# Pokhara University
## School of Engineering
## Internal Examination 2024

Level : Bachelor
Time: 3 hrs
Program: BCE
Course: Calculus I

Full Marks : 100
Pass Marks : 45

Candidates are requested to answer the in their own words as far as practicable. The marks in the margin indicate full marks

Attempt all the questions.

1. a. Define differentiability of a function at a point. Show that the function differentiable at a point is necessarily continuous at that point. [5]

   b. If $y = (x^2-1)^n$, show that
   $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ [5]

   c. State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[0, 4]$. [5]

2. a. Find the radius of curvature at the origin of the curve $y = x^4 - 4x^3 - 18x^2$ [5]

   b. Trace the curve $x^2y^2 = x^2 - a^2$ [3]

   c. Prove that $\int_0^a \frac{1}{x+\sqrt{a^2-x^2}}\,dx = \frac{\pi}{4}$ [5]

3. a. Find the reduction formula for $\int \cot^n x\,dx$ and hence evaluate $\int \cot^5 x\,dx$ [8]

   b. Find the area bounded by the curve $y^2 - 4x = 4$ and the line $4x - y = 16$. [7]

4. a. Find the area of the surface obtained by rotating the curve $y = \frac{x^2}{2} + \frac{1}{2}$ between $0 \le x \le 1$ about y-axis

   OR

   Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2 + 1$ and the line $y = -x + 3$ about x-axis. [8]

   b. Define homogeneous function of two independent variables $x$ and $y$. State and prove Euler's theorem on homogeneous function of two independent variables. [7]

5. a. Find the extreme value of $x^2 + y^2 + z^2$ subject to the constraint $ax + by + cz = a + b + c$. [8]

   OR

   Find the dimension of the rectangular box open at the top with volume 32cc requiring least material for the construction.

   b. Show that the substitution $y = y_1 + u$, where $y_1$ is the solution of Riccati equation, reduces the Riccati equation to a Bernoulli's differential equation [7]

6. a. Suppose that you turn off the heater in your home at night 2 hours before you go to bed. If temperature of thr room is 66°F when you turn off the heater. The temperature of the room fall to 63°F at the time you go to bed. What will the temperature of the room in the morning after 8 hours you go to bed ? Assume that the outside atmospheric temperature is 32°F. [8]

b. Find the general solution of the differential equation $y'' - 5y' + 6y = 3e^x$ by method of variation of parameter. [7]

OR

Solve the second order differential equation of the series LCR Circuit $L\frac{d^2V_C}{dt^2} + R\frac{dV_C}{dt} + \frac{1}{C}V_C = \frac{V_{in}}{C}$, where $V_c(0) = 6V$, $V_C'(0) = 6A$, $V_{in} = 0$, $R = 10\Omega$, $L = 1H$, $C = 16 \times 10^{-4}$ F.

7. Attempt all the questions. [4 x 2·5 = 10]

a. Determine whether the integral $\int_2^{\infty} \frac{\sin^2 x}{x^2}\,dx$ is convergent or divergent.

b. Find the arc length of curve $y = x^2$, $-1 \le x \le 2$.

6.12

c. Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = 0$.

d. Find the equation of tangent plane to the surface $z = 2x^2 + y^2$ at (1, 1, 3).

*The End*

Gandaki College of Engineering and science
Lamachaur ,Pokhara
Final Assessment 2080

FM : 100
PM : 45
Time : 3 hrs

Program: CE/SE
Subject : Calculus I
Semester : 1st

Attempt all the questions .

1(a) show that the function is continuous at x=1 and x=2 and it is derivable at x=2 but not at x=1. $f(x) = \begin{cases} x \; for \; x < 1 \\ 2 - x \; for \; 1 \le x < 2 \\ -2 + 3x - x^2 \; for \; x \ge 2 \end{cases}$ (8)

OR State Leibnitz theorem and if $y=(x^2-1)^n$ ,show that

(i) $(x^2-1)y_2$-2(n−1)$xy_{1-2ny=0}$

(ii) $(x^2-1)y_{n+2}$+2x$y_{n+1}$-n(n+1)$y_n$=0

(b) State and prove Rolle's Theorem .Verify the theorem for the function f(x)=$log\left(\frac{x^2+ab}{(a+b)x}\right)$ $for \; x \in [a,b]$ ,a> 0OR

State Maclaurin's series of infinite form .Prove the following using Maclaurin's series

$e^x log(1+x) = x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{9x^5}{5!} + \cdots \dots \dots$ (7)

2(a)$Define \; asymtotes \; of \; a \; curve \; with \; different \; types. Find \; asymptotes \; of \; x^3 + y^3 = 3axy$ (8)

(b) Find radius of curvature at (0,0) of the curve

$4x^2 - 3xy + y^2 - 3y = 0$

3.Integrate (any 3)

(a) $\int_0^{\pi/2} \frac{sin\theta \; d\theta}{sin\theta + cos\theta}$

(b) $\int_0^{\infty} \frac{xdx}{1+x^2}$

(c)obtain the reduction formula for $\int tan^n x$ dx and hence find the value of $\int tan^4 x$ dx

(d) $\int_0^{\pi/6} cos^4 3\theta sin^2 6\theta \; d\theta$

4(a) Find the volume of the solid generated by revolving the region between curves and lines x=$y^2$,x=0,y= - 1 revolve about y-axis

(b)State and prove Euler theorem on homogeneous $function \; of \; 2 \; independent \; variables \; of \; degree \; n. If \; u = cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right). \; Show \; that \; x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{cotu}{2} = 0$

5(a)$Maximize \; the \; function \; 3x + 5y + z - x^2 - y^2 - z^2 \; subject \; to \; x + y + z = 6$

10 marks

(b) $Solve : \frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x}$

OR Show that the substitution y=$y_1$+u where $y_1$ is a solution of Riccati's equation reduces the Riccati's equation to a Bernoulli's equation.

(6)(a) Solve initial value problem $y'' + 5y' + 6y = 0, y(0) = 2, y'(0) = 3$

(b) Find the general solution using the method of variation of parameter. $y'' + y = sinx$ (7)

7.Attempt all the questions.(4x2.5=10)

(a) Find the length of the curve y=logsecx from x=0 to x=$\pi/3$

(b) Find partial derivatives of u=$\sqrt{x^2+y^2}$

6. a. The growth rate of a culture of bacteria is proportional to the number of bacteria present. After one day it is 1.5 times of original number. Find after how many days it will be (a) double (b) triple. [8]

b. Find the general solution of the differential equation $y'' - 4y' + 4y = 3e^{2x}$ by method of variation of parameter. [7]

OR

Solve the second order differential equation of the parallel LCR Circuit $\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$, $V(0) = 6$, $V'(0) = -12$, $R = 20\Omega$, $L = 50H$, $C = 6 \times 10^{-3}$ F.

7. Attempt all the questions. [4 x 2.5 = 10]

a. Determine whether the integral $\int_1^{\infty} \frac{dx}{x - e^{-x}}$ is convergent or divergent.

b. Find the value of the integral $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta \, d\theta$ using gamma function.

c. Solve the differential equation: $ydx = (e^x + 1)\,dy$.

d. Find the radius of the curvature for the curve $x = at^2, y = 2at$.

$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$

$\int_0^{\infty} e^{-x} x^{n-1}$

*The End*

(c) Find limiting value of f(x) at x=2 where f(x)=$\begin{cases} 4-x^2 \, for \, x<2 \\ x-2 \, for \, x \geq 2 \end{cases}$

(d) Evaluate $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

**The End**