STA380.18 - Homework 2

Principal Components Analysis

- Avani Sharma (as85253)

1. Create a SAS dataset called WORK.RATINGS that contains the data in the job ratings.txt file. Assign the SAS names JOB, KNOWHOW, PROBLEM_SOLVING, ACCOUNTABILITY, SALARY, respectively, to the five variables as they appear from left to right in the file. Extract the principal components of the three dimensions that were rated by the management consulting firm. Use the default (standardized) version of the extraction. Your answer for question 1 is your SAS code only.

1:

**Q1.;

data WORK.RATINGS:

input job knowhow problem_solving accountability salary; cards:

0 800 608 1056 102000

2 528 304 460 75740

3 460 264 460 75740

5 528 304 304 79172

4 460 264 400 70000

0 460 264 400 66536

0 528 304 264 70000

7 460 230 264 68000

10 400 200 350 73140

7 400 175 230 66016

7 400 200 200 66016

5 400 175 200 71840

Run:

**Principal Components;

PROC Princomp data= WORK.RATINGS; var knowhow problem solving accountability; RUN:

- 2. This question verifies the basic property of principal components transformations.
 - a. Write the equations of the principal components of the PCA in question 1.
 - b. Verify that the principal component transformation in question 1 is an orthonormal rotation of the (standardized) original three dimensions by showing that the rotation matrix satisfies the definition of an orthonormal transformation.
- 2: a. Equations of the principal components of PCA in question 1 are -

- **b.** To prove that principal component transformation is an orthogonal rotation of the original 3 dimensions, two conditions should be true: Vectors should be of unit length & orthogonal to each other
 - i. 3 Vectors (principal components) must be of unit length

Length (PC 1) =
$$0.576251 * 0.576251 + 0.584343 * 0.584343 + 0.571383 * 0.571383 = 1$$

Length (PC 2) = $-0.618121 * -0.618121 + -0.145758 * -0.145758 + 0.772451 * 0.772451 = 1$
Length (PC 3) = $0.534660 * 0.534660 + -0.798310 * -0.798310 + 0.277201 * 0.277201 = 1$

ii. 3 Vectors (principal components) must be orthogonal to each other (i.e. dot product of any 2 pair of vectors must be zero)

Dot Products:

On Excel:

Unit Vectors:

Eigenvectors				Eigenvector Squares				
	Prin1	Prin2	Prin3	Prin1*Prin1	Prin2*Prin2	Prin3*Prin3	Sum	
knowhow	0.576251	-0.61812	0.53466	0.332065215	0.382073571	0.285861316	1	
problem_solving	0.584343	-0.14576	-0.79831	0.341456742	0.021245395	0.637298856	1	
accountability	0.571383	0.772451	0.277201	0.326478533	0.596680547	0.076840394	1	

Dot Product:

	Sum			
Prin1.Prin2	0.336728	0.090096	-0.42682	0
Prin2.Prin3	0.333884	-0.11259	-0.22129	0
Prin3.Prin1	0.32926	-0.47747	0.148208	0

From the above calculations, the first & second conditions are met. Thus, principal component transformation is an orthonormal transformation.

3. This question partially verifies the geometry-preserving property of principal components transformations.

- a. Rotate the first two jobs in the text file by calculating their principal component scores.
- b. The rotated scores for the two jobs in part (a) are each a vector of three scores. Verify that the lengths of these two vectors are the same as the lengths of the original (but standardized) ratings vectors of the two jobs.
- c. Verify that the angle between these two rotated vectors is the same as the angle between the original unrotated vectors.

3:

a. Principal Components for the first 2 jobs in the text file are shown below -

** 03:

PROC princomp data = WORK.RATINGS OUT= WORK.RATINGS_PCA; var knowhow problem_solving accountability; RUN;

From SAS:

job	Prin1	Prin2	Prin3
0	9.089332156	1.2654300739	-0.19679536
2	3.7513631804	-0.059567149	0.0985589143
3	3.2058572773	0.3254455567	0.1501086816
5	3.1543849719	-0.866619424	-0.191059403

On Excel:

For Instance:

	Prin1
knowhow	0.576251
problem_solving	0.584343
accountability	0.571383

Prin 1:

standardised_knowhow*0.576+standardised_problemsolving*0.584+standardised_accountability*0.571

Similar calculations Applied,

		Standardised Xs	Calculated Principal Components			
job	knowhow	problem_solving	accountability	Prin1	Prin2	Prin3
0	4.35032492	5.283939792	6.116424944	9.089334	1.265435	-0.1968
2	2.25124045	2.122082877	2.124774933	3.751364	-0.05957	0.098558

b. Verifying that the lengths of the standardized vector and PCs is same

Standardised Xs			Calculate	d Principal	Original	Transforme		
job	knowhow	problem_solving	accountability	Prin1	Prin2	Prin3	Length	Length
0	4.350325	5.283939792	6.116424944	9.089334	1.265435	0.196798143	9.1791068	9.17910885:
2	2.25124	2.122082877	2.124774933	3.751364	-0.05957	0.098557974	3.7531304	3.75313127

c. In order to verify that the angle between the 2 rotated vectors and the original vectors is same, we will calculate cosine of the angle between the 2 vectors.

Cosine (angle) between 2 vectors = dot product (vec 1, vec 2) / length (vec 1) * length (vec 2)

Cosine (angle) between original vectors	0.987002391
Cosine (angle) between rotated vectors	0.9870024

The cosine of the angle between 2 original vectors is same as in the rotated vectors

4. Obtain the principal components scores for all 67 jobs. Calculate the variances of the three sets of scores and verify that the variances are equal to the eigenvalues of the PC transformation.

4)

The eigen values of the PC transformation are shown below -

	Eigenvalues of the Correlation Matrix								
	Eigenvalue	Difference	Proportion	Cumulative					
1	2.90808114	2.82438377	0.9694	0.9694					
2	0.08369737	0.07547588	0.0279	0.9973					
3	0.00822149		0.0027	1.0000					

The variance of the principal component score across all the 67 jobs are **2.90808**, **0.08370** and **0.00822** as obtained from the calculations done in excel. Clearly, variances are equal to the eigenvalues of PCs.

Standardised kh	Standardised ps	Standardised acc	Prin1	Prin2	Prin3		Variances
4.35	5.28	6.12	9.09	1.27	-0.20	Prin1	2.90808
2.25	2.12	2.12	3.75	-0.06	0 10	Prin2	0.08370
1.73	1.71	2.12	3.21	0.33	0.15	Prin3	0.00822
2.25	2.12	1.08	3.15	-0.87	-0.19	-	
1.73	1.71	1.72	2.98	0.02	0.04		
1.73	1.71	1.72	2.98	0.02	0.04		
2.25	2.12	0.81	3.00	-1.07	-0.27		
1.73	1.35	0.81	2.25	-0.64	0.07		
1.26	1.04	1.39	2.13	0.14	0.23		
1.26	0.78		1.52	-0.44	0.21		
1.26	1.04		1.56	-0.64	-0.05		
1.26	0.78		1.40	-0.60	0.16		
0.52	0.16		0.52	-0.18	0.21		
0.21	0.00		0.25	0.03	0.17		
0.21	0.00		0.25	0.03	0.17		
-0.05	0.00	The state of the s	-0.07	-0.03	-0.05		

5. Find the regression equation that results from regressing **PRIN1** on the three ratings knowhow, problem_solving, and accountability after the ratings have been standardized and without an intercept.2 Are you surprised by the equation?

5)

** Q5;

PROC STDIZE DATA=WORK.RATINGS OUT=WORK.STD;

VAR knowhow problem_solving accountability;

PROC princomp data = WORK.STD OUT= WORK.STD_PCA;
 var knowhow problem_solving accountability;
 RUN;

PROC REG DATA = WORK.STD_PCA;
 model Prin1 = knowhow problem_solving accountability / noint;
 RUN;

Parameter Estimates									
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t				
knowhow	1	0.57625	0	Infty	<.0001				
problem_solving	1	0.58434	0	Infty	<.0001				
accountability	1	0.57138	0	Infty	<.0001				

The regression equation is as follows.

Prin1 = 0.57625 * knowhow + 0.58434 * problem_solving + 0.57138 * accountability

The regression coefficients are equal to the loadings that we obtained for Prin1 in question 2, part b. The principal components are orthogonal unit vectors which contain the entire information conveyed by the variables. So, for each variable when we regress it with principal components, regressing it alone or in group would give the same coeffcients (no multicollineraity). This is not that surprising, since Principal components are a linear combination of the variables.

6. Find the regression equation that results from regressing (standardized) KNOWHOW on the three principal components without an intercept. Are you surprised by the equation?

6)

** Q6;

PROC REG DATA = WORK.STD_PCA; model knowhow = Prin1 Prin2 Prin3 / noint; RUN;

The results from SAS:

		Parameter	Estimates		
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Prin1	1	0.57625	0	Infty	<.0001
Prin2	1	-0.61812	0	-Infty	<.0001
Prin3	1	0.53466	0	Infty	<.0001

From questin 2 calculations:

	Eigenvecto	ors		Eigenvector Squares				
	Prin1	Prin2	Prin3	Prin1*Prin1	Prin2*Prin2	Prin3*Prin3	Sum	
knowhow	0.576251	-0.61812	0.53466	0.332065215				
problem_solving	0.584343	-0.14576	-0.79831	0.341456742				
accountability	0.571383	0.772451	0.277201	0.326478533		0.076840394		

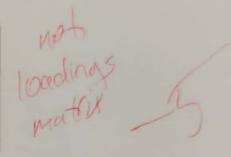
The regression equation is as follows.

Knowhow = 0.57625 * Prin1 - 0.61812 * Prin2 + 0.53466 * Prin3

The coefficients are just the loadings that we obtained for 'knowhow' when we ran a principal component analysis in question 1 (image picked from 2). The results are not surprising because original variables are also a linear combination of principal components. These PCs are orthogonal to each other and hence aren't multicollinear, running regression individually or with all PCs at a time would yield the same coefficients.

- 7. Write the loadings matrix, structured with components as columns and variables as rows. Using the loadings matrix, try to interpret meanings for the three principal components.
- 7) The loading matrix from SAS program:

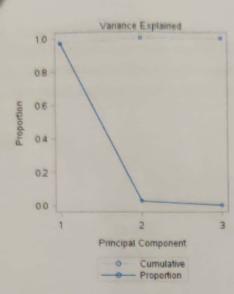
	Eigenvectors			
	Prin1	Prin2	Prin3	
knowhow	0.576251	618121	0.534660	
problem_solving	0.584343	145758	798310	
accountability	0.571383	0.772451	0.277201	



Interpretation -

- 1. Prin1: Loadings for all three variables are equally high for the first principal component implying an all rounded score for all three traits. Such can be the requirement of jobs with holiistic responsibilities starting from inception of an idea to implementation. It could represent job profiles which are manegerial or project owner level.
- 2. Prin2: This component is positively high on accountability but negatively loaded with knowhow (problem-solving to a small extent). These can be entry-level positions which are focussed highly on execution or assiting job profiles like personal assissants.
- 3. Prin3: This component has a high negative loading of problem-solving and smaller positive loading of knowhow, they can be associated with job profiles requiring research, sales which are required to know about the projects in and out but real world implementation is taken care of by others.
- 8. How many principal components would you retain ...
- 8) a. Kaiser Rule: The Kaiser rule is to drop all components with eigenvalues under 1.0 this being the eigenvalue equal to the information accounted for by an average single item. Thus we will only retain one PC.
 - b. Joliffe Rule: Retain all components that have eigen vale > 0.7. We will retain 1 PC
 - g. Using 80% rule Again we will retain the first PC. (from scree plot)

Eigenvalues of the Correlation Matrix					
	Eigenvalue	Difference	Proportion	Cumulative	
1	2.90808114	2.82438377	0.9694	0.9694	
2	0.08369737	0.07547588	0.0279	0.9973	
3	0.00822149		0.0027	1.0000	



9. Find the regression equation that results from regressing salary on the three principal components with intercept. How much explanatory power do the three PCs collectively have in explaining salary?

** Q9;

PROC REG DATA = WORK. STD_PCA;

model salary = Prin1 Prin2 Prin3;

RUN;

Regression equation of salary vs the 3 PC's is as follows, they have a collective power of explaining 90.03% of the variance in salary.

Salary = 63929 + 3557.20641* Prin1 + 2316.12408 * Prin2 + 3540.61136 * Prin3

Root MSE	2082.09165	R-Square	0.9003
Dependent Mean	63929	Adj R-Sq	0.8955
Coeff Var	3.25686		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t		
Intercept	1	63929	254.36798	251.33	<.0001		
Prin1	1/	3557.20641	150.28811	23.67	<.0001		
Prin2	1	2316.12408	885.87403	2.61	0.0112		
Prin3	1	3540.61136	2826.52316	1.25	0.2150		

- 10. In terms of explaining salary...
- a) Which component is most useful? Second most useful? Least useful?
- b) Is the usefulness of the PCs for explaining salary in the order PC1 > PC2 > PC3?
- c) How much explanatory power is lost if one uses only PRIN1 to explain salary?

10)

Analysis of Variance							
Source	DF	Sum of Squares	Mean Square	F Value	Pr>F		
Model	3	2465105931	821701977	189.55	<.0001		
Error	63	273111655	4335106				
Corrected Total	66	2738217587					

Root MSE	2082.09165	R-Square	0.9003
Dependent Mean	63929	Adj R-Sq	0.8955
Coeff Var	3.25686		

Parameter Estimates								
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Type I SS	Type II SS	Variance Inflation
Intercept	1	63929	254.36798	251.33	<.0001/	2.738263E11	2738263E11	0
Prin1	1	3557.20641	150.28811	23.67	<.0001	2428670447	2428670447	1.00000
Prin2	1	2316.12408	885.87403	2.61	0.0112	29633257	29633257	1.00000
Prin3	1	3540.61136	2826.52316	1.25	0.2150	6802227	6802227	1.00000

- a. Type I SS value for Prin1 is maximum, it is the most important component. Prin2 is the second most useful and Prin3 is the least useful component
- b. Yes, the usefulness of PCs in explaining salary is of the order PC1 > PC2 > PC3
- c. Using only Prin1, we are able to explain the following amount of variance in salary:

 0.9* 2428670447 / (2428670447 + 29633257 + 6802227) = 88.7%. Thus we have lost 90
 88.7 = 1.3% explanatory power if we only use Prin1. VIFs here are all 1 as PCs are not multicollinear.

In statistics, the variance inflation factor (VIF) is the ratio of variance in a model with multiple terms, divided by the variance of a model with one term alone. It quantifies the severity of multicollinearity in an ordinary least squares regression analysis. It provides an index that measures how much the variance (the square of the estimate's standard deviation) of an estimated regression coefficient is increased because of collinearity. VIF < 10. The square root of the variance inflation factor indicates how much larger the standard error is, compared with what it would be if that variable were uncorrelated with the other predictor variables in the model.

```
** Q1;
data WORK. RATINGS;
      input job knowhow problem_solving accountability salary;
      cards;
0
   800
        608
              1056
                    102000
         304
              460
2
   528
                    75740
        264
3
   460
              460
                    75740
              304
5
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                    79172
   460
        264
4
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                    70000
   460
         264
              400
0
                    66536
0
   528
         304
              264
                    70000
7
   460
         230
              264
                    68000
10
   400
         200
               350
                    73140
   400
         175
              230
7
                    66016
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   400
                    71840
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                    71580
   304
         115
              175
2
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                    65860
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                  61960
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3
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             100
                   63260
7
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         57
             100
                   59880
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   175
         57
             100
                   62480
3
   175
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             100
                   63000
2
   175
         57
             100
                   63260
3
   175
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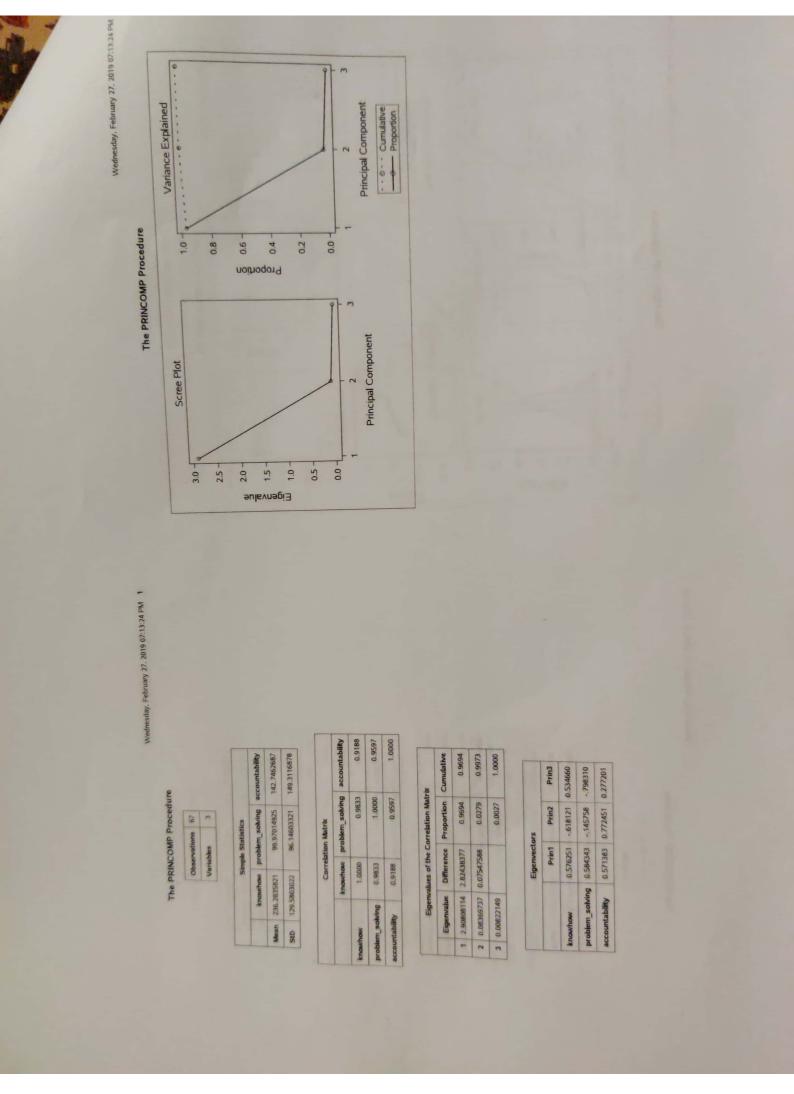
```
5
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          66
              76
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                  60200
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              76
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    175
         57
              76
                  61700
 5
    175
         66
                  60000
              66
 7
    152
         50
              87
                  60920
 7
    152
         50
              76
                  59100
 3
    152
         50
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                  59880
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                  59360
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2
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            57 59880
RUN;
**Principal Components;
PROC Princomp data= WORK.RATINGS;
     var knowhow problem_solving accountability;
     RUN;
** Q5;
PROC STDIZE DATA=WORK.RATINGS OUT=WORK.STD;
     VAR knowhow problem_solving accountability;
PROC princomp data = WORK.STD OUT= WORK.STD_PCA;
     var knowhow problem_solving accountability;
     RUN;
PROC REG DATA = WORK.STD_PCA;
     model Prin1 = knowhow problem_solving accountability / noint;
     RUN;
```

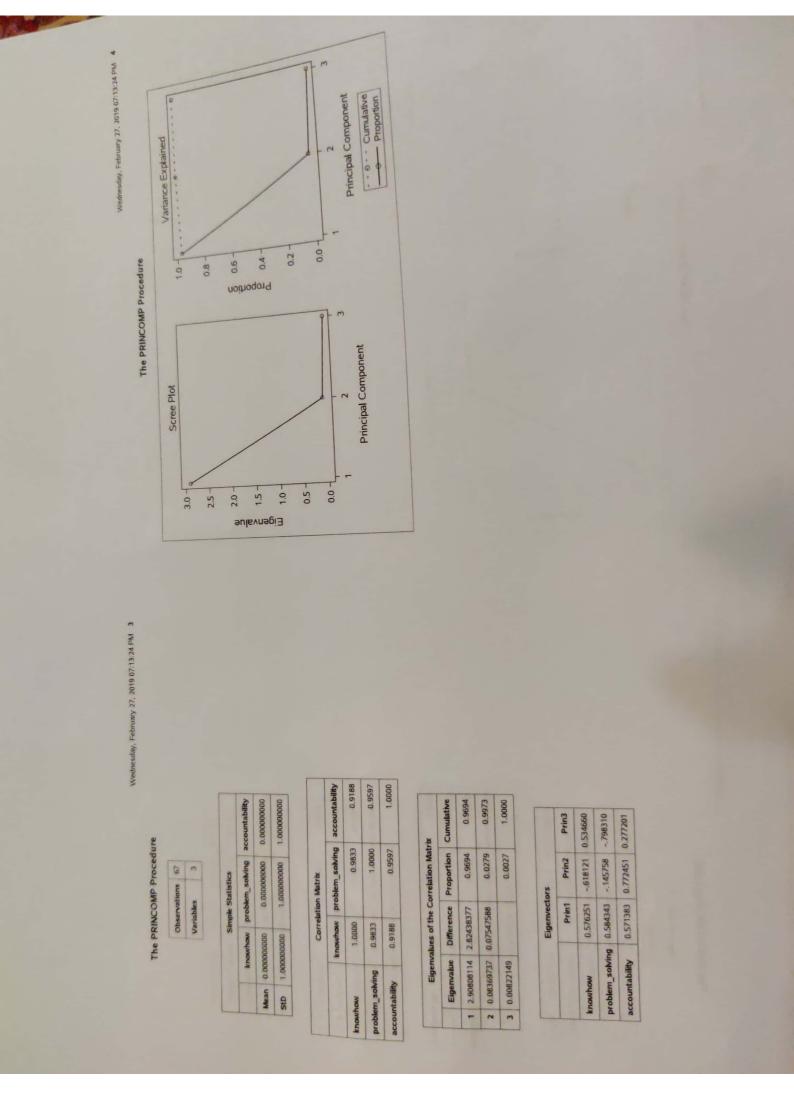
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** Q6;
pROC REG DATA=Work.STD_PCA;
    model knowhow = Prin1 Prin2 Prin3 / noint;

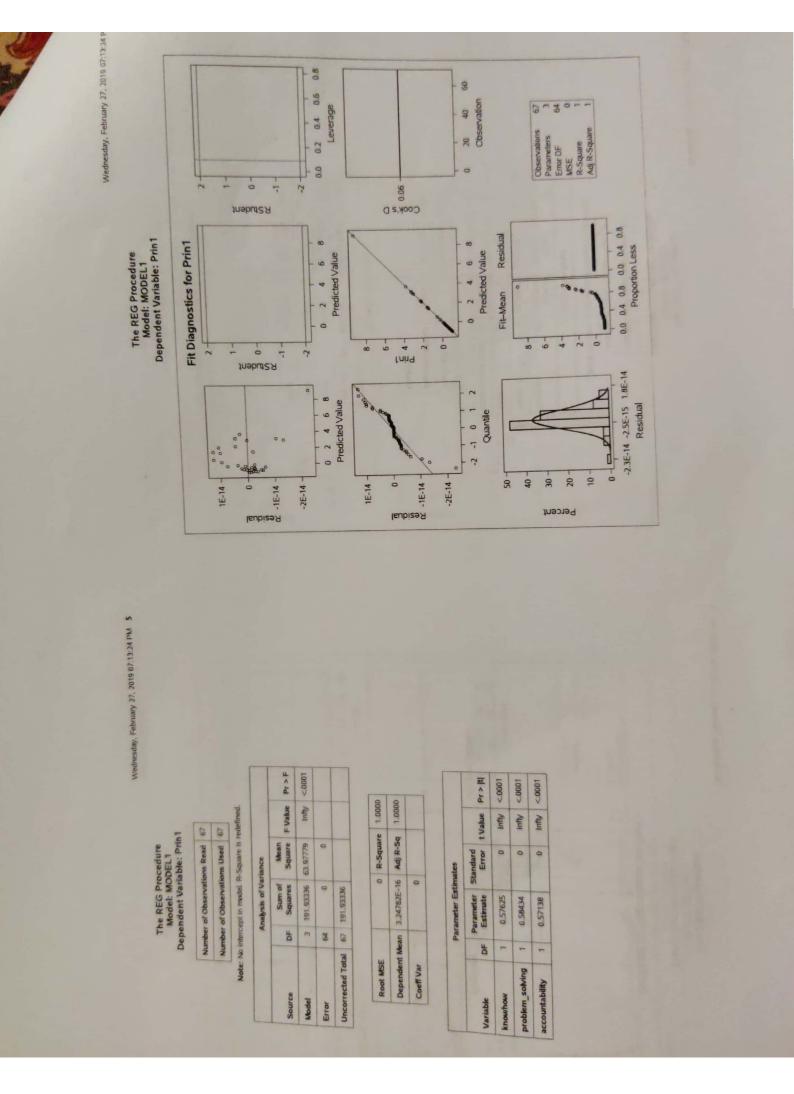
** Q9;

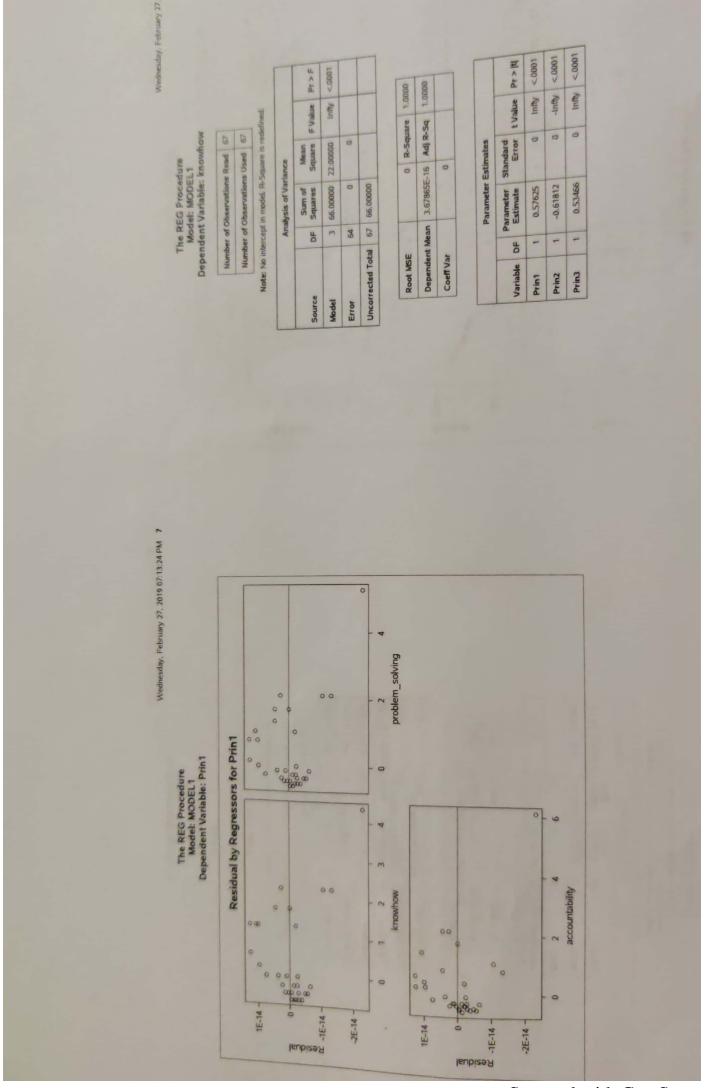
PROC REG DATA = WORK.STD_PCA;
    model salary = Prin1 Prin2 Prin3;
    RUN;

** Q10;
proc reg DATA=WORK.STd_PCA;
    model salary = PRIN1-PRIN3 / ss1 ss2 vif;
    RUN;
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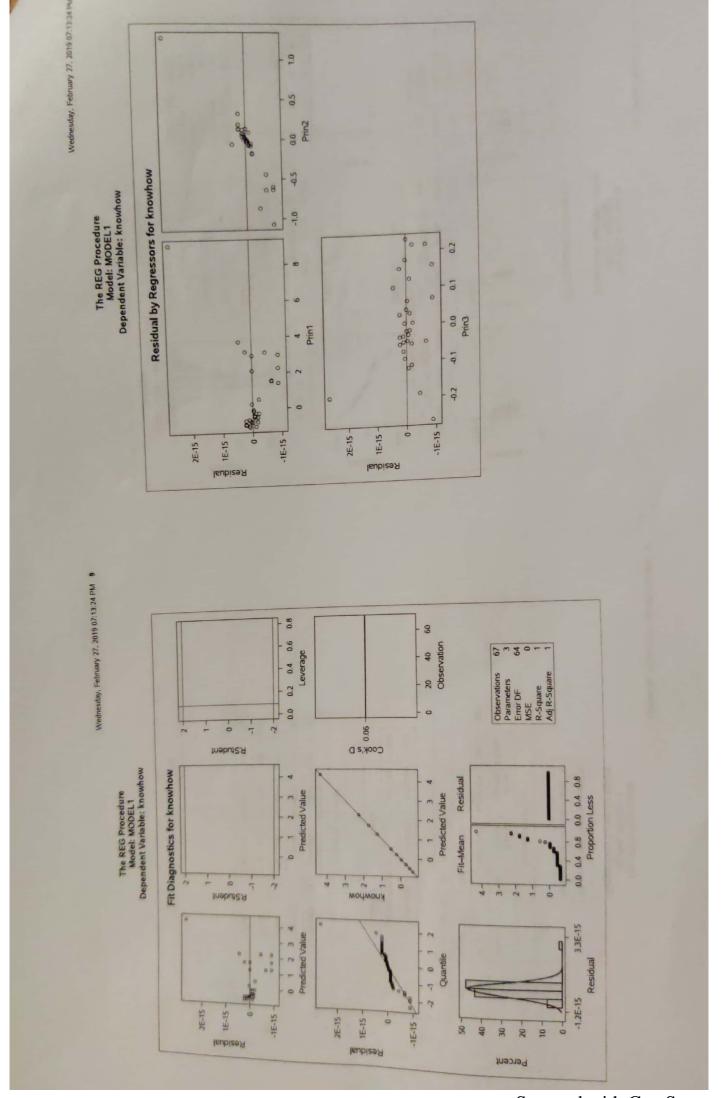








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