POKHARA UNIVERSITY
FACULTY OF SCINCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	Model Question		2079
Level	B. E.	F.M	100
Programme	Bachelor	PM	45
Year/Part	1st year/1st semester	Time	3 Hrs

Subject: Calculus I

Candidates are required to give answers in their own words as far as practicable.
The figure in the margin indicates full marks.
Attempt all the questions

1. a. Show that: f(x)= $\begin{cases} x^2+1 & for \quad x<1 \\ 3x+1 & for \quad x\geq 1 \end{cases}$ is continuous at x = 1 but not differentiable at x = 1. 5

   b. If $y=\left(\sin^{-1}x\right)^2$, prove that (i) $(1-x^2)y_2-xy_1-2=0$

   $(ii)(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$ 5

   c. State the Roll's theorem .Verify the Roll's Theorem y= f(x) =x^2 -4x+3 on [1,3] 4

2. a. Show that: log(1+x)= $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+$...................... by expanding Maclaurin's theorem . 5

   b. Find the asymptotes of the curves, $x^2(x-y)^2 - a^2(x^2+y^2)=0$ 5

   c. Find the radius of curvature of the curves $y^2 = 4x$ at the vertex (0,0). 4

3. a. Integrate, $\int\frac{dx}{2+\cos x+\sin x}$ 5

   b. Show that: $\int_0^1 \cot^{-1}(1-x+x^2)dx=\frac{\pi}{2}-\log 2$ 5

   c. Show that: $\int_0^1 x^6\sqrt{1-x^2}\,dx=\frac{5\pi}{256}$ 4

4. a. Find the area of the region and the circle $x^2+y^2=4$ cut off by the line x-2y =-2 in the first two quadrants. 5

   b. Find the volume of the solid generated by revolving the region in the first quadrant bounded on the left by the circle $x^2+y^2 =3$ and on the right by the line x = $\sqrt{3}$ and above the line y = $\sqrt{3}$ about y-axis. 5

   c. Let U = f(x , y, z)be a homogeneous function of three independent variables x, y, and z of degree n. then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=nu$ 4

5. a. Solve: $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$ 5

b. Solve the given initial value problem: $x^2y''-2xy'+2y=0, \quad y(1)=\frac{3}{2}, \quad y'(1)=1.$ 5

c. Solve by using the method of variation of parameters, $y''+4y=3\operatorname{cosec}2x$. 4

6. a. Use Lagrange's multiplier to find the minimum value of $x^2+y^2+z^2$ subjects to constraint

$ax+by+cz=p.$ 7

b. A tank initially contains 4lb of salt dissolved in 100 litre of water. Suppose that salt solution 2lb of salt per litre is allowed to enter the tank at the rate of 5 lire /min and the uniform solution is drained from the tank at the same rate .find the amount of salt in the tank after 10 minutes. 7

7 . Attempt all question. $8\times2=16$

a. Sketch the graph of parametric equations

$x=t^2-2t \qquad y=t+1 \qquad \text{for} \qquad 0\le t\le 4$

b. If $u=\log(x^3+y^3+z^3-3xyz)$ then prove that: $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$.

c. Find the arc length of the curves $y=x^2;\ -1\le t\le 2$.

d. Solve: $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

e. Find the particular integral of $y''+4y=2\sin 2x$.

f. Find the area enclosed by x-axis and the curve $y=3x-5x^2$.

g. Evaluate: $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

h. What are the condition of saddle point and undecided information?

Best of Luck The End

UNIVERSITY
FACULTY OF SCINCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all questions.

1. a) The function is defined by $f(x) = \begin{cases} x^2 - 2, & \text{for } x \le 2 \\ 2x^2 - 4, & \text{for } x > 2 \end{cases}$ 5

   Show that f (x) is continuous at x = 2 but not differentiable at x = 2.

   b) If $y = a\cos(\log x) + b\sin(\log x)$, then prove that: $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1) y_n = 0.$ 5

   c) Verify that mean value theorem for the function f(x) defined by f(x) $= Ax^2 + Bx + C,\ A \neq 0$, on (a, b). 4

2. a) Assuming the validity of expansion, find Maclaurin's series expansion of $f(x) = \log(\sec x)$. Find expansion of tan x. 5

   5

   b) Find all the asymptotes of the curve $(y-a)^2 (x^2 - a^2) = x^4 + a^4$. 4

   c) Trace the curve: $y^2 = (x - 2a)^3$.

3. a) Evaluate: $\int \frac{1}{5\sin x + 4} dx$. 5

   5

   b) Show that: $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx = \frac{a}{2}$. 4

   c) Obtain the reduction formula for $\int \sin^n x \, dx$.

4. a) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2 + 1$ and the line $y = x + 3$ about x-axis. 5

   b) Show that the area of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$. 5

   c) State and prove Euler's theorem for homogeneous function of two variables with degree n. 4

5. a) Solve the Riccati equation $\frac{dy}{dx} = y^2 - \frac{y}{x} - \frac{1}{x^2}, x > 0, y(1) = 2.$ 5

   b) Solve: $y'' - 4y' + 5y = 0$, given that $y(0) = 1, y'(0) = 2.$ 5

   c) Find the general solution of the differential equations by the method of variation of parameters $y'' - 2y' + y = \frac{e^x}{x^3}$ 4

6. a) Use Lagrange's multiplier to find the minimum value of $x^2 + y^2 + z^2$ subject to the constraint $ax + by + cz = p$. 7

b) A tank initially contains 40 kg of salt dissolved into 200 litres of water. A solution of 2 kg of salt per litre is allowed to enter the tank at the rate of 5 litres per minute and uniform solution is drained from the tank at the same rate. Find the amount of salt at any time. Also determine the salt in tank in 15 minutes. 7

7. Attempt all the questions:

a) Find the radius of curvature at any point (x, y) of the curve $y^2 = 4ax$. 2

b) Find first order partial derivatives of the function $u = xe^y + y\sin x$. 2

c) Find the perimeter of a circle $x^2 + y^2 = a^2$.

d) Solve: $\sqrt{1 - x^2}\, dy + \sqrt{1 - y^2}\, dx = 0$ 2

e) Solve the differential equation $x^2y'' + xy' + y = 0$. 2

f) Find the area of the curve $y^2 = 4x$ and line $y = x$.

g) Apply the comparison of the integrals, prove that integral $\int_a^\infty \frac{\sin^2 x}{x^2}\, dx \ (a > 0)$ is convergent. 2

h) Find the equation of tangent plane to the surface of $z = 2x^2 + y^2$ at (1, 1, 3). 2

*********The End *********