chapter-1 —5hrs

Signals, Systems and Signal Processing

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1.1 Basic elements of Digital Signal Processing
1.2 Energy signal, Power signal
1.3 Need of Digital signal Processing over Analog signal processing
1.4 Sampling continuous signals and spectral properties of sampled signals.

Introduction of signal:-

A signal is defined as any physical quantity that varies with respect to time, space or any other independent variables. eg:- voice signal, audio/video signal.

Classification of signals:-

Based upon their nature and characteristics in the time domain, the signals may be broadly classified as:

1) continuous time signals
2) Discrete time signals

1) continuous time signal:-

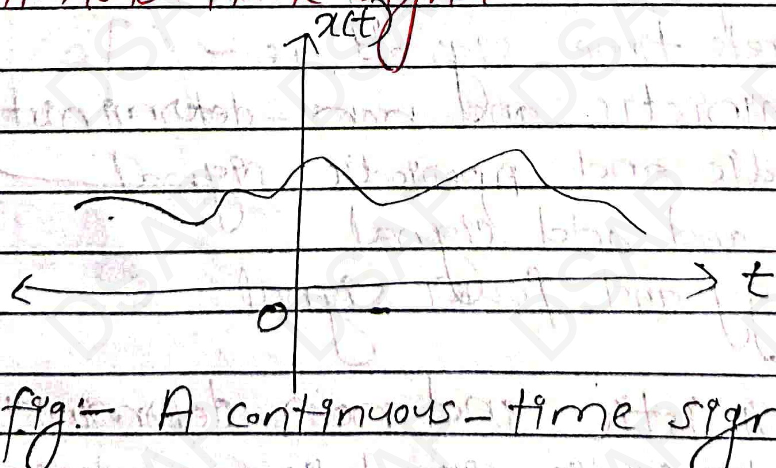

fig:- A continuous-time signal

A continuous time signal may be defined as a signal which move (vary) continuously in time domain. The independent variable is time (t) and a continuous time signal is represented by $x(t)$. (In above figure, $x(t)$ is the continuous time signal in time domain 't'.

for eg:- sine wave, cosine wave, etc.

2) Discrete time signal:-

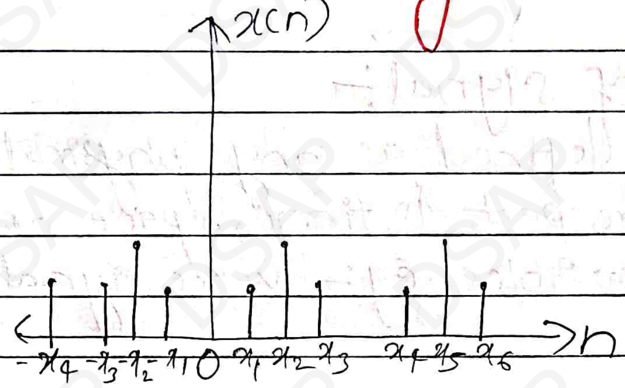

fig:- A discrete time signal

A discrete time signal is defined only at certain time instant. for discrete time signal, the independent variable is time (n) and is denoted by $x(n)$.

We may further classify both continuous and discrete time signals as:-

i) Deterministic and non-deterministic signal
ii) Periodic and Aperiodic signal
iii) Even and odd signal
iv) Energy and power signal

i) Deterministic and non deterministic signal:-

The deterministic signal is one whose future value can be predicted from the knowledge of present and past values.

A signal is said to be deterministic signal

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if it can be described without any uncertainity.
The pattern of this type of signal is regular and can be characterized mathematically.
Also, the nature and amplitude of such a signal at any time can be predicted.

Eg:- i) $x(t) = bt$.
This is ramp signal whose amplitude increases linearly with time 't' and slope 'b'.

ii) $x(t) = a \sin \omega t$

iii) $x(n) = \begin{cases} 2 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

A non deterministic signal is one whose occurence is always random in nature. The pattern of such signal is quite irregular, they are also known as random signals. Example: thermal noise generated in an electric circuit. Such a noise has probabilistic behavior.

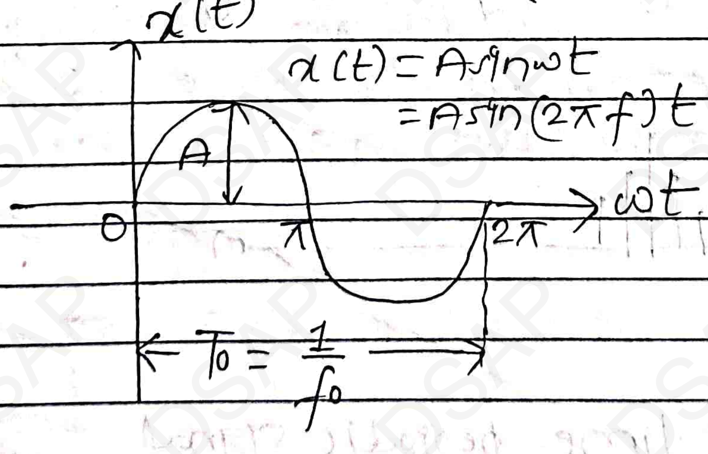

fig:- Deterministic signal

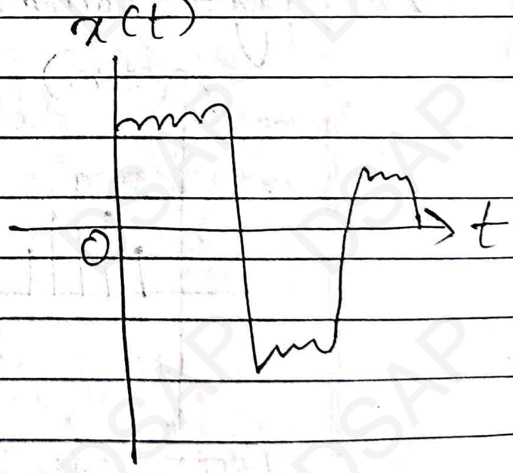

fig:- Non-deterministic/random signal

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ii) Periodic and Aperiodic signal :-

A periodic signal is that type of signal which have fixed patterns and repeats over and over with same time period 't'.

A signal is said to be periodic if it satisfies the following conditions:

$x(t) = x(t+T)$ for all t $(-\infty < t < \infty)$
where T is +ve constant value known as time period.

$x(n) = x(n+N)$ for discrete time signal.
where N is +ve integer called period of $x(n)$.

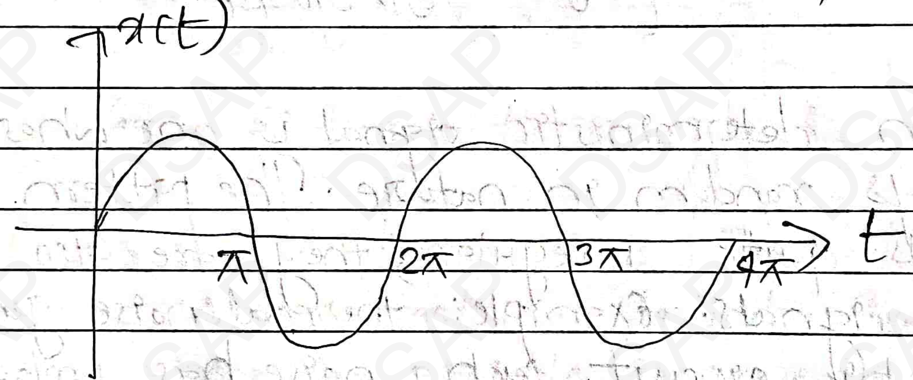


fig :- Continuous time periodic signal

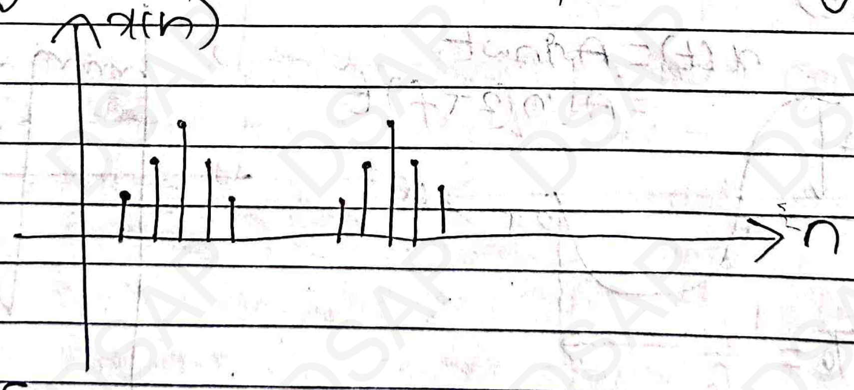


fig :- discrete time periodic signal

A signal whose set of value or pattern does not repeat after certain interval of time is known as aperiodic signal.

for aperiodic signal,

$x(t) \neq x(t+T)$ for all t, $(-\infty < t < \infty)$

$x(n) \neq x(n+N)$ for all n, $(-\infty < n < \infty)$

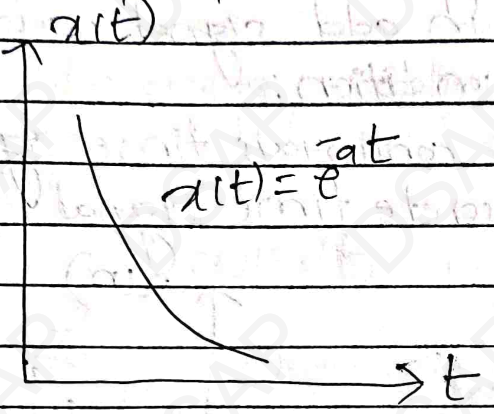


fig:- Continuous time aperiodic signal

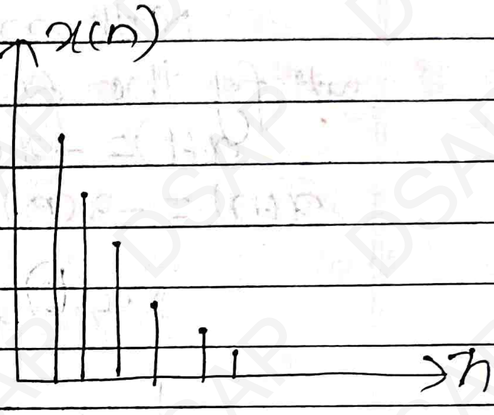


fig: Discrete time aperiodic signal

iii) Even and odd signal:- (symmetric or antisymmetric)

An even signal is that type of signal which exhibits symmetry in time domain.

This type of signal is identical about the origin. Mathematically, an even signal must satisfy the following condition:

$x(t) = x(-t)$ for continuous time signal

$x(n) = x(-n)$ for discrete time signal

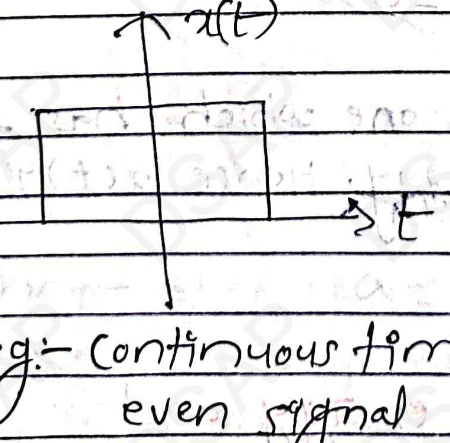


fig:- Continuous time even signal

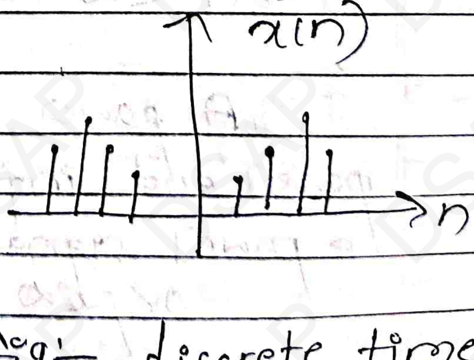


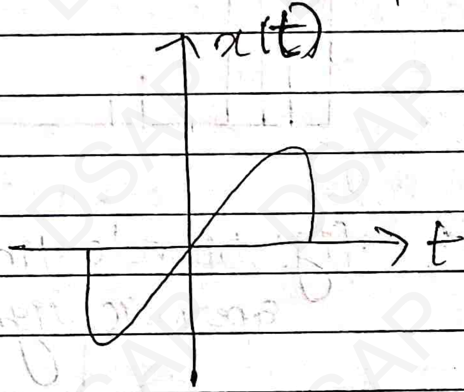
fig:- discrete time even signal

Similarly, an odd signal is that type of signal which exhibit antisymmetry ie odd signal are not identical about origin.

Mathematically, an odd signal must satisfy the following condition:

$x(-t) = -x(t)$ for continuous time signal

$x(-n) = -x(n)$ for discrete time signal



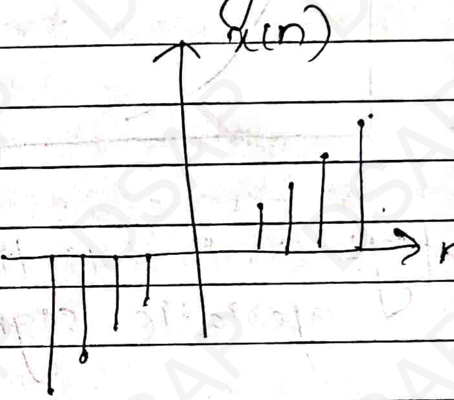


fig:- Continuous time odd signal

fig:- discrete time odd signal

iv) Energy and Power signal:-

The energy signal is one which has finite energy and zero average power. Hence $x(t)$ is an energy signal if

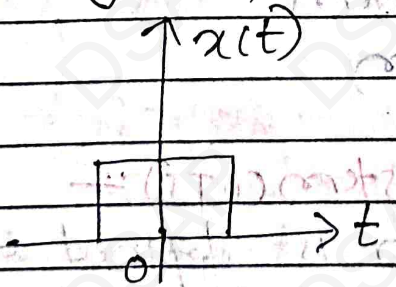
$0 < E < \infty$ and $P = 0$.

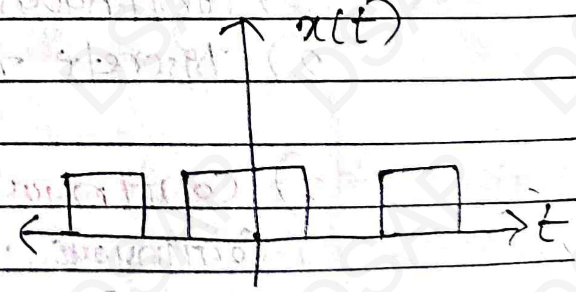
A power signal is one which has finite power and infinite energy. Hence $x(t)$ is a power signal if

$0 < P < \infty$ and $E = \infty$

*) Difference between energy and power signal:-

Energy signal	Power signal
1) Total normalized energy is finite and non-zero.	1) The normalized average power is finite and non zero.
2) The energy is obtained by $$E=\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt$$ for continuous time signal $x(t)$.	2) The average power is given by $$P=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\lvert x(t)\rvert^2\,dt$$
3) Nonperiodic or aperiodic signals are energy signals.	3) Practical periodic signals are power signal.
4) These signals are time limited.	4) These signals exist over infinite time.
5) Power of energy signal is zero.	5) Energy of the power signal is infinite.
6) for example - a single rectangular pulse	6) Example - a periodic pulse train





The energy of discrete time signal $x(n)$ is given by $$E=\sum_{n=-\infty}^{\infty}\lvert x(n)\rvert^2$$	The power of discrete time signal $x(n)$ is given by $$P=\lim_{N\to\infty}\frac{1}{N}\sum_{-N/2}^{N/2}\lvert x(n)\rvert^2$$

Introduction of system:-

A system is defined as a physical device that performs an operation on a signal to give a desired output. The system is characterized by the type of operation that it performs on a signal. for example:- A filter is used to reduce the noise and interference.

mathematically,

$$y(t) = f[x(t)]$$

where, $y(t)$ = output of system
$x(t)$ = Input of system
f = functioning of system

$$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$$

fig:- Block diagram of system

other examples: amplifiers, communication channel, etc.

*) Types of system:

1) continuous time system
2) Discrete time system

1) continuous time system (CTS):-

Continuous time system is defined as the system in which the associated signals are continuous in nature.

This means, if the continuous signal $x(t)$ is given as input to the system and the output is also a continuous with time.

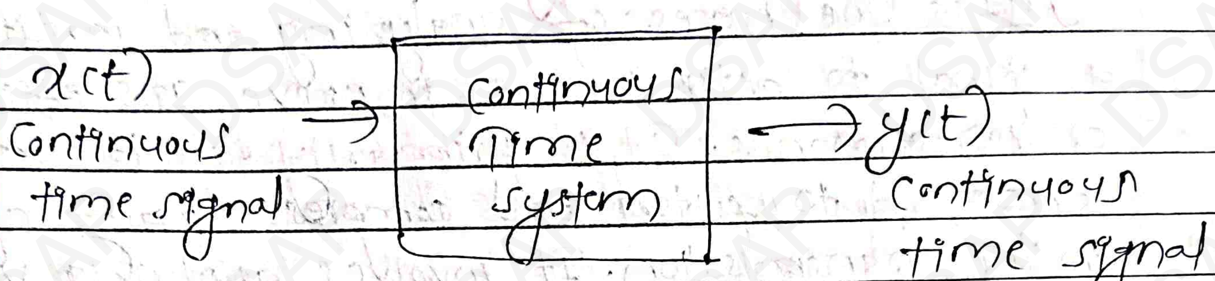


fig:- Block diagram of continuous time system

$$y(t) = f[x(t)]$$

Eg:- power supply, audio/video amplifier

2) Discrete Time system (DTS):-

Discrete time system may be defined as those system in which the associated signals are discrete time signal ie input and output of discrete time system are both discrete signal.

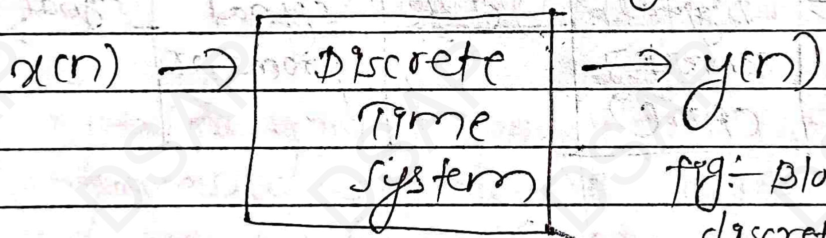


fig:- Block diagram of discrete time system

$$y(n) = f[x(n)]$$

eg:- microprocessor, flip flops, shift registers, etc.

Signal Analysis:-

The signal analysis describes the field of study whose goal is to collect, understand and deduce information and intelligence from various signals.

Signal processing is a method of extracting information from the signal which, in turn, depends upon the type of signal and the nature of information it carries.

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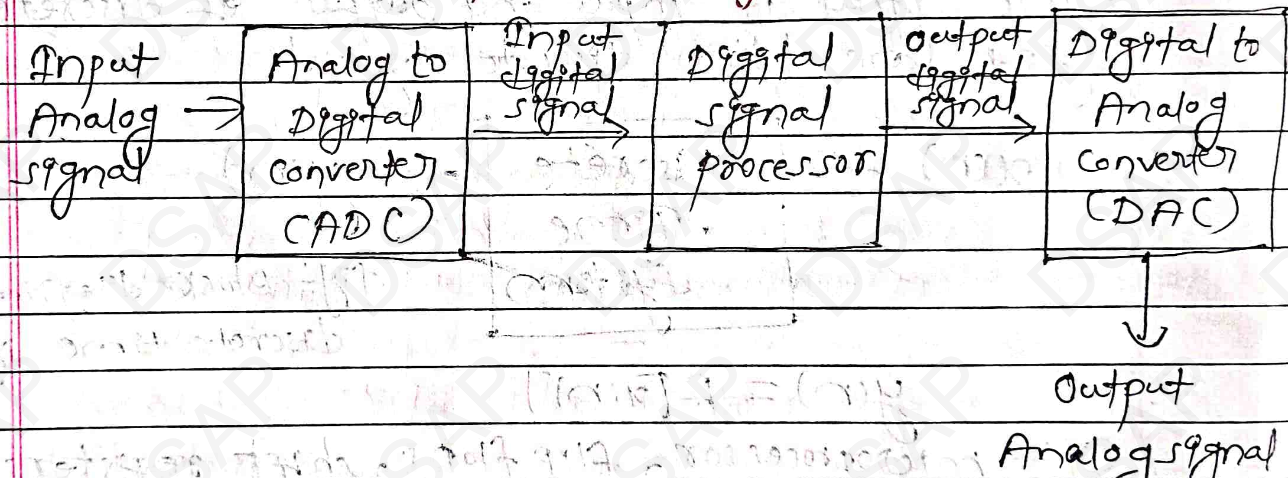
Digital signal Processing (DSP) :-

It is the process of analyzing and modifying a signal to optimize or improve its efficiency or performance. It is primarily used to detect error, and to filter and compress analog signals in the transmission. It involves applying various mathematical and computational algorithms to analog and digital signals to produce a signal of higher quality than the original signal. It facilitates the sharing of a single processor among a number of signals by time sharing.

Q) what are basic elements of Digital signal processing?

Basic elements of Digital signal Processing :-



Block diagram of
fig :- Basic elements of DSP.

Most of the signals generated are analog in nature for example ; sound, video, temperature, pressure etc. If such signals are processed by a digital signal processing system, then the signals must be digitized called digital form. Hence, input is given through ADC and output is obtained through DAC. The basic

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elements of DSP are as shown in above block diagram.

1) Analog to digital converter (ADC) :- It converts analog input to digital form which is the appropriate input to the digital processor. The ADC determines sampling rate and quantization error in digitizing operation.

2) Digital signal processor :- It is also called DSP processor. The digital signal processor may be a large programmable digital computer or a small microprocessor ~~microcontroller~~ programmed to perform the desired operations on the input signal. It performs amplification, regeneration, attenuation, filtering, spectral analysis, etc. operations on digital data.

3) Digital to analog converter (DAC) :- Some of the processed signals are required back in their analog original form. for eg :- sound, image, video, etc. Hence the output of digital signal processor is given to DAC which converts digital output of digital signal processor to its equivalent analog form.

Q) What are the advantages of digital signal processing over analog signal processing?

Comparison between DSP & ASP

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Need of D.S.P over ASP :- / Advantages of D.S.P over ASP :-

The digital signal processing (DSP) offers many advantages over analog signal processing. These advantages are as follows:

1) Flexibility :- The DSP signals are flexible. The system can be reconfigured for some other operations by simply changing the software program.

Example: High pass digital filter can be changed to low pass digital filter by changing the software.

2) Accuracy :- Accuracy of digital signal processing systems is much higher than analog systems. The analog systems suffer from component tolerances, their breakdown problems, etc. But in DSP systems these problems are absent.

So, DSP systems are more accurate.

3) Easy storage :- The digital signals can be easily stored on the storage media such as magnetic tapes, disks, etc whereas analog signals suffer from the storage problems like noise, distortion, etc.

4) Cost :- When there is large complexity in the application, then digital signal processing systems are cheaper compared to analog systems. The software control algorithm can be complex, but it can be implemented accurately with less efforts.

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5) Repeatability:- The processing of signals is completely digital in DSP system. Hence, the performance of these systems is exactly repeatable. Example: The lowpass filtering operation performed by digital filter today will exactly same even after 10 years.
But the performance may deteriorate in analog systems because of noise effects and life of components, etc.

6) Adaptability: The DSP systems are easily upgradable since they are software controlled. But such easy upgradation is not possible in analog systems.

7) Universal Compatibility:- The operation of the DSP is decided mainly by software program. Hence universal compatibility is possible in digital signal processing systems whereas it is not possible in analog systems.

8) Size and Reliability:- The DSP systems are small in size, more reliable and less expensive compared to the analog systems.

9) Mathematical processing:- mathematical operations can be accurately performed on digital signals compared to analog signals. Hence, mathematical signals processing algorithms can be routinely implemented on DSP systems. whereas such algorithms are difficult to implement on analog systems.

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*) Disadvantages of DSP :-

Even though the DSP systems have many advantages, they have few drawbacks as:

i) When the analog signals have wide bandwidth, then high speed A/D converters are required. Such high speeds of A/D conversion are difficult to achieve for some signals. For such applications, analog systems must be used.

ii) The digital signal processing systems are expensive for small applications. Hence, the selection is done on the basis of cost complexity and performance.

iii) Sampling quantization errors are occured due to which original signals may be altered.

Applications of DSP :-

There are various application areas of DSP. Some are listed below:

1) Speech processing: Speech is a one dimensional signal. DSP is applied to wide range of speech applications like

→ Speech analysis → Speech coding
→ Speech synthesis → Speech enhancement
→ Speech compression → channel vocoders
→ Speech recognition → Text to speech conversion
→ Speech recognition
→ Equalization

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2) **Image processing :-** Any two dimensional pattern is called an image. Image processing is used in
→ pattern recognition, → image enhancement
→ Image compression → Animation
→ Robotic vision

Digital processing of image requires 2-D DSP tools such as DFT, FFT, and z-transforms.

3) **Radar signal processing :-** Radar stands for "Radio Detection and Ranging". Improvement in signal processing is possible by digital technology. Development of DSP has led to greater sophistication of radar tracking algorithms. Radar system consists of transmit-receive antenna, digital processing system and control unit.

4) **~~Tele~~ Digital communications :-** Application of DSP in digital communications comprises of digital transmission using PCM, digital switching using TDM (Time Division multiplexing), echo control and digital tape-recorders. DSP in telecommunication systems are found to be cost effective due to availability of medium and large scale digital ICs. These ICs have desirable properties such as small size, low cost, low power, immunity to noise and reliability.

5) **Spectral Analysis :-** Frequency-domain analysis is easily and effectively possible in digital signal processing using Fast Fourier Transform (FFT) algorithms. These algorithms reduce the computational complexity and also reduce the computational time.

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6) Sonar signal processing :- Sonar stands for "Sound Navigation and Ranging". Sonar is used to determine the range, velocity and direction of targets that are remote from the observer. Sonar uses sound waves at lower frequencies to detect objects under water. DSP can be used to process sonar signals, for the purpose of navigation and ranging.

7) Biomedical engineering :-
→ X-ray storage and enhancement
→ Ultrasound equipment
→ CT scanning equipments
→ ECG analysis
→ EEG brain mappers
→ Hearing aids
→ Patient monitoring systems
→ Diagnostic tools etc

8) Entertainment applications :-
→ CD/ VCD/ DVD players
→ MPEG, MP3, music, sound and video compression
→ Speech recognition
→ Phone communications

9) Industrial applications :-
→ Robotics
→ Computerized numerically controlled (CNC) machines
→ Security access
→ Power line monitors, etc.

Analog to digital (A/D) and Digital to Analog (D/A) converter:-

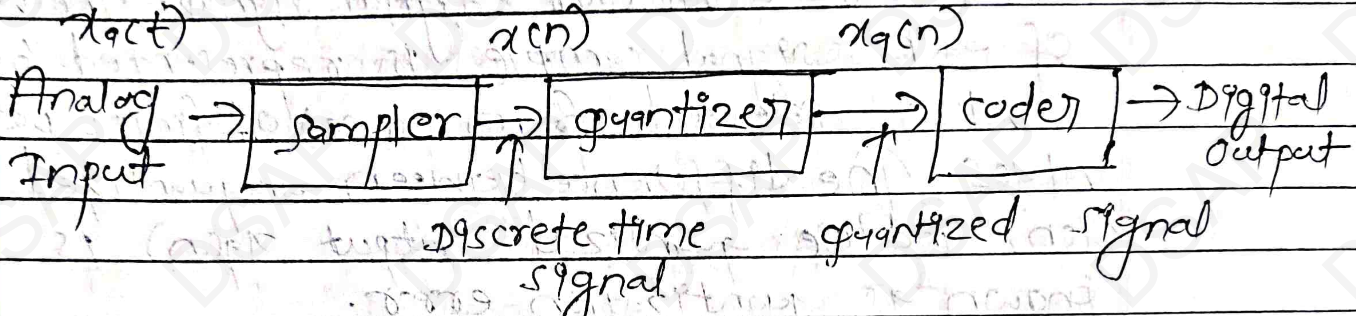

fig:- Block diagram of ADC

Most of the signals of practical interest are analog in nature such as voice signal, radar signal, biological, various communication signals, etc. To process analog signal by digital means, it is first necessary to convert them into digital form. This procedure is known as analog to digital converter. Analog to digital conversion is a three step process as shown in above block diagram.

1) Sampling:-

It converts continuous time signal into a discrete time signal obtained by taking "samples" of continuous time signals at discrete time instants. If $x_a(t)$ is the input to ~~the~~ the sampler, the output is obtained as

$$x(n) = x_a(nT)$$

where, $T \rightarrow$ samping interval

The sampling frequency or sampling rate, f_s is the average number of samples obtained in one second ie $\boxed{f_s = \frac{1}{T}}$

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2) Quantization :- This is the conversion of the discrete time continuous valued signal into a discrete time, discrete valued or digital signal. The value of each signal sample is represented by a value selected from a set of finite possible values. The difference between unquantized sample $x(n)$ and the quantized output $x_q(n)$ is known as quantization error.

3) Coding :- In this process, each discrete value $x_q(n)$ is represented by a binary sequence (b-bit).

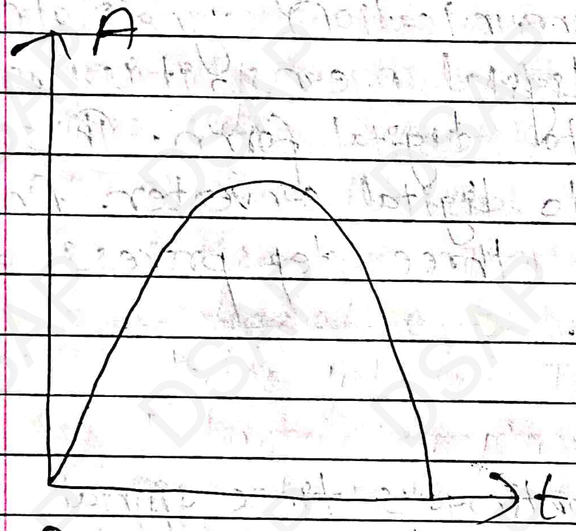


fig:- Analog signal

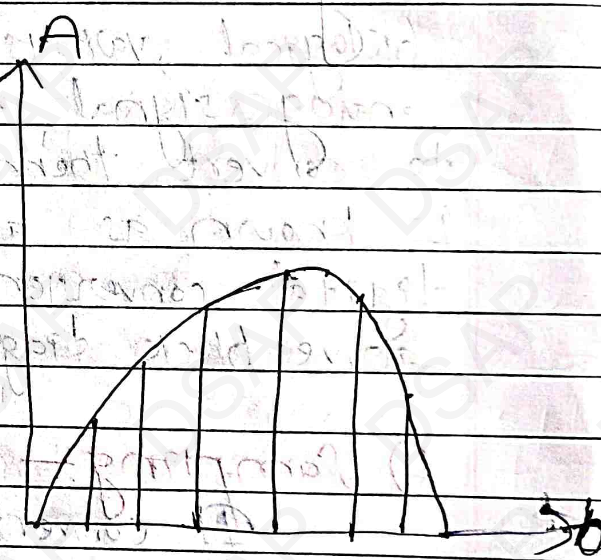


fig:- sampling

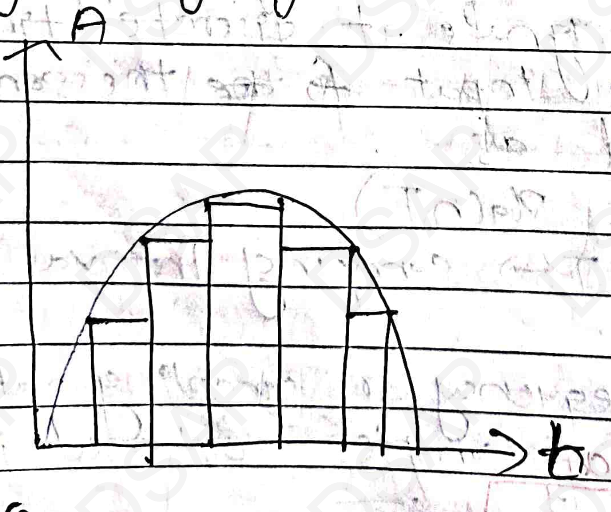


fig:- Quantization

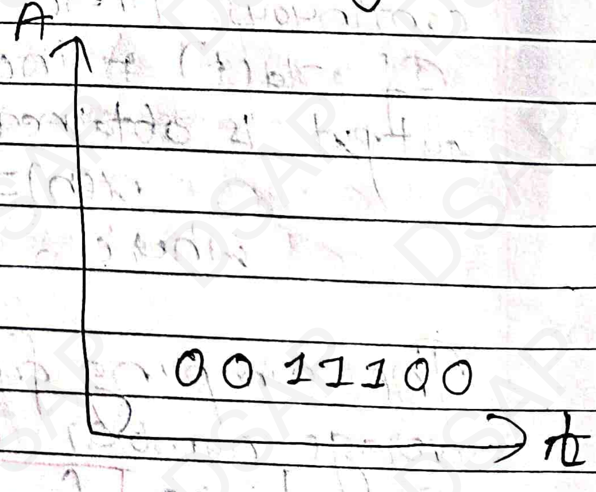


fig:- Encoding

*) Digital to Analog conversion :-

In many cases, it is desirable to convert the digital processed signal into an analog form. This process is known as digital to analog conversion.

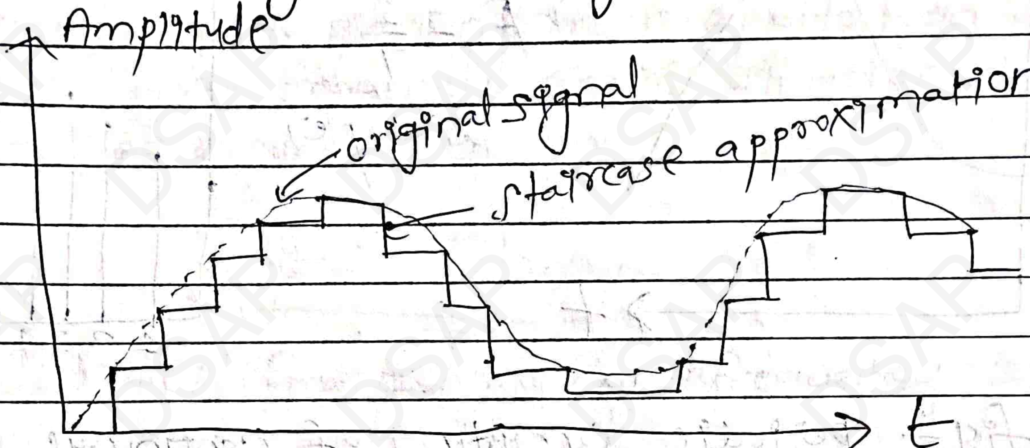


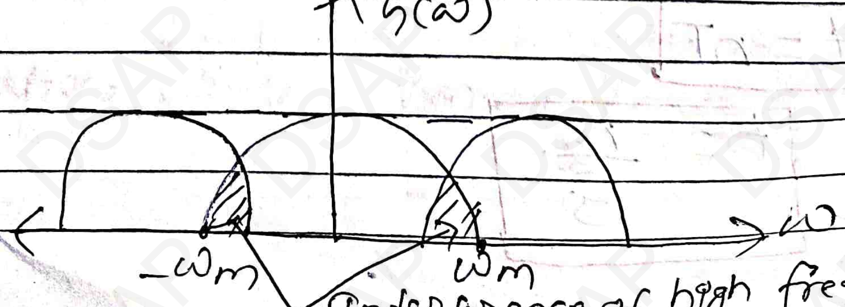
fig :- Digital to Analog converter

The analog signal can be reconstructed from the samples without any loss of information nor any distortion provided that the sampling rate is sufficiently high to avoid the problem known as aliasing.

→ 2 signals overlap

Aliasing is a phenomenon where the high frequency components of the sampled signal interference with each other because of inadequate sampling. It leads to the distortion in the received signal. This is the reason that sampling frequency should be at least twice the bandwidth of the signal.

$$\boxed{f_s \geq 2f_m}$$



Interference of high frequency signals

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Sampling of continuous signals :-

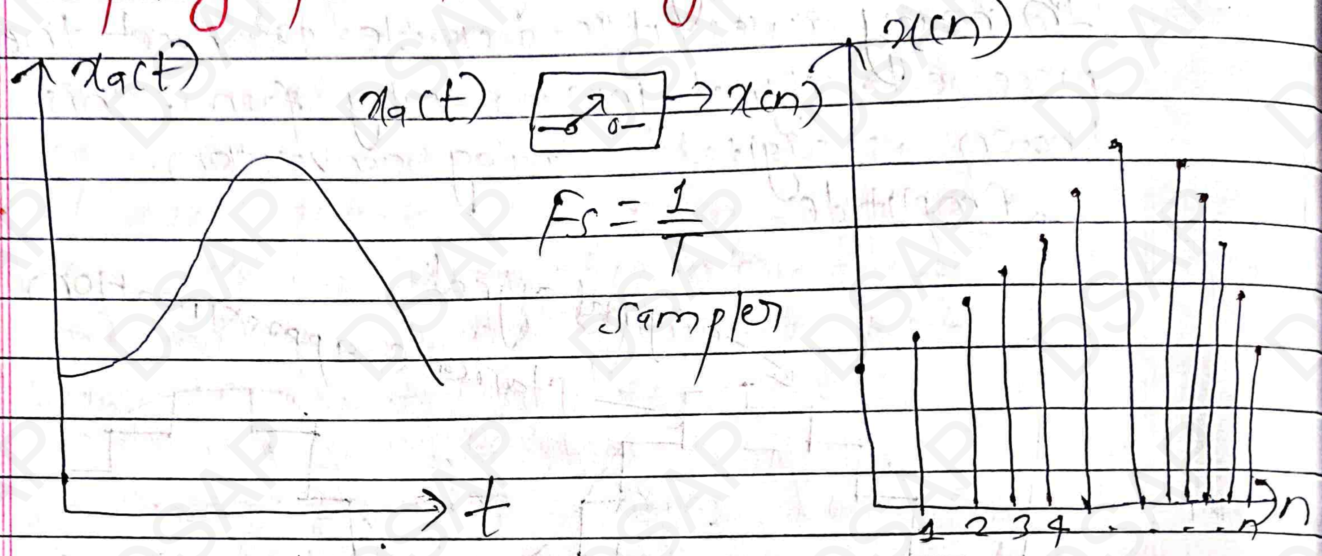

fig :- periodic sampling of continuous signal

Sampling is the process of conversion of continuous time signal $x_a(t)$ into discrete time signal $x(n)$ ie

$$\boxed{x(n) = x_a(nT)} \quad -\infty < n < \infty$$

The time interval 'T' between successive samples is known as sampling period of interval and it's reciprocal is known as sampling rate or sampling frequency.

Sampling frequency $\boxed{(F_s) = \frac{1}{T} \text{ Hz}}$ (sample per second)

The periodic sampling establish a relationship between the time variable 't' and 'n' number of continuous time and discrete time signal as $\boxed{t = nT}$

$$\boxed{t = \frac{n}{F_s}}$$

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Sampling Theorem:-

It states that a continuous time signal can be completely represented in its samples and recover that into an original form if the sampling frequency is greater than or equal to twice of the maximum frequency. In this situation aliasing does not occur.

ie

$$\boxed{F_s \geq 2F_{max}}$$

Where, F_{max} = maximum frequency of analog signal

F_s = Sampling frequency

The boundary condition value required to avoid aliasing is called Nyquist Rate ie

Nyquist rate $\boxed{F_N = 2F_{max}}$

$$\boxed{F_N = F_s}$$

Nyquist interval $\boxed{T_N = \frac{1}{2F_{max}}}$

The folding frequency is given by:

$$\boxed{\text{Folding frequency} = \frac{F_s}{2}}$$

Resolution, $\boxed{\Delta = \frac{X_{max} - X_{min}}{L-1}}$

where, $X_{max} = A_1 + A_2$

$X_{min} = -X_{max}$

L = no. of levels

A_1 and A_2 are amplitude of signal

Sampling of Analog signals:-

let us consider an analog signal,

$$x_a(t) = x_a(nT) \quad -\infty < n < \infty$$

where, $x(t)$ = i/p signal

T = sampling interval

$\frac{1}{T}$ = sampling frequency (F_s)

In discrete time,

$$t = nT = \frac{n}{F_s}$$

Consider $x_a(t) = A\cos(2\pi F t + \theta)$

Replace 't' by 'nT'

$$x_a(nT) = A\cos(2\pi F nT + \theta)$$

$$x(n) = A\cos(2\pi f n + \theta)$$

where, $f = \frac{F}{F_s}$ = frequency of digital signal

F = frequency of analog (original) signal

F_s = sampling frequency

Aliasing Effect is said to occur if $f > 1$ and Aliasing effect can be avoided if $f \leq 1$

or, $\frac{F}{F_s} \leq 1 \Rightarrow F \leq F_s$

$$x(n) = A\cos(2\pi F n \tfrac{1}{F_s} + \theta)$$

$$\rightarrow x(n) = A\cos\left(2\pi \frac{F}{F_s} n + \theta\right)$$

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Q1) For the given continuous time signal obtain its discrete form for sampling frequency
(i) $F_s = 1000\ Hz$
(ii) $F_s = 500\ Hz$

$x(t) = 2\cos 800\pi t$

Soln

i) $F_s = 1000\ Hz$

$x_a(t) = 2\cos 800\pi t$

Replacing 't' by nT

$$x_a(nT) = 2\cos 800\pi nT$$
$$= 2\cos 800\pi \frac{n}{F_s}$$
$$= 2\cos 800\pi \frac{n}{1000}$$
$$= 2\cos \frac{4\pi n}{5}$$

Here, $\frac{4}{5} < 1$, so, there is no aliasing effect

ii) 500 Hz

$x_a(t) = 2\cos 800\pi t$

Replacing 't' by nT

$$x_a(nT) = 2\cos 800\pi nT$$
$$= 2\cos 800\pi \frac{n}{F_s}$$
$$= 2\cos 2\pi \times \frac{400}{500} n$$
$$= 2\cos \frac{8}{5}\pi n$$

Here, $\frac{8}{5} > 1$ so there is an aliasing effect.

To eliminate it,

$$x(n) = 2\cos \frac{8\pi n}{5}$$

$$= 2\cos\left(2\pi - \frac{2\pi}{5}\right)n$$

$$= 2\cos \frac{2\pi}{5} n$$

Q2) Consider an analog signal $x_a(t) = 3\cos 100\pi t$

a) Determine minimum sampling rate required to avoid aliasing effect.

b) Suppose that the signal is sampled at the rate $F_s = 200Hz$, what is the discrete time signal obtained after sampling.

c) Suppose that the signal is sampled at the rate $f_s = 75Hz$, what is the discrete time signal obtained after sampling.

Soln

Here,

a) $x_a(t) = 3\cos 100\pi t$

$$x_a(nT) = 3\cos 100\pi nT$$

$$= 3\cos 100\pi \frac{n}{F_s}$$

$$x(n) = 3\cos \frac{100}{f_s}\pi n$$

To avoid aliasing effect we must have

$$\frac{100}{f_s} \leq 1$$

$$100 \leq F_s$$

So, minimum value of f_s to avoid aliasing effect is 100 Hz.

Then, the signal would be $x(n) = 3\cos \pi n$.

b) $f_s = 200\,Hz$

$$x(t) = 3\cos 100\pi t$$

$$x(nT) = 3\cos 100\pi nT$$

$$= 3\cos 100\pi \frac{n}{f_s}$$

$$= 3\cos 100\pi \frac{n}{200}$$

$$= 3\cos \frac{1}{2}\pi n$$

Here, $\frac{1}{2} < 1$, so there is no aliasing effect.

$\therefore$ Required signal is $\boxed{x(n) = 3\cos \frac{\pi}{2} n}$

c) $f_s = 75\,Hz$

$$x(t) = 3\cos 100\pi t$$

$$x(nT) = 3\cos 100\pi nT$$

$$= 3\cos 100\pi \frac{n}{f_s}$$

$$= 3\cos \frac{100}{75}\pi n$$

$$= 3\cos \frac{4}{3}\pi n$$

Here, $\frac{4}{3} > 1$, so there is aliasing effect.

Now, to eliminate it:

$$x(n) = 3\cos \frac{4\pi}{3} n$$

$$= 3\cos\left(2\pi - \frac{2\pi}{3}\right) n$$

$$= 3\cos \frac{2\pi}{3} n$$

$\frac{2}{3} < 1$, so aliasing effect is eliminated.

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$\therefore$ Required signal is: $\boxed{x(n) = 3\cos \frac{2\pi}{3} n}$

Q3) Consider the analog signal:
$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$
What is the Nyquist Rate and sampling Rate for the signal?

Soln

Here, $x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$

We have : $\omega = 2\pi f$

Now,

$\omega_1 = 50\pi$	$w_2 = 300\pi$	$w_3 = 100\pi$
$2\pi f_1 = 50\pi$	$2\pi f_2 = 300\pi$	$2\pi f_3 = 100\pi$
$f_1 = \frac{50}{2}$	$f_2 = \frac{300\pi}{2\pi}$	$f_3 = \frac{100\pi}{2\pi}$
$f_1 = 25\,Hz$	$f_2 = 150\,Hz$	$f_3 = 50\,Hz$

Here, $f_{max} = 150\,Hz$

Sampling frequency $(F_s) \geq 2 f_{max}$
$\geq 2 \times 150$
$\geq 300\,Hz$

Nyquist Rate, $F_N = 2 f_{max}$
$= 2 \times 150$
$= 300\,Hz$

Sampling Rate $(f_s) = 300\,Hz$

(4) Consider the analog signal:

$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$

a) What is the Nyquist rate for the signal?

Soln

We have: $\omega = 2\pi F$

$\omega_1 = 2000\pi$	$\omega_2 = 6000\pi$	$\omega_3 = 12000\pi$
$2\pi f_1 = 2000\pi$	$2\pi f_2 = 6000\pi$	$2\pi f_3 = 12000\pi$
$f_1 = \frac{2000\pi}{2\pi}$	$f_2 = \frac{6000\pi}{2\pi}$	$f_3 = \frac{12000\pi}{2\pi}$
$F_1 = 1000\,Hz$	$F_2 = 3000\,Hz$	$F_3 = 6000\,Hz$

Here, $F_{max} = 6000\,Hz$

$\therefore$ Nyquist Rate, $F_N = 2 \times F_{max}$
$= 2 \times 6000\,Hz$
$= 12{,}000\,Hz$

b) Assume now that we sample the signal using sampling rate $F_s = 5000$ samples/second. What is the discrete time signal obtained after sampling?

Soln

Here, $F_s = 5000\,Hz$

$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$

$x(nT) = 3\cos 2000\pi nT + 5\sin 6000\pi nT + 10\cos 12000\pi nT$

$x(n) = 3\cos 2000\pi \frac{n}{F_s} + 5\sin 6000\pi \frac{n}{F_s} + 10\cos 12000\pi \frac{n}{F_s}$

$= 3\cos \frac{2000}{5000}\pi n + 5\sin \frac{6000}{5000}\pi n + 10\cos \frac{12000}{5000}\pi n$

$= 3\cos \frac{2}{5}\pi n + 5\sin \frac{6}{5}\pi n + 10\cos \frac{12}{5}\pi n$

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$$x(n) = 3\cos\frac{2}{5}\pi n + 5\sin\left(2\pi - \frac{4\pi}{5}\right)n +$$

$$10\cos\left(2\pi + \frac{2\pi}{5}\right)n$$

$$= 3\cos\frac{2}{5}\pi n - 5\sin\frac{4\pi}{5}n + 10\cos\frac{2\pi}{5}n$$

$$\therefore x(n) = 13\cos\frac{2\pi}{5}n - 5\sin\frac{4\pi}{5}n$$

This is the required discrete time signal.

c) What is the analog signal $y_a(t)$ that we can reconstruct from the samples if we use ideal interpolation.

Soln

$$x(n) = 13\cos\frac{2\pi}{5}n - 5\sin\frac{4\pi}{5}n$$

We have: $t = nT$

or, $n = \frac{t}{T}$

or, $n = t \cdot F_s$

Now,

$$x(n) = 13\cos\frac{2\pi}{5}n - 5\sin\frac{4\pi}{5}n$$

$$y_a(t) = 13\cos\frac{2\pi}{5}\cdot t\cdot F_s - 5\sin\frac{4\pi}{5}\cdot t\cdot F_s$$

$$= 13\cos\frac{2\pi}{\cancel{5}}t \times \overset{1000}{\cancel{5000}} - 5\sin\frac{4\pi}{\cancel{5}}t \times \overset{1000}{\cancel{5000}}$$

$$\therefore y_a(t) = 13\cos 2000\pi t - 5\sin 4000\pi t$$

This is the required analog signal

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c) What is the folding frequency?

soln

$$\text{folding frequency} = \frac{f_s}{2}$$

$$= \frac{5000}{2}$$

$$= 2500 \text{ Hz}$$

Ans#

Q5) A digital communication link carries binary coded words representing samples of an input signal.

$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$

The link is operated at 10,000 bits/sec and each input sample is quantized into 1024 different voltage levels.

a) What is the sampling frequency and folding frequency?

soln

given, No. of quantization level (L) = 1024

No. of bits per sample (b) = $\log_2(L)$

$= \log_2(1024)$

$= 10$ bits/sample

bit rate = 10,000 bits/sec

$\therefore$ Sampling frequency (F_s) = $\frac{\text{Bit rate}}{\text{bits/sample}}$

$= \frac{10{,}000 \text{ bits/sec}}{10 \text{ bits/sample}}$

$= 1000$ sample/sec

$= 1000$ Hz

$\boxed{\text{Bit rate} = \underset{(F_s)}{\text{sample/sec}} * \text{bits/sample}}$

$\therefore$ folding frequency $= \frac{F_s}{2} = \frac{1000}{2} = 500\,Hz$

b) What is the Nyquist rate for the signal $x_a(t)$?

Soln

We have, $\omega = 2\pi F$

$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$

$\omega_1 = 600\pi$	$\omega_2 = 1800\pi$
$2\pi F_1 = 600\pi$	$2\pi f_2 = 1800\pi$
$F_1 = \frac{600\pi}{2\pi}$	$f_2 = \frac{1800\pi}{2}$
$f_1 = 300\,Hz$	$f_2 = 900\,Hz$

$\therefore f_{max} = 900\,Hz$

Nyquist Rate $(F_N) = 2 \times f_{max}$
$= 2 \times 900$
$= 1800\,Hz$

Nyquist Interval $= \frac{1}{2f_{max}} = \frac{1}{1800}$ sec

c) What are the frequencies in resulting discrete time signal $x(n)$?

Soln

Here, $x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$

$x(nT) = 3\cos 600\pi nT + 2\cos 1800\pi nT$

$= 3\cos 600\pi \frac{n}{F_s} + 2\cos 1800\pi \frac{n}{F_s}$

$= 3\cos \frac{600\pi n}{1000} + 2\cos \frac{1800\pi}{1000}$

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$$x(n) = 3\cos\frac{3\pi}{5}n + 2\cos\frac{9\pi}{5}n$$

$$= 3\cos\frac{3\pi}{5}n + 2\cos\left(2\pi - \frac{\pi}{5}\right)n$$

$$= 3\cos\frac{3\pi}{5}n + 2\cos\frac{\pi}{5}n \quad \text{--- (i)}$$

The frequencies in the signal are determined by:

$$x(n) = 3\cos 2\pi n\left(\frac{3}{5\times 2}\right) + 2\cos 2\pi n\left(\frac{1}{2\times 5}\right)$$

$$= 3\cos 2\pi n\left(\frac{3}{10}\right) + 2\cos 2\pi n\left(\frac{1}{10}\right)$$

Therefore, Required frequencies are

$$f_1 = \frac{3}{10} \quad \text{and} \quad f_2 = \frac{1}{10}$$

d) What is the resolution (Δ)?

Soln

$$\text{Resolution } (\Delta) = \frac{\text{Dynamic Range}}{L-1}$$

$$= \frac{X_{max} - X_{min}}{L-1}$$

$$= \frac{(A_1+A_2) - (-(A_1+A_2))}{1024-1}$$

$A_1 = 3$ and $A_2 = 2$, $X_{min} = -X_{max}$

$$= \frac{5+5}{1023}$$

$$= \frac{10}{1023}$$

$$= 9.77\times 10^{-3} \text{ units}$$

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OR

from above eqn (9), the value of n ranges from 0 to 5.

$$\text{Now, } x(0) = 3\cos\frac{3\pi}{5}\cdot 0 + 2\cos\frac{\pi}{5}\cdot 0$$

$$= 3\cos 0 + 2\cos 0$$

$$= 3+2$$

$$= 5\ (X_{max})$$

$$x(5) = 3\cos\frac{3\pi}{5}\cdot 5 + 2\cos\frac{\pi}{5}\cdot 5$$

$$= 3\cos 3\pi + 2\cos\pi$$

$$= -3-2$$

$$= -5\ (X_{min})$$

$$\therefore \text{Resolution } (\Delta) = \frac{X_{max} - X_{min}}{L-1}$$

$$= \frac{5-(-5)}{1024-1}$$

$$= \frac{10}{1023}$$

$$= 9.77\times 10^{-3} \text{ units}$$

e) Also, obtain reconstructed continuous time signal.

soln

$$x(n) = 3\cos\frac{3\pi}{5}n + 2\cos\frac{\pi}{5}n$$

we have: $t = nT$

$$n = \frac{t}{T}$$

$$\boxed{n = t\cdot F_s}$$

$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

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Then,

$y(t) = 3\cos\frac{3\pi}{5} t \cdot F_s + 2\cos\frac{\pi}{5} t \cdot F_s$

$= 3\cos\frac{3\pi}{\cancel{5}} t \cdot \cancel{1000}^{200} + 2\cos\frac{\pi}{\cancel{5}} t \cdot \cancel{1000}^{200}$

$\therefore y(t) = 3\cos 600\pi t + 2\cos 200\pi t$

Ans

Q6) Find Nyquist rate and Nyquist interval for the given continuous time signal.

$x(t) = \frac{1}{2\pi} \cos 4000\pi t \cdot \cos 1000\pi t$

Soln

Here, $x(t) = \frac{1}{2\pi} \cos 4000\pi t \cdot \cos 1000\pi t$

$= \frac{1}{2\pi} \times \frac{1}{2} \times (2\cos 4000\pi t \cdot \cos 1000\pi t)$

$= \frac{1}{4\pi} [\cos(4000+1000)\pi t + \cos(4000-1000)\pi t]$

$= \frac{1}{4\pi} [\cos 5000\pi t + \cos 3000\pi t]$

$\omega_1 = 2\pi f_1$	$\omega_2 = 3000\pi$
or, $5000\pi = 2\pi f_1$	or, $2\pi f_2 = 3000\pi$
or, $f_1 = \frac{5000\pi}{2\pi}$	or, $f_2 = \frac{3000\pi}{2\pi}$
or, $f_1 = 2500$ Hz	or, $f_2 = 1500$ Hz

$\therefore F_{max} = 2500$ Hz

$\therefore$ Nyquist Rate, $F_N = 2 \times F_{max} = 2 \times 2500$
$= 5000$ Hz

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Nyquist interval $= \frac{1}{f_N}$

$= \frac{1}{5000}$

$= 2\times10^{-4}$ sec

Q7) Consider the signal $x(t) = 10\cos 2\pi\times1000t + 5\cos 2\pi\times5000t$ is to be sampled

(i) Determine the Nyquist rate for the signal.

(ii) If the signal is sampled at 4 KHZ, will the signal be recovered from its sample.

Soln

Given, $x(t) = 10\cos 2\pi\times1000t + 5\cos 2\pi\times5000t$

i) $\omega_1 = 2\pi\times1000$ $\qquad$ $\omega_2 = 2\pi\times5000$

$2\pi f_1 = 2\pi\times1000$ $\qquad$ $2\pi f_2 = 2\pi\times5000$

$f_1 = 1000$ HZ $\qquad$ $f_2 = 5000$ HZ

$\therefore f_{max} = 5000$ HZ

Nyquist Rate, $f_N = 2\times f_{max}$

$= 2\times5000$

$= 10{,}000$ HZ

Ans

ii) Sampled frequency $(f_s) = 4$ KHZ $= 4000$ HZ

$x(t) = 10\cos 2\pi\times1000t + 5\cos 2\pi\times5000t$

$x(nT) = 10\cos 2\pi\times1000\times nT + 5\cos 2\pi\times5000\times nT$

$x(n) = 10\cos 2\pi\times\frac{1000n}{f_s} + 5\cos 2\pi\times\frac{5000n}{f_s}$

$= 10\cos 2\pi\times\frac{1000}{4000}n + 5\cos 2\pi\times\frac{5000n}{4000}$

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$$x(n) = 10\cos 2\pi \frac{n}{4} + 5\cos 2\pi \frac{5n}{4}$$

$$= 10\cos 2\pi \frac{n}{4} + 5\cos 2\pi \left(1 + \frac{1}{4}\right)n$$

$$= 10\cos 2\pi \frac{n}{4} + 5\cos 2\pi \frac{n}{4}$$

$$= 15\cos 2\pi \frac{n}{4}$$

$$n = t \times F_s = t \times 4000$$

$$g(t) = 15\cos 2\pi \times \frac{t \times 4000}{4}$$

$$= 15\cos 2\pi \times 1000\, t$$

We observed that the reconstructed signal contains only one frequency of 1,000 Hz and amplitude of 15. Thus, the effect of the signal of frequency 5,000 Hz is completely lost. This shows that with the sampling rate of 4,000 Hz, the original signal is not recovered from its sample. Hence, the minimum sampling frequency ~~of~~ for the given signal to avoid aliasing is 10,000 Hz.

2017 Fall

Q8) The discrete time signal $x(n) = 6.5\cos(0.1\pi)n$ is quantized with a resolution of 0.01. How many bits are required in the analog to digital converter if the maximum frequency of the signal is 500 Hz. Also, determine the reconstructed continuous time signal.

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Soln

given, $x(n) = 6.5\cos(0.1\pi)n$

resolution $(\Delta) = 0.01$

bits/sample = ?

$f_{max} = 500Hz$

Reconstructed continuous time signal = ?

$x(n) = 6.5\cos(0.1\pi)n$

$= 6.5\cos 2\pi \times \left(\frac{0.1}{2}\right)n$

$= 6.5\cos 2\pi \times 0.05n$

i-ans) $\therefore f = 0.05\,\cancel{Hz}$

$A = 6.5$

$L = ?$

$\Delta = \frac{X_{max} - X_{min}}{L-1}$

or, $0.01 = \frac{6.5+6.5}{L-1}$

or, $L = \frac{13}{0.01} + 1$

or, $L = 1301$

$\therefore$ bit/sample $= \log_2(L)$

$= \log_2(1301)$

$= 10.34$ Ans

ii-ans) ~~xxxx~~ sampling frequency $(f_s) = 2 \times f_{max}$

$= 2 \times 500$

$= 1000Hz$

$x(n) = 6.5 \cos(0.1\pi)n$

put $n = t \times F_s$

$y(t) = 6.5 \cos(0.1\pi) t \cdot F_s$
$= 6.5 \cos(0.1\pi) t \cdot 1000$
$= 6.5 \cos 2\pi \times 50t$

2015 fall

$f = 50Hz$

Q9) 1.b) A digital communication link binary coded rod representing samples of an input signal $x(t) = 3\cos 5000\pi t + 2\sin 6000\pi t$. The length is operated at 18,000 bits/sec and each inputs sample is quantized into 512 well different voltage level.

① what is the discrete time signal obtained after sampling.

② What is the nyquist rate and resolution of the signal.

soln

No. of quantization level (L) = 512

No. of bits/sample $(b) = \log_2(L)$
$= \log_2(512)$
$= 9$ bits/sample

bits/sec = 18,000 bits/sec

∴ Sampling frequency $(F_s) = \dfrac{\text{bits/sec}}{\text{bits/sample}}$
$= \dfrac{18,000}{9}$
$= 2,000$ sample/sec
$= 2,000$ Hz

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i-ans) $x(t) = 3\cos 5000\pi t + 2\sin 6000\pi t$

$x(nT) = 3\cos 5000\pi nT + 2\sin 6000\pi nT$

$x(n) = 3\cos 5000\pi \frac{n}{F_s} + 2\sin 6000\pi \frac{n}{F_s}$

$= 3\cos 5000\pi \frac{n}{2000} + 2\sin 6000\pi \frac{n}{2000}$

$= 3\cos \frac{5\pi}{2} n + \cancel{2\sin 3\pi n} \to 0$

$\therefore x(n) = 3\cos \frac{2\pi 5}{4} n + 2\sin 2\pi(1.5)n$

$= 3\cos 2\pi(1.25)n + 2\sin 2\pi(1.5)n$

This is the discrete time signal obtained after sampling.

ii-ans) $x(t) = 3\cos 500\pi t + 2\sin 6000\pi t$

$\omega_1 = 500\pi$ $\qquad$ $\omega_2 = 6000\pi$

$2\pi f_1 = 500\pi$ $\qquad$ $2\pi f_2 = 6000\pi$

$f_1 = \frac{500\pi}{2\pi}$ $\qquad$ $f_2 = 3000$

$f_1 = 250\ Hz$ $\qquad$ $\therefore f_2 = 3000\ Hz$

Here, $f_{max} = 3000\ Hz$

$\therefore$ Nyquist rate, $f_N = 2 \times f_{max}$
$= 2 \times 3000$
$= 6000\ Hz$

Resolution $(\Delta) = \frac{X_{max} - X_{min}}{L-1}$

$= \frac{(A_1 + A_2) - (-(A_1 + A_2))}{L-1}$

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$$= \frac{(3+2)+(3+2)}{512-1}$$

$$= \frac{5+5}{511}$$

$= 0.019$ units

2017 spring

1.b) For a digital system with a max audio input frequency of 6 KHz, determine the min. sample rate and the alias frequency produced if a 7 KHz audio signal were allowed to enter the analog to digital converter.

soln

given, let f_{max} be the frequency of audio

$\therefore f_{max} = 6 KHz$

$\therefore$ min sample rate $\geq 2 f_{max} = 2 f_{max} = 2\times6$
$= 12$ KHz

sampling frequency (FS) 7 KHz