We'll start with finishing the proof of Corollary 2. We have that for $\alpha = r + s\sqrt{m}$, $r, s \in \mathbb{Q}$, for $s \neq 0$, the monic irreducible polynomial over $\mathbb{Q}$ having α as a root is

$$x^2 - 2rx + r^2 - ms^2.$$

Hence, α is an algebraic integer if and only if $2r$ and $r^2 - ms^2$ are integers. Now if m is squarefree, then we must have $m \equiv 1, 2, 3 \pmod 4$.

In the proof of Theorem 2, the polynomial is monic, since it's the characteristic polynomial of M (in α).