Poisson_Distribution

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1 Probability Distributions

1.1 Poisson Distribution

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```
In [59]: import numpy as np
         import matplotlib as mpl
         import matplotlib.pyplot as plt
         from scipy import stats
         from scipy.stats import poisson
```

The Poisson distribution is a very common probability distribution to describe stochastic processes. Hence it is often used as a noise model for physical processes. We call a discrete random variable K Poisson distributed, if its probability mass function (PMF) is given by

$$p(K = k) = \frac{e^{-\mu}\mu^k}{k!}, \tag{1}$$

where μ is the only shape parameter of this distribution. The random variable K can take any discrete integer value $k \geq 0$.

Using a python environment we use scipy's built in functionality to work with Poisson distributions. For this purpose use the

```
from scipy.stats import poisson
```

statement. Then you can access the probability mass function in the following way

```
poisson.pmf(k, mu, loc)
```

where k is the discrete random variable, μ the shape parameter and `loc` the location parameter of the Poisson distribution.

```
In [60]: def plot_pmfs(X, muVals, labels):
             """
             plot Poisson probability mass functions
             """

             pColors = ['#CCCCCC', 'C0', 'C1', 'C2']
```

```
f, ax = plt.subplots(1)
f.set_size_inches(5.5, 3.5)

ax.set_xlabel(r'$k$', fontsize = 18.0)
ax.set_ylabel(r'$p(k\, ; \mu)$', fontsize = 18.0)
ax.xaxis.labelpad = 4.0
ax.yaxis.labelpad = 4.0

labelfontsize = 15.0

for tick in ax.xaxis.get_major_ticks():
    tick.label.set_fontsize(labelfontsize)
for tick in ax.yaxis.get_major_ticks():
    tick.label.set_fontsize(labelfontsize)

lineWidth = 1.5

ax.plot([-5.0, 35.0], [0.0, 0.0],
         color = pColors[0],
         alpha = 1.0,
         lw = lineWidth,
         zorder = 2,
         dashes = [4.0, 2.0])

for i in range(len(muVals)):

    ax.plot(X[:, 0], X[:, i + 1],
             color = pColors[i + 1],
             alpha = 1.0,
             lw = lineWidth,
             zorder = 2,
             label = r'')

    ax.scatter(X[:, 0], X[:, i + 1],
                s = 20.0,
                lw = lineWidth,
                facecolor = pColors[i + 1],
                edgecolor = 'None',
                zorder = 11,
                label = labels[i])

leg = ax.legend(handlelength = 0.25,
                scatterpoints = 1,
                markerscale = 1.0,
                ncol = 1,
                fontsize = 14.0)
leg.draw_frame(False)
plt.gca().add_artist(leg)
```

```
        ax.set_xlim(-0.5, 19.0)

        return None
```

In [61]:
```
%%time
# create Poisson distribution
muVals = [1.0, 5.0, 9.0]
xVals = np.arange(0, 30, 1)

X = np.zeros((len(xVals), len(muVals) + 1))
X[:, 0] = xVals

for i, mu in enumerate(muVals):

    yVals = poisson.pmf(xVals, mu)
    assert xVals.shape == yVals.shape, "Error: Shape assertion failed."

    X[:, i + 1] = yVals

labels = [r'$\mu = 1$',
          r'$\mu = 5$',
          r'$\mu = 9$']

plot_pmfs(X, muVals, labels)
```

```
CPU times: user 74.9 ms, sys: 6.57 ms, total: 81.4 ms
Wall time: 71 ms
```