HW7

Khyathi Balusu

4/27/2020

Question 1

The Bellman equation is:

$$V(i,t) = min(c_{i,j} + V(j,t+1)$$

where i = node number at time t t = columns in the picture (we have 4 columns) c = cost of that path V(i,t) = the shortest path from node i at column t to the node 10

t = 4

$$V(10,4) = 0$$

t = 3

$$V(6,3) = c_{6,10} + V(10,4) = 3$$
$$V(7,3) = c_{7,10} + V(10,4) = 4$$

$$V(8,3) = c_{8,10} + V(10,4) = 2$$
$$V(9,3) = c_{9,10} + V(10,4) = 3$$

t = 2

$$V(3,2) = min(c_{3,6} + V(6,3), c_{3,7} + V(7,3)) = min(6,6) = 6$$
$$V(4,2) = min(c_{4,6} + V(6,3), c_{4,8} + V(8,3, c_{4,9} + V(9,3)) = min(7,4,8) = 4$$
$$V(5,2) = min(c_{5,7} + V(7,3), c_{5,8} + V(8,3)) = min(6,4) = 4$$

t = 1

$$V(1,1) = min(c_{1,3} + V(3,2), c_{1,4} + V(4,2)) = min(9,6) = 6$$
$$V(2,1) = min(c_{2,3} + V(3,2), c_{2,4} + V(4,2), c_{2,5} + V(5,2)) = min(10,6,8) = 6$$

Shortest path from:

Node 1 = 1 - 4 - 8 - 10 Node 2 = 2- 4 -8- 10.

Shortest distance for both the paths are 6

Question 2

The Bellman equation is:

$$V(s,t) = min(c_s + V(s+1,t+1), p - rv_s + c_1 + V(1,t+1))$$

where c(s) = operating cost of a car(used for s years) p = price of car rv(s) = resale value of a car (used for s years) V(s,t) = lowest cost at year t for a car (used for s years)

```
rm(list = ls())

NP = 20000
RV = c(14000,12000,8000,6000,4000,2000)
c = c(600,1000,1600,2400,3200,4400)
T = 6

V = matrix(NA,nrow = T, ncol = T)
## columns- year
##rows - number of years for which the car has been used

tvalues = seq(1,6)
svalues = seq(1:6)

rownames(V) = svalues
colnames(V) = tvalues

names(c) = svalues
names(RV) = svalues

U =V

t = tvalues[T]

V[,paste(t)] = -RV

for (j in rev(tvalues[1:T-1])){
  for (i in svalues[1:j]){
    v_no_r = V[paste(i+1),paste(j+1)] + c[paste(i+1)]
    v_r = V[paste(1),paste(j+1)]+ NP - RV[paste(i)] + c[paste(1)]
    V[paste(i),paste(j)] = min(c(v_no_r,v_r))
    U[paste(i),paste(j)] = which.min(c(v_no_r,v_r))
  }
}

print(U)
```

```
##    1  2  3  4  5  6
## 1  1  1  1  1  1 NA
## 2 NA  2  1  2  1 NA
## 3 NA NA  1  1  1 NA
## 4 NA NA NA  2  1 NA
## 5 NA NA NA NA  1 NA
## 6 NA NA NA NA NA NA
```

We should keep the car for the first 2 years, replace and keep again for 2 years (year 4) and sell it off at the end of year 6. The total cost would be 8800(not considering the intial investment of 20000)

Question 3

The Bellman equation is:

$$V(i,t) = max_{j=1,2,3}(V(j,t+1) + E_i - TC_{i,j})$$

where e = earnings c = travelling cost from i to j V(i,t) = value when he is at place i(on day t) j = 1 Indianapolis 2 = Blomington 3 = Chicago

t = 4

He is required at IndianaPolis, hence negative(infinity) is assigned V(2,4) and V(3,4)

t= 3

$$V(1,3) = max_{j=1,2,3}(V(j,4) + E_1 - TC_{1,j}) = 120$$
$$V(2,3) = max_{j=1,2,3}(V(j,4) + E_2 - TC_{2,j}) = 160 - 50 = 110$$
$$V(3,3) = max_{j=1,2,3}(V(j,4) + E_3 - TC_{3,j}) = 170 - 20 = 150$$

t = 2

$$V(1,2) = max_{j=1,2,3}(V(j,3) + E_1 - TC_{1,j}) = max(120 + 120, 120 - 50 + 110, 120 - 20 + 150) = 250$$
$$V(2,2) = max_{j=1,2,3}(V(j,3) + +E_2 - TC_{2,j}) = max(160 - 50 + 120, 160 + 110, 160 - 70 + 150) = 270$$
$$V(3,2) = max_{j=1,2,3}(V(j,3) + E_3 - TC_{3,j}) = max(170 - 20 + 120, 170 - 70 + 110, 170 + 150) = 320$$

t = 1

$$V(1,1) = max_{j=1,2,3}(V(j,2) + E_1 - TC_{1,j}) = max(120 + 250, 120 - 50 + 270, 120 - 20 + 320) = 420$$
$$V(2,1) = max_{j=1,2,3}(V(j,2) + E_2 - TC_{2,j}) = max(160 - 50 + 250, 160 + 270, 160 - 70 + 320) = 430$$
$$V(3,1) = max_{j=1,2,3}(V(j,2) + E_3 - TC_{3,j}) = max(170 - 20 + 250, 170 - 70 + 270, 170 + 320) = 490$$

If he is at Bloomington, we have,

$$V(2,0) = max_{j=1,2,3}(V(j,1) + TC_{2,j}+) = max(370, 430, 420) = 430$$

It is not precisely Bellman's equation.

Closest optimum solution will be to be at Bloomington, stay there for the first 3 days and then go to Indianapolis

Question 4

states

$$statevariable = [s, t]$$

where t = year s = time used by machine in years at time t

Bellman equation is:

Condition 1: s+1 < n

$$v(s, t) = max(-c + p(0) + v(1, t + 1), p(s) + v(s + 1, t + 1))$$

Condition 2:

$$v(s, t) = -c + p(0) + v(1, t + 1)$$

Question 5

SuperMarket chain purchases 6 gallons of milk - sells at three stores HEB purchase rate - \$1 HEB selling rate - \$2 Unused milk - 50cents

Cost price - \$1 Selling price - \$2 unused- 50 cents

The Bellman equation is:

$$V(x, s) = max_{xin(0to6)}(min(x, d(s)) * 1 - max(0, x - d(s)) * 0.5 + V(6 - x, s + 1))$$

where V(x,s) - x gallons of milk sent to store s d(s) - demand at store s

```
rm(list = ls())

dem = c(1.8,1.9,1.9) ## expected demand-value

profGains = rep(0,6)
opt = rep(0,3)
optProfit = rep(0,3)

for (s in 1:3){
  for (x in 0:6){

      v = (min(x,dem[s])*1) - (max(0,x-dem[s])*0.5)
      profGains[x] = v
      opt[s] = which.max(profGains)
      optProfit[s] = max(profGains)
```

```
}
}

sum(optProfit)
```

```
## [1] 5.4
```

Optimal solution - alloting 2 gallons of milk at each store @ profit of $5.4