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1 (A)

- Signal processing is the process of analysing a signal and performing an operation on signal.

→

Analog signal processing	Digital signal processing
1 ASP operation can not be change by changing of program in programmable system.	1. DSP operation can be change by change of program in programmable system.
2. It is not flexible	2. It is flexible
3. It uses more bandwidth compare to DSP.	3. It uses less bandwidth.
4. It can not be transmitted over long distance.	4. It can be transmitted over long distance
5. In this processing, Electromagnetic Interference are more than DSP.	5. In this processing, Electromagnetic interference are less.
6. Analog signal processing have less control of accuracy than Digital signal processing.	6. They have better control of accuracy compare to Analog signal processing

→ It can not be stored in magnetic media.	→ It can be stored in magnetic media with minimum error.

(B) Sol^n

given, $x(n) = \{1, 2, 1, 2\}$

$h(n) = \{2, 1, 2, 1\}$

2

(A)

The z- transform of discrete time signal $x(n)$ is denoted by $Z[x(n)]$ or $X(z)$ and defined by

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

The range of value of z for which X(z) converges is called ROC.
That means, X(z) has finite value

Mathematically,

$$\sum_{n=-\infty}^{\infty} \left| x(n) \cdot z^{-n} \right| < \infty.$$

given,

$$x(n) = r^n \sin wn \; u(n)$$

By defn. of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} r^n \cdot \sin wn \, z^{-n}$$

$$= \sum_{n=0}^{\infty} r^n * \left(\frac{e^{jwn} - e^{-jwn}}{2j} \right) * z^{-n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} r^n e^{jwn} z^{-n} - r^n e^{-jwn} z^{-n}$$

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$$= \frac{1}{2j}\left[\sum_{n=0}^{\infty}\left(re^{j\omega}z^{-1}\right)^n - \sum_{n=0}^{\infty}\left(re^{-j\omega}z^{-1}\right)^n\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1-re^{j\omega}z^{-1}} - \frac{1}{1-re^{-j\omega}z^{-1}}\right]$$

$$= \frac{1}{2j}\left[\frac{z}{z-re^{j\omega}} - \frac{z}{z-re^{-j\omega}}\right]$$

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$$= \frac{1}{2j}\left(\frac{z^2 - rze^{-j\omega} - z^2 + rze^{j\omega}}{z^2 - rze^{-j\omega} - rze^{j\omega} + r}\right)$$

$$= \frac{1}{2j}\left[\frac{+rz\left(e^{-j\omega} - e^{+j\omega}\right)}{z^2 + r - rz\left(e^{j\omega} + e^{-j\omega}\right)}\right]$$

$$= \frac{1}{-2}\left(\frac{rz\,2\sin\omega}{\frac{z^2}{2} + \frac{r}{2} - rz\,2\cos\omega}\right)$$

$$= \frac{2rz\sin\omega}{-z^2 - r + rz\cos\omega}$$

$$= \frac{2rz\sin\omega}{rz\cos\omega - z^2 - r}$$

$z[x(n-k)] = z^{-k}[X(z) + \sum_{n=1}^{k} x(-n) z^{n}]$

(8) given,

$$y(n) - \frac{3}{2} y(n-1) + \frac{1}{2} y(n-2) = \left(\frac{1}{4}\right)^n$$

taking z-transform

$$\Rightarrow Y(Z) - \frac{3}{2}\left[z^{-1}\left(Y(z) + y(-1)Z\right)\right] + \frac{1}{2}\left[z^{-2}\left(Y(z) + y(-1)Z + y(-2)Z^2\right)\right] = \frac{Z}{Z - 1/4}$$

$$= Y(z) - \frac{3}{2} z^{-1} Y(z) - \frac{3}{2} y(-1) z^0 + \frac{1}{2} z^{-2} Y(z) + \frac{1}{2} y(-1) z^{-1} + \frac{1}{2} y(-2) z^0 = \frac{Z}{Z - 1/4}$$

$$= Y(z) - \frac{3}{2} z^{-1} Y(z) - \frac{3}{2} * 4 + \frac{1}{2} z^{-2} Y(z) + \frac{1}{2} * 4 z^{-1} + \frac{1}{2} * 10 = \frac{Z}{Z - 1/4}$$

$$= Y(z)\left[1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}\right] - 6 + 2z^{-1} + 5 = \frac{Z}{Z - 1/4}$$

$$= Y(z) = \frac{1}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}} * \frac{Z}{Z - 1/4} + 1 - \frac{2}{Z}$$

$\frac{1}{4}$ 1

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$$Y(z) = \frac{1}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} * \frac{z^2+z^2-\frac{z}{4}-2z+\frac{1}{2}}{z(z-\frac{1}{4})}$$

$$\therefore Y(z) = \frac{1}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} * \frac{4z^2+4z^2-z-8z+2}{4(z(z-\frac{1}{4})}$$

$$Y(z) = \frac{z^2}{z^2-\frac{3}{2}z+\frac{1}{2}} * \frac{8z^2-9z+2}{4z(z-\frac{1}{4})}$$

$$= \frac{z^2}{(z-\frac{1}{2})(z-1)} * \frac{8z^2-9z+2}{4z(z-\frac{1}{4})}$$

$$= \frac{z^2(8z^2-9z+2)}{4z(z-\frac{1}{2})(z-1)(z-\frac{1}{4})}$$

Let $F(z) = \frac{Y(z)}{z} = \frac{(8z^2-9z+2)}{4(z-\frac{1}{2})(z-1)(z-\frac{1}{4})}$

$$= \frac{A}{z-\frac{1}{2}} + \frac{B}{z-1} + \frac{C}{z-\frac{1}{4}}$$

where,

$$A = F(z)\cdot(z-\tfrac{1}{2})\Big|_{z=\frac{1}{2}} = 1.$$

$$B = F(z)\,(z-1)\Big|_{z=1} = 2/3$$

$$C = F(z)\left(z - \tfrac{1}{4}\right)\Big|_{z=1/4} = \tfrac{1}{3}.$$

Now,

$$\frac{Y(z)}{z} = \frac{1}{z-1/2} + \frac{2/3}{z-1} + \frac{1/3}{z-1/4}$$

$$\therefore Y(z) = \frac{z}{z-1/2} + \frac{2}{3}\frac{z}{z-1} + \frac{1}{3}\frac{z}{z-1/4}$$

Taking Inverse z-transform,

$$\boxed{y(n) = \left(\frac{1}{2}\right)^n u(n) + \frac{2}{3}\,u(n) + \frac{1}{3}\left(\frac{1}{4}\right)^n u(n).}$$

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3 (a) Sol'n

$$H(z) = \frac{0.56z^{-1} + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

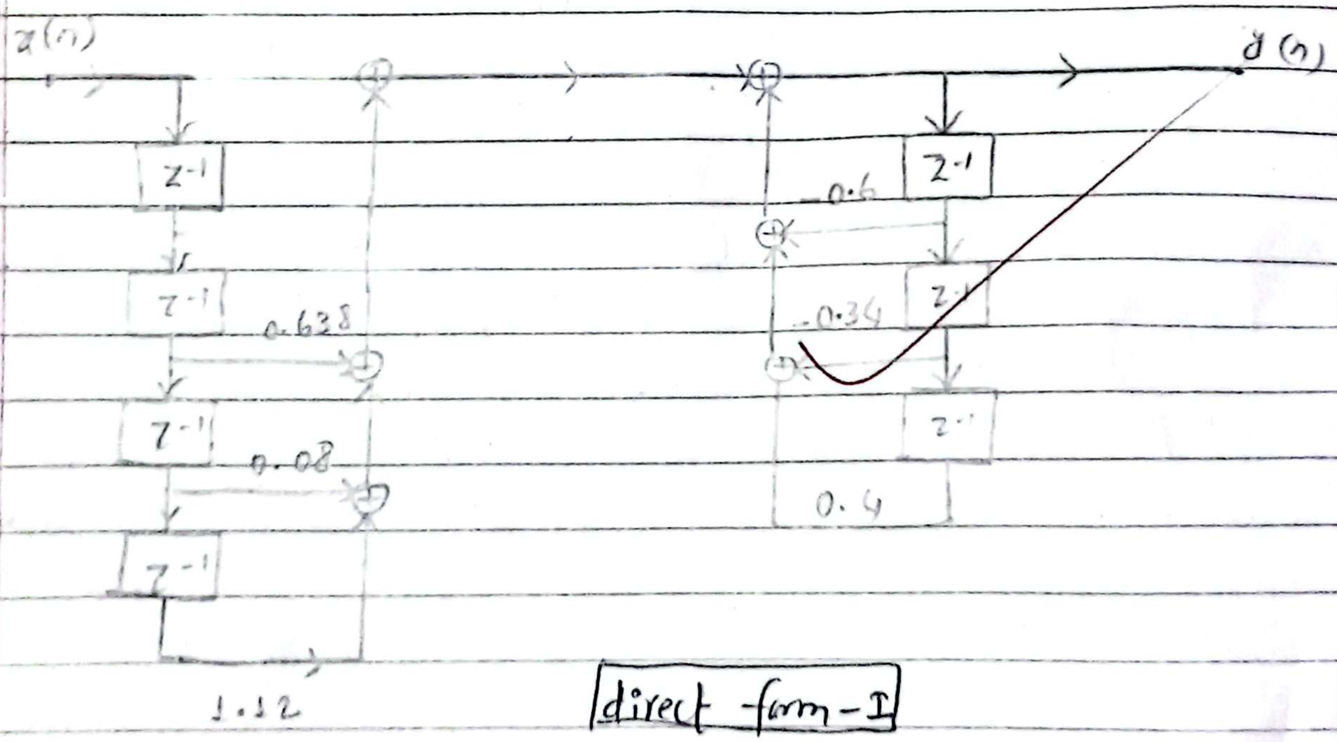
Dividing denominator and numerator by $0.5z^3$

$$H(z) = \frac{1.12z^{-4} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$

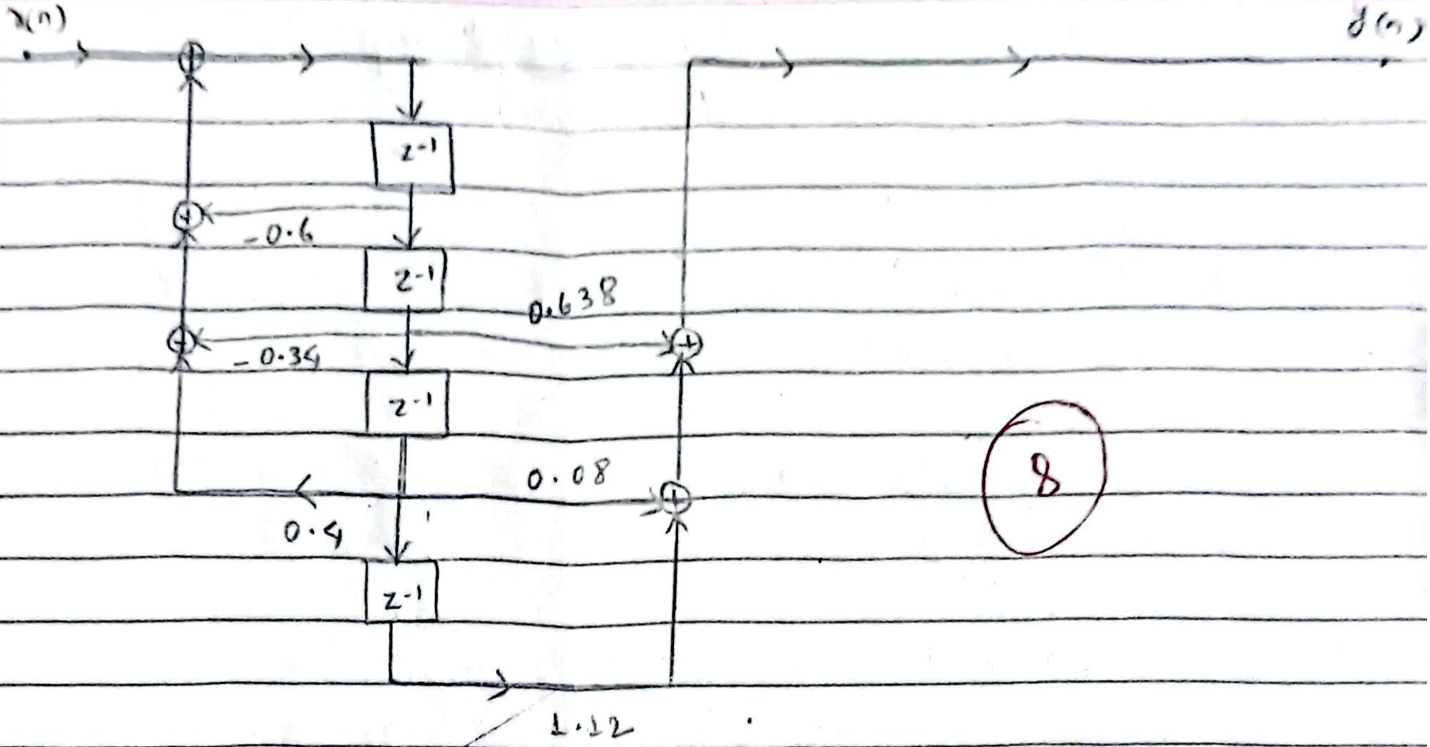
$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{0.638z^{-2} + 0.08z^{-3} + 1.12z^{-4}}{1 + 0.6z^{-1} + 0.34z^{-2} + 0.4z^{-3}}$$

Compare with.

$$\frac{b_0 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3}}.$$



direct form-I



direct form-II

(b) soln

$$H(z) = A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$

$\therefore$ $a_3(0) = 1$

$a_3(1) = \frac{13}{24}$

$a_3(2) = \frac{5}{8}$

$a_3(3) = \frac{1}{3}$

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$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)\, a_m(m-k)}{1 - a_m^2(m)}$$

$k_1 = a_1(1)$
$k_2 = a_2(2)$
$k_3 = a_3(3)$

$\therefore k_3 = 1/3$

<u>for $a_1(1)$</u>

$m = 2, k = 1$

$$k_1 = a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(1)}{1 - a_2^2(2)}$$

$$= \frac{3/8 - 1/2 * 3/8}{1 - (1/2)^2} = \frac{3/16}{3/4} = 1/4.$$

<u>for $a_2(2)$</u>

$m = 3, k = 2$

$$a_2(2) = \frac{a_3(2) - a_3(3) * a_3(1)}{1 - a_3^2(3)}$$

$$= \frac{5/8 - 1/3 * 13/24}{1 - (1/3)^2} = \frac{4/9}{8/9} = 1/2$$

<u>for $a_2(1)$</u>

$m = 3, k = 2$

$$\therefore a_2(1) = \frac{a_3(1) - a_3(3) \cdot a_3(2)}{1 - a_3^2(3)}$$

$$= \frac{13/24 - 1/3 * 5/8}{1 - (1/3)^2} = \frac{1/3}{8/9} = 3/8$$

$\therefore k_1 = 1/4$

$k_2 = 1/2$

$\therefore k_3 = 1/3$

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All values are less than one so, system is stable.

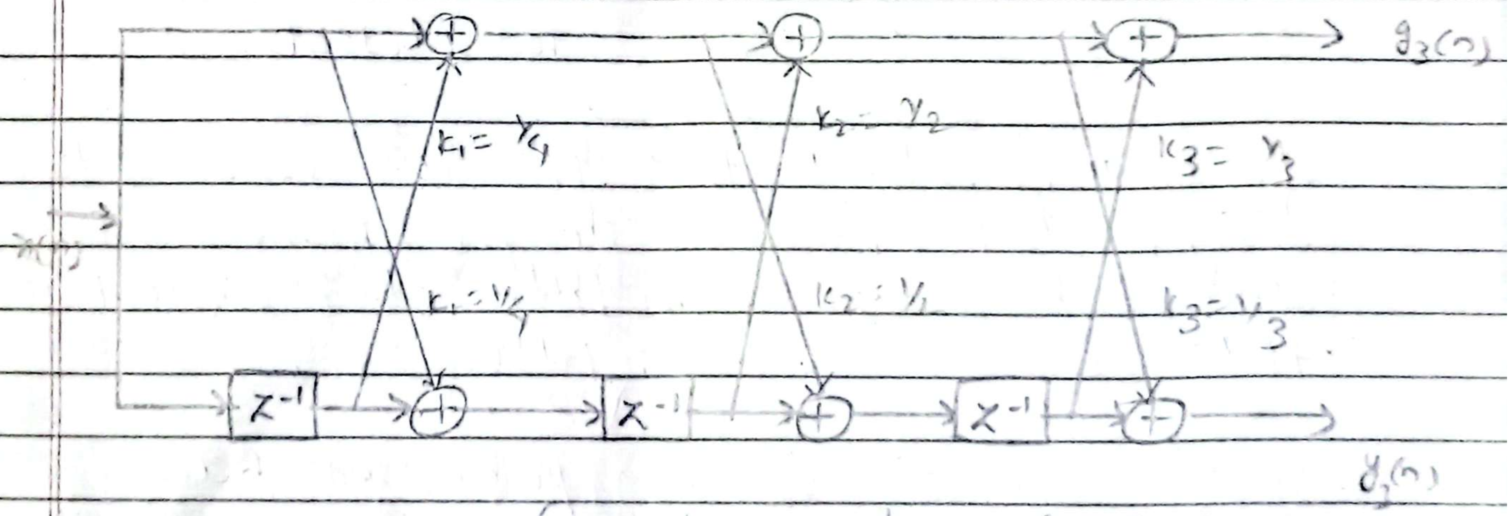

fig. Lattice structure for FIR filter

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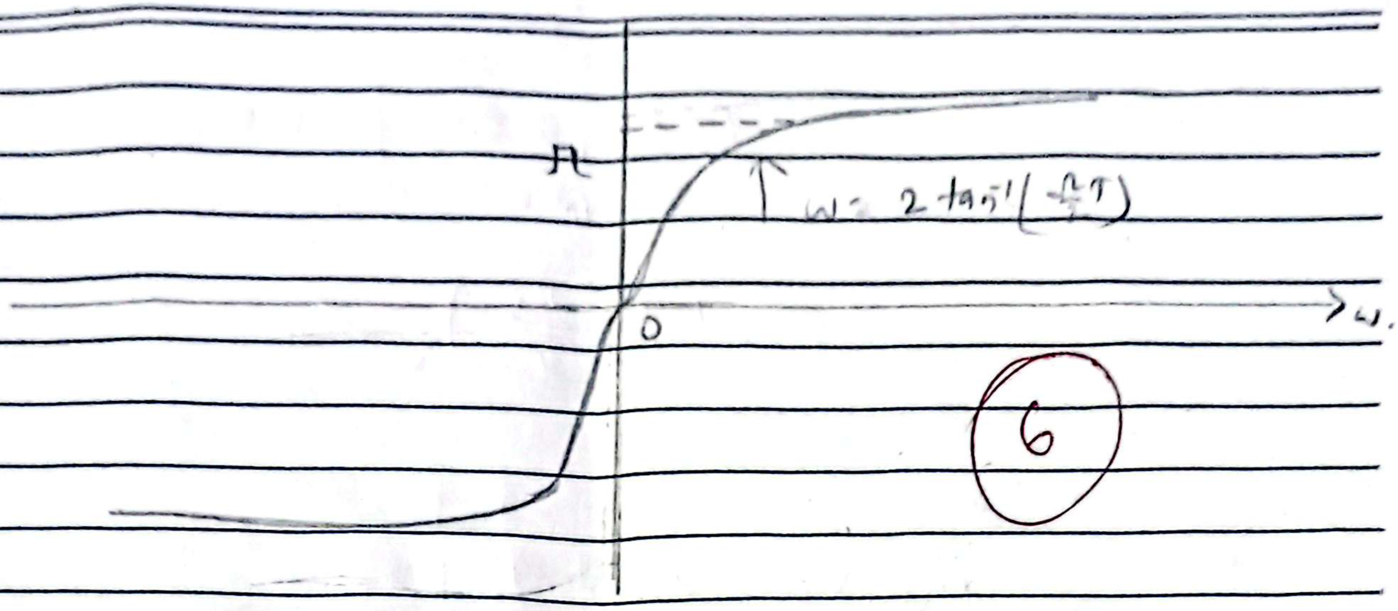
4 (a)

IIM	BLT
→ poles can be expressed (mapped) as the expression $\frac{1}{s-p_i} \rightarrow \frac{1}{1-e^{p_i T} z^{-1}}$	→ poles can be mapped by using expression $S = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$
→ Mapping is Many-one	→ Mapping is one-one
→ present aliasing effect	→ No aliasing effect is present.
→ It is not suitable to design high pass filter, band reject filter etc.	→ It is suitable to design high pass filter, band reject filter
→ Only poles of system can be mapped	→ poles as well as zeros are mapped
→ No frequency wrapping effect is present	→ Frequency wrapping effect is present



→ In BLT, the mapping is one-one but non-linear due to this non-linear mapping, there occur frequency wrapping effect.

4 (b)

Soln

given, $A_p = 0.89$ $\quad$ $\omega_p = 0.4\pi$

$A_s = 0.18$ $\quad$ $\omega_s = 0.6\pi$

step 1:

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) \qquad \Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

$$= 2\tan\left(\frac{0.4\pi}{2}\right) \qquad \Omega_s = 2\tan\left(\frac{0.6\pi}{2}\right)$$

$$\boxed{\Omega_p = 1.453} \qquad \boxed{\Omega_s = 2.752}$$

step 2:

$$N = \frac{1}{2} \times \frac{\log\left[\frac{1/A_s^2 - 1}{1/A_p^2 - 1}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

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$$= \frac{1}{2} \times \frac{\log\left[\frac{1/0.18^2 - 1}{1/0.89^2 - 1}\right]}{\log\left(\frac{2.75}{1.453}\right)}$$

$$= \frac{1}{2} \times \frac{2.056}{0.277} = 3.711 \simeq 4.$$

$\boxed{\therefore N = 4}$

step 3:

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2} - 1\right)^{1/2N}} = \frac{1.453}{0.846}$$

$$= 1.717$$

step 4:

$$p_i = \pm \Omega_c \cdot e^{j(N+2i+1)}$$

$i = 0$

$$p_0 = \pm \Omega_c \cdot e^{j(4+1)}$$

$$= \pm 1.717\, e^{j5}$$

$$= \pm 1.717(0.283 + j0.958)$$

$$= \pm(0.485 - j1.194)$$

$i=1 \quad P_1 = \pm \Omega_c \cdot e^{j(4+3)} = \pm 1.717 \cdot e^{j7^\circ}$

$= \pm (1.2944 + j1.128)$

$i=2 \quad P_2 = \pm \Omega_c e^{j(9)} = \pm 1.717 \, e^{9j^\circ}$

$= \pm(-1.564 + j0.707)$

$i=3 \quad P_3 = \pm \Omega_c e^{j(11)} = \pm 1.717 \, e^{11j^\circ}$

$= \pm(7.59\times10^{-3} - j1.716)$

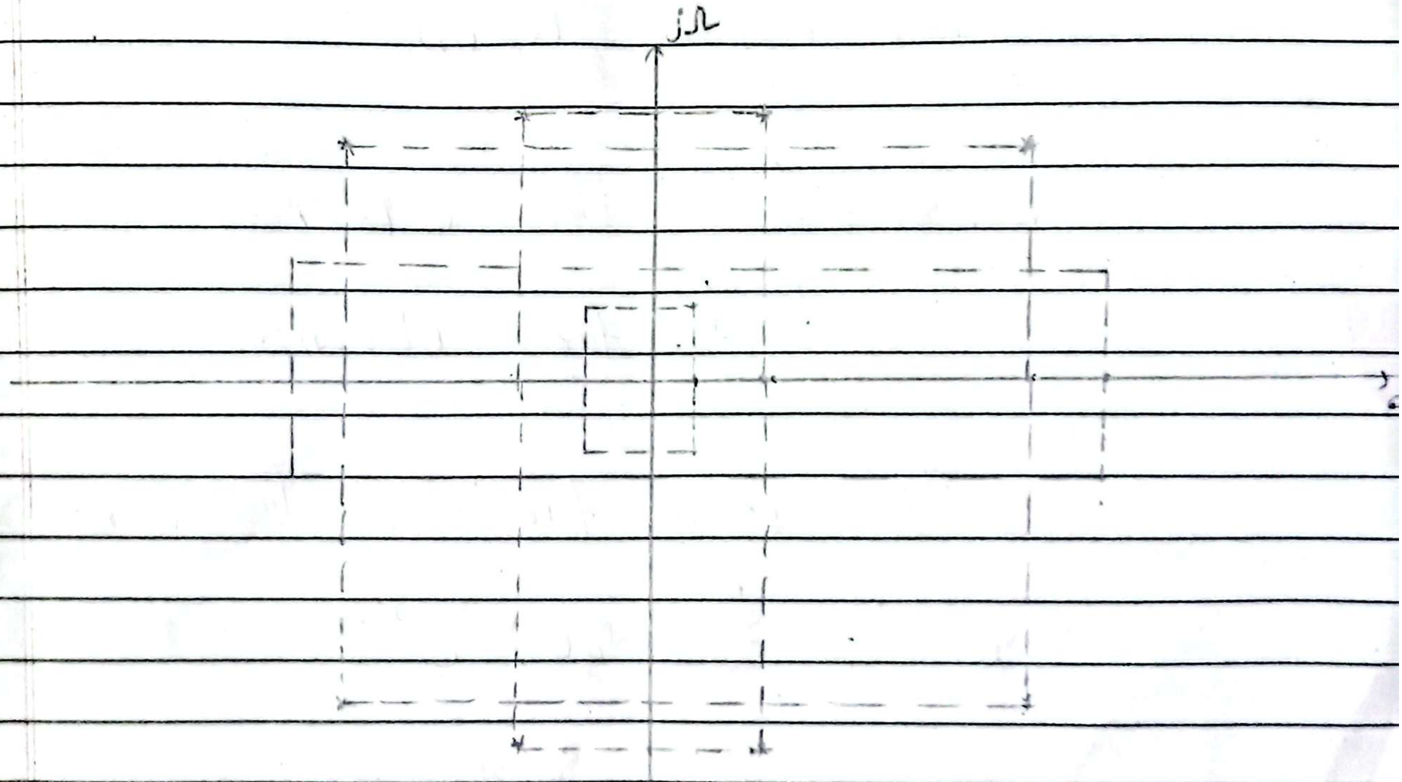
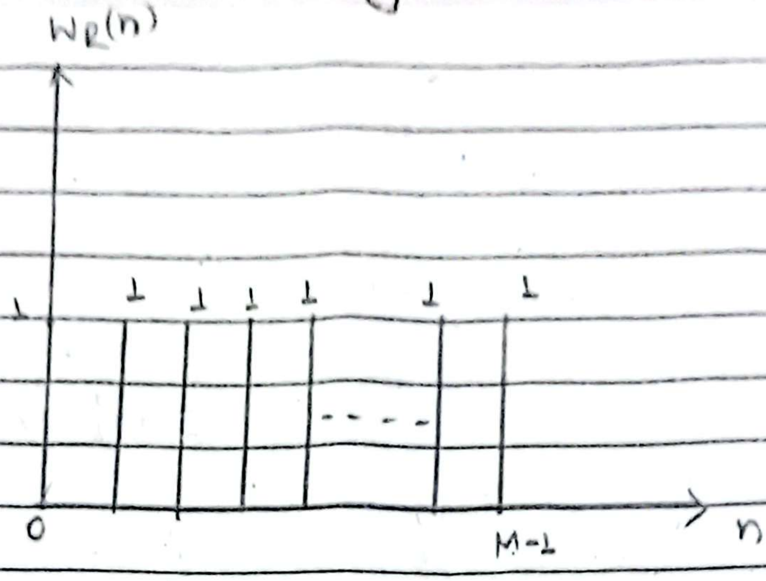


fig:- poles mapping

Rectangular Window.

5 (a) Sol



→ It is denoted by $W_R(n)$
→ Magnitude is 1 for range 0 to M-1.
→ Since, The shape of window function is rectangular, so it is called rectangular window $W_R(n)$.

→

$$W_R(n) = \begin{cases} 1 & \text{for } 0 \text{ to } M-1 \\ 0 & \text{for otherwise} \end{cases}$$

let consider fourier transform of $W_R(n)$

$$W_R(\omega) = \sum_{n=0}^{M-1} W_R(n)\, e^{-j\omega M}$$

$$= \sum_{n=0}^{M-1} e^{-j\omega M}$$

Name: Now.

$$W_R(\omega) = \frac{1-e^{-j\omega M}}{1-e^{-j\omega}}$$

$$= \frac{e^{-j\omega M/2} \cdot e^{j\omega M/2} - e^{-j\omega M/2} \cdot e^{-j\omega M/2}}{e^{-j\omega/2} \cdot e^{+j\omega/2} - e^{-j\omega/2} \cdot e^{-j\omega/2}}$$

$$= \frac{e^{-j\omega M/2}\left[e^{j\omega M/2} - e^{-j\omega M/2}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{j\omega/2}\right]}$$

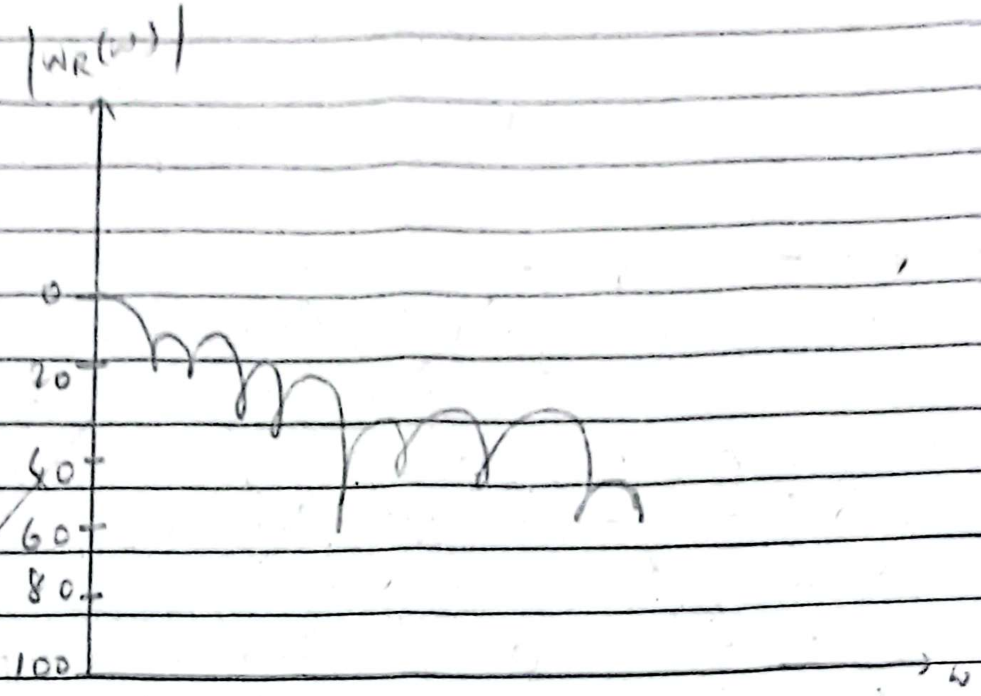
$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \times \frac{\cancel{2} \sin \omega M/2}{\cancel{2} \sin \omega/2}$$

$$W_R(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)} \left(\frac{\sin \omega M/2}{\sin \omega/2}\right)$$

Also,

$$|W_R(\omega)| = \frac{|\sin \omega M/2|}{|\sin(\omega/2)|}$$

Magnitude



6. (a) Sol

$$x(n) = n+1 \quad \text{for} \quad 0 \le n \le 7$$

$x(0) = 1$

$x(1) = 2$

$x(2) = 3$

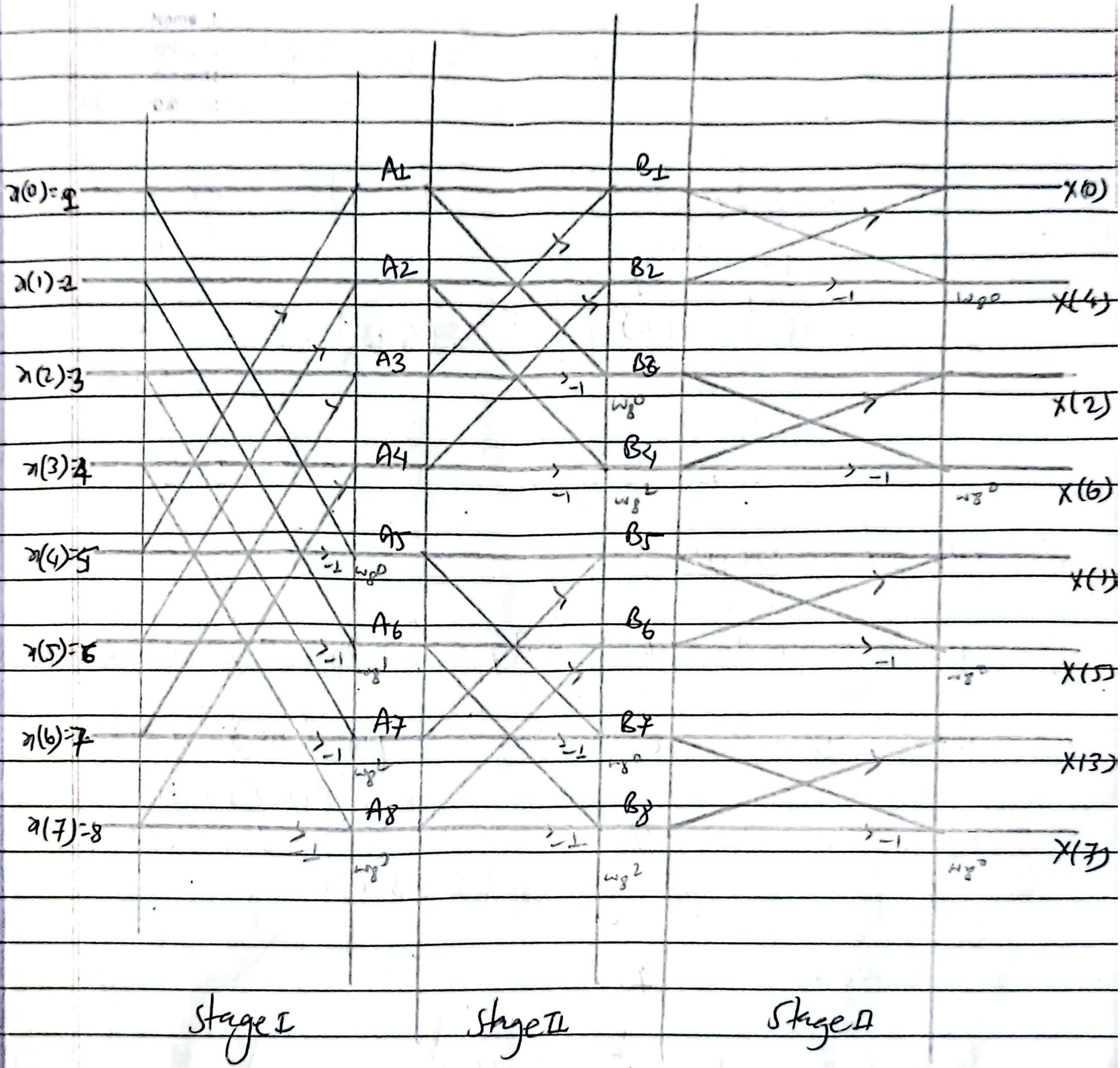
$x(3) = 4$

$x(4) = 5$

$x(5) = 6$

$x(6) = 7$

$x(7) = 8$



o/p of stage I

$A_1 = x(0) + x(4) = 1+5=6$

$A_2 = x(1) + x(5) = 8$

$A_3 = x(2) + x(6) = 10$

$A_4 = x(3) + x(7) = 12$

$W_8 = e^{-j\left(\frac{2\pi}{8}\right)}$ $\quad e^{\left(\frac{\pi}{4}\right)1}$ $= -0.707 - j0.707$

$= e^{-j\left(\frac{\pi}{4}\right)}$

$= 0.707 - j0.707$

$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$
1	2	3	4	5	6	7	8

$W_8^2 = e^{-j\frac{\pi}{2}}$

$= 0 - j1$

$= -j$

$A_5 = x(0) - x(4) = 1 - 5 = -4$

$A_6 = [x(1) - x(5)]\, W_8^1 = (2-6) W_8^1 = -4 * W_8^1$

$= -2.828 + j\,2.828$

$A_7 = [x(2) - x(6)]\, W_8^2 = (3-7) W_8^2 = -4 * -j$

$= 4j$

$A_8 = [x(3) - x(7)]\, W_8^3 = (4-8) W_8^3 = -4 * W_8^3$

$= 2.828 + j\,2.828$

<u>O/p of stage A</u>

$B_1 = A_1 + A_3 = 16,$ $\quad B_7 = A_5 - A_7 = -4 - 4j$

$B_2 = A_2 + A_4 = 20$ $\quad B_8 = (A_6 - A_8)\, W_8^2$

$= -2.828 + j\,2.828 - 2.828$

$j\,2.828$

$= -5.656$

$B_3 = A_1 - A_3 = -4$

$B_4 = [A_2 - A_4]\, W_8^2$

$= -4 * W_8^2 = 4j$

$B_5 = (A_5 + A_7) = -4 + 4j$

$B_6 = (A_6 + A_8) = (-2.828 + j\,2.828) + (2.828 + j\,2.828)$

$= 5.656j$

o/p of stage III :

$X(0) = B_1 + B_2 = 36$

8

$X(4) = B_1 - B_2 = -4$

$X(2) = B_3 + B_4 = -4 + 4j$

$X(6) = B_3 - B_4 = -4 - 4j$

$X(1) = B_5 + B_6 = -4 + 4j + 5.656j = -4 + 9.656j$

$X(5) = B_5 - B_6 = -4 + 4j - 5.656j = -4 - j1.656$

$X(3) = B_7 + B_8 = -4 - 4j - 5.656 = -9.656 - 4j$

$X(7) = B(7) - B_8 = -4 - 4j + 5.656 = 1.656 - 4j$

$X(k) = \{ X(0), X(1)\ X(2)\ X(3)\ X(4)\ X(5)\ X(6)\ X(7) \}$

$= \{ 36, -4 + 9.656j, -4 + 4j, -9.656 - 4j, -4,$
$-4 - j1.656, -4 - 4j, 1.656 - 4j \}$

6(b)

It is the process of adding the zero to original sequence of efficient DFT is called zero padding.

Eg. $x_1(n) = [1, 2, 5]$, $l = 3$

$x_2(n) = [1, 5, 7, 8]$, $l = 4$

After zero padding.

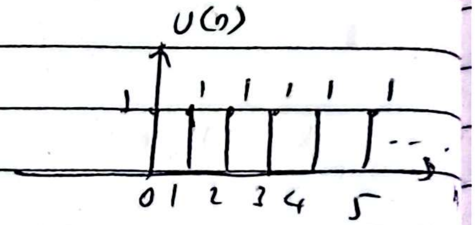
$x_1(n) = [1 \quad 2 \quad 5 \quad 0]$, $l = 4$

$x_2(n) = [1 \quad 5 \quad 7 \quad 8]$, $l = 4$.

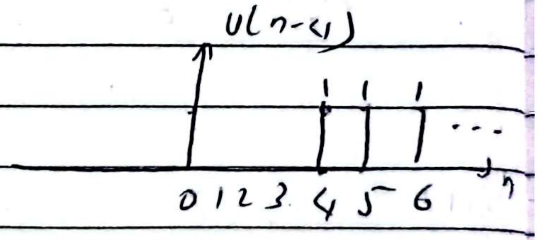
$x_1(n) = \{1, 2\}$

$x_2(n) = u(n) - u(n-4)$

$= \{1, 1, 1, 1\}$



$x_1(n) = \{1, 2, 0, 0\}$



$y(n) = x_1(n) \; (N) \; x_2(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1+2+0+0 \\ 1+2+0+0 \\ 1+2+0+0 \\ 1+2+0+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

$\therefore\ y(n) = \{3, 3, 3\ 3\}$

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1 (B) Soln given as,

$$x(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$$

$$h(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

By using overlapp & Add method,

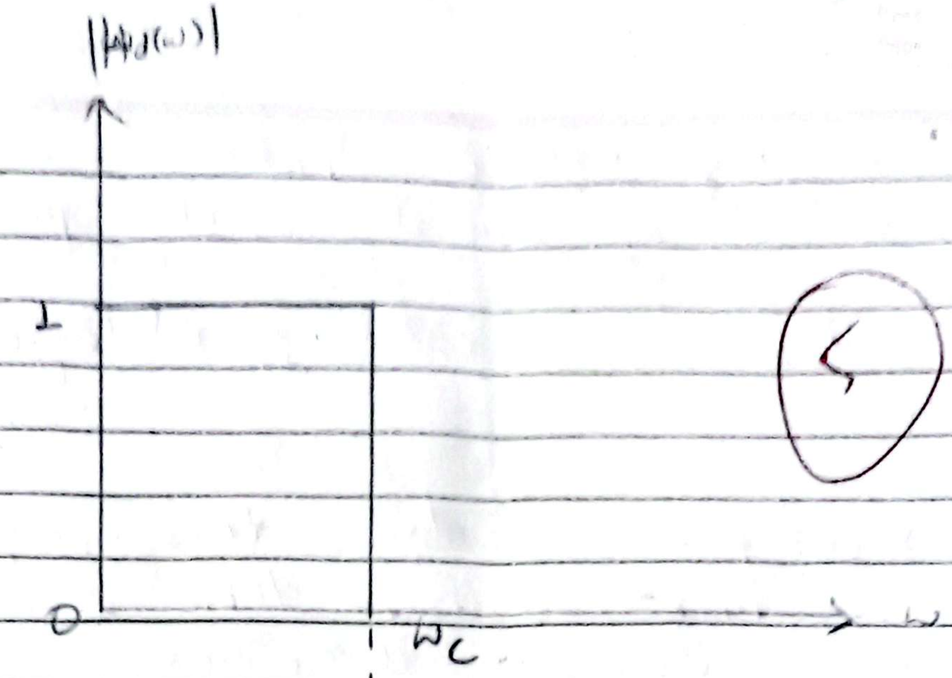
$x(n)$ / $h(n)$	2	1	2	1
1	2	1	2	1
2	4	2	4	2
1	2	1	2	1
2	4	2	4	2

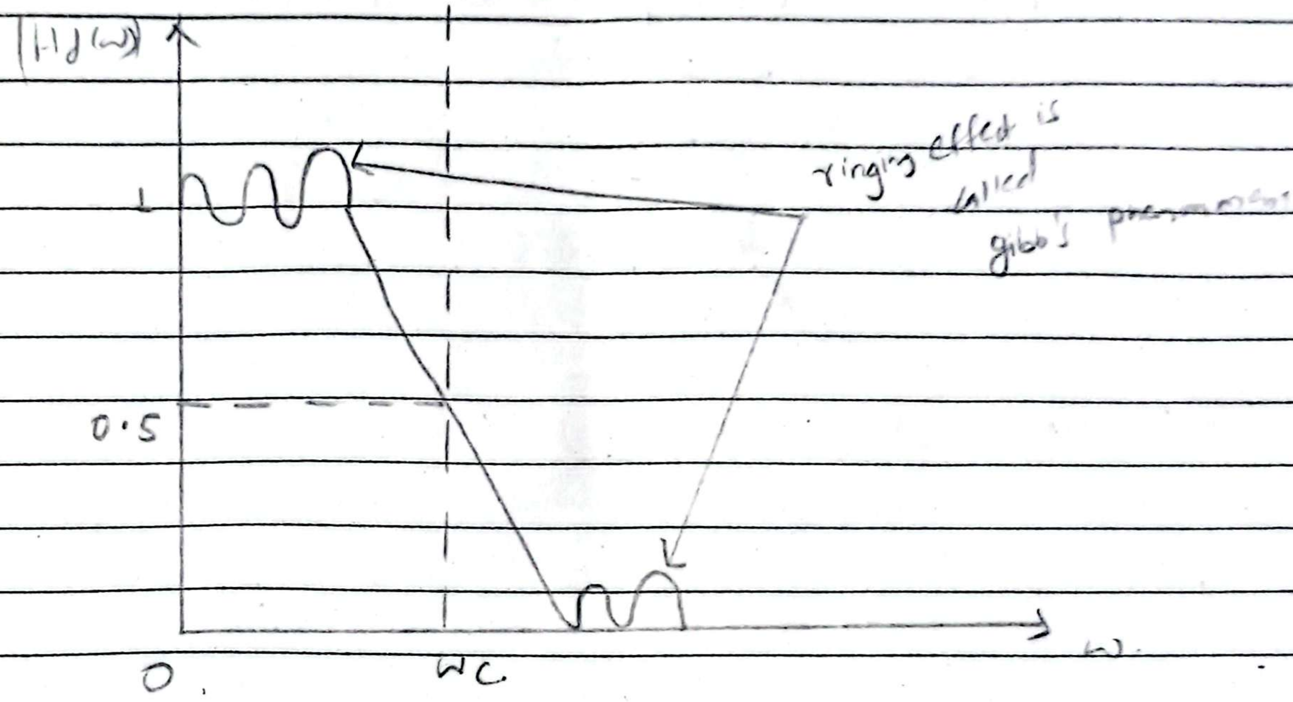
$$\therefore y(n) = \{\underset{\uparrow}{2}, 5, 6, 10, 6, 5, 2\}$$

(origin.)

7. (b)

Let Consider a low pass filter having desired frequency Response $H_d(\omega)$ and cut off frequency ω_c.





→ The ringing effect takes near the bandwidth ω_c of filter.

→ The ringing effect occur due to sidelobes in window function at ω_c (or near ω_c)

→ The sidelobes occur due to discontinities disrupt in $W(\omega)$.

→ The ringing effect near w_c due to sidelobes is called Gibbs phenomenon.

→ This effect is max^m. in Rectangular window.

→ In rectangular window, this ringing effect is max^m because there is max^m sidelobes due to discontinuities.

Everest Engineering C...

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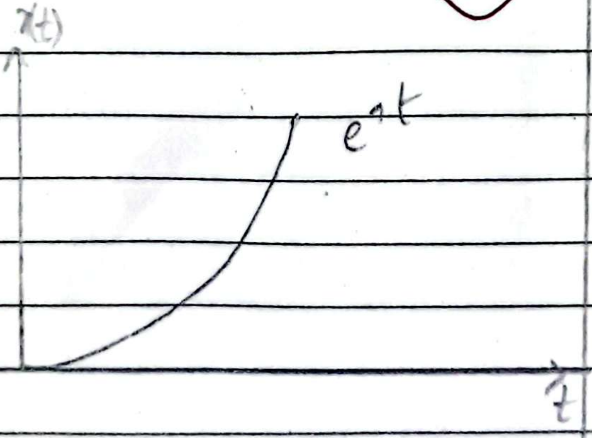
7 (c) 5

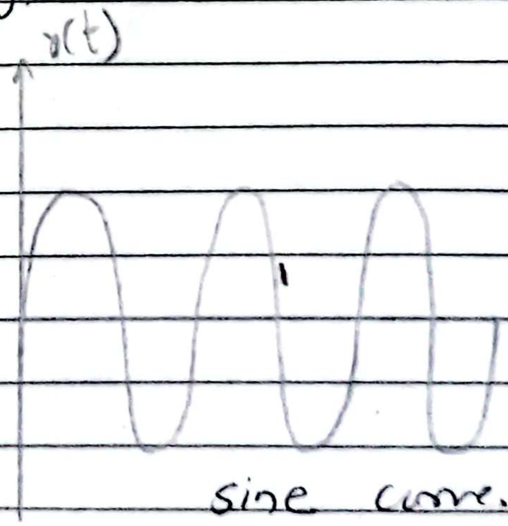
Energy	Power
→ Total normalized energy is non zero and finite.	→ Total normalized average power is non zero and finite
→ All Aperiodic signals are power signals.	→ All periodic signals are power signal.
→ It is expressed as $E = \sum_{n=-\infty}^{\infty} \lvert x(n) \rvert^2$	→ It is expressed as $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n) \rvert^2$

→ Eg.



exponential

Eg.



sine curve.

→ $0 < E < \infty$ & $P = 0$	$0 < P < \infty$ & $E = \infty$
→ In this signal, power is zero.	→ In this signal Energy is infinite (∞)