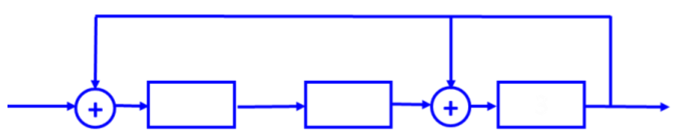
Cpr E 489 Spring 2023
Homework #2 Solution

1. (30 points) Consider the 2-out-of-5 error detection code. In this code, each codeword is 5-bit long; 2 out of 5 bits are 1's and the others are 0's. For example, 01001 is a valid codeword, but 11110 is not.
   a. (10 points) List all the codewords.
   Answer:
   Total number of codewords = $\binom{5}{2}$ = 10. The codewords are:
   00011 00101 01001 10001 00110 01010 10010 01100 10100 11000

   b. (10 points) What fraction of errors is undetectable by this code, i.e., what is FUE of this code? Justify your answer.
   Answer:
   Total number of valid errors = $2^5 - 1 = 31$. Total number of codewords = $\binom{5}{2}$ = 10.
   Therefore, FUE = (10 – 1) / 31 = 9/31.

   c. (10 points) What fraction of 4-bit errors is undetectable by this code, i.e., what is FUE(M = 4) of this code? Justify your answer.
   Answer:
   Total number of 4-bit errors= $\binom{5}{4}$ = 5.
   For a 4-bit error to be undetectable, it needs to flip both 1's to 0's, and 2 out of 3 0's to 1's.
   This means that the total number of undetectable 4-bit errors = $\binom{2}{2}\binom{3}{2}$ = 3.
   Therefore, FUE(M = 4) = 3/5.

2. (70 points) Consider a CRC code with a generator polynomial of $g(x) = x^3 + x^2 + 1$.
   a. (10 points) Show the shift-register circuit that implements this CRC code.
   Answer:



   b. (10 points) Show step by step (using the longhand division) how to find the codeword that corresponds to five information bits of 11111.
   Answer:
   We know: $g(x) = x^3 + x^2 + 1$
   Information bits of 11111 $\Rightarrow$ $i(x) = x^4 + x^3 + x^2 + x + 1$
   $\Rightarrow$ dividend polynomial = $x^3 * i(x) = x^7 + x^6 + x^5 + x^4 + x^3$
   Next, perform the long-hand division:

$$
\begin{array}{r|llllllll}
 & x^4 & & +x^2 & +x & & & \\
\cline{2-8}
x^3+x^2+1 & x^7 & +x^6 & +x^5 & +x^4 & +x^3 & & \\
 & x^7 & +x^6 & & +x^4 & & & \\
\cline{2-7}
 & & & x^5 & & +x^3 & & \\
 & & & x^5 & +x^4 & & +x^2 & \\
\cline{3-8}
 & & & & x^4 & +x^3 & +x^2 & \\
 & & & & x^4 & +x^3 & & +x \\
\cline{4-8}
 & & & & & & x^2 & +x
\end{array}
$$

   So, we have: $r(x) = x^2 + x$ $\Rightarrow$ $b(x) = x^3 * i(x) + r(x) = x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x$
   $\Rightarrow$ codeword is (11111 110)

c. (10 points) Suppose the codeword length is 8. What fraction of errors is undetectable by this code, i.e., what is FUE of this code? Justify your answer.

Answer:
Total number of valid errors = $2^8 - 1 = 255$. Total number of codewords = $2^5 = 32$.
Therefore, FUE = $(32 - 1)/255 = 31/255$.

d. (10 points) Suppose the codeword length is 8. What fraction of error bursts of length 6 is undetectable by this code, i.e., what is FUE(L = 6) of this code? Justify your answer.

Answer:
Since $L = 6 > (n - k) + 1 = 4$, it is a long error burst.
Hence, $FUE(L = 6) = ½^{(n-k)} = ½^3 = 1/8$.

e. Suppose the codeword length is 8. Answer the following questions, with proper justifications.

i. (10 points) Give an example error vector of undetectable error burst of length 6 (L = 6).

Answer:

$$\left.\begin{array}{r} e(x) \text{ is an error burst of length 6} \Rightarrow e(x) = x^i(x^5+\ldots+1) \\ e(x) \text{ is undetectable} \Rightarrow e(x) = x^i g(x)c(x) \\ g(x) = x^3+x^2+1 \end{array}\right\} \Rightarrow c(x) = x^2+\ldots+1$$

Let's pick i = 1 and $c(x) = x^2 + 1$. Then, we have:

$$e(x) = x^1(x^3+x^2+1)(x^2+1) \Rightarrow \underline{e} = [01110010]$$

ii. (10 points) Give an example error vector of undetectable 6-bit error (M = 6).

Answer:
One such an example is $\underline{e} = [11011101]$ because:
The corresponding $e(x) = x^7 + x^6 + x^4 + x^3 + x^2 + 1 = g(x) * (x^4 + 1)$.

iii. (10 points) Give an example error vector of undetectable error that is both a 3-bit error and an error burst of length 7 (M = 3 and L = 7).

Answer:

$$\left.\begin{array}{r} e(x) \text{ is an error burst of length 7} \Rightarrow e(x) = x^i(x^6+\ldots+1) \\ e(x) \text{ is undetectable} \Rightarrow e(x) = x^i g(x)c(x) \\ g(x) = x^3+x^2+1 \end{array}\right\} \Rightarrow c(x) = x^3+\ldots+1$$

We can generate one such an example error vector as follows.
Let's pick i = 0 and $c(x) = x^3+x^2+1$. Then, we have:
$e(x) = x^0(x^3+x^2+1)(x^3+x^2+1) \Rightarrow \underline{e} = [01010001]$ which is an undetectable error that is both a 3-bit error and an error burst of length 7.