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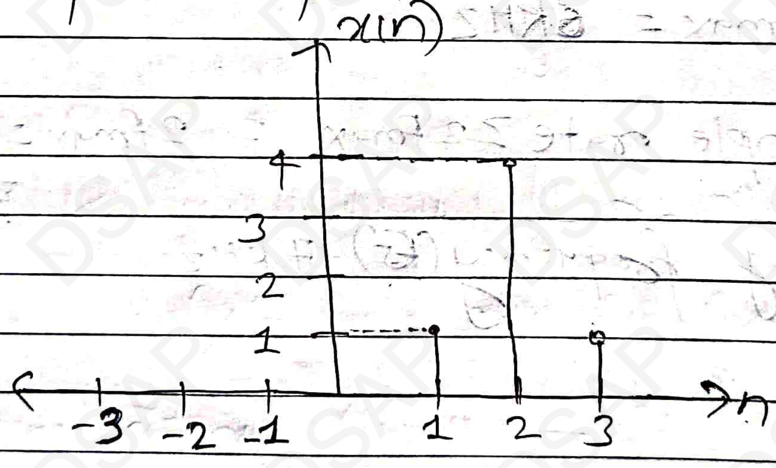
Chapter-2 (-9hrs)

Discrete-time signals and System

2.1 Elementary discrete-time signals
2.2 Discrete time fourier series and properties
2.3 Discrete time fourier transform and properties
2.4 Discrete time system properties
2.5 Properties of Linear Time-Invarient systems (LTI)
2.6 LTI convolution sum characterized by constant coefficient difference equations
2.7 stability of LTI systems, Implementation of LTI system.
2.8 frequency response of LTI systems

2.1 Elementary discrete-time signals

i) Graphical representation:



ii) functional representation

$$x(n) = \begin{cases} 1 & \text{for } n = 1 \text{ and } 3 \\ 4 & \text{for } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

iii) Sequence representation

$$x(n) = \{ \ldots 0,0,0,1,4,1,0,0,0, \ldots \}$$

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iv) Tabular representation

n	-3	-2	-1	0	1	2	3
$x(n)$	0	0	0	0	1	4	1

A discrete time signal $x(n)$ is a function of independent variable 'n' which is an integer. It is defined only at certain time instant ie $-\infty < n < \infty$.

Some elementary discrete time signals are:

1) unit sample signal / unit impulse signal / ~~delta~~ dirac delta signal
2) unit step signal
3) unit ramp signal
4) unit signum function
5) exponential signal
6) sinusoidal signal

1) Unit sample signal :- It is also known as unit impulse or dirac-delta signal. It is denoted by $\delta(n)$ and defined as

$$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{otherwise} \end{cases}$$

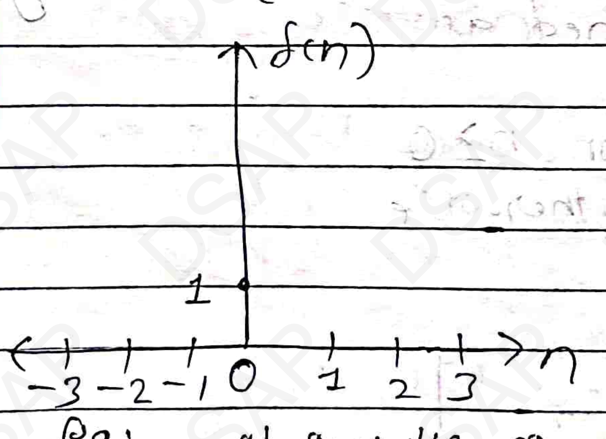
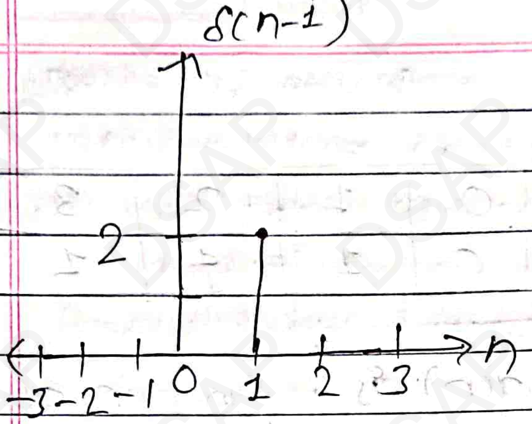
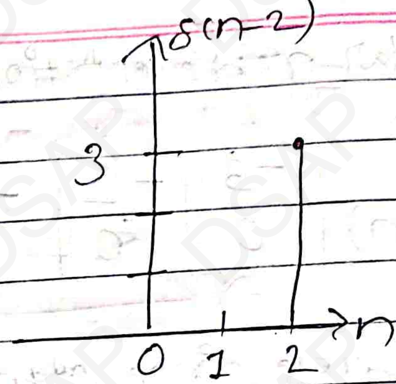


fig:- unit impulse signal





2) unit step signal :- It is denoted by u(n) and defined as

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

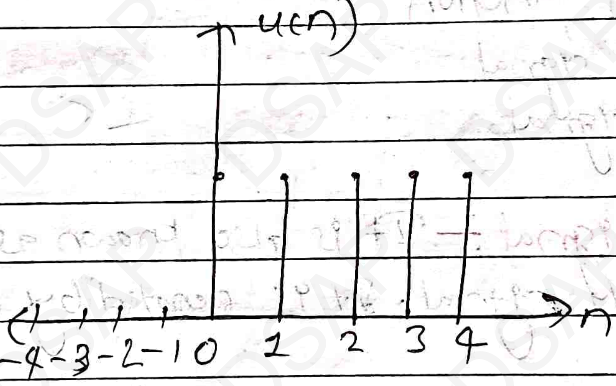


fig :- unit step function

3) unit ramp function :- It is denoted by $u_r(n)$ and defined as

$$u_r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

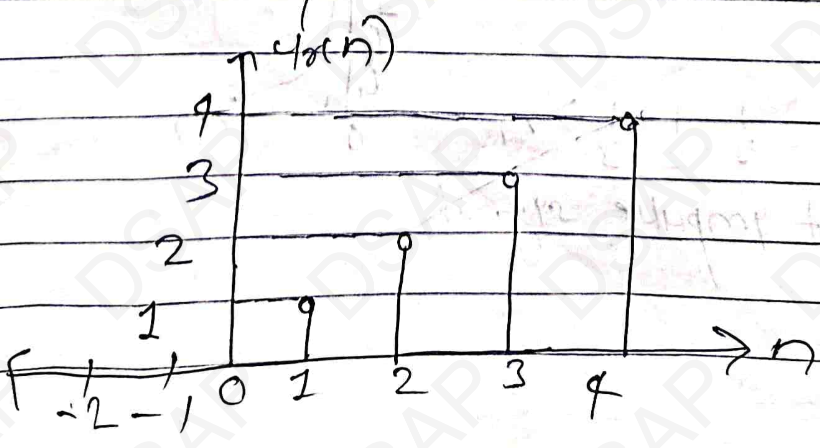


fig :- unit ramp function

4) **Unit signum function:-** It is denoted by sign(n) and defined as

$$\text{sign}(n) = \begin{cases} 1 & \text{for } n>0 \\ 0 & \text{for } n=0 \\ -1 & \text{for } n<0 \end{cases}$$

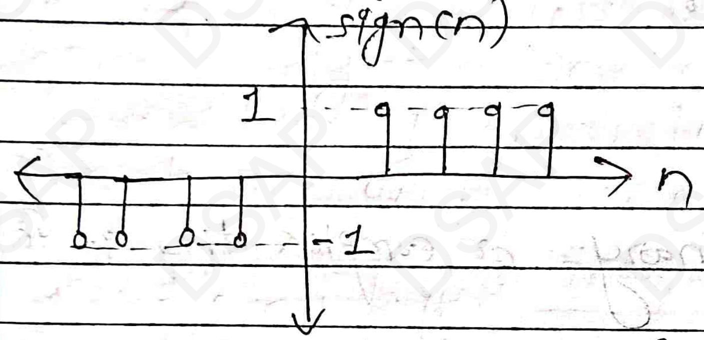


fig:- unit signum function

5) **Exponential signal:-** A discrete time exponential signal is expressed as $x(n) = a^n$ for all n

where $a = re^{j\theta}$

ie $x(n) = (re^{j\theta})^n$

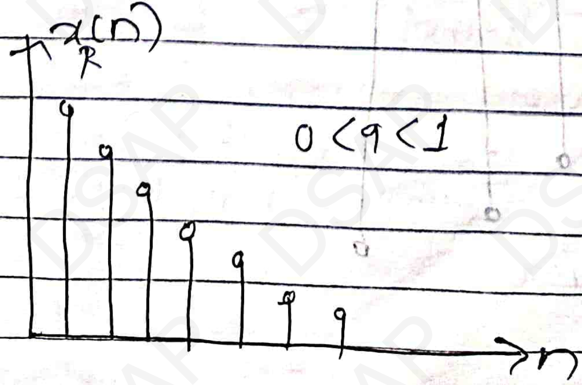
$= r^n e^{jn\theta}$

$x(n) = r^n(\cos\theta n + j\sin\theta n)$

$x_R(n) = r^n \cos\theta n$

$x_I(n) = r^n \sin\theta n$

If a is real then $x(n)$ is a real signal as shown below:-



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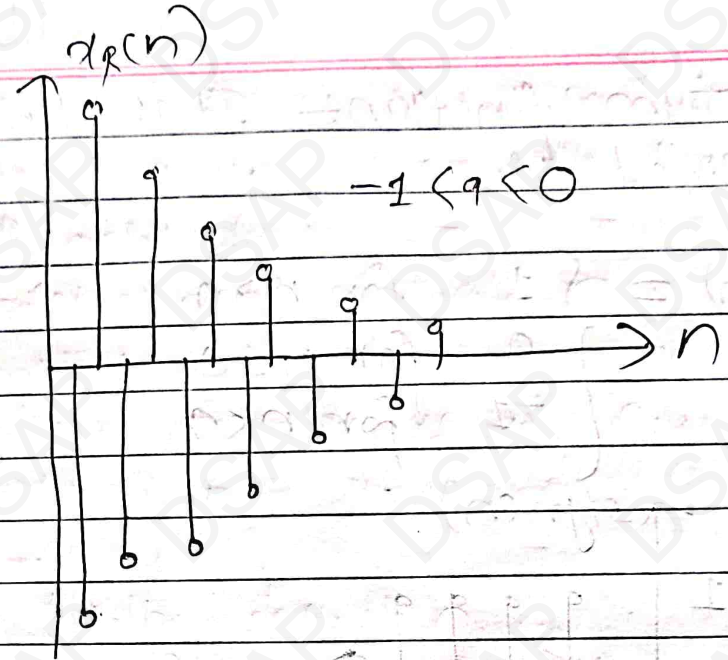
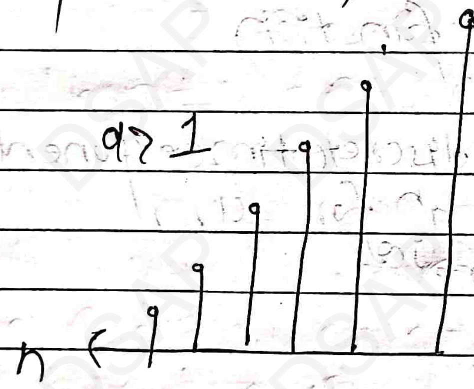
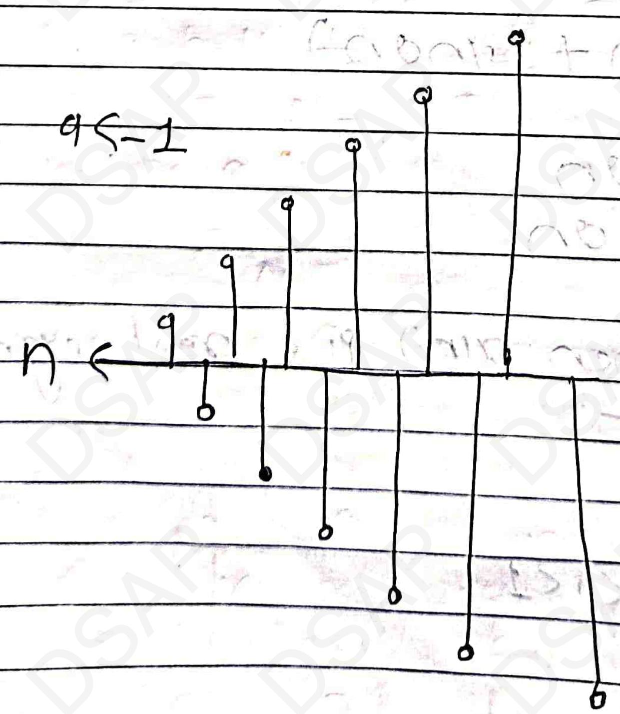


If a^n is imaginary or complex then it is expressed as,





$x_I(n) = r^n \sin n\theta$

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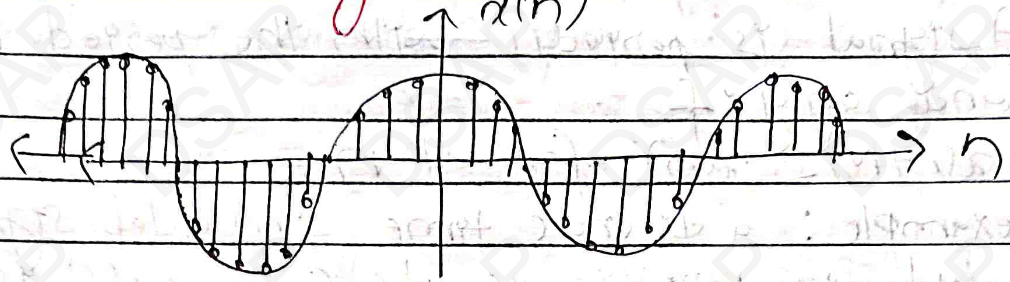
6) sinusoidal signal :-



$$x(n) = A\cos(\omega n + \phi)$$

*) Relationship between unit step, unit impulse and ramp function :-

$$\delta(t) \xrightarrow{\text{Integration}} u(t) \xrightarrow{\text{Integration}} r(t)$$

$$r(t) \xrightarrow{\text{differentiation}} u(t) \xrightarrow{\text{differentiation}} \delta(t)$$

classification of discrete time signals :-

2018 Fall 1) Energy and power signal :-

The energy of signal x(n) is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

If E is finite ie $0 < E < \infty$ then x(n) is an energy signal.

The average power of x(n) is given by

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

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2) Periodic and Aperiodic signals :-

A signal is periodic with the period $N > 0$ if and only if

$x(n+N) = x(n)$ for all n

For example : a discrete time sinusoidal signal or sinusoid is periodic if its frequency 'f' is a rational number.

Proof : By the definition,

$x(n+N) = x(n)$ for all n

let us consider a sinusoid with frequency f_0,

$x(n) = A\cos(2\pi f_0 n + \theta)$

and $x(n+N) = A\cos(2\pi f_0(n+N) + \theta)$

for a sinusoid with frequency f_0 to be periodic =

$x(n+N) = x(n)$

ie $A\cos(2\pi f_0 n + 2\pi f_0 N + \theta) = A\cos(2\pi f_0 n + \theta)$

which is true if and only if there exists an integer 'K' such that,

$2\pi f_0 N = 2\pi K$

$\therefore f_0 = \frac{K}{N}$ where $K = 0, \pm1, \pm2, \ldots$

$\therefore$ frequency f_0 must be rational number

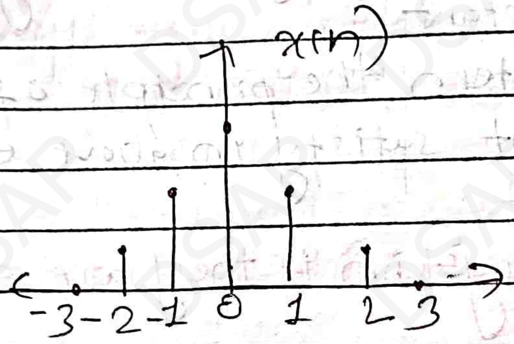
eg, $f_0 = \frac{2}{3}$

3) Symmetric (even) and Asymmetric (odd) :-

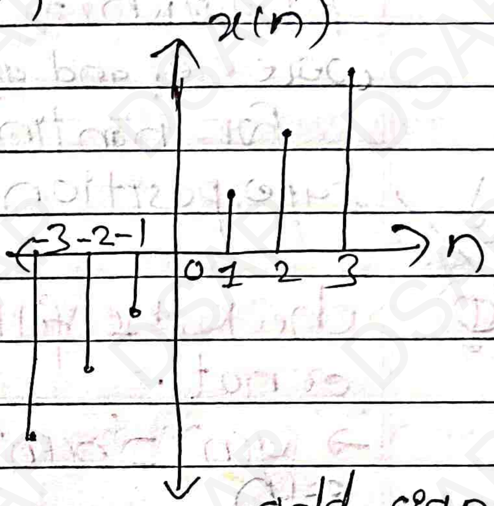
A real valued signal $x(n)$ is called symmetric or even digital if

$x(-n) = x(n)$ for all n

and a signal is asymmetric (odd) if it satisfies $x(-n) = -x(n)$ for all n



Even signal



odd signal

Properties of discrete time system /

classification of discrete time system :-

1) Linear systems & non-linear systems (Linearity)
2) Time-invarient & Time-varient systems (Time-variance)

1) Linear and non-linear systems (Linearity)
2) causal and non-causal systems (causality)
3) stable and unstable systems (stability)
4) Time-invarient & Time-varient system (Time-invarient)
5) static & dynamic systems (system with memory)

1) Linearity :- A system is said to be linear if superposition principle applies to that system ie it is defined as a system whose response to the sum of the weighted input is same as the sum of the weighted output
Example: filters, communication channels, etc.
let us consider two systems

$$y_1(n) = F[x_1(n)]$$
$$y_2(n) = F[x_2(n)]$$

where $x_1(n)$ and $x_2(n)$ are inputs and $y_1(n)$ and $y_2(n)$ are output or response.

$y(n)$ $x^2(n)$ → not

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Now for discrete time linear system
$F[a_1x_1(n) + a_2x_2(n)] \longrightarrow a_1y_1(n) + a_2y_2(n)$
were a_1 and a_2 are constant
For nonlinear system, the principle of superposition does not satisfy in above equations.

2012 fall

Q) Check the fillowing system if they are linear or not.

→ $y(n) = x(n^2)$

Soln

Here, $y(n) = x(n^2)$
The output of the system to two inputs $x_1(n)$ and $x_2(n)$ will be;
$y_1(n) = x_1(n^2)$
$y_2(n) = x_2(n^2)$

Now, the linear combination of the two outputs will be,
$y_3(n) = a_1y_1(n) + a_2y_2(n)$
$= a_1x_1(n^2) + a_2x_2(n^2)$ --- (1)

Also, the response to the linear combination of inputs will be
$y_4(n) = f[a_1x_1(n) + a_2x_2(n)]$
$= a_1f[x_1(n)] + a_2f[x_2(n)]$
$= a_1x_1(n^2) + a_2x_2(n^2)$ --- (2)
(∵ the linear system satisfy the additive property)
Here (1) and (2) are equal.
Hence, the given system is linear.

$y(n) = x(2n)$
put $n=1$
$y(1) = x(2)$. It depends on future value so non-causal.

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2) Causality :-
A system is said to be causal if the output at any time depends only on the values of the input at the present and past time but not in the future value.
A register is a causal system is voltage across resistance depends upon the present time current.
Eg:- Real time systems, resistors, etc.

If the output or response of the system to inputs depends on the future values of that input then the system is called non-causal system.
Eg:- Image processing signal
$y(n) = x(n) - x(n+1) \rightarrow$ non-causal
$y(n) = x(n) - x(n-1) \rightarrow$ causal

Causal system is also called non-anticipatory system.
Non-causal system is also called anticipatory system.

3) Stability :-
A signal having finite magnitude value is known as bounded signal like

If $|x(n)| \leq M_x < \infty$ then
$|y(n)| \leq M_y < \infty$
where M_x and M_y are finite numbers

Bounded I/P $\rightarrow$ $\boxed{\text{System}}$ $\rightarrow$ Bounded o/p

A system is called stable if different bounded input results in bounded output (BIBO), then the output of such system does not divert or does not grow unnecessarily large.

The systems not satisfying the above conditions are unstable.

(i) A discrete time system is described as,

$$y(n) = y^2(n-1) + x(n)$$

Now, a bounded input of $x(n) = 2\delta(n)$ is applied to this system. Assume that the system is initially relaxe. check whether the system is stable or unstable.

Soln.

We know, $\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$

$\therefore x(n) = \begin{cases} 2 & \text{for } n=0 \\ 0 & \text{for } n\neq 0 \end{cases}$

Again, we have

$$y(0) = y^2(0-1) + x(0)$$
$$= y^2(-1) + 2$$
$$= 2 \qquad [\because y^2(-1) = 0 \text{ - initially relaxed}]$$

$$y(1) = y^2(1-1) + x(1)$$
$$= y^2(0) + x(1)$$
$$= 2^2 \qquad [\because x(1) = 0]$$

$$y(2) = y^2(2-1) + x(2)$$
$$= y^2(1) + x(2)$$
$$= (2^2)^2$$
$$= 2^4$$

$$y(3) = y^2(3-1) + x(3)$$
$$= y^2(2) + x(3)$$
$$= (2^4)^2$$
$$= 2^{2^3}$$

$$\therefore y(n) = 2^{2^n}$$

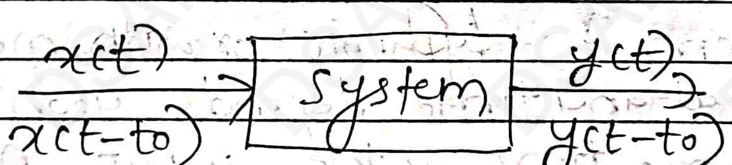
Here, as $n \to \infty$, $y(n) \to \infty$.

Hence, the input $x(n) = 2\delta(n)$ is bounded for all 'n'.

$\therefore$ system is unstable.

4) Time Invarient :-

A system is called time invarient if the input, output relationship does not vary with time ie the behaviour and input-output (i/o) characteristics do not change with time.



$$y[n, k] = y(n-k)$$

where

$$y(n, k) = F[x(n-k)]$$

ie o/p of the system for delayed input by k unit

and $y(n-k) \to$ delay the o/p by k ie simply place $(n-k)$ at the place of n.

Q) Check whether the following systems are time invariance or time-variance.

$$y(n) = n x(n)$$

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Soln

We have, $y(n) = n x(n) = f[x(n)]$ --- (1)

Now, if the input to the system ie $x(n)$ is delayed by K, then the o/p of the system is given by

$$y[n,K] = f[x(n-K)]$$
$$= n\,x(n-K) \quad \text{---} (2)$$

Again, delay the o/p, $y(n)$ by K, we get,

$$y(n-K) = (n-K)\,x(n-K) \quad \text{---} (3)$$

from (2) and (3)

$$y[n,K] \neq y(n-K)$$

Hence, the given system is not time invariant.

5) System with memory: not → present (present + past)

→ A system is dynamic or with memory if its output at any time depends upon the input at the same time or the input has previous/past time.

eg:- $y(n) = x(n-1) + x^2(n)$

→ A system is said to be static or memory less, if its output at the given time depends only on the input at the same time.

→ A register is an example of memory with system.

The function of memory less is as:

$$y(n) = 2x(n) - x^2(n)$$

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(1) A system has i/p and o/p relationship given by the expression, $y = ax + b$ where a and b are constant. Does this system satisfy linearity property?

Soln

Here, $y = ax + b$

let x_1 and x_2 be two inputs. Then

$$y_1 = ax_1 + b$$
$$y_2 = ax_2 + b$$

Now, the linear combination of two o/ps will be

$$y_3 = a_1 y_1 + a_2 y_2$$
$$= a_1 (ax_1 + b) + a_2 (ax_2 + b)$$
$$= a(a_1 x_1 + a_2 x_2) + b(a_1 + a_2)$$

Now, the response of linear combination of two i/ps will be

$$y_4 = f[a_1 x_1 + a_2 x_2]$$
$$= a(a_1 x_1 + a_2 x_2) + b$$

Here, $y(3) \neq y(4)$

So, the system is not linear.

Numericals on signal types:-

1) Determine whether the following signals are periodic or not.

(i) $x(t) = \sin 15\pi t$

(ii) $x(t) = \sin \sqrt{2}\pi t$

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i) Soln

Here, $x(t) = \sin 15\pi t = \sin \omega t$

Now, $x(t+T) = \sin 15\pi(t+T)$

$= \sin(15\pi t + 15\pi T)$ --- ①

Again, $T = \frac{2\pi}{\omega}$

$2\pi = T\omega$

$2\pi = 15\pi T$ --- ②

from eqn ①

$x(t+T) = \sin(2\pi + 15\pi t)$

$= \sin 15\pi t$

$= x(t)$

$\because x(t) = x(t+T)$

Hence the given function is periodic.

ii) Soln

Here, $x(t) = \sin\sqrt{2}\pi t = \sin\omega t$

Now, $x(t+T) = \sin\sqrt{2}\pi(t+T)$

$= \sin(\sqrt{2}\pi T + \sqrt{2}\pi t)$ --- ①

Again, $T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{\sqrt{2}\pi}$

$2\pi = \sqrt{2}\pi T$ --- ②

from eqn ①

$x(t+T) = \sin(2\pi + \sqrt{2}\pi t)$
$= \sin\sqrt{2}\pi t$
$= x(t)$
$\because x(t+T) = x(t)$
Hence the given function is periodic.

d) Determine $x(n) = 2n^2$ is periodic or not.
soln
Here, $x(n) = 2n^2$
$x(n+N) = 2(n+N)^2$
$= 2(n^2 + 2nN + N^2)$
$= 2n^2 + 4nN + 2N^2$
Since, $x(n) \neq x(n+N)$
so, the given signal is periodic.

Q) Prove that the exponential signal is non-periodic.
soln
mathematically, exponential signal is expressed as,
$x(t) = e^{-at}$
substituting $t = (t+T_0)$, we get
$x(t+T_0) = e^{-a(t+T_0)}$
$= e^{-at} \cdot e^{-aT_0}$

put $T_0 = \infty$
$x(t+T_0) = e^{-at} \times e^{-\infty}$
$= e^{-at} \times 0$
$= 0$
$\because x(t) \neq x(t+T_0)$
Hence exponential signal is non periodic signal.

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Q) State whether the following signals are periodic or not. If periodic determine their period. $x(n) = A\sin(\pi n) + Bt$.

Soln

Here, $x(n) = A\sin(\pi n) + Bt$

$x(n+N) = A\sin(\pi n + \pi N) + Bt$

At $N = \frac{2\pi}{\omega}$

$2\pi = N\omega$

$2\pi = \pi N$

Now, $x(n+N) = A\sin(\cancel{2\pi} + \pi n) + Bt$

$= A\sin \pi n + Bt$

$= x(n)$

$\therefore x(n+N) = x(n)$

So, The signal is periodic.

For period, $2\pi = \pi N$

$N = 2$

$\therefore$ period $= 2$

Recursive and non recursive system :-

A system whose output $y(n)$ at time 'n' depends on any number of past output values $y(n-1)$, $y(n-2)$, $y(n-k)$ is known as recursive system.

ie $y(n) = F[y(n-1), y(n-2) \ldots]$

A system whose output $y(n)$ depends only on the present and past values (input), then the system are known as non-recursive system ie

$y(n) = F[x(n), x(n-1) \ldots]$

past

past + present

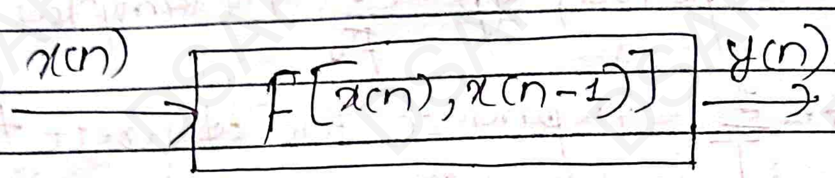

fig:- non-recursive system

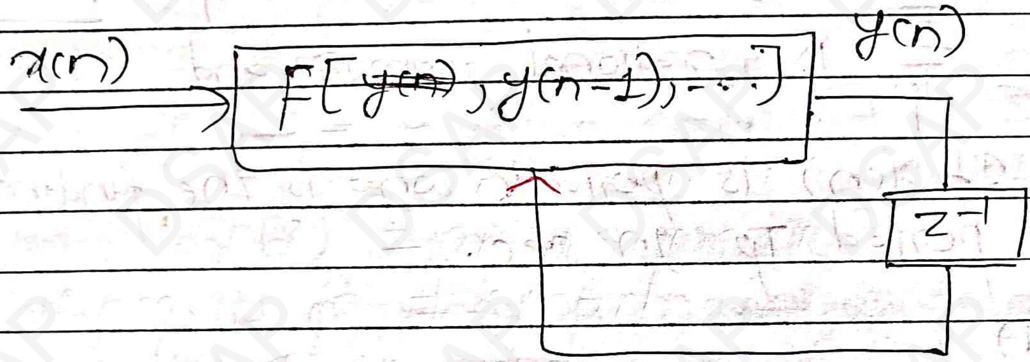

fig:- recursive system

Q) How would you define a linear time invariant system? or LTI system

Linear Time Invarient (LTI) System:-

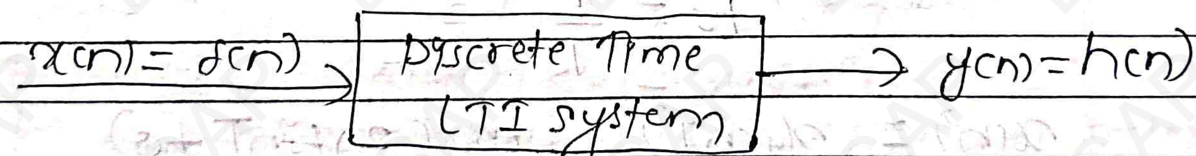

fig:- LTI system

If the system has both the linearity and time invariance property, then the system is said to be LTI system.

In the above block diagram, δ(n) is the unit impulse input in discrete time system and h(n) or y(n) is the unit impulse response of the discrete time LTI system. Hence, any LTI system can be completely characterized in terms of unit impulse response.

2013 fall

Numerical

Q1) A continuous time sinusoid $x_a(t)$ with fundamental period, $T_p = \frac{1}{F_0}$ is sampled at rate of $F_s = \frac{1}{T}$ to produce the discrete time sinusoid

$$x(n) = x_a(nT).$$

1) show that $x(n)$ is periodic if $\frac{T}{T_p} = \frac{k}{N}$ ie $\frac{T}{T_p}$ is a rational number and

2) If $x(n)$ is periodic what is the fundamental period T_p in sec.

Soln

Here, let the sinusoid be,

$$x_a(t) = \cos(2\pi f_0 t + \theta) \quad \text{---(1)}$$

$$x_a(n) = \cos(2\pi f_0 nT + \theta) \quad \text{---(2)}$$

After sampling,

$$F_s = \frac{1}{T}$$

$$x(n) = x_a(nT) = \cos(2\pi f_0 nT + \theta)$$

and

$$x(n+N) = \cos(2\pi f_0 (n+N) T + \theta)$$

By the definition of periodicity,

$$x(n+N) = x(n)$$

$$\therefore \cos(2\pi f_0 (n+N)T + \theta) = \cos(2\pi f_0 nT + \theta)$$

This relation is true if there exist an integer 'k' such that

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$$2\pi f_0 NT = 2\pi K$$

or, $\frac{1}{T_p} NT = K$ $\left[\because T_p = \frac{1}{F_0} \Rightarrow F_0 = \frac{1}{T_p}\right]$

or, $\frac{T}{T_p} = \frac{K}{N}$ ---- (3)

or equivalently, $f_0 = \frac{F_0}{F_s} = \frac{K}{N}$

$\because F_s = \frac{1}{T}$ and $F_0 = \frac{1}{T_p}$

from eqn (3), $x(n)$ is periodic only if its frequency f_0 or $\frac{1}{T_0}$ or $\frac{1}{T_p}$ can be expressed as the ratio of two integers ie f_0 or $\frac{T}{T_p}$ is a rational.

Again from eqn (iii) $T_p = N$
ie fundamental period of $x(n)$ is N.

Convolution sum and Impulse response:-

let us consider a relaxed LTI system. A relaxed system is one in which if the input $x(n)=0$, then the output $y(n)$ is also zero (0). let us consider a unit impulse function $\delta(n)$ is applied to the system, then its output is denoted by $h(n)$ where $h(n)$ = impulse response of the system.

i) $\delta(n) \xrightarrow{T} y(n) = h(n)$

since the system is time invarient if we delay input by 'k' samples then output should be delayed by same amount.

ii) $\delta(n-k) \xrightarrow{T} y(n) = h(n-k)$

now multiplying both sides by $x(k)$ we get

iii) $x(k)\,\delta(n-k) \longrightarrow y(n) = x(k)\,h(n-k)$

since the system is linear we can apply superposition theorem,
Hence taking summation on both sides we get,

$$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \xrightarrow{T} y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

Thus we have,

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

which is known as convolution sum or linear convolution.

$$\therefore x(n) \cdot h(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

Thus the behavior of LTI system is completely characterized by its impulse response and the process of convolution.

1) Folding
2) Shifting
3) multiplication
4) Summation

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Q1) find convolution sum of :-

$$x(n) = \begin{cases} 1 & -1 \le n \le 1 \\ 0 & \text{otherwise} \end{cases} \quad L = 3$$

and

$$h(n) = \begin{cases} 1 & -1 \le n \le 1 \\ 0 & \text{otherwise} \end{cases} \quad M = 3$$

soln

from convolution sum :

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

Here, limits of $x(n)$ are :

$$x_r = 1, \quad x_l = -1$$

Limits of $h(n)$ are :

$$h_r = 1, \quad h_l = -1$$

Total length of $y(n) = L+M-1 = 5$

$\therefore$ Limits of $y(n)$ are obtained as

$$y_r = x_r + h_r = 1+1 = 2 \quad \&$$
$$y_l = x_l + h_l = (-1) + (-1) = -2$$

Now, ~~when~~

$$y(n) = \sum_{k=-2}^{2} x(k) \cdot h(n-k)$$

when $n = -2$

$$y(-2) = \sum_{k=-2}^{2} x(k)\, h(-2-k)$$

$$= x(-2)\, h(0) + x(-1)\, h(-1) + x(0)\, h(-2) + x(1)\, h(-3) + x(2)\, h(-4)$$

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$= 0 + 1 \times 1 + 0 + 0 + 0$
$= 1$

when $n = -1$

$$y(-1) = \sum_{k=-2}^{2} x(k)\, h(-1-k)$$

$= x(-2)h(1) + x(-1)h(0) + x(0) \cdot h(-1) + x(1)h(-2) + x(2)h(-3)$

$= 0 + 1 \times 1 + 1 \times 1 + 0 + 0$

$= 2$

when $n = 0$

$$y(0) = \sum_{k=-2}^{2} x(k)\, h(0-k)$$

$= x(-2)h(2) + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2)$

$= 0 + 1 \times 1 + 1 \times 1 + 1 \times 1 + 0$

$= 3$

when $n = 1$

$$y(1) = \sum_{k=-2}^{2} x(k)\, h(1-k)$$

$= x(-2)h(3) + x(-1)h(2) + x(0)h(1) + x(1)h(0) + x(2)h(-1)$

$= 0 + 0 + 1 \times 1 + 1 \times 1 + 0$

$= 2$

when $n = 2$

$$y(2) = \sum_{n=-2}^{2} x(k)\, h(2-k)$$

$= x(-2)h(4) + x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)h(0)$

$= 0 + 0 + 0 + 1 + 0$

$= 1$

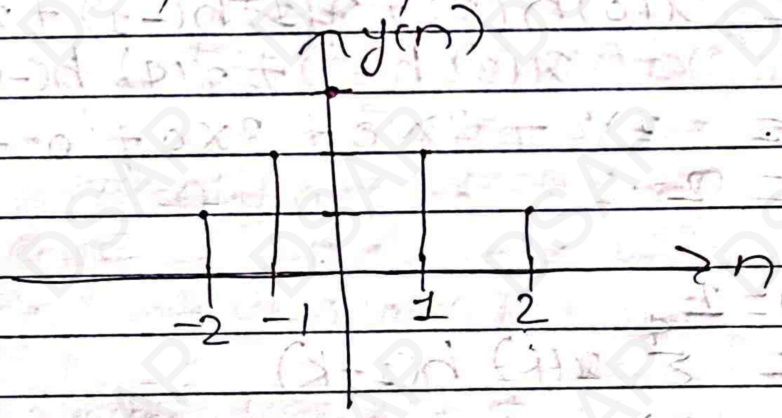
linear convolution

	1	1	1
1	1	1	1
-1	-1	-1	-1
1	1	1	1

Hence, convolution sum is :

$y(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

The graphical representation is :



Q2) Find the convolution between $x(n) = \{\underset{\uparrow}{1}, 2, 3\}$ and $h(n) = \{\underset{\uparrow}{2}, 1, 0\}$

Soln

Here, $x(n) = \{\underset{\uparrow}{1}, 2, 3\}$

$h(n) = \{\underset{\uparrow}{2}, 1, 0\}$

From convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Here, limits of $x(n)$ are: $x_r = 2$, $x_l = 0$

limits of $h(n)$ are : $h_r = 2$, $h_l = 0$

$\therefore$ limits of $y(n)$ are obtained as

$y_r = x_r + h_r = 2+2 = 4$

$y_l = x_l + h_l = 0+0 = 0$

Now, $y(n) = \sum_{k=0}^{4} x(k)\, h(n-k)$

when $n = 0$

$y(0) = \sum_{k=0}^{4} x(k)\, h(0-k)$

$= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3) + x(4)h(-4)$

$= 1\times 2 + 2\times 0 + 3\times 0 + 0 + 0$

$= 2.$

when $n = 1$

$y(1) = \sum_{k=0}^{4} x(k)\, h(1-k)$

$= x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) + x(4)h(-3)$

$= 1\times 1 + 2\times 2 + 3\times 0 + 0 + 0$

$= 1+4$

$= 5$

when $n = 2$

$y(2) = \sum_{k=0}^{4} x(k)\, h(2-k)$

$= x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1) + x(4)h(-2)$

$= 1\times 0 + 2\times 1 + 3\times 2 + 0 + 0$

$= 8$

when $n=3$

$$y(3) = \sum_{k=0}^{4} x(k)\, h(3-k)$$

$$= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) + x(4)h(-1)$$

$$= 0 + 2\times 0 + 3\times 1 + 0 + 0$$

$$= 3$$

when $n=4$

$$y(4) = \sum_{k=0}^{4} x(k)\, h(4-k)$$

$$= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0)$$

$$= 0 + 0 + 3\times 0 + 0 + 0$$

$$= 0$$

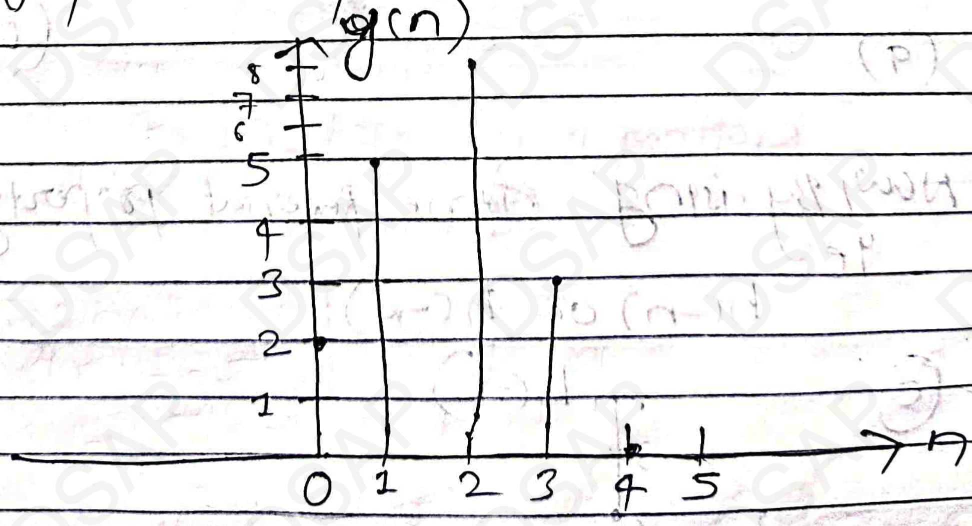
Hence, convolution sum is :

$$y(n) = \{\underset{\uparrow}{2}, 5, 8, 3, 0\}$$

Linear convolution

	1	2	3
2	2	4	6
1	1	2	3
0	0	0	0

The graphical representation is :



*) Graphical convolution method to find y(n):-

y(n):-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k)$$

i) $h(k) \rightarrow h(-k)$ [Time Reversal property]

ii) $h(-k) \rightarrow h(n-k)$ [Time shifting property]

iii) $x(k)\cdot h(n-k)$ [obtaining product]

iv) $\sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ [summation of product]

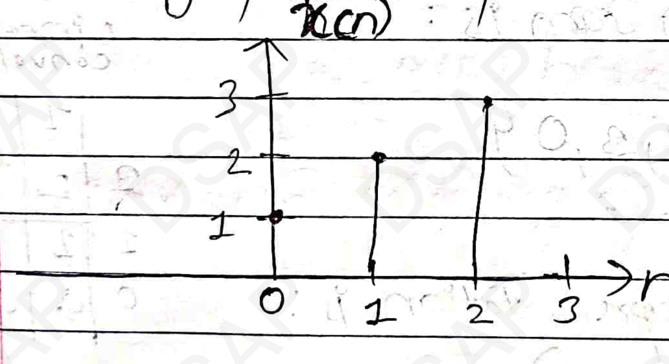
$L=3$

Q) Find y(n) at $x(n) = \{\underset{\uparrow}{1},2,3\}$ and $h(n) = \{\underset{\uparrow}{2},1,2\}$ from graphical convolution.

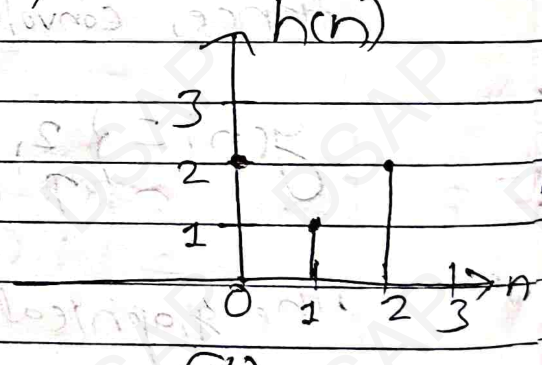
$M=3$ Total length $= L+M-1 = 3+3-1 = 5$

soln

The graphical representation of x(n) and h(n) is:



(a)

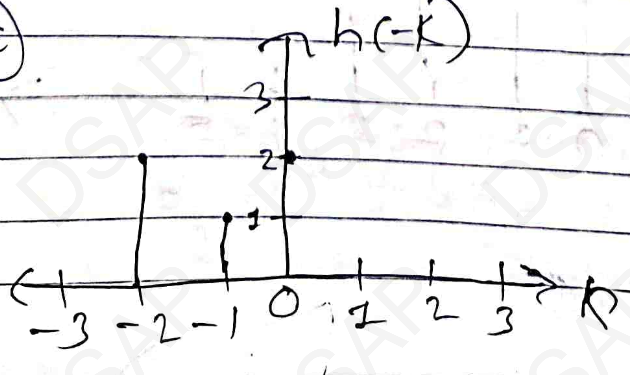


(b)

Now, By using Time Reversal property on h(n) ie

h(-n) or h(-k)

(c)



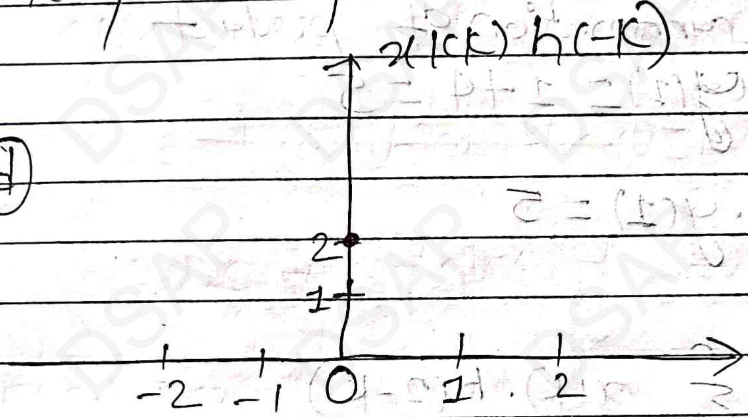
Now, obtaining the product ie
$x(k)\, h(n-k)$.

At $n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k)$$

The product of (a) and (c) is:

(d)



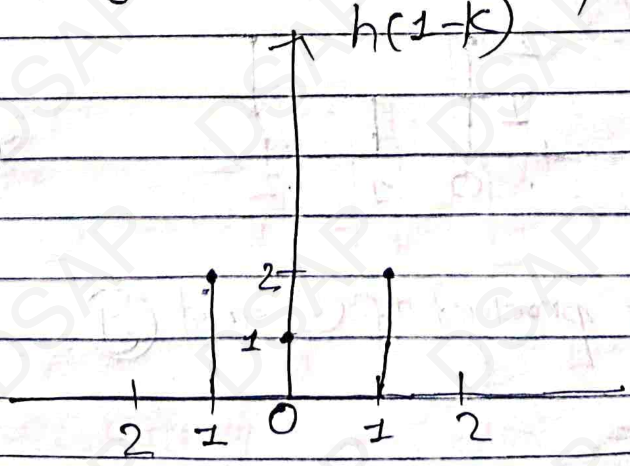
$\therefore\ y(0) = 2$

At $n=1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

Now, In fig (c), shift by 1 position

(e)



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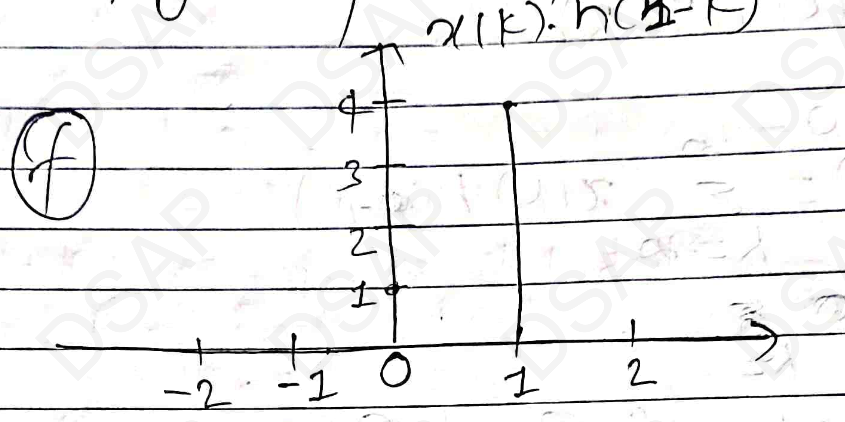
obtaining the product of (a) and (e)



Now, using summation of product

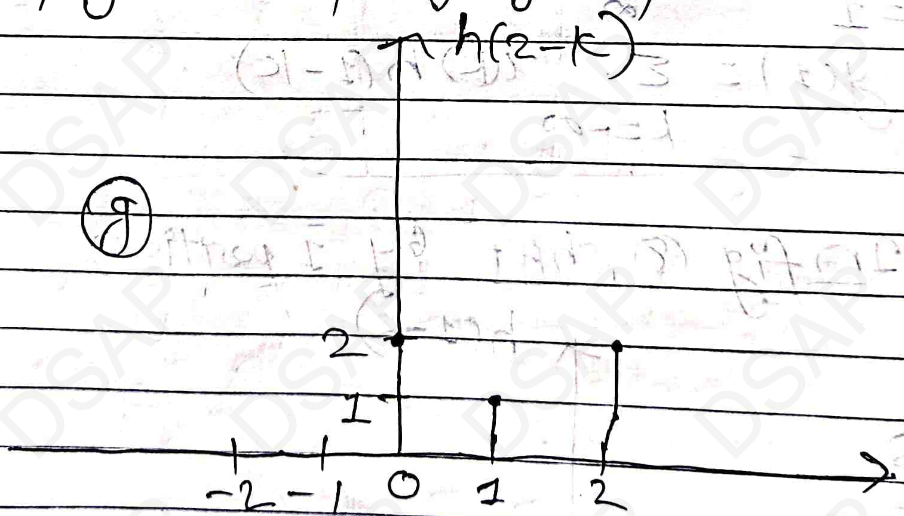
ie $y(1) = 1+4 = 5$

$\therefore y(1) = 5$

At $n=2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(2-k)$$

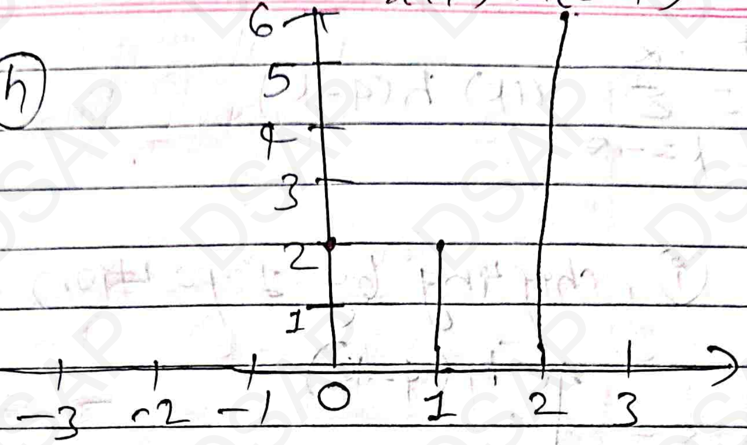
In fig (e), shifting by 1 position



obtaining the product of (a) and (g)

$x(k) \cdot h(2-k)$

(h)



Now, using summation of product

ie $y(2) = 2+2+6$

$= 10$

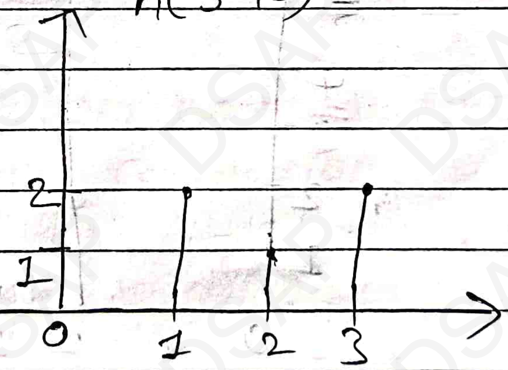
At $n = 3$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$$

In fig (f), shifting by 1 position

$h(3-k)$

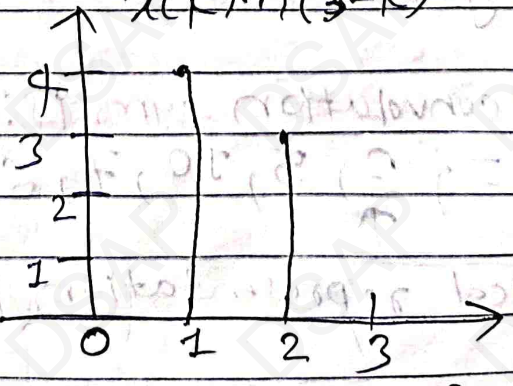
(i)



obtaining the product of (a) and (i)

$x(k) \cdot h(3-k)$

(g)



Now, using summation of product

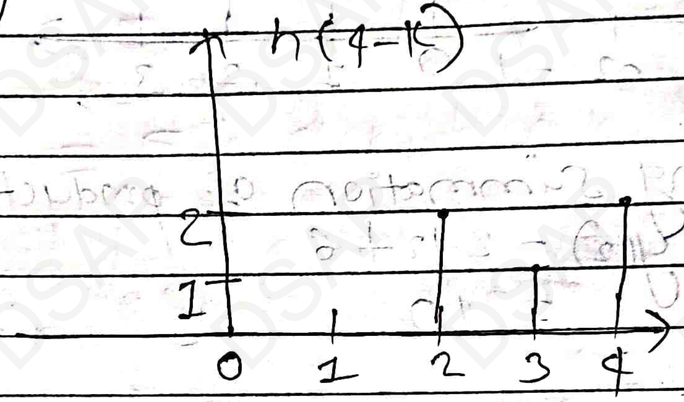
$y(3) = 4+3 = 7$

At $n = 4$

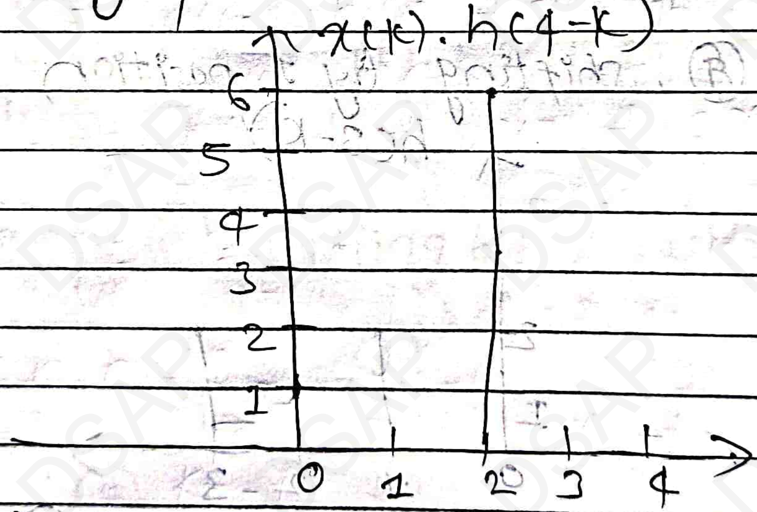
$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$$

In fig (i), shifting by 1 position

(k)



Obtaining product of (a) and (k)



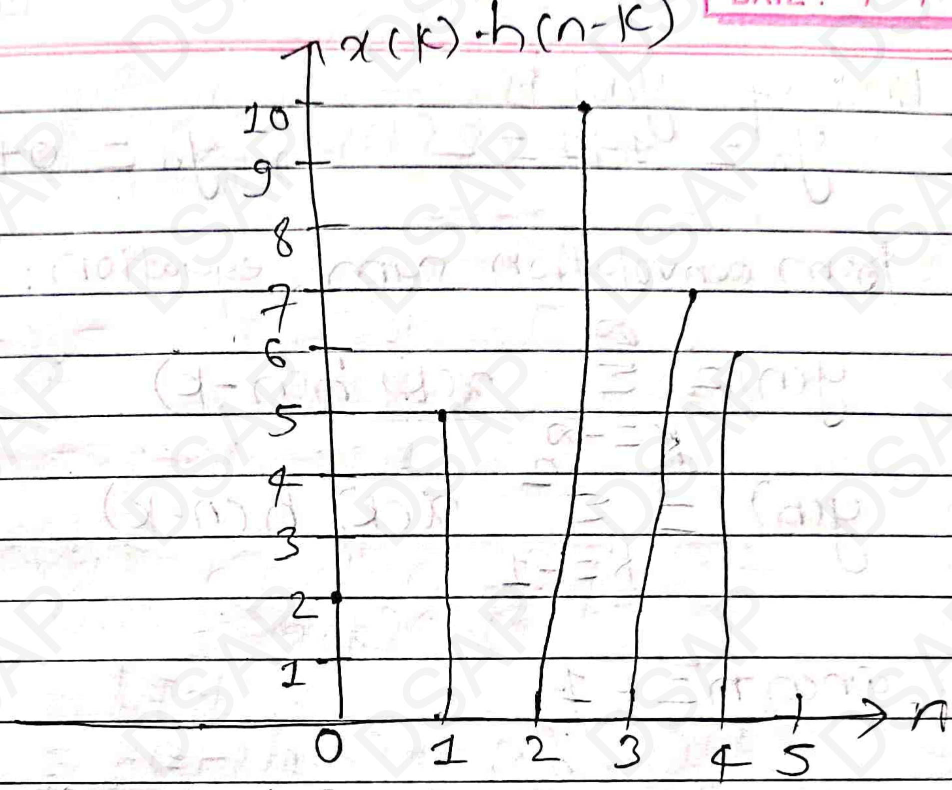
using summation of product

$\therefore y(4) = 6$

Hence, the convolution sum is:

$$y(n) = \{\underset{\uparrow}{2}, 5, 10, 7, 6\}$$

The graphical representation is:



(i) for a system defined by the impulse response $h(n) = \{1, -1\}$.
obtain the response of the system with an input signal $x(n) = 2^n$ for $-1 \leq n \leq 1$ is applied to it.
use: (i) convolution sum equation.
(ii) Graphical convolution.

i) Convolution sum equation:

soln

Here, $h(n) = \{1, -1\} \Rightarrow h(0) = 1, \ h(1) = -1$

limit of $h(n)$ is $h_r = 1$, $h_l = 0$

$x(n) = 2^n$ for $-1 \leq n \leq 1$

$x(-1) = 2^{-1} = \frac{1}{2}$

$x(0) = 2^0 = 1$

$x(1) = 2^1 = 2$

limit of $x(n)$ is $x_r = 1$, $x_l = -1$

limit of $y(n)$ is

$y_r = 1+1 = 2$, $y_l = 0+(-1) = -1$

from convolution sum equation:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$y(n) = \sum_{k=-1}^{2} x(k)\,h(n-k)$$

when $n = -1$

$$y(-1) = \sum_{k=-1}^{2} x(k)\,h(-1-k)$$

$$= x(-1)h(0) + x(0)h(-1) + x(1)h(-2) + x(2)h(-3)$$

$$= \frac{1}{2}\times 1 + 1\times 0 + 2\times 0 + 0$$

$$= \frac{1}{2}$$

when $n = 0$

$$y(0) = \sum_{k=-1}^{2} x(k)\,h(0-k)$$

$$= x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2)$$

$$= \frac{1}{2}\times -1 + 1\times 1 + 2\times 0 + 0$$

$$= +\frac{1}{2}$$

when $n = 1$

$$y(1) = \sum_{k=-1}^{2} x(k)\, h(1-k)$$

$$= x(-1)\,h(2) + x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1)$$

$$= 0 + 1\times -1 + 2\times 1 + 0$$

$$= 1$$

when $n = 2$

$$y(2) = \sum_{k=-1}^{2} x(k)\, h(2-k)$$

$$= x(-1)\,h(3) + x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0)$$

$$= 0 + 0 + 2\times -1 + 0$$

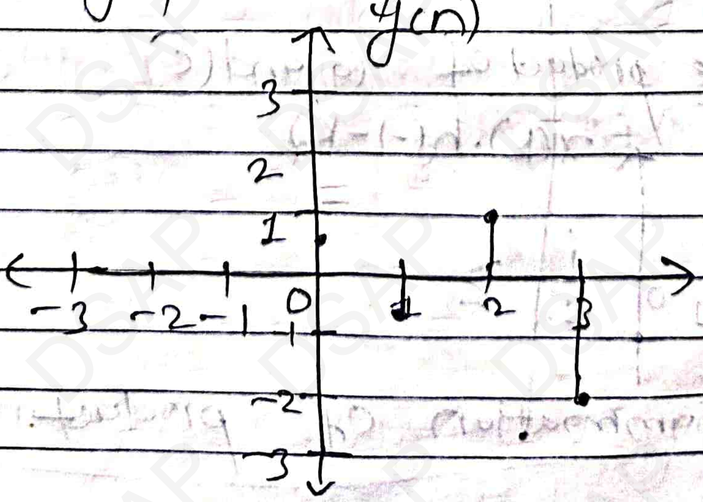
$$= -2$$

	$\frac{1}{2}$	1	2
1	$1/2$	1	2
-1	$-1/2$	-1	-2

Hence, convolution sum is

$$y(n) = \{ \underset{\uparrow}{\tfrac{1}{2}}, +1/2, 1, -2 \}$$

The graphical representation is



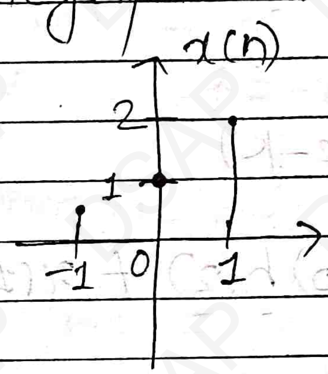
ii) Graphical convolution:-

$$h(n) = \{\underset{\uparrow}{+1}, -1\}$$

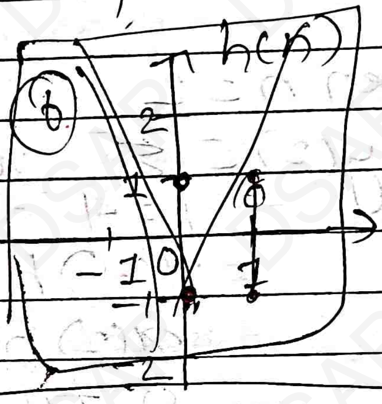
$$x(n) = \{\tfrac{1}{2}, \underset{\uparrow}{1}, 2\} \qquad y_r = 4, \; h_l = -1$$

The graphical representation of $x(n)$ and $h(n)$ is:

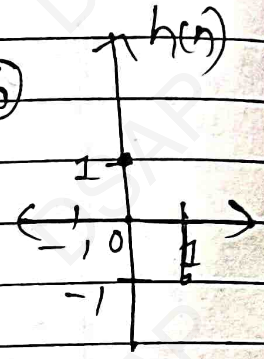
(a)



(b)



(b)

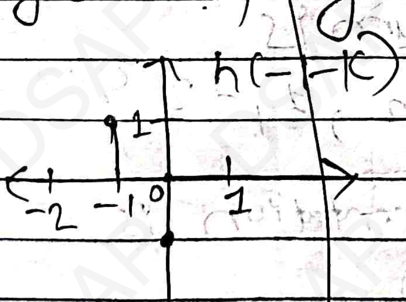


At $n = -1$

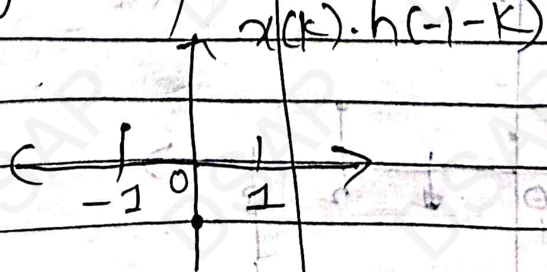
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$$

In fig (b), shifting by -1 position

(c)



obtaining the product of (a) and (c)



Now, using summation of product

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$\boxed{\cancel{y(-1) = -1 + \frac{1}{2} = -\frac{1}{2}}}$

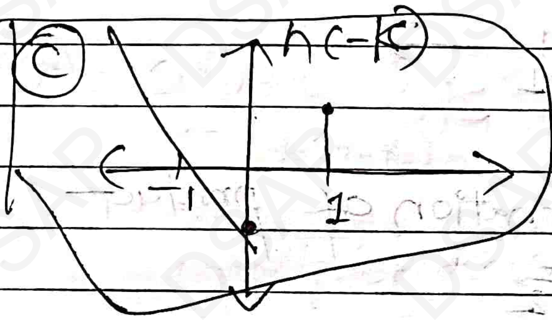
Now,

By using time reversal property on $h(n)$
ie

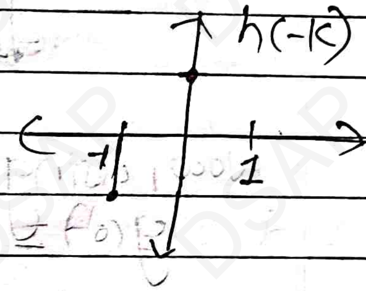
$$h(k) \rightarrow h(-k)$$

$\Rightarrow h(n) \rightarrow h(-n)$

(c)



(c)



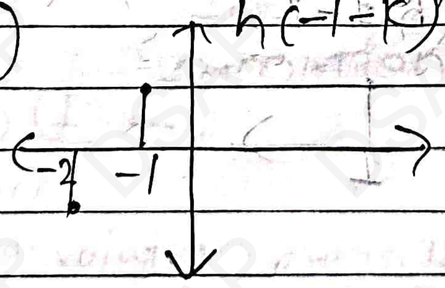
Now, obtaining the product
ie $x(k)\, h(n-k)$

At $n = -1$

$$y(-1) = \sum_{k=-\infty} x(k)\, h(-1-k)$$

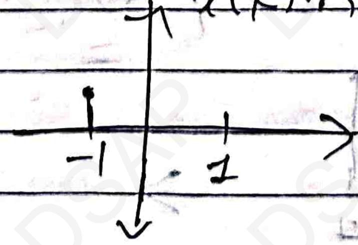
In fig (c), shift by -1 position

(d)



Obtaining the product of (a) and (d)

(e)



Now, using summation of product

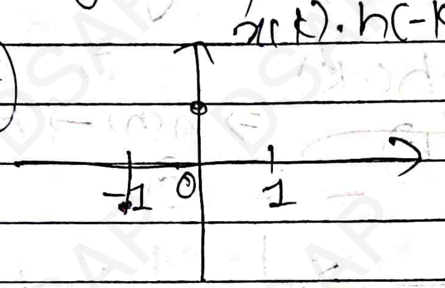
$$y(-1) = 1/2$$

At $n=0$

$$y(0) = \sum_{k=-1}^{2} x(k)\,h(-k)$$

obtaining the product of (a) and (c)

(f)



Now, using summation of product

$$y(0) = 1 - \frac{1}{2}$$

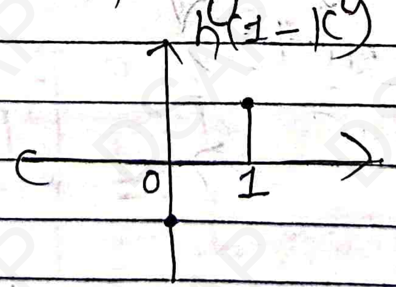
$$= \frac{1}{2}$$

At $n=1$

$$y(1) = \sum_{k=-1}^{2} x(k)\,h(1-k)$$

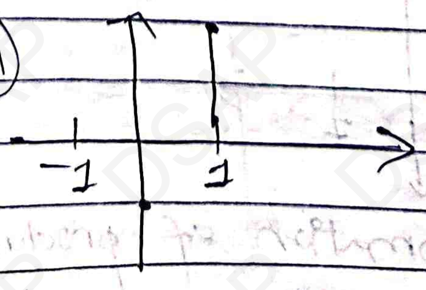
In fig (c), shifting by 1 position

(g)



obtaining the product of (a) and (g)

(h)



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Now, using summation of product

$y(1) = 2 - 1 = 1$

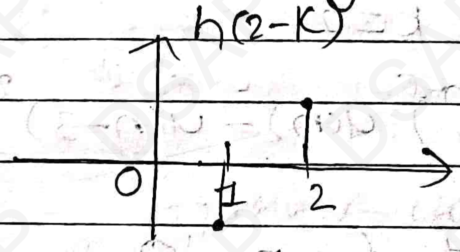
In c fig :

At $n=2$

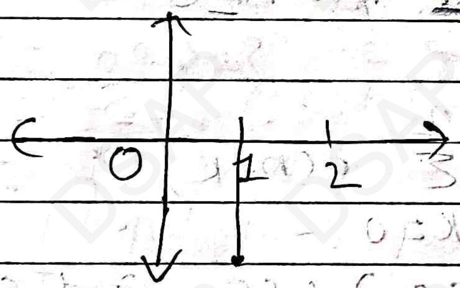
$$y(2) = \sum_{k=-1}^{2} x(k)\, h(2-k)$$

In fig (8), shift by 1 position

(9)



obtaining the product of (8) and (9)



Now, using summation of product

$y(2) = -2$

Hence, convolution sum is :

$$y(n) = \left\{ \frac{1}{2}, \frac{1}{2}, 1, -2 \right\}$$

2013 spring

$\delta(n-1)$ for $n=1$
$\delta(1-1) = \delta(0) = 1$

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(1) Obtain linear convolution of the given discrete time signals :-

$$x(n) = \sum_{k=0}^{2} \delta(n-k)$$

$$h(n) = 2^n \{ u(n) - u(n-3) \}$$

Soln

Here, $x(n) = \sum_{k=0}^{2} \delta(n-k)$

$h(n) = 2^n \{ u(n) - u(n-3) \}$

we know,

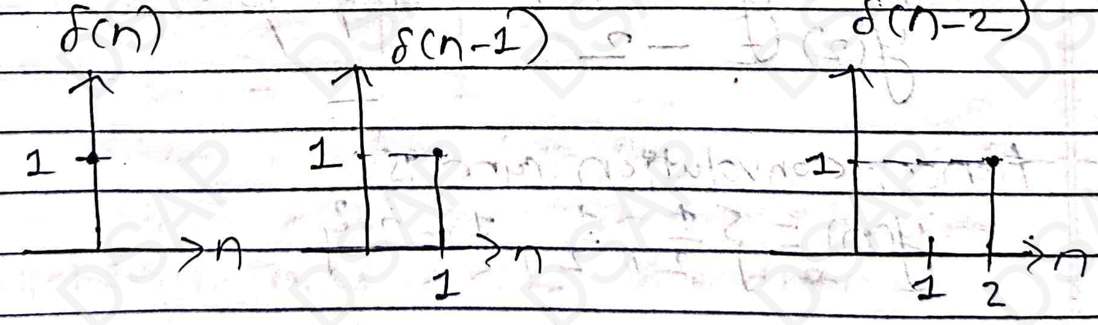
$\delta(n) = 1$ for $n = 0$

$u(n) = 1$ for $n \geq 0$

Now,

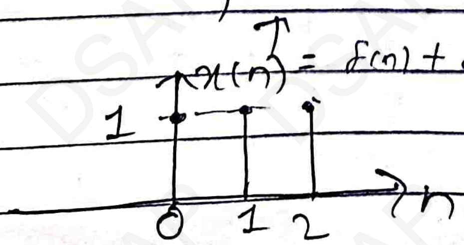
$$x(n) = \sum_{k=0}^{2} \delta(n-k)$$

$$= \delta(n) + \delta(n-1) + \delta(n-2)$$



$x(n) = \{ \underset{\uparrow}{1}, 1, 1 \}$

$x(n) = \delta(n) + \delta(n-1) + \delta(n-2)$



$$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1} - a^{N_2+1}}{1-a}$$

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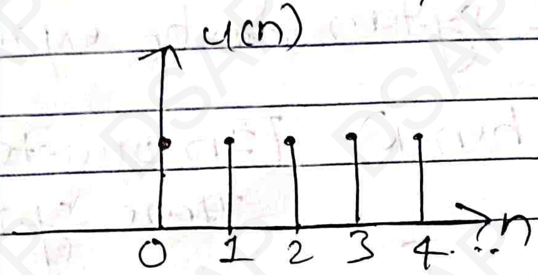
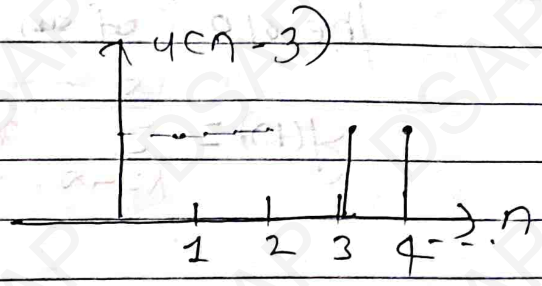
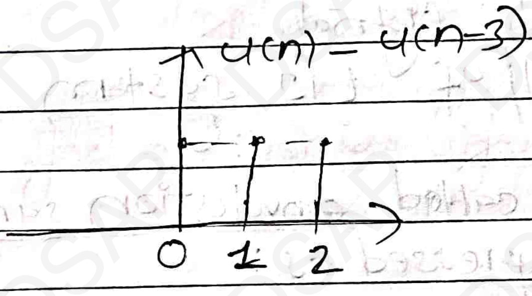
$$h(n) = 2^n \{u(n) - u(n-3)\}$$



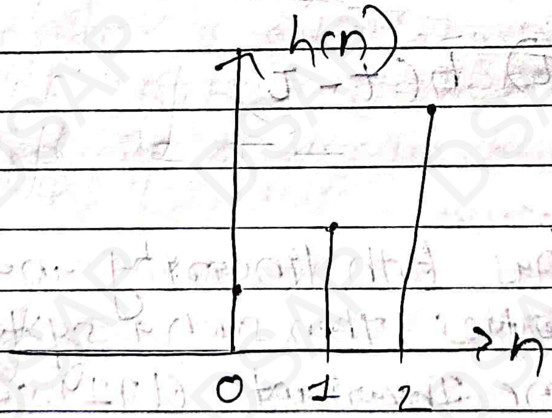




$$h(n) = 2^n \{u(n) - u(n-3)\}$$

$$h(0) = 2^0\{1\} = 1$$
$$h(1) = 2^1\{1\} = 2$$
$$h(2) = 2^2\{1\} = 4$$

$$h(n) = \{\underset{\uparrow}{1}, 2, 4\}$$



we know

$$y(n) = x(k) \circledast h(n-k)$$
$$= \sum_{k=0}^{4} x(k) \cdot h(n-k)$$

By linear convolution

tabular method

	1	1	1
1	1	1	1
2	2	2	2
4	4	4	4

$$y(n) = \{\underset{\uparrow}{1}, 3, 7, 6, 4\}$$

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Convolution :-

The o/p of an LTI system can be expressed as:-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$ [In Discrete Time system]

where, $x(k)$ = i/p signal
$h(n-k)$ = o/p of LTI system

This equation is called <u>convolution sum</u> and it can be also expressed as :-

$$y(n) = x(k) \otimes h(n-k)$$

similarly, In continuous time system,

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) \cdot h(t-\tau)$$

and $y(t) = x(\tau)\ h(t-\tau)$

LTI System :-

If a system has both linearity and time invariance properties then such a system is called Linear Time Invariant (LTI) system.

→ Most of the practical and physical process can be modeled in the form of LTI system.

→ LTI system can be analyzed very easily ie

$$x(n) \rightarrow \boxed{\text{LTI system}} \rightarrow y(n)$$

for example:

In LTI system, if the i/p $x(n) = \delta(n)$, then the resulting o/p is $y(n) = h(n)$.

where, $h(n)$ = Impulse Response

$\delta(n)$ = unit Impulse Input

ie

$$\delta(n) \rightarrow \boxed{\text{LTI system}} \rightarrow h(n)$$

Q) Prove that "a LTI system response is absolutely summable". What is BIBO stable? (7)

2013

Show the necessary and sufficient condition for stability.

Ans A system is called Bounded Input Bounded output (BIBO) stable if and only if every bounded i/p results in bounded o/p.

The output of such system does not diverge or grow unnecessarily large.

from covolution sum:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$ [By using commutative property]

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$$|y(n)| = \left|\sum_{K=-\infty}^{\infty} h(K)\cdot x(n-K)\right|$$

Applying swartz's inequality principle

(ie $ab = |a||b|$)

ie $|y(n)| = \sum_{K=-\infty}^{\infty} |h(K)|\,|x(n-K)|$

Now, If the i/p is bounded :

$|x(n-K)| \le M_x \le \infty$

Then,

$$|y(n)| \le M_x \sum_{K=-\infty}^{\infty} |h(K)|$$

Hence, the output will be bounded if

$$\boxed{\sum_{K=-\infty}^{\infty} |h(K)| < \infty}$$

This means that the impulse response of the system should be absolutely summable for stability of LTI system.

Thus, $\sum_{K=-\infty}^{\infty} |h(K)| < \infty$ is the necessary and sufficient condition for stability.

Q) Check the BIBO stability

$y(n) = x(n) + e^{a}\cdot y(n-1)$

[Given : $h(-1) = 0$]

soln.

If $x(n) = \delta(n)$

Then

$y(n) = h(n)$

Thus,

$h(n) = \delta(n) + e^{a}\cdot h(n-1)$

when $n = 0$

$h(0) = \delta(0) + e^{a}\cdot h(-1)$

$= 1 + e^{a}\cdot 0$

$= 1$

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$\therefore h(0) = 1 \Rightarrow e^{0.\alpha}$

when $n = 1$

$$h(1) = \delta(1) + e^{\alpha}.h(1-1)$$
$$= \delta(1) + e^{\alpha}.h(0)$$
$$= 0 + e^{\alpha}.1$$
$$= e^{1.\alpha}$$

$\therefore h(1) = e^{1.\alpha}$

when $n = 2$

$$h(2) = \delta(2) + e^{\alpha}.h(2-1)$$
$$= \delta(2) + e^{\alpha}.h(1)$$
$$= 0 + e^{\alpha}.e^{\alpha}$$
$$= e^{2\alpha}$$

$\therefore h(2) = e^{2\alpha}$

similarly,

$$h(n) = e^{n\alpha}$$

Now, the condition for stability is :

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

For causal system :- $h(n) = 0, n < 0$

$$\sum_{k=0}^{\infty} h(k) < \infty$$

Now, we have :

$$\sum_{k=0}^{\infty} h(k) = h(0) + h(1) + h(2) + \cdots$$
$$= 1 + e^{\alpha} + e^{2\alpha} + \cdots$$
$$= \sum_{k=0}^{\infty} |e^{k\alpha}|$$
$$= \left|\frac{1}{1-e^{\alpha}}\right|$$

Analysis of LTI system using Constant coefficient Difference (CCD) Equation:-

Paley – Wiener criteria:

It states that 'A system is causal if its impulse response is causal ie

$$h(n) = 0 \quad , \quad n < 0$$

Proof:

We have,

The convolution sum is given by:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

Using commutative property:-

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

Now,

output at any instant n_0 is:

$$y(n)\Big|_{n=n_0} = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n_0-k)$$

$$= \sum_{k=-\infty}^{-1} h(k)\, x(n_0-k) + \sum_{k=0}^{\infty} h(k)\, x(n_0-k)$$

$$= (\cdots + h(-2)\, x(n_0+2) + h(-1)\, x(n_0+1)) + (h(0)\, x(n_0) + h(1)\, x(n_0-1) + h(2) \cdot x(n_0-2) + \cdots)$$

$$= a_1 + a_2$$

Here, $a_1 = \cdots + h(-2) \cdot x(n_0+2) + h(-1) \cdot x(n_0+1)$

It consist of future values of i/p [ie $x(n)$]

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$a_2 = h(0).x(n_0) + h(1).x(n_0-1) + h(2).x(n_0-2) + \dots$
It consists of past values of i/p and present value of i/p [ie $x(0)$]

Now, To make causal, a_1 should be cancelled out for that,

$$h(n) < 0$$

so, limit of Σ changes for causal system as:

$$\boxed{y(n) = \sum_{k=0}^{\infty} h(k).x(n-k)}$$

Homogeneous and Particular solution:-

The general form of CCD equation is:

$$\sum_{k=0}^{N} a_k.y(n-k) = \sum_{k=0}^{M} b_k.x(n-k) \quad \dots (1)$$

We assume $a_0 = 1$ then,

$$a_0.y(n) + \sum_{k=1}^{N} a_k.y(n-k) = \sum_{k=0}^{M} b_k.x(n-k)$$

or, $$y(n) + \sum_{k=1}^{N} a_k.y(n-k) = \sum_{k=0}^{M} b_k.x(n-k)$$

or, $$y(n) = \sum_{k=0}^{M} b_k.x(n-k) - \sum_{k=1}^{N} a_k.y(n-k) \quad \dots (2)$$

The total soln of this equation is:

$$y(n) = y_h(n) + y_p(n)$$

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where, $y_h(n)$ = zero i/p Response (Homogeneous solution)

$y_p(n)$ = zero state Response (particular solution)

ie $\boxed{y(n) = y_{zi} + y_{si}}$

1) Homogeneous solution:-
(zero i/p Response)

For Homogeneous solution, we assume $x(n) = 0$.

Then eqn. ① becomes:

$$\sum_{K=0}^{N} a_K . y(n-K) = 0 \quad \cdots (3)$$

Assume an exponential form of solution of eqn ③ as:

$$y_h(n) = \lambda^n$$

We get:

$$\sum_{K=0}^{N} a_K . \lambda^{n-K} = 0$$

or, $a_0 . \lambda^{n-0} + a_1 . \lambda^{n-1} + a_2 . \lambda^{n-2} + \cdots + a_N \lambda^{n-N} = 0$

or, $\lambda^{n-N} (a_0 \lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} + \cdots + a_N) = 0$

It is the polynomial of order N
so, there are N complex roots of the equation.

let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N$ be the roots then the homogenous solution is:

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n + \cdots c_N \lambda_N^n$$

where,

$c_1, c_2, c_3, \cdots c_N$ are constant coefficient terms.

$$\boxed{\therefore y_h(n) = \sum_{k=1}^{N} c_k \cdot \lambda_k^n = y_{ZIR}(n)}$$

Q) Obtain the zero Input Response (ZIR) of the system described by homogeneous equation:

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

Assume, initial state of this system as:

$$y(-1) = 1 \quad \text{and} \quad y(-2) = 1$$

Soln

Given:

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

let $y(n) = \lambda^n$

Now, The equation becomes:

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

or, $\lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$

or, $\lambda^2 - 3\lambda - 4 = 0$

or, $\lambda^2 - 4\lambda + \lambda - 4 = 0$

or, $\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

or, $(\lambda - 4)(\lambda + 1) = 0$

$\therefore \lambda = -1$ and $\lambda = 4$

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Now, The homogeneous solution is given by:

$$y_h(n) = \sum_{k=1}^{2} c_k \lambda_k^n$$

$$= c_1\lambda_1^n + c_2\lambda_2^n$$

$$\Rightarrow y_h(n) = c_1.(-1)^n + c_2.(4)^n \quad \text{---} \quad (1)$$

Now, The difference equation is
$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

At $n=0$

$$y(0) - 3y(-1) - 4y(-2) = 0$$
or, $y(0) - 3\times1 - 4\times1 = 0$
or, $y(0) = 7$

At $n=1$

$$y(1) - 3y(0) - 4y(-1) = 0$$
or, $y(1) - 3\times7 - 4\times1 = 0$
or, $y(1) = 25$

We have, the homogeneous solution is:
$$y_h(n) = c_1\cdot(-1)^n + c_2(4)^n$$

At $n=0$,

$$y(0) = c_1(-1)^0 + c_2(4)^0$$
$$7 = c_1 + c_2 \quad \text{---} \quad (a)$$

At $n=1$

$$y(1) = c_1(-1)^1 + c_2(4)^1$$
or, $25 = -c_1 + 4c_2 \quad \text{...} \quad (b)$

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solving (a) and (b)

$$c_1 + c_2 = 7$$
$$-c_1 + 4c_2 = 25$$
$$5c_2 = 32$$
$$c_2 = 6.4$$

from (a)

$$c_1 = 7 - c_2 = 7 - 6.4 = 0.6$$

Now,

$$y_h(n) = 0.6(-1)^n + 6.4(4)^n$$

2) Particular solution :-
(zero state Response)

Any solution $y_p(n)$ is a particular solution if it satisfies the following equation:

$$\sum_{k=0}^{N} a_k \cdot y(n-k) = \sum_{k=0}^{M} b_k \cdot x(n-k) \quad [a_0 = 1]$$

We assume $y_p(n)$ on a form that depends upon the form of input $x(n)$.

Q) Determine particular solution of:

$$y(n) + a_1 \cdot y(n-1) = x(n)$$
where, $x(n) = u(n)$

soln

let, particular eqn be:

$$y_p(n) = K \cdot x_n$$

$$\Rightarrow \boxed{y_p(n) = K \cdot u(n)} \quad [\because x(n) = u(n)]$$

where, K is constant.

Now, The solution becomes:

$$k \cdot u(n) + a_1 \cdot k\, u(n-1) = u(n)$$

To determine k we must evaluate the equation for any $n \geq 0$.

where,

$$k + a_1 \cdot k = 1$$

$$k(1+a_1) = 1$$

$$\therefore k = \frac{1}{1+a_1}$$

Now, The particular soln is:

$$\boxed{y_p(n) = \frac{1}{1+a_1} u(n)}$$ Ans

q) For a system described by CCD equation where,

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \quad , x(n) = u(n)$$

with input signal $x(n) = 2^n \cdot u(n)$ for $n \geq 0$.
Obtain zero state Response.
(particular soln)

soln

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$$\text{or, } y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = x(n) \quad \ldots (1)$$

For homogeneous solution:

let $y_h(n) = \lambda^n$ and $x(n) = 0$

Then eqn (1) becomes,

$$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0$$

or, $\lambda^{n-2}\left(\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right) = 0$

or, $6\lambda^2 - 5\lambda + 1 = 0$

or, $6\lambda^2 - 3\lambda + 2\lambda + 1 = 0$

or, $3\lambda(2\lambda - 1) - 1(2\lambda - 1) = 0$

or, $(2\lambda - 1)\ (3\lambda - 1) = 0$

$\therefore \lambda_1 = \frac{1}{2}$ and $\lambda_2 = \frac{1}{3}$

Then, the homogeneous soln is given by:

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n$$

$$\Rightarrow y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$$

Input is given by

$x(n) = 2^n . u(n)$, for $n \geq 0$

At $n=0$, $x(0) = 2^0 . u(0)$

$= 1 . 1$

$= 1 \neq \lambda_1$ or λ_2

At $n=1$, $x(1) = 2^1 u(1)$

$= 2 . 1$

$= 2 \neq \lambda_1$ or λ_2

Thus, the i/p signal of particular soln is not equal to roots of homogeneous solution.

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So, the particular solution is given by:

$$y_p(n) = K \cdot x(n)$$
$$= K \cdot 2^n \cdot u(n)$$

for $n \geq 0$, $u(n) = 1$

$$\therefore y_p(n) = K \cdot 2^n$$

The CCD equation becomes: (eqn i)

$$K \cdot 2^n \cdot u(n) - \frac{5}{6} K \cdot 2^{n-1} u(n-1) + \frac{1}{6} K \cdot 2^{n-2} \cdot u(n-2) = 2^n u(n)$$

Assume minimum value of 'n' such that no product is attenuated.
ie
for $n = 2$

$$K \cdot 2^2 \cdot u(2) - \frac{5}{6} K \cdot 2^1 u(1) + \frac{1}{6} K \cdot 2^0 u(0) = 2^2 \cdot u(2)$$

or, $4K - \frac{10}{6} K + \frac{1}{6} K = 4$

or, $24K - 10K + K = 24$

or, $15K = 24$

or, $K = \frac{8}{5}$

Hence, the particular solution is given by:

$$y_p(n) = K \cdot 2^n u(n)$$
$$\therefore y_p(n) = \frac{8}{5} 2^n u(n)$$

Ans

Q) A Discrete Time system is defined by CCD equation:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 0.5x(n-1)$$

with the initial condition $y(-1) = y(-2) = 1$

obtain the total response of the system when i/p $x(n) = 4^n u(n)$ is applied to it.

soln

given,

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 0.5x(n-1) \quad \text{---(1)}$$

for Homogeneous solution:

Take $y_h(n) = \lambda^n$ and $x(n) = 0$

Then eqn (1) becomes:

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

or, $\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$

or, $\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$

or, $\lambda^2 - 3\lambda - 4 = 0$

or, $\lambda^2 - 4\lambda + \lambda - 4 = 0$

or, $\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

or, $(\lambda - 4)(\lambda + 1) = 0$

$\therefore \lambda_1 = 4$ and $\lambda_2 = -1$

The homogeneous solution is given by

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

$$\Rightarrow y_h(n) = C_1 \cdot (4)^n + C_2 \cdot (-1)^n$$

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Input is given by
$x(n) = 4^n . u(n)$

At $n=0$, $x(0) = 4^0 . u(0)$
$= 1.1$
$= 1 \neq \lambda_1$ or λ_2

At $n=1$, $x(1) = 4^1 . u(1)$
$= 4 = \lambda_1$

Thus, the i/p signal of particular solution is same as the root (λ_1) of the homogeneous solution.

So, the particular solution is:

$$y_p(n) = n.K.x(n)$$

$$\Rightarrow y_p(n) = n.K.4^n.u(n) \quad [\because x(n) = 4^n.u(n)]$$

Now, the total response is given as:

$$y(n) = y_h(n) + y_p(n)$$

$$\Rightarrow y(n) = [c_1.(4)^n + c_2(-1)^n] + [n.K.4^n.u(n)]$$

Now, The CCD eqn becomes (ie eqn(1)):

$$n.K.4^n.u(n) - 3(n-1).K.4^{n-1}.u(n-1) - 4.(n-2).K.4^{n-2}.u(n-2) = 4^n.u(n) + 0.5.4^{n-1}u(n-1)$$

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Assume the minimum value of 'n' such that no product is attenuated.
ie

for $n=3$

$$3K\cdot 4^3\cdot u(3) - 3\times 2\cdot K\cdot 4^2\cdot u(2) - 4\cdot 1\cdot K\cdot 4\cdot u(1)$$
$$= 4^3\cdot u(3) + 0.5\times 4^2\times u(2)$$

or, $192K - 96K - 16K = 64 + 8$

or, $80K = 72$

or, $K = \frac{9}{10}$

$\therefore K = 0.9$

so, the particular solution is

$$y_p(n) = 0.9\, n\, 4^n u(n)$$

Then, Total response is

$$y(n) = \left[c_1(4)^n + c_2(-1)^n\right] + \left[0.9\, n\, 4^n u(n)\right] \quad \text{--- (2)}$$

Now, The difference eqn is

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 0.5\, x(n-1)$$

At $n=0$

$y(0) - 3y(-1) - 4y(-2) = x(0) + 0.5\, x(-1)$

or, $y(0) - 3\times 1 - 4\times 1 = 1 + 0$

or, $y(0) = 8$

At $n=1$

$y(1) - 3y(0) - 4y(-1) = x(1) + 0.5\, x(0)$

or, $y(1) - 3\times 8 - 4\times 1 = 4 + 0.5$

or, $y(1) = 32.5$

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Now, using eqn ② :-

At $n=0$:

$y(0) = c_1 (4)^0 + c_2(-1)^0 + 0$

or, $8 = c_1 + c_2$ ---- ⓐ

At $n=1$

$y(1) = c_1(-1)^1 + c_2 (4)^1 + 1 \times 0.9 \times 4^1 . u(1)$

or, $32.5 = -c_1 + 4c_2 + 3.6$

or, $28.9 = -c_1 + 4c_2$ ---- ⓑ

solving ⓐ and ⓑ

$$8 = \cancel{c_1} + c_2$$
$$28.9 = -\cancel{c_1} + 4c_2$$
$$36.9 = 5c_2$$

or, $c_2 = 7.38$

from eqn ⓐ,

$c_1 = 8 - 7.38 = 0.62$

Hence, Total Response is

$y(n) = 0.62 (4)^n + 7.38 (-1)^n + 0.9 n 4^n u(n)$

Imp

Q) For the system described by the difference equation:

$y(n) + 6y(n-1) + 5y(n-2) = x(n) + 2x(n-1)$

with initial condition $y(-1) = 1$, $y(-2) = 2$.

Obtain the total response of the system when an i/p $x(n) = 3^n$ for $n \geq 1$ is applied to it.

soln

Given equation is :

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$y(n) + 6y(n-1) + 5y(n-2) = x(n) + 2x(n-1)$ ---(1)

For Homogeneous solution :
Take $y_h(n) = \lambda^n$ and $x(n) = 0$

Then eqn (1) becomes :

$\lambda^n + 6\lambda^{n-1} + 5\lambda^{n-2} = 0$

or, $\lambda^{n-2}[\lambda^2 + 6\lambda + 5) = 0$

or, $\lambda^2 + 5\lambda + \lambda + 5 = 0$

or, $\lambda(\lambda+5) + 1(\lambda+5) = 0$

or, $(\lambda+5)(\lambda+1) = 0$

$\therefore \lambda_1 = -5$ and $\lambda_2 = -1$

The homogeneous solution is given as

$y_h(n) = c_1(-5)^n + c_2(-1)^n$

Now, I/p is given by

$x(n) = 3^n$ for $n \geq 1$

At $n = 0$, $x(0) = 3^0 = 1 \neq \lambda_1$ or λ_2

At $n = 1$, $x(1) = 3^1 = 3 \neq \lambda_1$ or λ_2

Thus, the i/p signal of particular solution is not same as the roots of homogeneous solution.

So, the particular solution is :- $y_p(n) = K \cdot x(n)$

$\Rightarrow y_p(n) = K \cdot 3^n$ for $n \geq 1$

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Now, The CCD eqn becomes : [ie eqn (1)]

$$K \cdot 3^n + 6K\,3^{n-1} + 5K\,3^{n-2} = 3^n + 2\times 3^{n-1}$$

Assume the minimum value of 'n' such that no product is attenuated.

ie

for $n=2$

$K \cdot 3^2 + 6K \cdot 3^1 + 5K \cdot 3^0 = 3^2 + 2\times 3^1$

or, $9K + 18K + 5K = 9+6$

or, $32K = 15$

or, $K = \frac{15}{32}$

Hence, $y_p(n) = \frac{15}{32} 3^n$

Thus, the total response is

$$y(n) = y_h(n) + y_p(n)$$

$$\Rightarrow y(n) = [c_1(-5)^n + c_2(-1)^n] + \left(\frac{15}{32}3^n\right) \quad \text{---(2)}$$

Now, from difference equation [eqn eqn (1)]

At $n=0$

$y(0) + 6y(-1) + 5y(-2) = x(0) + 2x(-1)$

or, $y(0) + 6\times 1 + 5\times 2 = 0+0$

or, $y(0) + 6 + 10 = 0$

$\therefore y(0) = -16$

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At $n=1$

$$y(1)+6y(0)+5y(-1)=x(1)+2x(0)$$

or, $y(1)+6\times-16+5\times1=3^1+0$

or, $y(1)-96+5=3$

or, $y(1)=94$

Now, using eqn (5)

At $n=0$

$$y(0)=c_1+c_2+\frac{15}{32}$$

or, $-16=c_1+c_2+\frac{15}{32}$

or, $c_1+c_2=\frac{527}{32}$ ... (a)

At $n=1$

$$y(1)=c_1\times(-5)+c_2\times(-1)+\frac{15}{32}\times3$$

or, $94-\frac{45}{32}=-5c_1-c_2$

or, $-5c_1-c_2=\frac{2963}{32}$ ... (b)

solving (a) and (b)

$$c_1+c_2=\frac{527}{32}$$

$$-5c_1-c_2=\frac{2963}{32}$$

$$-4c_1=\frac{1745}{16}$$

$$c_1=-27.26$$

from eqn (9)

$$c_2 = \frac{527}{32} + 27.26$$

$$= 43.73$$

Hence, Total Response is :

$$y(n) = -27.26\,(-5)^n + 43.73\,(-1)^n + \frac{15}{32}\,3^n$$