Final Internal Examination 2081

Faculty of Science and Technology

School of Engineering, Pokhara University

Full Marks: 100

Pass Marks: 45

Program: BE (Computer)

Course: Applied Mathematics Time: 3 hrs.

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all questions.

Section A

1. a) Show that the function $u = \cos x \cos hy$ is harmonic. 2.5

b) State the Cauchy Integral formula. 2.5

c) Derive the formula of $Z[a^n]$. 2.5

d) Write the Fourier cosine and sine integral formula for the function $f(x)$. 2.5

Section B

2. a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series for the region $0 < |z+1| < 2$. 5

b) Find the poles and residues of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$. 5

c) Find the fixed point and the normal form of the bilinear transformation $w = \frac{z-1}{z+1}$. 5

3. a) Obtain the inverse Z-transform of $\frac{z}{z^2+9z+20}$. 5

b) Solve the partial differential $u_{xy} - u = 0$ by separating the variable. 5

c) Find Fourier cosine transform of $f(x) = e^{-mx}$, where $m > 0$. 5

Section C

4. a) Evaluate the integral $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. 7

b) What do you mean by analyticity of function $f(z)$. State Cauchy Riemann equation and show that it is the necessary condition for the function to be analytic. 2+4

50

5. a) State and prove first shifting theorem for Z-transform using it to find the value of $Z(\cos hat \sin bt)$. 7

b) Solve the differential equation by using Z-transform. $y_{n+2} - y_n = 2^n$ with $y_0 = 0, y_1 = 1$.

[2+3+2]

6. a) Examine the suitable function show that: 7

$$\int_0^\infty \left[\frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2}\right] d\omega = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$$

OR

Find the Fourier sine transform of $e^{-x}, x \geq 0$ and hence by Parseval's identity, show that $\int_0^\infty \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{4}$.

b) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$u(x,t) = \begin{cases} x; & 0 \leq x \leq \frac{L}{2} \\ L - x; & \frac{L}{2} \leq x \leq L \end{cases}$$ 8

7. a) Derive one dimensional wave equation of a string of length L which is fixed in two end points with required assumptions. 7

OR

Find the solution of one-dimensional heat equation, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with initial temperature f(x) and boundary conditions $u(x,0) = 0 = u(l,t)$.

b) Express Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinate system. 8

THE END