DSAP

1 (b) $h[n] = \{1, \underset{\uparrow}{4}, 2, -1\}$, $y[n] = \{1, \underset{\uparrow}{4}, 13, 15, 4, -3\}$

Here length of $h[n]$ is $M = 4$ & let length of $x[n]$ be N

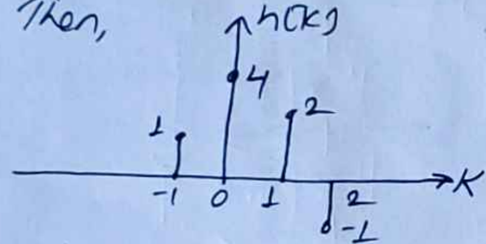
Then length of $y[n]$ is $N+M-1 = 6 \Rightarrow N = 6+1-M$
$= 6+1-4 = 3$

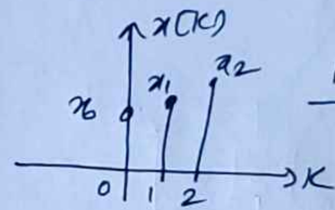
Also $h[n]$ starts at -1 & $y[n]$ also starts at $n=-1$

$\therefore$ $x[n]$ should starts at $n=0$

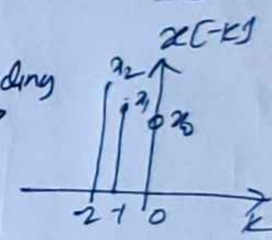
Let $x[n] = \{\underset{\uparrow}{x_0}, x_1, x_2\}$

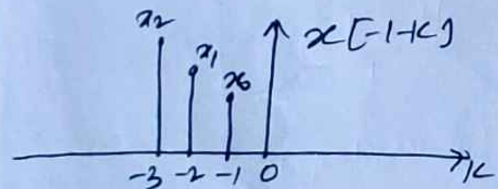
Then,

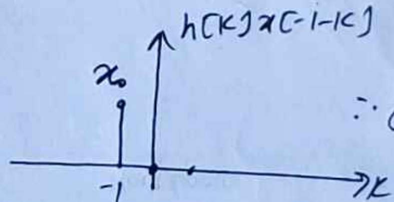




Folding $\longrightarrow$

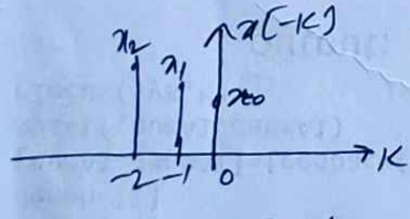


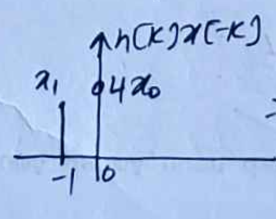




$\therefore y[-1] = x_0$

$\Rightarrow \boxed{1 = x_0}$

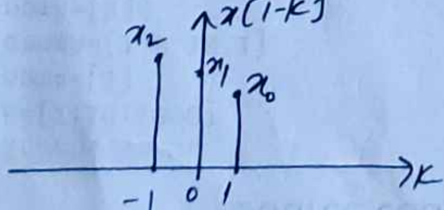


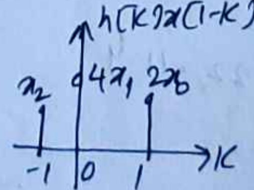


$\therefore y[0] = x_1 + 4x_0$

$\Rightarrow 4 = x_1 + 4\times1$

$\therefore \boxed{x_1 = 4-4 = 0}$





$\therefore y[1] = x_2 + 4x_1 + 2x_0$

$\Rightarrow 13 = x_2 + 4\times0 + 2\times1$

$\therefore \boxed{x_2 = 13-2 = 11}$

$\therefore x[n] = \{\underset{\uparrow}{1}, 0, 11\}$ or $x[n] = \{\underset{\uparrow}{1}, 2, 3\}$

(from forward shifting) i.e. shifting from left to right

(from backward shifting)

2 a. $\Rightarrow y[n] = n\,x[n^2]$

(i) <u>For linearity</u>

Let $x[n] = a x_1[n] + b x_2[n]$

Then, $y[n] = n\,x[n^2] = n\{a x_1[n^2] + b x_2[n^2]\}$

$a y_1[n] + b y_2[n] = a n x_1[n^2] + b n x_2[n^2]$

Here, $y[n] = a y_1[n] + b y_2[n]$

$\therefore$ The system is <u>linear</u>

(ii) For <u>causality</u>

(v) & for <u>memory</u>

$y[2] = 2x[4]$

o/p depends on future value of i/p

$\therefore$ The system is <u>non-causal</u> & <u>with memory</u>

(iii) <u>For stability</u>

Let the i/p be bounded, i.e. $|x[n]| \le M_x < \infty$

The o/p, $|y[n]| = |n\,x[n^2]|$

$= |n|\,|x[n^2]|$

$\le |n|\,M_x$

as $n \to \infty$, $|y[n]| \to \infty$ Here o/p is not bounded

$\therefore$ The system is <u>not stable</u>

(iv) <u>For time invariant</u>

Response of delayed i/p, $y[n,k] = n\,x[n^2-k]$

Delayed response, $y[n-k] = (n-k)\,x[(n-k)^2]$

Here, $y[n,k] \neq y[n-k]$

$\therefore$ The system is <u>time-variant</u>

2 (b) $x[n] = n\, 0.4^n u[n-2]$

$$n a^n u[n] \xleftrightarrow{z} \frac{a z^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$$

$$(n-2) a^{n-2} u[n-2] \xleftrightarrow{z} z^{-2} \cdot \frac{a z^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$$

$$2 a^{n-2} u[n-2] \xleftrightarrow{z} 2 \cdot z^{-2} \cdot \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

$$n a^{n-2} u[n-2] \xleftrightarrow{z} z^{-2} \frac{a z^{-1}}{(1-az^{-1})^2} + \frac{2 z^{-2}}{1-az^{-1}}, \quad |z| > |a|$$

$$n a^n u[n-2] \xleftrightarrow{z} a^2 \left[\frac{z^{-2} \cdot a z^{-1}}{(1-az^{-1})^2} + \frac{2 z^{-2}}{1-az^{-1}} \right], \quad |z| > |a|$$

$$= a^2 z^{-2} \frac{a(z^{-1}) + 2(1-az^{-1})}{(1-az^{-1})^2}, \quad |z| > |a|$$

$$= a^2 z^{-2} \times \frac{2 - az^{-1}}{(1-az^{-1})^2}, \quad |z| > |a|$$

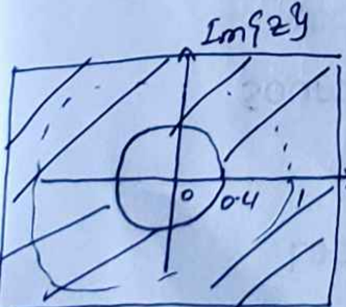
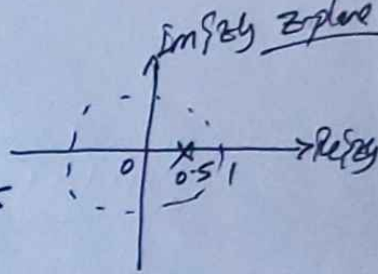


Fig:- ROC of given function

3 (a) $H(z) = \dfrac{2}{1-0.5z^{-1}}$



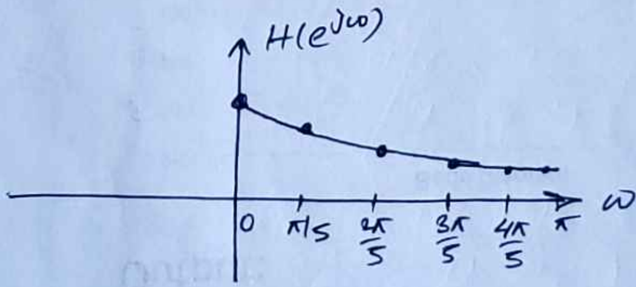
Here pole is at $z=0.5$

For magnitude response,

$$H(e^{j\omega}) = \frac{2}{1-0.5e^{-j\omega}}$$

$$|H(e^{j\omega})| = \frac{2}{|1-0.5e^{-j\omega}|} = \frac{2}{\sqrt{(1-0.5\cos\omega)^2+(0.5\sin\omega)^2}}$$

ω	0	$\pi/5$	$2\pi/5$	$3\pi/5$	$4\pi/5$	π
$\lvert H(e^{j\omega})\rvert$	4	3.012	2.062	1.602	1.394	1.333



3 (b) $X(z) = \dfrac{2z^{-1}+1}{3z^{-2}-4z^{-1}+1} = \dfrac{1+2z^{-1}}{1-4z^{-1}+3z^{-2}}$

$= \dfrac{1+2z^{-1}}{(1-z^{-1})(1-3z^{-1})}$

Here, zero is at $z_1 = -2$

& poles are at $P_1 = 1$ & $P_2 = 3$

∴ The possible ROCs are, $|z| < 1 \rightarrow$ Anticausal

$1 < |z| < 3 \rightarrow$ Non causal

$|z| > 3 \rightarrow$ Causal

For inverse z-transform

$$X(z) = \frac{1+2z^{-1}}{(1-z^{-1})(1-3z^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1-3z^{-1}}$$

$$\therefore A = (1-z^{-1})X(z)\Big|_{z=1} = \frac{1+2z^{-1}}{1-3z^{-1}}\Big|_{z=1} = -1.5$$

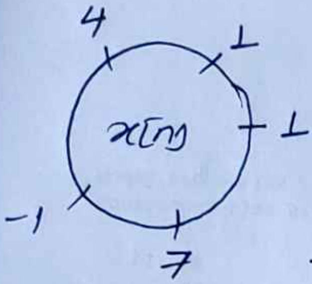
$$B = (1-3z^{-1})X(z)\Big|_{z=3} = \frac{1+2z^{-1}}{1-z^{-1}}\Big|_{z=3} = 2.5$$

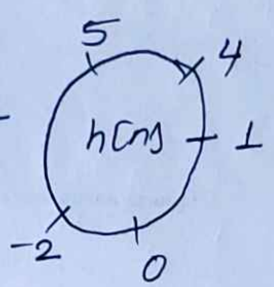
$$\therefore X(z) = -\frac{1.5}{1-z^{-1}} + \frac{2.5}{1-3z^{-1}}$$

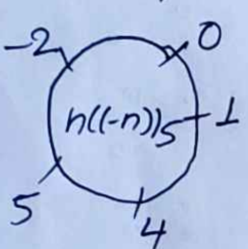
Taking inverse z-transform (causal), ~~of~~ we get

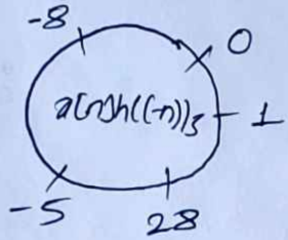
$$x[n] = -1.5\,u[n] + 2.5(3)^n u[n].$$

4 (a) $x[n] = \{1, 1, 4, -1, 7\}$, $h[n] = \{1, 4, 5, -2, 0\}$
(zero padding)

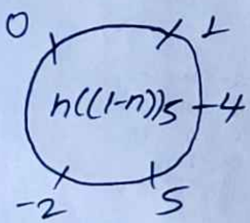


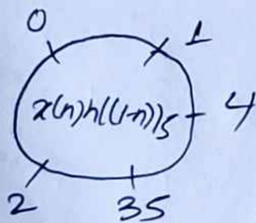






$\therefore y[0] = 1+0-8-5+28 = 16$





$\therefore y[1] = 4+1+0+2+35 = 42$

similarly

$y[2] = 5+4+4+0-14 = -1$

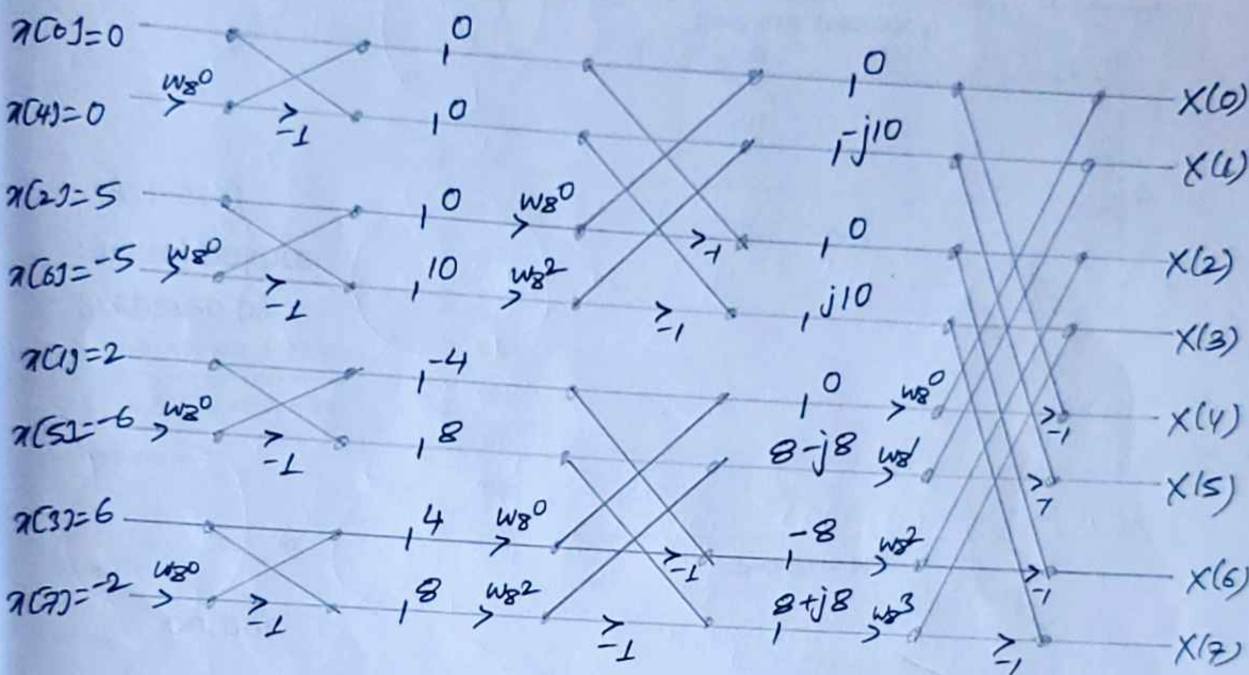
$y[3] = -2+5+16-1+0 = 18$

$y[4] = 0-2+20+\overset{-4}{\cancel{16}}+7 = 21$

$\therefore y[n] = \{16, 42, -1, 18, 21\}$ or $y[n] = \{1, 5, 13, 18, 21, 15, 37, -14\}$

4 (b) $x[n] = \{0, 2, 5, 6, 0, -6, -5, -2\}$ Radix-2 DIT FFT

$W_8^0 = 1$, $W_8^1 = 0.707 - j0.707$, $W_8^2 = -j$, $W_8^3 = -0.707 - j0.707$



$\therefore X(0) = 0 + W_8^0 \cdot 0 = 0$

$X(1) = (-j10) + (0.707 - j0.707)(8 - j8)$
$= -j21.312$

$X(2) = 0 + (-j)(-8) = j8$

$X(3) = j10 + (-0.707 - j0.707)(8 + j8)$
$= -j1.312$

$X(4) = 0 - W_8^0 \cdot 0 = 0$

$X(5) = (-j10) - (0.707 - j0.707)(8 - j8)$
$= j1.312$

$X(6) = 0 - (-j)(-8) = -j8$

$X(7) = j10 - (-0.707 - j0.707)(8 + j8)$
$= j21.312$

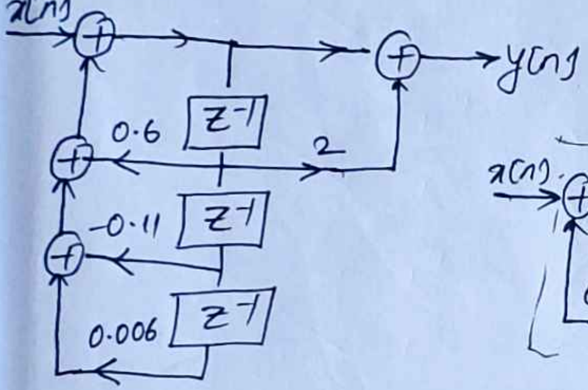
$\therefore X(K) = \{0, -j21.312, j8, -j1.312, 0, j1.312, -j8, j21.312\}$

5 (a) $y[n] = 0.6y[n-1] - 0.11y[n-2] + 0.006y[n-3] + x[n] + 2x[n-1]$

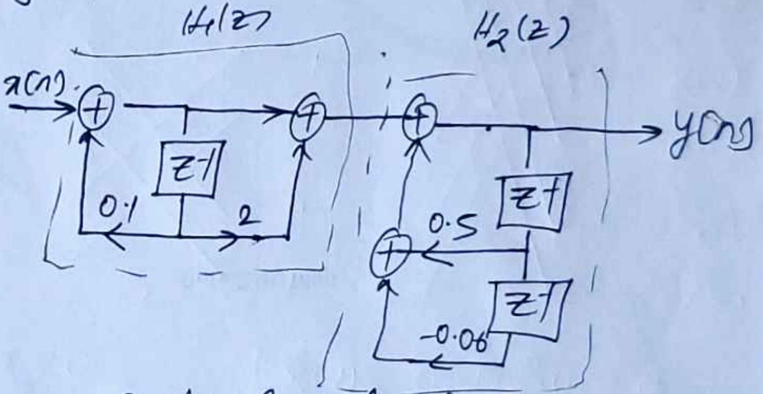
$$H(z) = \frac{1+2z^{-1}}{1-0.6z^{-1}+0.11z^{-2}-0.006z^{-3}}$$

Here, poles are at $P_1 = 0.1$, $P_2 = 0.3$, $P_3 = 0.2$

$$\therefore H(z) = \frac{1+2z^{-1}}{(1-0.1z^{-1})(1-0.5z^{-1}+0.06z^{-2})} = H_1(z)H_2(z)$$



Fig(i) : Direct form-II



Fig(ii) : Cascade structure

(b) $$H(z) = \frac{1-0.8z^{-1}+2z^{-2}}{1-0.8z^{-1}-0.86z^{-2}} = \frac{C_2(z)}{A_2(z)}$$

Draw lattice-ladder structure yourself.

For lattice parameter

$A_2(z) = 1-0.8z^{-1}-0.86z^{-2}$

$\boxed{\therefore K_2 = -0.86}$

$$A_1(z) = \frac{A_2(z) - K_2B_2(z)}{1-K_2^2}$$

$$= \frac{1-0.8z^{-1}-0.86z^{-2}-(-0.86)(-0.86-0.8z^{-1}+z^{-2})}{1-(-0.86)^2}$$

$$= 1 + \frac{-0.8-(-0.86)(-0.8)}{0.2604}z^{-1}$$

$\boxed{\therefore K_1 = -5.714}$ → unstable

For ladder parameter

$C_2(z) = 1-0.8z^{-1}+2z^{-2}$

$\boxed{\therefore v_2 = 2}$

$C_1(z) = C_2(z) - v_2B_2(z)$

$= 1-0.8z^{-1}+2z^{-2}-2(-0.86-0.8z^{-1}+z^{-2})$

$= 2.72+0.8z^{-1}$

$\boxed{\therefore v_1 = 0.8}$

$C_0(z) = C_1(z) - v_1B_1(z)$

$= 2.72+0.8z^{-1} - 0.8(-5.714+z^{-1})$

$\boxed{\therefore v_0 = 7.291}$

6 (a) Butterworth filter, Bilinear transformation method

Given $\omega_p = 0.15\pi \quad \alpha_{max} = 3\,dB$

$\omega_s = 0.55\pi \quad \alpha_{min} = 21\,dB$

$F_s = 3000$ samples/s

Sol: For BLT, $\Omega = \frac{2}{T}\tan(\omega/2)$

$\therefore \Omega_p = \frac{2}{T}\tan(0.15\pi/2) = \frac{0.48}{T}$

$\Omega_s = \frac{2}{T}\tan(\omega_s/2) = \frac{2}{T}\tan(0.55\pi/2) = \frac{2.342}{T}$

$\varepsilon = \sqrt{10^{0.1\alpha_{max}}-1} = \sqrt{10^{0.3}-1} = 0.998$

$\delta = \sqrt{10^{0.1\alpha_{min}}-1} = \sqrt{10^{2.1}-1} = 11.176$

$\therefore N = \frac{\log_{10}(\delta/\varepsilon)}{\log_{10}(\Omega_s/\Omega_p)} = \frac{\log_{10}(11.176/0.998)}{\log_{10}(2.342/0.48)} = 1.52 \cong 2$

For $N=2$ & $\Omega_c = 1$

$H(s) = \frac{1}{s^2+0.707s+1}$

At stopband

$\Omega_c = \frac{\Omega_s}{\delta^{1/N}} = \frac{\frac{2.342}{T}}{11.176^{1/2}}$

$= \frac{0.7}{T}$

$\therefore$ at Ω_c,

$H(s/\Omega_c) = \frac{\Omega_c^2}{s^2+0.707\Omega_c s+\Omega_c^2}$

BLT is $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

$\therefore H(z) = \frac{\Omega_c^2}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+0.707\Omega_c\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\Omega_c^2}$

6 (b) Linear phase FIR filter

$\omega_c = 0.6\pi$, $\tau = 4$ $\quad \tau = \frac{M-1}{2} \Rightarrow M = 2\tau + 1 = 9$

$\alpha_s = 48$ dB $\Rightarrow$ Hamming Window

The desired frequency response is given by

$$H_d(\omega) = \begin{cases} e^{-j\omega\tau} & \text{for } |\omega| \leq \omega_c \\ 0 & \text{elsewhere} \end{cases}$$

In time-domain

$$h_d[n] = \begin{cases} \dfrac{\sin\{\omega_c(n-\tau)\}}{\pi(n-\tau)} & , \text{for } n \neq \tau \\ \dfrac{\omega_c}{\pi} & , \text{for } n = \tau \end{cases}$$

$$= \begin{cases} 0.6 & \text{for } n = 4 \\ \dfrac{\sin\{0.6\pi(n-4)\}}{\pi(n-4)} & , \text{for } n \neq 4 \end{cases}$$

For $\alpha_s = 48$ dB, we will have Hamming window

$\therefore \quad w[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right) \quad 0 \leq n \leq M-1$

$\therefore \quad h[n] = h_d[n]\, w[n]$

n	0	1	2	3	4	5	6	7	8
$h_d[n]$	0.076	−0.062	−0.094	0.303	0.6	0.303	−0.094	−0.062	0.076
$w[n]$	0.08	0.215	0.54	0.865	1	0.865	0.54	0.215	0.08
$h[n]$	0.00608	−0.013	−0.051	0.262	0.6				