Level: Bachelor Semester –FALL Year : 2023
Programme: BE Marks: 100
Course: Calculus I Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all the questions.

1.a) Define continuity and differentiability of a function. Show that Differentiability of a function f(x) at x=a implies continuity but Converse may not be always true. [7]

b) State and prove Role's theorem. Interpret Geometrically. [8]

OR

a) If $y=e^{\tan^{-1}x}$, show that [7]

i) $(1+x^2)y_2 +(2x-1) y_1 = 0$

ii) $(1+x^2)y_{n+2}+(2nx+2x-1)y_{n+1}$ n(n+1)y_n =0

b) Trace the curve: $y^2(a-x) = x^2(a+x)$ [8]

2.a) Find the asymptotes of the curve:

$x^3+2x^2y- xy^2-2y^3+4y^2+2xy-5y+6=0$ [8]

b) Find the perimeter of the asteroid: $x^{2/3}+y^{2/3} = a^{2/3}$ [7]

3. Integrate any THREE of the following: 5*3=15

a) $\int \frac{x^3}{(x-2)(x-3)} dx$

b) $\int \frac{1}{3\, sinx+4\sin x} dx$

c) Prove that: $\int_0^{\pi/4} \log(1+tanx)\, dx = \frac{\pi}{8}\, log2$

d) $\int_0^{\frac{\pi}{2}} sin^3xcos^4xdx$

4.a) Find the volume of the solid in region bounded by the curve y= x^2 +1 and the line y= -x+3 revolved about the X-axis. [8]

b) State and prove Euler's theorem on homogeneous function of Three independent variables of degree n.

If sin u =, $\frac{x^2y^2}{x+y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$ [7]

5.a) Find the extreme values of the function f(x,y,z)= $x^2+y^2+z^2$ subject to the constraints ax+by+cz=k. [7]

b) Show that the substitution y=y_1+u where y_1 is a solution of Riccati's Equation reduces the Riccati's equation to a Bernoulli's equation. [8]

6.a) Find the general solution of the differential equation
y"-y'-2y= $3e^{2x}$, y(0)=0 , y'(0)= -2 [7]

b) Find general solution of differential equation by using method of Parameters: y"+2y$'$ +y= e^{-x}cosx [8]

OR

a) Solve Second order differential equation of the series RLC circuit

$$L\frac{d^2Vc}{dt^2} + R\frac{dVc}{dt} + \frac{1}{C}Vc = \frac{Vin}{C}$$, [7]

where R=10 Ω ,L=1 H, C=$16\times10^{-4}F$ V_{in}=0,V_C(0)=6V, V'_c (0)=6A

b) Solve the following initial value problem:

x^2 y"- 2xy'+2y= 0 , y(1) = $\frac{3}{2}$, y'(1) = 1 [8]

Attempt all the questions: 4*2.5=10

a) Find y_n if x^n ,where n is positive integ

b)Find the radius of curvature:y^2= 4ax

c) Show that the function f(x,y) = x^3+y^3-3xy has a saddle point at (0,0).

d) Solve: $\frac{dy}{dx} + \frac{1-cos2y}{1-cos2x} = 0$