Q 1)

a) correlation matrix of manifest variables

$$R_{3\times3} = \begin{bmatrix} 1 & (0.8\times0.6)+(0.4\times0.6) & (0.8\times0.4)+(0.4\times0.8) \\ (0.8\times0.6)+(0.4\times0.6) & 1 & (0.6\times0.4)+(0.6\times0.8) \\ (0.8\times0.4)+(0.4\times0.8) & (0.6\times0.4)+(0.6\times0.8) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.72 & 0.64 \\ 0.72 & 1 & 0.72 \\ 0.64 & 0.72 & 1 \end{bmatrix}$$

b) communality of $X_1 = (0.8)^2 + (0.4)^2 = 0.64 + 0.16 = 0.8$

communality of $X_2 = (0.6)^2 + (0.6)^2 = 0.36 + 0.36 = 0.72$

communality of $X_3 = (0.4)^2 + (0.8)^2 = 0.16 + 0.64 = 0.8$

Q 2)

a) variance of X_1 explained by

factor1 $= (0.8)^2 = 0.64$

factor 2 $= (0.4)^2 = 0.16$

variance of X_2 explained by

factor1 $= (0.6)^2 = 0.36$

factor2 $= (0.6)^2 = 0.36$

Variance of X_3 explained by

factor1 $= (0.4)^2 = 0.16$

factor 2 $= (0.8)^2 = 0.64$

b) PCA is a special case of factor analysis where the uniqueness is zero. But here, we have non-zero uniqueness.

Hence, this model cannot be computationally equivalent to a principal component analysis.

Q3)

a) the rotated factor pattern $F^* = \begin{bmatrix} 0.8 & 0.4 \\ 0.6 & 0.6 \\ 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$$= \begin{bmatrix} 1.2 \times 1/\sqrt{2} & 0.4 \times 1/\sqrt{2} \\ 1.2 \times 1/\sqrt{2} & 0 \times 1/\sqrt{2} \\ 1.2 \times 1/\sqrt{2} & -0.4 \times 1/\sqrt{2} \end{bmatrix}$$

the correlation matrix for the F^* is:

$$\begin{bmatrix} 1 & (1.2)^2 \times 1/2 & (1.2)^2 \times 1/2 - (0.4)^2 \times 1/2 \\ (1.2)^2 \times 1/2 & 1 & (1.2)^2 \times 1/2 \\ (1.2)^2 \times 1/2 - (0.4)^2 \times 1/2 & (1.2)^2 \times 1/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.72 & 0.64 \\ 0.72 & 1 & 0.72 \\ 0.64 & 0.72 & 1 \end{bmatrix}$$

$\therefore$ we can see that both the correlation matrix are the same

b) communalities from the rotated two-factor model:

for x_1: $(1.2)^2 \times 1/2 + (0.4)^2 \times 1/2 = (1.44 + 0.16)/2 = 0.8$

for x_2 $= (1.2)^2 \times 1/2 + 0 = (1.44)/2 = 0.72$

for x_3 $= (1.2)^2 \times 1/2 + (-0.4)^2 \times 1/2 = (1.44 + 0.16)/2 = 0.8$

$\therefore$ The communalities are the same as in question 1(b)

Since the uniqueness of the manifest variables remain the same before and after the rotation, the communalities also remain the same.

angles

Q4) a) with the one-factor model given,

the correlation matrix is $\begin{bmatrix} 1 & ab & ac \\ ab & 1 & bc \\ ac & bc & 1 \end{bmatrix}$

to get the same correlation matrix as before

$ab = 0.72 \quad ac = 0.64 \quad bc = 0.72$

the values of a,b,c which satisfy these conditions are

$\boxed{a = 0.8 \quad b = 0.9 \quad c = 0.8}$

b) the solutions for 3(a) and 4(a) tells us that given a set of manifest variables there is no 'unique' set of factors.

we can get different factors based on the method, initialization, rotation or number of factors mentioned.

Question5:

Part a:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=one MINEIGEN=0
NFACTORS=6;
  VAR beefs previlege newlearn raises critical advance;
  TITLE "PC style factor analysis Factor Analysis - 6 factors ";
RUN;
```

Part b:

6 factors will be retained by the NFACTOR criterion.

Factor Pattern						
	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
beefs	0.78219	-0.31363	0.38883	-0.23490	-0.10787	0.26797
previlege	0.70268	-0.30973	0.18990	0.60569	-0.02123	-0.08333
newlearn	0.82140	-0.21777	-0.23756	-0.16709	0.43688	-0.05153
raises	0.87704	0.11590	0.00490	-0.27139	-0.25930	-0.27649
critical	0.40022	0.80479	0.39938	0.07429	0.16271	0.02533
advance	0.67791	0.32172	-0.59975	0.15293	-0.14347	0.18237

Variance Explained by Each Factor					
Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
3.1692232	1.0063467	0.7629087	0.5525165	0.3172465	0.1917584

I would retain the first **two factors** as they are the only factors with eigen value greater than 1

Part c:

The first factor descirbes factors which contribute to most of the good attributes of a supervisor. A person who settles complaints well, encourages merit, new learning and advances the employees career but this supervisor shows favoritsm as well.

The second factor addresses the not too good characteristic of an supervisor. Someone who is very critical, doesn't encourage new learnings and doesn't solve employee complaints.

Question6:

To see how each of the 6 factors explain the overall supervisor rating, we ran 7 regression models.

First 6 models is for each factor on the 'overall' rating column

And seventh model uses all the six factors

Model1: Overall ~ Factor1 **R2 = 0.457**

Model2: Overall ~ Factor2 **R2 = 0.0788**

Model3: Overall ~ Factor3 **R2 = 0.0878**

Model4: Overall ~ Factor4 **R2 = 0.089**

Model5: Overall ~ Factor5 **R2 = 0.006**

Model6: Overall ~ Factor6 **R2 = 0.0139**

Model7: Overall ~ Factor7 **R2 = 0.73**

Question7:

Part a:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=SMC NFACTORS=6
OUT=q7_factors;
  VAR beefs previlege newlearn raises critical advance;
  TITLE "Rsquare style factor analysis Factor Analysis - 6 factors ";
RUN;
```

Part b:

3 factors will be retained by the MINEIGEN criterion.

Factor Pattern			
	Factor1	Factor2	Factor3
beefs	0.74755	-0.36273	0.12483
previlege	0.61091	-0.17725	-0.06404
newlearn	0.76629	-0.05146	-0.21483
raises	0.84947	0.11042	0.13720
critical	0.32091	0.25308	0.26760
advance	0.61147	0.39882	-0.15045

The first factor descirbes factors which contribute to most of the good attributes of a supervisor. A person who settles complaints well, encourages merit, new learning and advances the employees career but this supervisor shows favoritsm as well.

Second factor describes a supervisor who is 'critical' and helps employees 'advance' their careers but isn't rated high in terms of solving employers complaints (lower 'beefs' score)

Part c:

In the factor analysis for Q5, we assume the uniqueness of each manifest variable is zero and assume that the factors explain all the variance. So the results are same as the results from principal component analysis.

And for Q7, we assume that the uniqueness is non zero. So to get the initial values of the commonalities, we regress each variable on the other variables and get the R2. These values would be the initial values.

Question8:

Part a:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=SMC NFACTORS=2
ROTATE=varimax OUT=work.evaluatesupervisors_scores;
  VAR beefs previlege newlearn raises critical advance;
  TITLE "Rsquare style factor analysis Factor Analysis - 2 factors ";
RUN;
```

Part b:

2 factors will be retained by the NFACTOR criterion.

Factor Pattern		
	Factor1	Factor2
beefs	0.74755	-0.36273
previlege	0.61091	-0.17725
newlearn	0.76629	-0.05146
raises	0.84947	0.11042
critical	0.32091	0.25308
advance	0.61147	0.39882

Yes, the first two factors for this question are not the same as the first 2 factors for the previous question

Part c:

The VARIMAX factor rotation matrix is:

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix		
	1	2
1	0.79912	0.60117
2	-0.60117	0.79912

Checking for orthonormality:

VARIMAX factor rotation matrix			VARIMAX factor rotation matrix			Product of both matrices	
0.79912	0.60117		0.79912	-0.60117		0.999998	0
-0.60117	0.79912		0.60117	0.79912		0	0.999998

Hence we see that the matrix is orthonormal

Part d:

Rotated Factor Pattern		
	Factor1	Factor2
beefs	0.81544	0.15954
previlege	0.59475	0.22561
newlearn	0.64329	0.41955
raises	0.61245	0.59891
critical	0.10431	0.39516
advance	0.24889	0.68630

The first factor descirbes factors which contribute to a supervisor who solves employee complaints efficiently, encourages new learnings and recognizes the employees by merit. Although these employees also tend to show favoritsm.

The second factor describes supervisors who tends to focus on the improvement of employees – either by recongnizing merit or encouraing new learnings or helping employees advance in their roles.

Question9:

Part a:

Simple Statistics						
Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
Factor1	30	0	0.85285	0	-1.86118	1.36525
Factor2	30	0	0.77645	0	-1.31002	2.01667

Pearson Correlation Coefficients, N = 30 Prob > |r| under H0: Rho=0		
	Factor1	Factor2
Factor1	1.00000	0.29041 0.1195
Factor2	0.29041 0.1195	1.00000

The statistics shows us that both the factors have standard deviations which are not one. This is surprising as the factors generally tend to be standardized with zero mean and unit standard deviation.

Also the correlations between the factors is non zero which is surprising as well.

Part b:

	beefs	previlege	newlearn	raises	critical	advance
Actual Values	51.0	30.0	39.0	61.0	92.0	45.0
Mean	66.6	53.1	56.4	64.6	74.8	42.9
Standard Deviation	13.3	12.2	11.7	10.4	9.9	10.3
Standardized values	-1.2	-1.9	-1.5	-0.3	1.7	0.2
Rotated Factor Pattern (Factor 1)	0.6	0.2	0.2	0.1	-0.1	-0.1
Rotated Factor Pattern (Factor 2)	-0.3	0.0	0.1	0.5	0.2	0.4
Factor Scores (Factor 1)	-1.463					
Factor Scores (Factor 2)	0.456					

Question10:

```
PROC FACTOR DATA=WORK.evaluate_supervisors METHOD=ML PRIORS=smc ULTRAHEYWOOD;

VAR BEEFS--ADVANCE;

TITLE 'Maximum likelihood factors with SMC for communality, 6 factors, VARIMAX rotation -- Evaluation Data';

RUN
```

Part a:

Significance Tests Based on 30 Observations			
Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	15	65.5127	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	4	2.8155	0.5892
HA: More factors are needed			

For the Null Hypothesis: No common factors, the p-value is less than 0.05, which suggests that we can reject the null hypothesis. Hence, the output suggests that common factors exist.

Part b:

For the Null Hypothesis: 2 Factors are sufficient, the p-value is greater than 0.05. This suggests that we cannot reject the null hypothesis. This suggests that default number of extracted factors is adequate.