1) Define harmonic function, verify given function is harmonic and find its harmonic conjugate

ⓐ $u = y^3 - 3x^2y$ ⓑ $u =$ ~~sin~~ $\cos x \cosh y$

2) ⓐ Integrate: $\int_c \frac{dz}{z^2+1}$ where c is (i) $|z+i| = 1$ (ii) $|z-i| = 1$ ccw

ⓑ integrate $\int_c \frac{1}{z^2+4}$ over c where $c: 4x^2+(y-2)^2=4$

3) state cauchy residue theorem & evaluate

ⓐ $\int \frac{dz}{(z^2+4)^3}$ where c is the circle $|z-i| = 2$

ⓑ $\int_c \frac{z+1}{z^4-2z^3} dz$ where $c: |z| = \frac{1}{2}$ counterclockwise

4 Find the Taylor and Laurent series of

ⓐ $f(z) = \frac{2z-3i}{z^2-3iz-2}$ in the region (i) $0<|z|<1$ (ii) $|z|>2$

ⓑ $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region $2<|z|<3$.

5 ⓐ Find the z-transform of $e^{in\pi/2}$ and then deduce $z\left(\cos \frac{n\pi}{2}\right)$ and $z\left(\sin \frac{n\pi}{2}\right)$

ⓑ state and prove 1st and 2nd shifting theorem

ⓒ state and prove initial and final value theorem

6 ⓐ Find the inverse z-transform of

(i) $F(z) = \frac{z^2-3z}{(z-5)(z+2)}$ (ii) $F(z) = \frac{3z^2+2z+1}{z^2+3z+2}$

ⓑ use z-transform to solve

(i) $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$

(ii) $y_{n+2} - 3y_{n+1} + 2y_n = 4^n$ given $y_0 = 0$, $y_1 = 1$

7) find the bilinear ~~map~~ transformation which maps the points $z_1 = 2$, $z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$ and $w_3 = -1$.

8) Derive ⓐ one dimensional wave eqn.

ⓑ one dimensional heat eqn.

ⓒ two dimensional heat eqn.

9) Find the solution of one dimensional heat eqn with initial temperature $f(x)$ and boundary conditions $u(0,t)=0$ and $u(L,t)=0$

10 ~~Find the~~ verify ~~that~~ $u = e^{-4t}\cos 3x$ to satisfy one dimensional heat equation.

(11) show that

(a) $\int_0^\infty \left(\frac{1-\cos\pi\omega}{\omega}\right) \sin x\omega \, d\omega = \begin{cases} \pi/2 & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$

(b) $\int_0^\infty \left[\frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2}\right] d\omega = \begin{cases} 0 & \text{if } x<0 \\ \pi/2 & \text{if } x=0 \\ \pi e^{-x} & \text{if } x>0 \end{cases}$

12) (a) Find the Fourier sine transform of e^{-x} for $x>0$ and show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ for $m>0$

(b) Find the Fourier cosine transform of $f(x) = e^{-mx}$ and then show that

$$\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$$

13) Express Laplacian in polar coordinates from Cartesian Coordinates (i.e. show that $\nabla^2 u = u_{rr} + \frac{1}{r} u_r + u_{\theta\theta} \frac{1}{r^2}$)

14) Determine the Location and zeros of

(a) $\tan \pi z$ (b) $(z^2+1)(e^z-1)$ (c) $\sin\left(\frac{1}{1-z}\right)$