Cpr E 489 Spring 2023
Homework #1 Solution

1. (15 points) A baseband channel with a bandwidth of 2.5 KHz is used by a digital transmission system. Suppose the ideal pulses are sent at the Nyquist rate, and the pulses can take 128 different levels. There is no noise in the system. What is the bit rate of this system? Justify your answer.

   Answer:
   Nyquist pulses can be sent over this channel at a rate of 2.5 K * 2 = 5 K pulses per second. Each pulse carries $\log_2(128)$ = 7 bits of information. Thus, the bit rate is 5 K * 7 = 35 Kbps.

2. (15 points) Suppose that multi-level square pulses are used in a digital transmission system, and the maximum pulse amplitude is ±2.35 Volts. Suppose that the amplitude of the additive noise is uniformly distributed between (–0.1, +0.2) Volts. What is the maximum number of levels of pulses this transmission system can use before the noise may start introducing errors? Justify your answer.

   Answer:
   If two adjacent signal levels are separated by more than +0.2 – (–0.1) = 0.3 Volts, then it is impossible for the noise to translate one signal level into another.

   As the maximum range that the signal can span is +2.35 – (–2.35) = 4.7 Volts, the maximum number of levels is $\lfloor 4.7 / 0.3 \rfloor + 1 = 16$.

3. (15 points) Suppose we wish to transmit at a bit rate of 160 Kbps reliably over a noisy AWGN (Additive White Gaussian Noise) communication channel with a bandwidth of 20 KHz. What is the minimum SNR (Signal to Noise Ratio) (**in dB**) required to accomplish this? Justify your answer.

   Answer:
   The Shannon channel capacity formula in binary transmission system gives: $C = W \log_2 (1 + SNR)$

   We know that R = 160 Kbps and W = 20 KHz. What we need to find is SNR_{min}.

   $C \geq C_{min} = R = 160$ Kbps

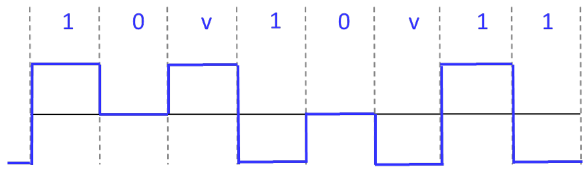
   $C_{min} = W \log_2 (1 + SNR_{min}) \Rightarrow \log_2 (1 + SNR_{min}) = 160/20 \Rightarrow 1 + SNR_{min} = 2^8$

   $\Rightarrow SNR_{min} = 255$

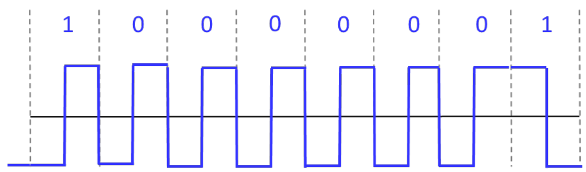
   $\Rightarrow$ in dB: $SNR_{min} = 10 \log_{10}(255)$ dB ≈ 24 dB

4. (25 points) For bit stream 10000001, sketch the waveform for each one of the following line coding schemes that we learned in the class. (Assume that the waveform in the bit interval prior to 10000001 ends at a negative voltage level.)

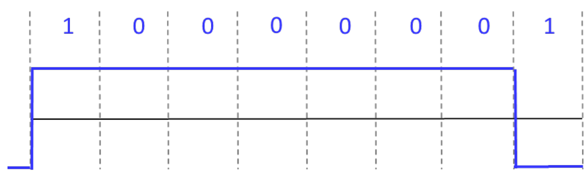
Answer:



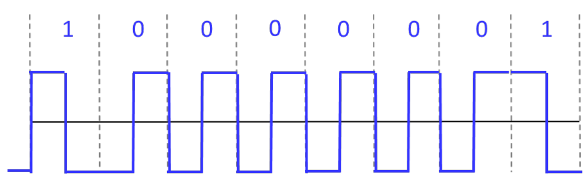
B6ZS: 10000001 → 10v10v11



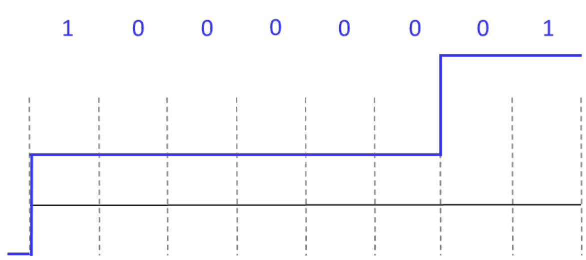
Differential Manchester



NRZ Inverted



1B2B (i.e., Manchester)



2B1Q

5. (30 points) Suppose that two check bits are added to four information bits. The first check bit is the even parity check of the first two information bits, and the second check bit is the even parity check of the second two information bits.

   a. What fraction of errors is undetectable? Justify your answer.

   Answer:
   Since each error vector is 6-bit long, there are 2^6 = 64 different error vectors in total; 63 out of them represent valid errors while e = [000000] represents no error.

   Let's call the first two information bits and the first check bit Group 1, and the second two information bits and the second check bit Group 2. The table below summarizes all cases of undetectable errors:

	# error bits In Group 1	# error bits In Group 2	Total # error bits	# such errors
Case 1	0	2	2	$\binom{3}{0}\binom{3}{2} = 3$
Case 2	2	0	2	$\binom{3}{2}\binom{3}{0} = 3$
Case 3	2	2	4	$\binom{3}{2}\binom{3}{2} = 9$
				Total: 15

   Thus, the fraction of undetectable errors is 15/63 = 5/21.

   b. What fraction of 2-bit errors is undetectable? Justify your answer.

   Answer:
   As for 2-bit errors, there are a total of $\binom{6}{2} = 15$ possibilities.
   Thus, from the above table, we know the fraction of undetectable 2-bit errors is (3+3)/15 = 6/15 = 0.4.