Optimal Bayes Classifier

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Question 1

Question 1.a

We have

$$\text{Optimal Bayes Classifier: } \hat{Y}(x) = I(\frac{P(X = x|Y = 1)L(0,1)P(Y = 1)}{P(X = x|Y = 0)L(1,0)P(Y = 0)} > 1 \tag{1}$$

But $P(Y = 0) = P(Y = 1) = 0.5$ because $Y \sim Bern(0.5)$. $L(0, 1) = L(1, 0)$ because they are symmetric loss function. Thus, terms in the numerator are canceled out with terms in the denominator, leaving us

$$\hat{Y}(x) = I(\frac{P(X = x|Y = 1}{P(X = x|Y = 0}) > 1) \tag{2}$$

The equation for multivariate normal distribution is

$$f_X(x) = \frac{1}{(2\pi)^{d/2}\sqrt{|\Sigma|}}\exp(-\frac{(x-\mu)^t\Sigma^-1(x-\mu)}{2} \tag{3}$$

We have $d = 2$, thus we could write the equation on the left hand side of (2) as

$$\begin{aligned} log\frac{P(X = x|Y = 1)}{P(X = x|Y = 0)} &= \frac{-(x-\mu_1)^t\Sigma^{-1}(x-\mu_1)}{2} + \frac{(x-\mu_0)^t\Sigma^-1(x-\mu_0)}{2} \\ &= \frac{\Sigma^{-1}(\mu_1-\mu_0)x}{2} \end{aligned}$$

Hence the classifier is of the form:

$$\hat{Y}(x) = I(l^t x > 1), \qquad l = \Sigma^{-1}(\mu_1 - \mu_0) \tag{4}$$