Assignment-2

* 1-D unsteady State Conduction:-

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\Gamma \cdot \frac{\partial T}{\partial x}\right)$$

Taking $\Gamma = \frac{K}{\rho c_p}$ as constant

$$\frac{\partial T}{\partial t} = \Gamma \cdot \frac{\partial^2 T}{\partial x^2} = \frac{K}{\rho c_p} \cdot \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \frac{\delta t}{\delta t} = \frac{T'(x) - T(x)}{\delta t} \quad (1)$$

$$\therefore \frac{\delta T}{\delta t} = \frac{K}{\rho c_p}\left[\frac{T(x+h) - 2T(x) + T(x-h)}{h^2}\right] \quad (2)$$

In (2), we have discritize, the $\frac{\partial T}{\partial t}$ and used Taylor's expansion for Second order.

$$T(x+h) = T(x) + h\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \cdot \frac{h^2}{2} + \ldots$$

$$T(x-h) = T(x) - h\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \cdot \frac{h^2}{2} + \ldots$$

$$\therefore T(x+h) + T(x-h) = 2T(x) + \frac{\partial^2 T}{\partial x^2} \cdot h^2$$

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$$\therefore \frac{\partial^2 T}{\partial x^2} = \frac{T(x+h) + T(x-h)}{h^2} \quad — \text{③}$$

from eqⁿ ① & ③

$$\frac{T'(x) - T(x)}{\Delta t} = \frac{K}{\rho c_p} \left[\frac{T(x+h) - 2T(x) + T(x-h)}{h^2} \right]$$

$$\therefore T'(x) = \frac{\Delta t \cdot K}{\rho c_p} \left[\frac{T(x+h) - 2T(x) + T(x-h)}{h^2} \right]$$