Some Important Question of vector Calculus (near about* 35 *marks)

1 If $\vec{V} = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2+y^2+z^2}}$ Show that: $\nabla.\vec{V} = \dfrac{2}{\sqrt{x^2+y^2+z^2}}$ and $\nabla \times \vec{V} = 0$.

2. If $\phi = \log(x^2+y^2+z^2)$ then find div(grad ϕ) and curl(grad ϕ) .

3. Prove that: $\vec{F} = r^2\vec{r}$, Show that $\vec{F}$ is a conservative vector field and scalar potential is

$$\phi = \frac{r^4}{4} + \text{Constant.}$$

4. Define directional derivative of the function in the direction a. Find the directional Derivative of F $=xy^2+yz^3$ at (2, -1, 1) along the direction of the normal to the surface

S : $x \log z - y^2 + 4 = 0$ at (-1,2 ,1).

5. Find the directional derivate of F at p in the direction $\vec{a}$ where at P (3,0,4) ; $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

$$F = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

6. Calculate $\oint_c \vec{F}.\vec{dr}$ where $\vec{F}$ = (cosh x, sinh y, e^z) and C be a path given by

$\vec{r}$ = $(t, \quad t^2, \quad t^3)$ From (0,0, 0) to (2,4, 8).

7. Prove that if $\vec{F} = (2xz^3 + 6y \;\; , 6x - 2yz \;\; , 3x^2z^2 - y^2)$ $\vec{F}$ is a conservative vector field

Also find the scalar potential.

8. A particle moves along the curve $x= t^3+1$, $y= t^2$, $z= 2t+5$.Find the component of velocity and acceleration at t= 1 in the direction $\vec{i} + \vec{j} + \vec{k}$.

9. Calculate $\oint_c \vec{F}.\vec{dr}$ where $\vec{F}$ = $(e^x, \; e^{-y} , \; e^z)$ and C be a path given by

$\vec{r}$ = $(t, \quad t^2, \quad t)$ From (0,0, 0) to (1,1, 1).

10. Find the work done in moving a particle in the force field $\vec{F}$ = $(3x^2, \; 2xz - y , \; z)$ along the curve defined by $x^2=4y$, $3x^3=8z$ from x=0 to x=2.

11. The necessary and sufficient condition for the vector value function $\vec{a}$ of the scalar variable t to have a constant direction is $\vec{a} \times \dfrac{d\vec{a}}{dt} = 0$.

12. The necessary and sufficient condition for the vector value function $\vec{a}$ of the scalar variable t to have a constant magnitude is $\vec{a} . \dfrac{d\vec{a}}{dt} = 0$.

13. Evaluate: a. $\int_{(0,0,0)}^{(4,1,2)} [3ydx + 3xdy + 2zdz]$

b. $\int_{(0,1)}^{(2,3)} [(2x + y^3)dx + (3xy^2 + 4)dy]$

c. $\int_{\left(0,1,\frac{1}{2}\right)}^{\left(\frac{\pi}{2},3,2\right)} [y^2 \cos x dx + (2y \sin x + e^{2z})dy + 2ye^{2z} dz]$

B Green's Theorem

1. Evaluate by using Green's Theorem of $\oint_c [(y - \sin x)dx + \cos x dy]$ where C is the triangle with vertices (0,0), $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$.

2. Evaluate by using Green's Theorem of $\oint_c (\sqrt{y}dx + \sqrt{x}dy)$ where C is the triangle with vertices (1,1), $(2,2)$ and $(3,1)$.

3. Evaluate by using Green's Theorem of $\oint_c (5xydx + x^3 dy)$ where C is the closed curve consisting of the graph of y = x^2 and y = 2x between the points (0,0) and (2,4).

4. Evaluate by using Green's Theorem of $\oint_c (x^2 + y^2)\vec{i} - 2xy\,\vec{j}).d\vec{r}$ along the rectangle bounded by y=0, y=b, x=0, x=a.

C Surface Integral

1. Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$, $\vec{r} = (u\cos v,\quad u\sin v,\ 3v); 0 \le u \le 1, 0 \le v \le 2\pi$.

2. Define the surface integral of $\vec{F}$, on the surface S. Evaluate $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = x^2\,\vec{i} + e^x\,\vec{j} + \vec{k}$, Where S is the surface, x+ y+ z=1, $x \ge 0, y \ge 0, z \ge 0$.

3. Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = 4x\,\vec{i} + x^2y\,\vec{j} - x^2z\,\vec{k}$, ; S is the surface of the tetrahedron with vertices (0,0,0) , (1,0,0), (0,1,0) ,(0,0,1).

4. Find $\iint_s (\vec{F}.\vec{n})dA$, for $\vec{F} = (x^2, e^y, 1)$; S is the portion of the plane x+ y+ z=1 lying in the first Octant.

5. Find the flux integral of $\vec{F} = (x, y, z)$ through the surface S, Where S is the portion of the plane 2x+3y+z =6 in first octant.

6. Find the flux integral of $\vec{F} = (yz, zx, xy)$ through the surface S, Where S is the portion of the sphere, $x^2+y^2+z^2=1$ in first octant.

7. Find the flux integral of $\vec{F} = (3x, 3y, z)$ through the surface S, Where S is the part of the graph $z=9-x^2-y^2$.

D Stoke's Theorem

1. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = y\,\vec{i} + xz^3\,\vec{j} - zy^3\,\vec{k}$, and

   C: $x^2+y^2 = 4$, $z = -3$.

2. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = -3y\,\vec{i} + 3x\,\vec{j} + z\,\vec{k}$, and

   C: $x^2+y^2 = 4$, $z = 1$.

3. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = y^3\,\vec{i} + x^3\,\vec{k}$, and C is the boundary of the triangle with vertices (1,0,0), (0,1,0), (0,0,1).

4. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (2x-y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$, and S is the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on xy –plane.

5. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (z^2, 5x, 0)$ and S is the square

   $0 \le x \le 1,\ 0 \le y \le 1,\quad z = 1$

6. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (y^2, z^2, x^2)$ and S is the first portion of the plane x+ y+ z =1 .

7. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (y^2, 2xy + \sin x, 0)$ where c is the boundary of the of $0 \le x \le \frac{\pi}{2}$, $0 \le y \le 2$.

Gauss Divergence Theorem

1. Using the divergence theorems to find $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = e^x\,\vec{i} + \vec{j} + e^z\,\vec{k}$ and

   S: $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1.$

2. Using the divergence theorems to find $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = y^2e^z\,\vec{i} - xy\,\vec{j} + x\tan^{-1}y\,\vec{k}$ and S is the surface of the region bounded by the coordinate planes and the plane x+y+z=1.

3. State Gauss divergence Theorem. Use it to evaluate $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = \left(4x,-2y^2, z^2\right)$, S is the surface bounding the region $x^2 + y^2 = 4$, $z = 3$, $z = 0$ ·

4. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = y^3\,\vec{i} + x^3\vec{j} + z^3\,\vec{k}$ and

   S: $x^2 + 4y^2 = 1, \quad x \geq 0, y \geq 0, 0 \leq z \leq h$,

5. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = 4x\,\vec{i} + 2y^2\,\vec{j} + z^2\,\vec{k}$ and

   S is the surface of the cube: $|x| \leq 1, |y| \leq 1, |z| \leq 1$.

6. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})ds,$ where $\vec{F} = x^2\,\vec{i} + e^y\,\vec{j} + 1\vec{k}$ and

   S: $x + y + z = 1$, $x \geq 0$, $y \geq 0$ $z \geq 0$.