Assignment - 1

* 1) 1-D Steady - State equation conduction

General Transport equation is

$$\frac{\partial \phi}{\partial t} + (u \cdot \nabla)\phi = \nabla(\Gamma \cdot \nabla)\phi + S$$

Here $\frac{\partial \phi}{\partial t}$ is time related term,

$(u \cdot \nabla)\phi$ is conduction term
$\nabla(\Gamma \cdot \nabla)\phi$ is diffusion term.
S is a source term

For 1-D Steady state conduction,

$\nabla(\Gamma \cdot \nabla)\phi$ will not be zero, where there is no Source term so it will be zero and conduction will ~~to~~ also be zero.

Also we are assuming Γ as constant

$$\therefore T \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = 0$$

But as we are taking 1-D only, x term is there and y and z are zero.

$$\therefore T \cdot \left(\frac{\partial^2 T}{\partial x^2}\right) = 0$$

as T depends only on x, we can rewrite as

$$\frac{\partial^2 T}{\partial x} \frac{d^2 T}{\partial x^2} = 0$$

using finite difference method

$$\frac{\partial^2 T}{\partial x^2} = 0 \text{ — (i)}$$

using Taylor's Expansion

$$T(x+h) = kT(x) + \frac{\partial T}{\partial x} \cdot h + \frac{\partial^2 T}{\partial x^2} \frac{h^2}{T} + \dots$$

$$T(x-h) = T(x) - \frac{\partial T}{\partial x} \cdot h + \frac{\partial^2 T}{\partial x} \cdot \frac{h^2}{2} - -$$

$$\therefore T(x+h) + T(x-h) = 2T(x) + \frac{\partial^2 T}{\partial x^2} \cdot h^2$$

$$\therefore \frac{\partial^2 T}{\partial x^2} = T(x+h) - \frac{2T(x)}{h^2} + T(x-h) \text{ — (2)}$$

From eqn (i) & (ii),

$$\therefore \frac{\partial^2 T}{\partial x^2} = T(x+h) - 2T(x) + T(x-h) = 0$$

$$T(x) = T(x+h) - 2T(x) + T(x-h) = 0$$

VISION

$$\therefore \boxed{T(x) = \frac{T(x+h) + T(x-h)}{2}}$$