Question 1:

A: Manifest variables – Correlation matrix:

$$R_{3\times3} = \begin{bmatrix} 1 & (0.8\times0.6)+(0.4\times0.6) & (0.8\times0.4)+(0.4\times0.3) \\ (0.8\times0.8)+(0.4\times0.8) & 1 & (0.6\times0.4)+(0.6\times0.3) \\ (0.8\times0.4)+(0.4\times0.3) & (0.6\times0.4)+(0.6\times0.8) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.72 & 0.64 \\ 0.72 & 1 & 0.72 \\ 0.64 & 0.72 & 1 \end{bmatrix}$$

B:

Communality of $x_1 = (0.8)^2 + (0.4)^2$

$= 0.8$

Communality of $x_2 = (0.6)^2 + (0.6)^2$

$= 0.72$

Communality of $x_3 = (0.4)^2 + (0.8)^2$

$= 0.8$

Question 2:

A:

variance of X_1 explained by

factor 1 = $(0.8)^2 = 0.64$

factor 2 = $(0.4)^2 = 0.16$

variance of X_2 explained by

factor 1 = $(0.6)^2 = 0.36$

factor 2 = $(0.6)^2 = 0.36$

variance of X_3 explained by

factor 1 = $(0.4)^2 = 0.16$

factor 2 = $(0.8)^2 = 0.64$

B: PCA $\Rightarrow$ factor analysis where uniqueness = 0

In this case, uniqueness $\neq 0$.

$\Rightarrow$ This model and PCA are not equivalent computationally.

Question 3:

A: F^* = rotated factor pattern

$$= \begin{bmatrix} 0.8 & 0.4 \\ 0.6 & 0.6 \\ 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1.2 \times 1/\sqrt{2} & 0.4 \times 1/\sqrt{2} \\ 1.2 \times \frac{1}{\sqrt{2}} & 0 \times \frac{1}{\sqrt{2}} \\ 1.2 \times \frac{1}{\sqrt{2}} & -0.4 \times \frac{1}{\sqrt{2}} \end{bmatrix}$$

Correlation matrix of F^* =

$$\begin{bmatrix} 1 & (1.2)^2 \times \frac{1}{2} & (1.2)^2 \times \frac{1}{2} - (0.4)^2 \times \frac{1}{2} \\ (1.2)^2 \times \frac{1}{2} & 1 & (1.2)^2 \times \frac{1}{2} \\ (1.2)^2 \times \frac{1}{2} & (1.2)^2 \times \frac{1}{2} & 1 \\ (0.4)^2 \times \frac{1}{2} & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.72 & 0.64 \\ 0.72 & 1 & 0.72 \\ 0.64 & 0.72 & 1 \end{bmatrix}$$

⇒ Both correlation matrices are same

B: Communalities for $X_1 = (1.2)^2 \times \frac{1}{2} + (0.4)^2 \times \frac{1}{2} = 0.8$

Communalities for $X_2 = (1.2)^2 \times \frac{1}{2} + 0 = 0.72$

Communalities for $X_3 = (1.2)^2 \times \frac{1}{2} + (0.4)^2 \times \frac{1}{2} = 0.8$

⇒ Communalities are same as in Q1.B

⇒ Uniqueness of manifest variables ⇒ same before and after rotation ⇒ communalities are same.

Question 4:

A: correlation matrix = $\begin{bmatrix} 1 & ab & ac \\ ab & 1 & bc \\ ac & bc & 1 \end{bmatrix}$

for previous correlation matrix:

$ab = 0.72,\ bc = 0.72,\ ac = 0.64$

⇒ to satisfy these,

$a = 0.8$

$b = 0.9$

$c = 0.8$

B: In Q3.A, Q4.A ⇒ no unique set of factors.

⇒ Factors based on initialization, method, number of factors or rotation can be set.

Question 5:

A:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=one MINEIGEN=0
NFACTORS=6;

 VAR beefs previlege newlearn raises critical advance;

TITLE "PC style factor analysis Factor Analysis - 6 factors ";

RUN;
```

B:

6 factors will be retained by the NFACTOR criterion.

Factor Pattern

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
beefs	0.78219	-0.31363	0.38883	-0.23490	-0.10787	0.26797
previlege	0.70268	-0.30973	0.18990	0.60569	-0.02123	-0.08333
newlearn	0.82140	-0.21777	-0.23756	-0.16709	0.43688	-0.05153
raises	0.87704	0.11590	0.00490	-0.27139	-0.25930	-0.27649
critical	0.40022	0.80479	0.39938	0.07429	0.16271	0.02533
advance	0.67791	0.32172	-0.59975	0.15293	-0.14347	0.18237

Variance Explained by Each Factor

Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
3.1692232	1.0063467	0.7629087	0.5525165	0.3172465	0.1917584

First **two factors** to be retained as they have eigen value greater than 1

C:

The first factor:

gives factors which contribute towards good attributes of a supervisor. A person who handles incoming complaints reasonably, identifies and gives a hand to merit, encourages learning and growth in the career. Thought inspite of it, this supervisor prefers a few over others.

The second factor:

gives the non good characteristic of an supervisor. Cynicism, no learning encouragement and no heed to complaints.

Question 6:

7 regression models were run.

First 6 models is on each factor on overall rating column

Seventh model uses all six factors

Model1: Overall ~ Factor1 R2 = 0.457

Model2: Overall ~ Factor2 R2 = 0.0788

Model3: Overall ~ Factor3 R2 = 0.0878

Model4: Overall ~ Factor4 R2 = 0.089

Model5: Overall ~ Factor5 R2 = 0.006

Model6: Overall ~ Factor6 R2 = 0.0139

Model7: Overall ~ Factor7 R2 = 0.73

Question 7:

A:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=SMC NFACTORS=6
OUT=q7_factors;

  VAR beefs previlege newlearn raises critical advance;

  TITLE "Rsquare style factor analysis Factor Analysis - 6 factors ";

RUN;
```

B:

3 factors will be retained by the MINEIGEN criterion.

Factor Pattern

	Factor1	Factor2	Factor3
beefs	0.74755	-0.36273	0.12483
previlege	0.61091	-0.17725	-0.06404
newlearn	0.76629	-0.05146	-0.21483
raises	0.84947	0.11042	0.13720
critical	0.32091	0.25308	0.26760
advance	0.61147	0.39882	-0.15045

The first factor:

gives factors which contribute towards good attributes of a supervisor. A person who
- handles incoming complaints reasonably,
- identifies and gives a hand to merit,
- encourages learning and growth in the career.

Thought inspite of it, this supervisor prefers a few over others.

Second factor:

Gives factors of supervisor who is :
- critical and
- helps in advancement in employees' career
- doesn't solve well the employees' complaints (low beefs score)

C:

In Q5,
Assumptions are made such that:
- the uniqueness of each manifest variable is zero
- factors explain all the variance.

Implies this results and PCA results are same

In Q5,
Assumptions are made such that:
- the uniqueness of each manifest variable is non zero

For initial values, each variable is regressed on other variables for R2. These will be initial values.

Question 8:

A:

```
PROC FACTOR DATA=work.evaluate_supervisors METHOD=principal PRIORS=SMC NFACTORS=2
ROTATE=varimax OUT=work.evaluatesupervisors_scores;
  VAR beefs previlege newlearn raises critical advance;

  TITLE "Rsquare style factor analysis Factor Analysis - 2 factors";

RUN;
```

B:

2 factors will be retained by the NFACTOR criterion.

Factor Pattern

	Factor1	Factor2
beefs	0.74755	-0.36273
previlege	0.61091	-0.17725
newlearn	0.76629	-0.05146
raises	0.84947	0.11042
critical	0.32091	0.25308
advance	0.61147	0.39882

Yes, the first two factors for this question are not the same as the first 2 factors for the previous question

C:

The VARIMAX factor rotation matrix is:

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2
1	0.79912	0.60117
2	-0.60117	0.79912

Orthonormality : Matrix is orthonormal

VARIMAX factor rotation matrix	
0.79912	0.60117
-0.60117	0.79912

VARIMAX factor rotation matrix	
0.79912	-0.60117
0.60117	0.79912

Product of both matrices	
0.999998	0
0	0.999998

D:

Rotated Factor Pattern		
	Factor1	Factor2
beefs	0.81544	0.15954
previlege	0.59475	0.22561
newlearn	0.64329	0.41955
raises	0.61245	0.59891
critical	0.10431	0.39516
advance	0.24889	0.68630

The first factor:

gives factors which contribute towards attributes of a supervisor. A person who
handles incoming complaints reasonably,
identifies and gives a hand to merit,
encourages learning and growth in career.

Thought inspite of it, this supervisor prefers a few over others.

Second factor:

Gives factors of supervisor who is :
Needs to recognize merit
Encourage learnings
Help employees advance in roles

Question 9:

A:

Simple Statistics						
Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
Factor1	30	0	0.85285	0	-1.86118	1.36525
Factor2	30	0	0.77645	0	-1.31002	2.01667

Pearson Correlation Coefficients, N = 30 Prob > |r| under H0: Rho=0		
	Factor1	Factor2
Factor1	1.00000	0.29041 0.1195
Factor2	0.29041 0.1195	1.00000

We can see that the factors have standard deviations that are not one. Generally speaking, factors are to be standardized with zero mean and unit standard deviation. Here, correlations are not one either.

B:

	beefs	previlege	newlearn	raises	critical	advance
Actual Values	51.0	30.0	39.0	61.0	92.0	45.0
Mean	66.6	53.1	56.4	64.6	74.8	42.9
Standard Deviation	13.3	12.2	11.7	10.4	9.9	10.3
Standardized values	-1.2	-1.9	-1.5	-0.3	1.7	0.2
Rotated Factor Pattern (Factor 1)	0.6	0.2	0.2	0.1	-0.1	-0.1
Rotated Factor Pattern (Factor 2)	-0.3	0.0	0.1	0.5	0.2	0.4
Factor Scores (Factor 1)	-1.463					
Factor Scores (Factor 2)	0.456					

Question 10:

```
PROC FACTOR DATA=WORK.evaluate_supervisors METHOD=ML PRIORS=smc ULTRAHEYWOOD;

VAR BEEFS--ADVANCE;

TITLE 'Maximum likelihood factors with SMC for communality, 6 factors, VARIMAX rotation -- Evaluation Data';

RUN;
```

A:

Significance Tests Based on 30 Observations			
Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	15	65.5127	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	4	2.8155	0.5892
HA: More factors are needed			

Null Hypothesis:

No common factors,

p-value < 0.05, implying rejection of the null hypothesis.

This shows that common factors exist.

B:

Null Hypothesis:

Two factors that are sufficient,

p-value > 0.05, implying no rejection of the null hypothesis.

This shows that extracted factors that are default in number are enough