independent nor mutually exclusive. We can also calculate conditional probability using Baye's theorem.

i.e. $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

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Other question

1) Explain diffrent approaches of probability and law of probability

-) Different approaches of probability are:

I) Mathematical /emperical approaches of probability

let 'n' be the total exhaustive and mutually enclusive and equally ~~to~~ likely events among them 'n' are favorable to happening events A then,

$$P(A) = \frac{\text{favorabale cases } (M)}{\text{Total no. of cases } (n)}$$

$$P = M/n$$

2) Statstical /Emperical approaches of probability

$$P = n \xrightarrow{\lim} \infty \; m/n$$

3) Modern /Axiomatic approaches of probability

$A_1, A_2, A_3 \ldots A_n$ be mutually enclusive and equally likely events in sample space s. then

a) $P(S) = 1$

b) $P(A_1 \cup A_2 \cup A_3 \ldots A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

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$$\bigcup_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} P(A_i)$$

Laws of probability are.

i) Probability of impossible events is zero
i.e. $P(\phi) = 0$

ii) $P(A) + P(\bar{A}) = 1$

iii) Additive law
a) When events are mutually exclusive
let A & B be two mutually exclusive and equally likely events then probability occurrance of A or B
i.e. $P(A \cup B) = P(A) + P(B)$

b) When event are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

iv) Multiplicative law
a) when events are dependant
let A and B be two dependant events, the probability occurrance of A effects, probability

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of occurrance of B and vice versa This is case of conditional probability.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B)$$

$$= P(A) \cdot P(A/B)$$

Similarly,

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = \cancel{P(A) \cdot P(B/A)}$$

$$= P(A) \cdot P(B/A)$$

b) When events are independent:

let A & B are two independent events. then probability of occurrance of A & B.

i.e. $P(A \cap B) = P(A) \cdot P(B)$

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2) State and Explain Baye's theorem

→ Statement.

let E_1, E_2, E_3 & $E_4 \ldots E_n$ be mutually exclusive & equally likely events in sample spaces (S) be any ~~arbi~~ arbitary events in S, then probability of rth events is defined as

$$P\left(\frac{E}{A}\right) = \frac{P(E_r) \cdot P\left(\frac{A}{E_r}\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)}$$

where, $i < n_r \leq n$

where $P\left(\frac{E}{A}\right)$ = Posterior probabilities

$P(E_r)$ = prior probabilities

$P\left(\frac{A}{E_r}\right)$ = liklihood probabilities obtained from experiments

Proof. Since $E_1, E_2, E_3 \ldots E_n$ be mutually exclusive and equally likely events in sample spaces (S), A be any ~~arbi~~ orbitary events in S, then

$$S = E_1 \cup E_2 \cup E_3 \cup \ldots E_n$$

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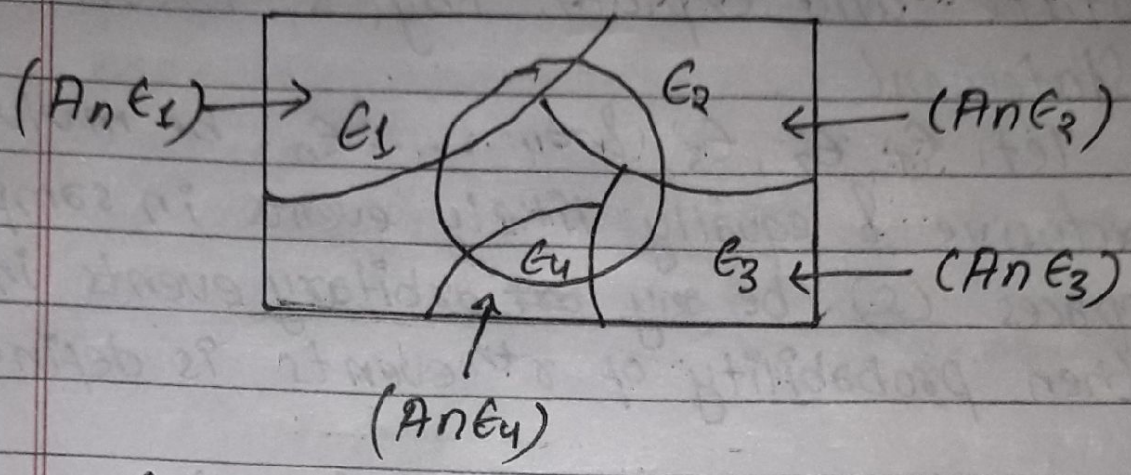

from figure,

$$A = A \cap S$$

$$= A \cap \{E_1 \cup E_2 \cup E_3 \ldots E_n\}$$

$$= (A \cap E_1) \cup (A \cap E_2) \ldots \cup (A \cap E_n)$$

Since

$(A \cap E_1) \cdot (A \cap E_2)\ (A \cap E_3) \ldots (A \cap E_n)$

are disjoint events

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

or, $$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

$$= \sum_{i=1}^{n} P(E_i) \cdot P\left(\frac{A}{E_i}\right)$$

~~For~~

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For any r^{th} events

$$P\left(\frac{E_r}{A}\right) = \frac{P(A \cap E_r)}{P(A)}$$

$$= \frac{P(E_r) \cdot P\left(A/E_r\right)}{\sum_{i=1}^{n} P(E_i) \cdot P\left(A/E_r\right)}$$

Ans.

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3. What is normal distribution explain its properties

→ A continuous random variable x is said to follow normal distribution with mean μ and variance σ^2 of its probability density function is given as

$$f(x) = \frac{1}{\sigma\sqrt{2x}} e^{-1/2\left(\frac{x-M}{\sigma}\right)^2}$$

if $-\infty < x < \infty$

where

μ = position mean

σ = position standard deviation

x = no. of observation

e = natural logarithm

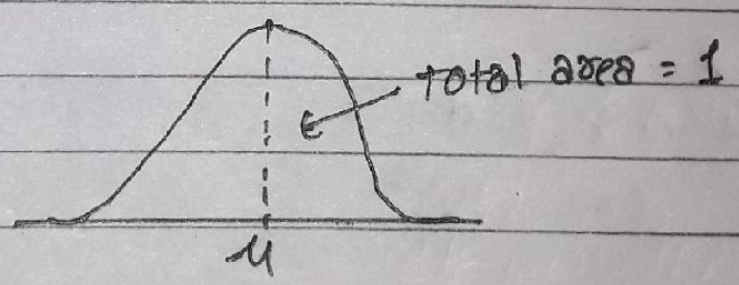

fig: Bell curve (Normal distribution)

Properties of normal distribution are:

i) The mean, median, mode are equal.

ii) The normal curve is bell-shaped and symmetric

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about the mean.

iii) The total area under the curve is equal to one.

iv) The normal curve approaches but never touches the x-axis as it extends farther and farther away from mean

iv) The shape and position of normal distribution curve depend on two parameters the mean (μ) and standard deviation (σ)

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4. State Chebyshev's inequality with suitable example.

→ Chebyshev's inequality states that if a probability distribution having mean (μ) and s.d (σ), then probability of setting a value which deviates μ is at least K_σ is at most $1/K^2$

Mathematically

$$P\{|x-\mu| \geq K\sigma\} \leq 1/K^2$$

$$P\{|x-\mu| \leq K\sigma\} \geq \left(1-\frac{1}{K^2}\right)$$

Example:

A fair die is thrown and x denotes the no then use chegshev's inequality to show that:

$$P\{|x-3.5| > 2.5\} < 0.47$$

Sol^n:

when a fair dice is thrown then x denotes the no. 1, 2, 3, 4, 5, 6 with equal probability $\frac{1}{6}$

$$E(x) = \Sigma x \cdot P(x) = 1\times 1/6 + 2\times 1/6 + 3\times 1/6 + 4\times 1/6 + 5\times 1/6 + 6\times 1/6$$

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$= 7/2 = 3.5$

$E(x)^2 = \sum x^2 (P(x)$

$= 1/6 \cdot 1^2 + 2^2 \cdot 1/6 + 3^2 \cdot 1/6 + 4^2 \cdot 1/6 + 5^2 \cdot 1/6 + 6^2 \cdot 1/6$

$= \frac{91}{6}$

$V(x) = E(x^2) - [E(x)]^2$

$= \frac{91}{6} - (3.5)^2$

$= 2.9167$

Using Chebyshev's inequality

$P\{ |x - 3.5| \geq 2.5 \} < \frac{2.916}{(2.5)^2}$

$P\{ |x - 3.5| > 2.5 \} < 0.47$

Proved

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5. State the operating characteristics of the following queue model

a) $M/M/1:\infty/FCFS$ Model

It means arrival departure process follows mark-ovian distribution. with single server having infinite capacity.

Operating characteristics

i) Server utilization or traffic intensity

$$\rho = \frac{\lambda}{\mu}$$

ii) Probability of n customer in the system

$$P_n = \rho^n (1-\rho)$$

iii) Average or expected no. of customer in system

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

iv) Probability of customer

$$P_0 = \rho^0 (1-\rho)$$

v) Average expected no. of customer in queue

$$L_q = \rho \times L_s = \frac{\rho}{1-\rho} \times \rho$$

$$= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

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b) M/M/1 : N/FCFS model

operating characteristics

i) Serving utilization of traffic intensity

$$\rho = \lambda / \mu$$

ii) Probability of customer

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}, \quad \rho < 1 \quad \text{but} \quad P_0 = \frac{\rho - 1}{\rho^{N+1} - 1}$$

if $\rho > 1$ where N is size of room

iii) Probability of n customer in system

$$P_n = \frac{(1-\rho) \cdot e^N}{1-\rho^{N+1}}$$

iv) Average expected no. of customer in system

$$L_s = \left\{ \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{K+1}} \right.$$

v) Average no. of customer in queue

$$L_q = L_s = \lambda_1 / M$$

vi) Effective arrival rate

$$\lambda^1 = M(1-P_0)$$

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c) c) M/M/S:N/FCFS model

operating characteristics

i) Server utilization or traffic intensity

$$\rho = \lambda / \mu s$$

ii) Probability of no. of customer in system

$$P_0 = \left\{ \sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \sum^{N} \left(\frac{\lambda}{\mu s}\right)^{n-s} \right\}^{-1}$$

iii) Expected no. of customer in queue

$$L_q = P_0 \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho)^2} \left[1 - \rho^{N-s} - (N-s)(1-\rho)\rho^{N-s} \right]$$

iv) Average no of customer in system

$$L_s = L_q + \left\{ s - \sum_{n=0}^{s-1} (s-n) P_n \right\}$$

$$= L_q + \frac{\lambda}{\mu}$$

v) Average waiting time of customer in queue

$$W_q = \frac{1}{\lambda'} \times L_q$$

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d) M/M/S : ∞/FCFS model
operating characteristics

i) Server utilization or traffic intensity

$$\rho = \frac{\lambda}{\mu_s}$$

ii) Probability of no. of customer in the system

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{(S\rho)}{n!} + \frac{(\rho^S)}{(1-\rho)} \frac{S^2}{S!} \right]^{-1}$$

$$P_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0, & \text{if } n = 0, 1, 2 \ldots S-1 \\ \frac{1}{S!} \cdot \frac{1}{S^{n-S}} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{if } n = S, S+1 \ldots \end{cases}$$

iii) Average no. of customer in queue

$$L_q = \frac{\rho (S\rho)^S}{S! (1-\rho)^2} P_0 = \frac{P_S \rho}{(1-\rho)^2}$$

iv) Average ~~no.~~ expected no of customer in ~~queue~~ system

$$L_s = L_q + S\rho = L_q + \lambda/\mu$$

v) Average waiting line of customer in a system

$$W_s = \frac{L_s}{\lambda} = W_q + \frac{1}{\mu}$$

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6. Explain central limit theorem and its applicatio

-> The central limit theorem states that the distribution of a sample mean (μ) that approximates the normal distribution, as the sample size becomes larger, assumming that all the sample are similar and no matter what the shape of population distribution.

Central limit theorem formula: It is applicable sufficiently large sample size ($n \geq 30$).

The formula can be

$$\mu_n = \mu$$

$$\sigma_n = \frac{\sigma}{\sqrt{n}}$$

where.

μ = population mean

σ = population standard deviation

μ_x = sample mean

σ_x = sample standard deviation

n = sample size.

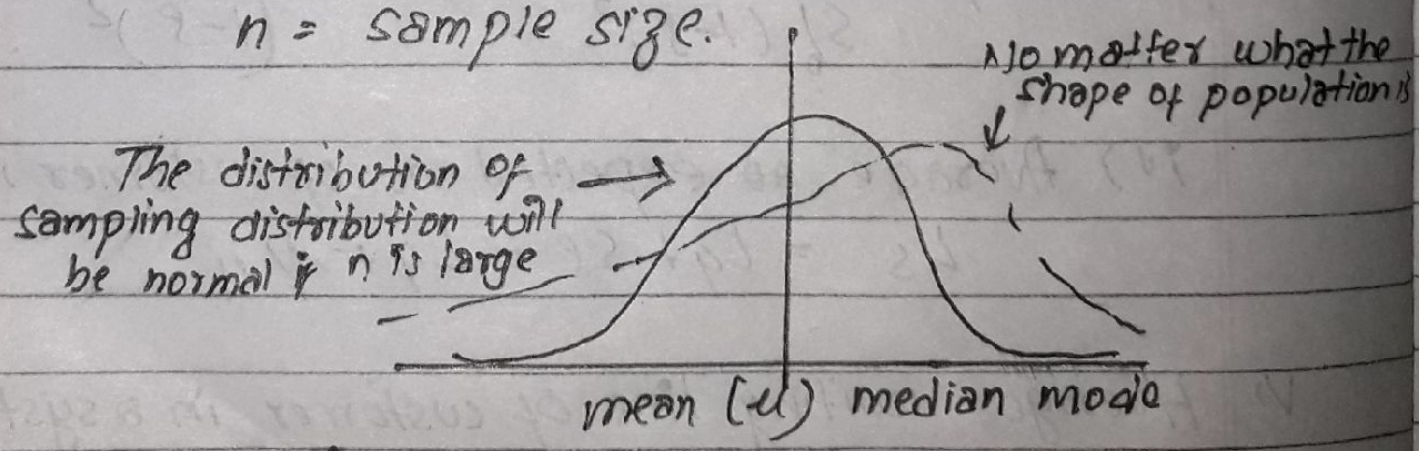

fig: Cental limit theorem

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Application

- If the distribution is not known or not normal we consider the sample distribution to be normal according to CTL. As this method assume that the population given is normally distributed. This helps in analyzing data in methods like constructing confidence.
- To estimate the population mean more accurately
- To create a range of values which is likely to include the population mean.