Pandit Deendayal Energy University
Computational Engineering Laboratory
Matlab Case study Report

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Case Study 1

Problem Statement :-

To Solve a problem on Heat Transfer of Natural Convection in H – Shaped Cavity.

Derivation :-

As Per the Source of paper

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x}+\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\begin{aligned}u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=&-\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial y}+\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\\&+g\beta_{T,hnf}(T-T_c)+g\beta_{c,hnf}(c-c_c)\end{aligned} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \tag{5}$$

Introducing the following dimensionless variables:

$$U=\frac{uL}{\alpha_f}, V=\frac{vL}{\alpha_f}, \theta=\frac{(T-T_c)}{\Delta T}, C=\frac{(c-c_c)}{\Delta c}, P=\frac{pL^2}{\rho_f\alpha_f^2} \tag{6}$$

The non-dimensional shape of governing equations becomes:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\rho_f}{\rho_{hnf}}\frac{\partial P}{\partial X}+\mathrm{Pr}\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$\begin{aligned}U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=&-\frac{\rho_f}{\rho_{hnf}}\frac{\partial P}{\partial Y}+\mathrm{Pr}\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)\\&+\frac{\beta_{hnf}}{\beta_f}Ra\mathrm{Pr}(\theta+NC)\end{aligned} \tag{8}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{(k_{hnf}/k_f)}{\left((\rho C_p)_{hnf}/(\rho C_p)_f\right)}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \tag{10}$$

Where;

$$Ra=\frac{g\beta_f\Delta TL^3}{\nu_f\alpha_f}, \mathrm{Pr}=\frac{\upsilon_f}{\alpha_f}, \mathrm{Le}=\frac{\alpha_f}{D} \tag{11}$$

Boundary conditions:

$$\text{Bottom wall: } \frac{\partial\theta}{\partial Y}=0, U=0, V=0, \Psi=0, \frac{\partial C}{\partial Y}=0$$

$$\text{Bottom rib: } \frac{\partial\theta}{\partial n}=-1, U=0, V=0, \Psi=0, \frac{\partial C}{\partial n}=-1$$

$$\text{Right and left walls: } \theta=0, U=0, V=0, \Psi=0, C=0$$

$$\text{Top wall: } \frac{\partial\theta}{\partial Y}=0, U=0, V=0, \Psi=0, \frac{\partial C}{\partial Y}=0$$

$$\text{Top rib: } \frac{\partial\theta}{\partial n}=-1, U=0, V=0, \Psi=0, \frac{\partial C}{\partial n}=-1$$

$$\text{Baffle: } \frac{\partial\theta}{\partial n}=-1, U=0, V=0, \Psi=0, \frac{\partial C}{\partial n}=-1 \tag{12}$$

The target physical parameters in this paper are the local Nusselt and Sherwood numbers. These parameters are defined as:

$$Nu=\left(\frac{k_{hnf}}{k_f}\right)\left(\frac{1}{\theta}\right) \tag{13}$$

$$Sh=\left(\frac{1}{C}\right) \tag{14}$$

And the average ones are:

$$Nu_{avg}=\frac{1}{s}\int Nu.ds \tag{15}$$

$$Sh_{avg}=\frac{1}{s}\int Sh.ds \tag{16}$$

Where s is the length of the bottom rib.

K, (ρC_p), μ, $(\rho\beta)$, and ρ of hybrid nanoliquid could be represented as:

$$\frac{k_{hnf}}{k_f}=\left[(1-\lambda)\varphi\left(k_f-k_s\right)+k_s-(1-\lambda)k_f\right]\times\left[k_s-(1-\lambda)k_f+\varphi\left(k_f-k_s\right)\right]^{-1} \tag{17}$$

$$\left(\rho C_p\right)_{hnf}=\varphi_{Cu}\left(\rho C_p\right)_{Cu}+\varphi_{Al_2O_3}\left(\rho C_p\right)_{Al_2O_3}+(1-\varphi)\left(\rho C_p\right)_f \tag{18}$$

$$\mu_{hnf}=(1-\varphi)^{-2.5}\mu_f \tag{19}$$

$$(\rho\beta)_{hnf}=(1-\varphi)(\rho\beta)_f+\varphi_{Cu}(\rho\beta)_{Cu}+\varphi_{Al_2O_3}(\rho\beta)_{Al_2O_3} \tag{20}$$

$$\rho_{hnf}=(1-\varphi)\rho_f+\varphi_{Cu}\rho_{Cu}+\varphi_{Al_2O_3}\rho_{Al_2O_3} \tag{21}$$

And the correlations of entropy generation would be as:

$$S_{gen}=S_{HT}+S_{FF}+S_{Mass} \tag{22}$$

In which

$$S_{HT}=\frac{k_{hnf}}{k_f}\left(\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2\right) \tag{23}$$

$$S_{FF}=\varphi_1\frac{\mu_{hnf}}{\mu_f}\left[2\left(\frac{\partial V}{\partial Y}\right)^2+2\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2\right] \tag{24}$$

$$S_{Mass}=\varphi_2\left(\left(\frac{\partial C}{\partial X}\right)^2+\left(\frac{\partial C}{\partial Y}\right)^2\right)+\varphi_3\left(\frac{\partial C}{\partial X}\frac{\partial\theta}{\partial X}+\frac{\partial C}{\partial Y}\frac{\partial\theta}{\partial Y}\right) \tag{25}$$

And the Bejan number is:

$$Be=\frac{S_{HT}+S_{Mass}}{S_{gen}} \tag{26}$$

Equations (Our Source (Handwritten))

Case Study -1 Equations

* The derivation ~~the~~ the equations we need :-

1) Governing equations :-

$\nabla \cdot V = 0$

Its represents conservation of Mass.

2) Navier Stokes equation :-

$$\rho \left(\frac{\partial v}{\partial t} + V \cdot \nabla v \right)$$

$$= -\nabla p + \mu \nabla^2 v + \rho g.$$

It's represents the Conservation Momentum

3) Energy equations :-

$$\rho c_p \left(\frac{\partial T}{\partial t} + V \cdot \nabla T \right) = k \nabla^2 T$$

It's represents the Conservation of energy.

* Assumptions and Simplifications :-

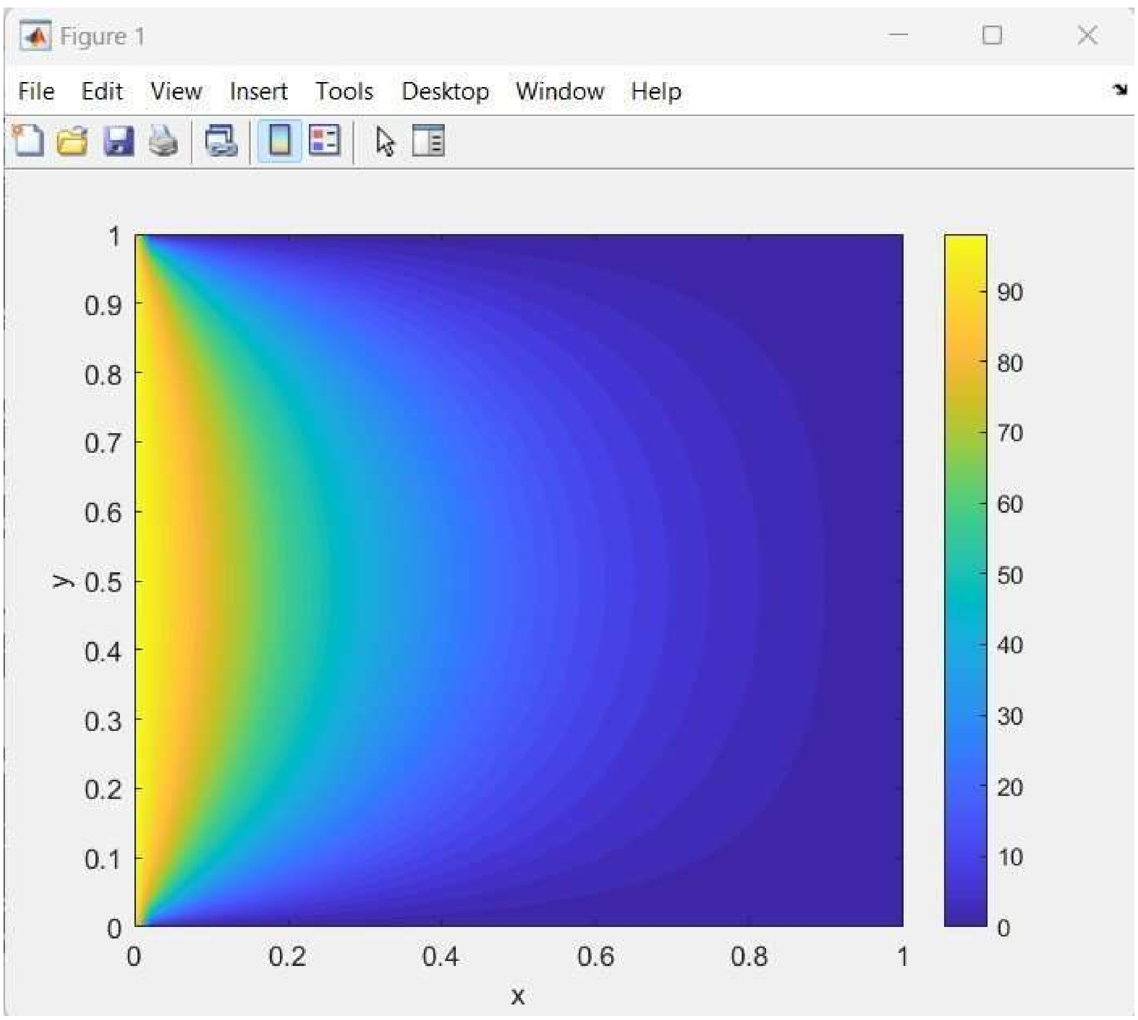
- Steady State Condition : $\frac{\partial}{\partial t} = 0$
- Incompressible flow : $\nabla \cdot V = 0$
- Boussinesq approximation (density variation with temperature) : $\rho = \rho_0 (1 - \beta (T - T_0))$
- Negligible viscous dissipation : Neglecting the viscous term $\mu \nabla^2 v$
- Constant fluid properties : ρ, μ, k and C_p are constant within the Domain.

Code

```
title('Temperature Distribution in H-Shaped Cavity');
% Parameters
L = 1; % Length of cavity
W = 1; % Width of cavity
H = 0.1; % Height of cavity
T_hot = 100; % Temperature of hot wall
T_cold = 0; % Temperature of cold walls
nx = 51; % Number of grid points in x-direction
ny = 51; % Number of grid points in y-direction
dx = L/(nx-1); % Grid spacing in x-direction
dy = W/(ny-1); % Grid spacing in y-direction
max_iter = 1000; % Maximum number of iterations
tolerance = 1e-5; % Convergence tolerance

% Initialization
T = ones(nx, ny) * T_cold; % Initialize temperature array

% Main loop (iterate until convergence)
for iter = 1:max_iter
    % Boundary conditions
    T(:,1) = T_cold; % Left cold wall
    T(:,end) = T_cold; % Right cold wall
    T(1,:) = T_hot; % Hot wall

    % Compute temperature field using simple averaging
    T_new = T;
    for i = 2:nx-1
        for j = 2:ny-1
            % Average of neighboring points
            T_new(i,j) = 0.25*(T(i+1,j) + T(i-1,j) + T(i,j+1) + T(i,j-1));
        end
    end

    % Check for convergence
    if max(abs(T_new(:) - T(:))) < tolerance
        disp(['Converged at iteration ', num2str(iter)]);
        break;
    end
```

```
    % Update temperature array
    T = T_new;
end

% Plotting the results
[X, Y] = meshgrid(linspace(0,L,nx), linspace(0,W,ny));
contourf(X, Y, T', 50, 'LineColor', 'none');
colorbar;
xlabel('x');
ylabel('y');
```

Results :-



Case Study 2

Problem Statement:-

To Update the code on the problem on Heat Transfer of Natural Convection in H – Shaped Cavity.

Derivations: -

Case Study 2 Equations

* To derive the equations :-

⇒ Governing equations :-

Start with the continuity equation :-

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By using Navier-Stokes equations (momentum equation):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g_x$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g_y$$

Then,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Code :-

```
% Parameters
L = 1;           % Length of cavity
W = 1;           % Width of cavity
H = 0.1;         % Height of cavity
T_hot = 100;     % Temperature of hot wall
T_cold = 0;      % Temperature of cold walls
nx = 51;         % Number of grid points in x-direction
ny = 51;         % Number of grid points in y-direction
dx = L/(nx-1);   % Grid spacing in x-direction
dy = W/(ny-1);   % Grid spacing in y-direction
max_iter = 100; % Maximum number of iterations
tolerance = 1e-5;% Convergence tolerance
alpha = 0.1;     % Thermal diffusivity

% Initialize temperature array
T = ones(nx, ny) * T_cold;

% Set up VideoWriter object

writerObj = VideoWriter('temperature_evolution.avi'); % Specify the file name
writerObj.FrameRate = 10; % Set the frame rate

% Open the video writer object
open(writerObj);

% Create a figure for animation
figure;

% Main loop (iterate until convergence)
for iter = 1:max_iter
    % Boundary conditions
    T(:,1) = T_cold;            % Left cold wall
    T(:,end) = T_cold;          % Right cold wall
    T(1,:) = T_hot;             % Hot wall
    T(ceil(end/2-round(H/(2*dy))):floor(end/2+round(H/(2*dy))), end) = T_cold; % Middle cold wall

    % Compute temperature field using finite difference method
    T_new = T;
    for i = 2:nx-1
        for j = 2:ny-1
            % Finite difference equation
            T_new(i,j) = T(i,j) + alpha * ( (T(i+1,j) - 2*T(i,j) + T(i-1,j))/dx^2 + (T(i,j+1) - 2*T(i,j) + T(i,j-1))/dy^2 );
        end
    end

    % Check for convergence
    if max(abs(T_new(:) - T(:))) < tolerance
        disp(['Converged at iteration ', num2str(iter)]);
        break;
    end

    % Update temperature array
    T = T_new;

    % Plot the current temperature distribution in 3D
    [X, Y] = meshgrid(linspace(0,L,nx), linspace(0,W,ny));
    surf(X, Y, T', 'EdgeColor', 'none');
    colorbar;
    xlabel('x');
    title(['Iteration: ', num2str(iter)]);
    axis([0 L 0 W 0 100]); % Set axis limits for consistency

    % Capture the frame
    frame = getframe(gcf);

    % Write the frame to the video file
    writeVideo(writerObj, frame);
end

% Close the video writer object
close(writerObj);
```

Results :-

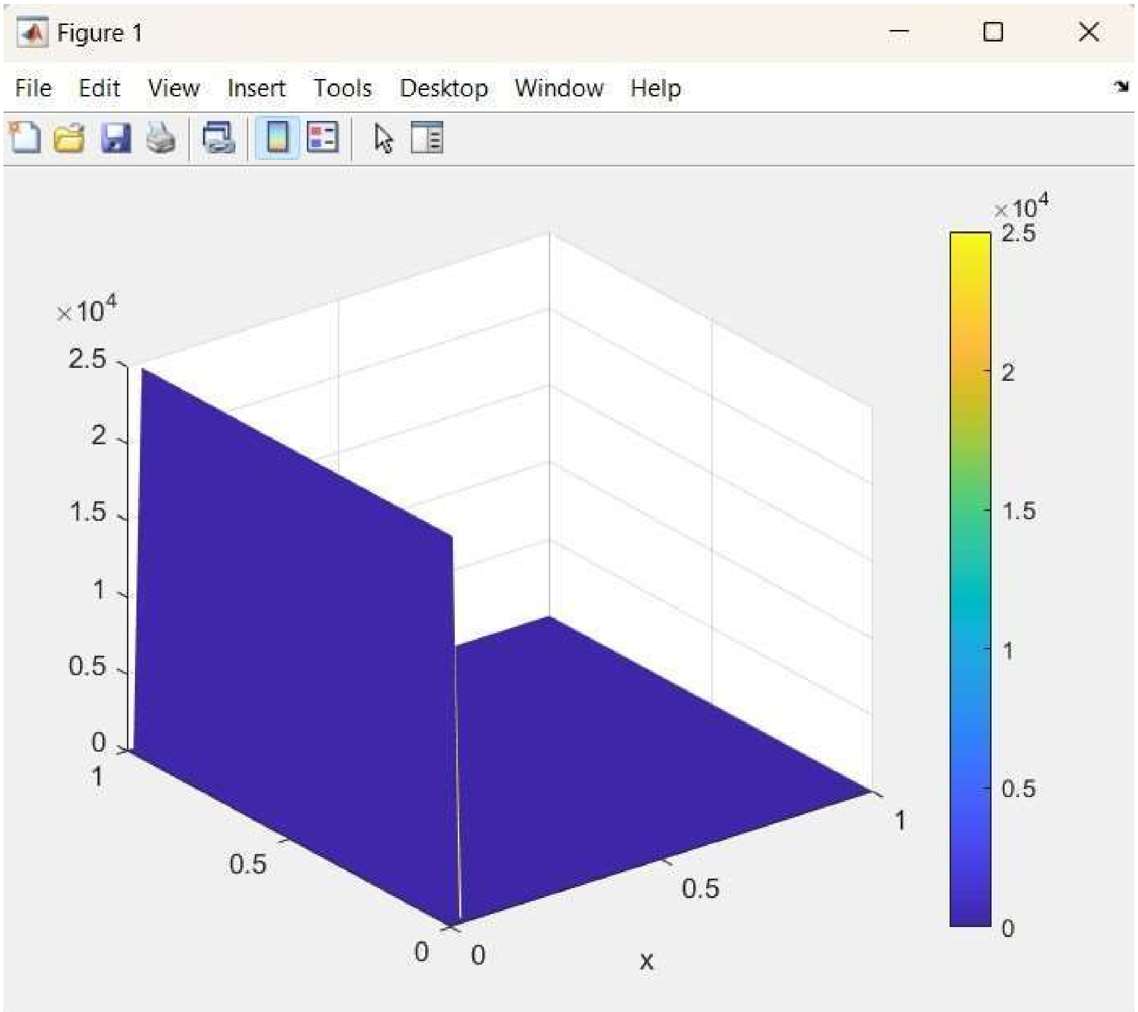


Figure 1
File Edit View Insert Tools Desktop Window Help
×10^4
2.5
2
1.5
1
0.5
0
1
0.5
0
0
0.5
1
x
×10^4
2.5
2
1.5
1
0.5
0