Homework_2

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Question 1

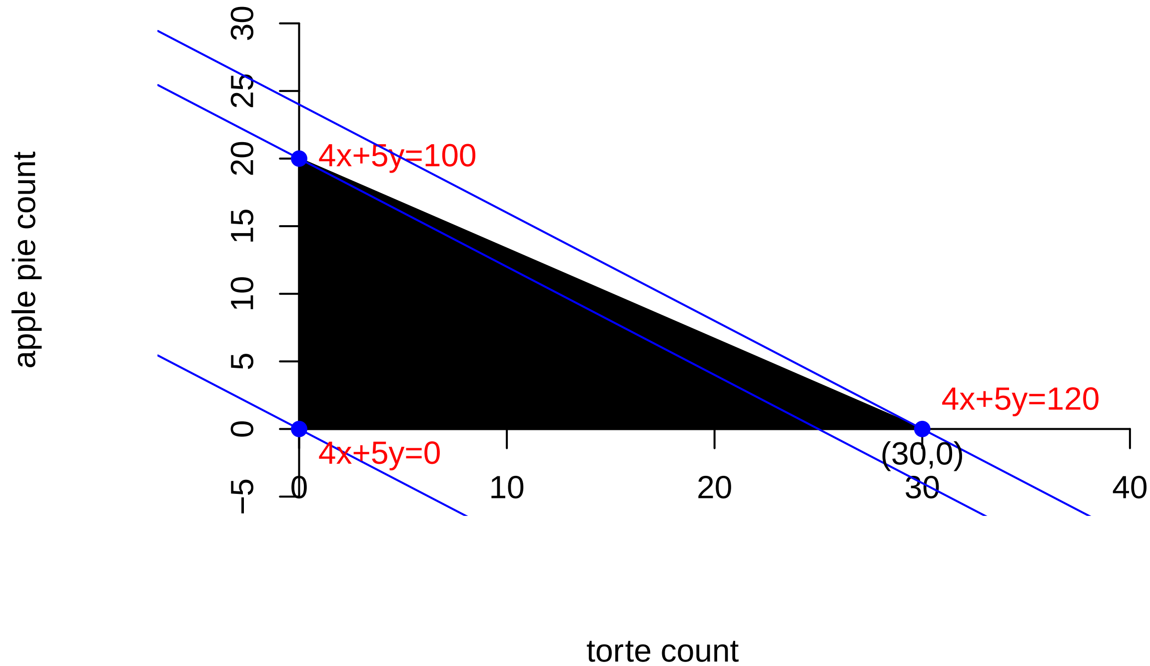
Max is in a pie-eating contest that lasts 1 hour. Each torte that he eats takes 2 minutes. Each apple pie that he eats takes 3 minutes. He receives 4 points for each torte and 5 points for each pie. What should Max eat to get the most points? Solve the problem using the graphical method.
Next, let's see what happens if he would like to stick to his preference of eating at least as many pies as tortes. That is; the number of pies he eats should be greater than or equal to the number of tortes. By how many points does this constraint decrease Max's total points?

$x =$ torte count $y =$ apple pie count

max of $4 * x + 5 * y$ constraint: $2 * x + 3 * y \leq 60$.

We can use graphical method:

```
x <- c(0:30)
y <- -2/3*x + 20
plot(x, y, type="l", xlim = c(-5,40), ylim=c(-5,30), axes = FALSE,
     xlab = "torte count", ylab = "apple pie count")
axis(1, pos=0); axis(2, pos=0)
polygon(x = c(0,0,30), y = c(0,20,0), col = "black")
abline(a=0, b=-4/5, col = "blue")
text(x=0, y = -2, labels = "4x+5y=0", col = "red",pos=4)
points(x=0, y=0, col="blue", pch=19)
abline(a=20, b=-4/5, col = 'blue')
text(x=0, y = 20, labels = "4x+5y=100", col = "red",pos=4)
points(x=0, y=20, col="blue", pch=19)
abline(a=24, b=-4/5, col = "blue")
text(x=30, y = 2, labels = "4x+5y=120", col = "red",pos=4)
points(x=30, y=0, col="blue", pch=19)
text(x=30, y = -2, labels = "(30,0)")
```

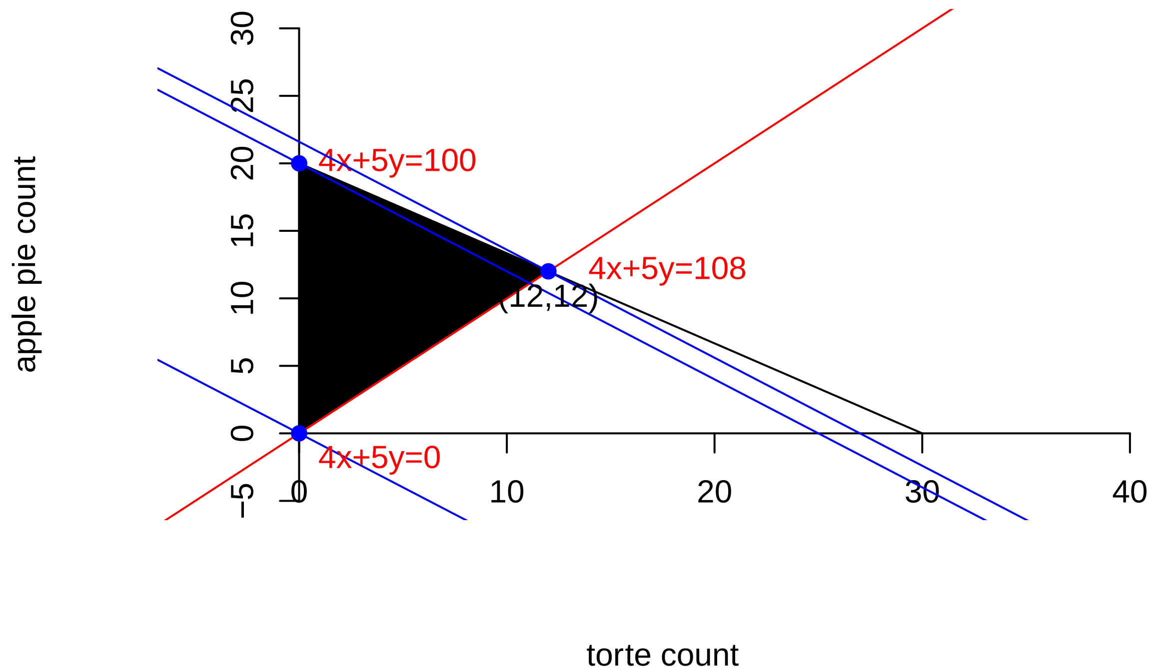


upon consuming 30 torte, Max gets 120 points

Additional constraint:that number of pies is at least as many as torte($y \geq x$),

Change in graph:

```
x <- c(0:30)
y <- -2/3*x + 20
plot(x, y, type="l", xlim = c(-5,40), ylim=c(-5,30), axes = FALSE,
     xlab = "torte count", ylab = "apple pie count")
axis(1, pos=0); axis(2, pos=0)
polygon(x = c(0,0,12), y = c(0,20,12), col = "black")
abline(a=0, b =1, col ="red")
abline(a=0, b=-4/5, col = "blue")
points(x=0, y=0, col="blue", pch=19)
text(x=0, y = -2, labels = "4x+5y=0", col = "red",pos=4)
abline(a=20, b=-4/5, col = 'blue')
points(x=0, y=20, col="blue", pch=19)
text(x=0, y=20, labels = "4x+5y=100", col = "red",pos=4)
abline(a=21.6, b=-4/5, col = "blue")
points(x=12, y=12, col="blue", pch=19)
text(x=13, y = 12, labels = "4x+5y=108", col = "red",pos=4)
text(x=12, y = 10, labels = "(12,12)")
```



MAx gets 108 points - 12 points decrease because of additional constraint

Question 2

A farmer in Iowa owns 450 acres of land. He is going to plant each acre with wheat or corn. Each acre planted with wheat (corn) yields \$2,000(\$3,000) profit, requires three (two) workers, and requires two (four) tons of fertilizer. There are currently 1,000 workers and 1,200 tons of fertilizer available.

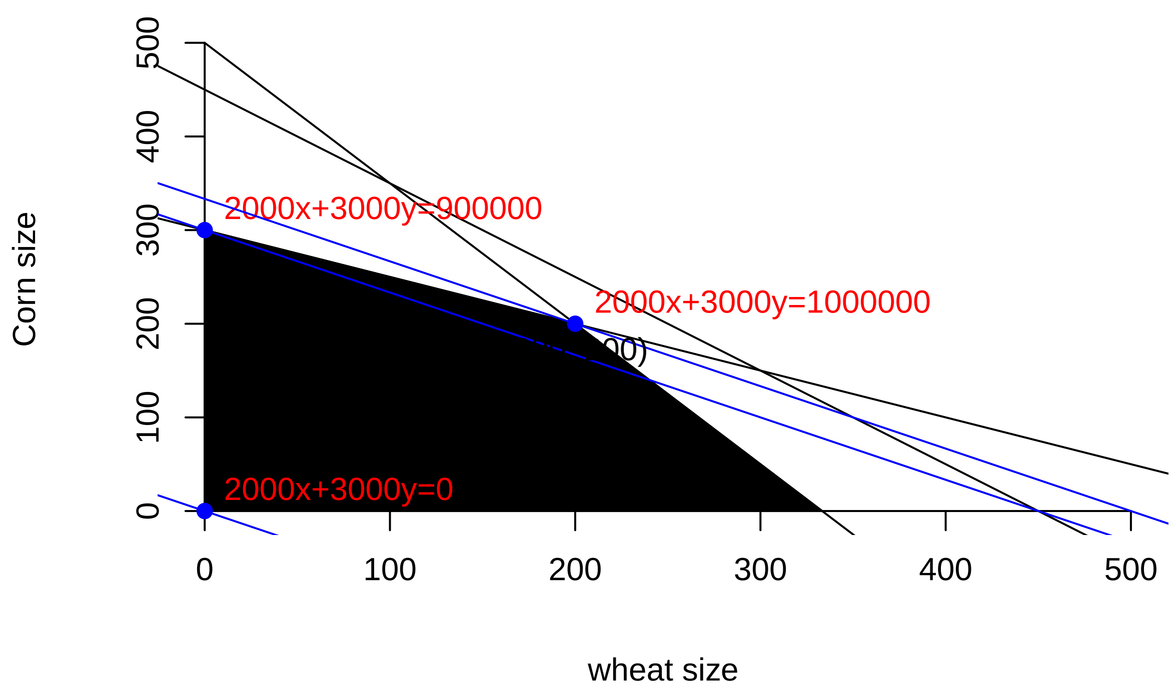
a) Formulate the optimization problem and solve the problem graphically.
b) Solve the problem in R and verify that the solutions are the same.
c) What happens to the decision variables and the total profit when the availability of fertilizer varies from 200 tons to 2200 tons in 100-ton increments? When does the farmer discontinue producing wheat? When does he stop producing corn? (Run a loop for different values of availability of fertilizer from 200 tons to 2200 tons).

a. Let x = wheat arc size and y = corn arc size. Constraints:

- $x + y \leq 450$
- $3x + 2y \leq 1000$
- $2x + 4y \leq 1200$

Graph:

```r
x <- c(0:500)
y <- -3/2*x + 500
plot(x, y, type="l", xlim = c(-5,500), ylim=c(-5,500), axes = FALSE,
     xlab = "wheat size", ylab = "Corn size")
axis(1, pos=0); axis(2, pos=0)
polygon(x = c(0,0,200,1000/3), y = c(0,300,200,0), col="black")
abline(a=450, b =-1)
abline(a=300, b=-1/2)
abline(a=0, b=-2/3, col="blue")
points(x=0, y=0, col="blue", pch=19)
text(x=0, y = 20, labels = "2000x+3000y=0", col = "red",pos=4)
abline(a=300, b=-2/3, col="blue")
points(x=0, y=300, col="blue", pch=19)
text(x=0, y = 320, labels = "2000x+3000y=900000", col = "red",pos=4)
abline(a=1000/3, b=-2/3, col="blue")
points(x=200, y=200, col="blue", pch=19)
text(x=200, y = 220, labels = "2000x+3000y=1000000", col = "red",pos=4)
text(x=200, y = 170, labels ="(200, 200)")
```



Solution: 1. 200 acres with wheat 2. 200 acres with corn 3. $1,000,000 profit

b. R solution

```r
library("lpSolve")
objective.in <- c(2000, 3000)
const.mat <- matrix(c(1,1,
                      3,2,
                      2,4), nrow = 3, ncol = 2, byrow = TRUE)
const.rhs <- c(450, 1000, 1200)
const.dir <- c("<=", "<=", "<=")
optimum <- lp(direction = "max", objective.in, const.mat, const.dir, const.rhs)
optimum$solution
```

```
## [1] 200 200
```

```r
optimum$objval
```

```
## [1] 1e+06
```

Solution: 1. 200 acres with wheat 2. 200 acres with corn 3. $1,000,000 profit

c. Dynamic fertilizer:

```r
df_raw <- data.frame()
df_int <- data.frame()
for(i in c(seq(200, 2200, 100))){
  objective.in <- c(2000, 3000)
  const.mat <- matrix(c(1,1,
                        3,2,
                        2,4), nrow = 3, ncol = 2, byrow = TRUE)
  const.rhs <- c(450, 1000, i)
  const.dir <- c("<=", "<=", "<=")

  optimum <- lp(direction = "max", objective.in, const.mat, const.dir, const.rhs)
  df_raw <- rbind(df_raw, data.frame(Fertilizer = i, Wheat=optimum$solution[1],
                  Corn = optimum$solution[2], Profit=optimum$objval))

  optimum_int <- lp(direction = "max", objective.in, const.mat, const.dir, const.rhs,
                    all.int = TRUE)
  df_int <- rbind(df_int, data.frame(Fertilizer = i, Wheat=optimum_int$solution[1],
                  Corn = optimum_int$solution[2], Profit=optimum_int$objval))
```

```r
}
df_raw    # Result without integer condition
```

```
##    Fertilizer Wheat  Corn  Profit
## 1         200   100   0.0  200000
## 2         300   150   0.0  300000
## 3         400   200   0.0  400000
## 4         500   250   0.0  500000
## 5         600   300   0.0  600000
## 6         700   325  12.5  687500
## 7         800   300  50.0  750000
## 8         900   275  87.5  812500
## 9        1000   250 125.0  875000
## 10       1100   225 162.5  937500
## 11       1200   200 200.0 1000000
## 12       1300   175 237.5 1062500
## 13       1400   150 275.0 1125000
## 14       1500   125 312.5 1187500
## 15       1600   100 350.0 1250000
## 16       1700    50 400.0 1300000
## 17       1800     0 450.0 1350000
## 18       1900     0 450.0 1350000
## 19       2000     0 450.0 1350000
## 20       2100     0 450.0 1350000
## 21       2200     0 450.0 1350000
```

```r
df_int    # Result with integer condition
```

```
##    Fertilizer Wheat Corn  Profit
## 1         200   100    0  200000
## 2         300   150    0  300000
## 3         400   200    0  400000
## 4         500   250    0  500000
## 5         600   300    0  600000
## 6         700   324   13  687000
## 7         800   300   50  750000
## 8         900   274   88  812000
## 9        1000   250  125  875000
## 10       1100   224  163  937000
## 11       1200   200  200 1000000
## 12       1300   174  238 1062000
## 13       1400   150  275 1125000
## 14       1500   124  313 1187000
## 15       1600   100  350 1250000
## 16       1700    50  400 1300000
## 17       1800     0  450 1350000
## 18       1900     0  450 1350000
## 19       2000     0  450 1350000
## 20       2100     0  450 1350000
## 21       2200     0  450 1350000
```

Solution: without integer condition

1. Fertilizer +> 1800 - Discontinue wheat
2. Wheat = 600 - Discontinue corn

Question 3

Star Oil Company is considering five different investment opportunities. Table 1 below gives the cash outflows and net present values in millions of dollars. Star Oil has $40 million available for investment now (time 0); it estimates that one year from now (time 1) $20 million will be available for investment. Star Oil may purchase any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly.

For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of $1/5 \times 5 = \$1$ million would be required at time 0, and a cash outflow of $1/5 \times 5 = \$1$ million would be required at time 1. The one-fifth share of investment 3 would yield an NPV of $1/5 \times 16 = \$3.2$ million. Star Oil wants to maximize the NPV that can be obtained by investing in investments 1-5. Formulate an LP that will help achieve this goal. Assume that any funds leftover at time 0 cannot be used at time 1.

Investment	1	2	3	4	5
Time 0 Cash Outflow	11	53	5	5	29
Time 1 Cash Outflow	3	6	1	34	
NPV	13	16	16	14	39

Function : * $13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

Constraint:
* $11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40$
* $3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20$

```
objective.in <- c(13, 16, 16, 14, 39)
const.mat <- matrix(c(11, 53, 5, 5, 29,
                      3, 6, 5, 1, 34), nrow = 2, ncol = 5, byrow = TRUE)
const.rhs <- c(40, 20)
const.dir <- c("<=", "<=")
optimum <- lp(direction = "max", objective.in, const.mat, const.dir, const.rhs)
optimum$solution
```

```
## [1] 0 0 3 5 0
```

```
optimum$objval
```

```
## [1] 118
```

Question 4

The goal of the diet problem is to select a set of foods that will satisfy a set of daily nutritional requirement at minimum cost. Suppose there are three foods available, corn, milk, and bread. Besides, there are restrictions on the number of calories (between 2000 and 2250) and the amount of Vitamin A (between 5000 and 50,000). The table below shows, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving. Also, suppose that the maximum number of servings for each food is 10.

Food	Cost Per Serving	Vitamin	Calories
Corn	$0.18	107	72
2% Milk	$0.23	500	121
Wheat Bread	$0.05	0	65

Variables
* x_1 : Corn
* x_2 : Milk
* x_3 : Bread

Function: * Minimize $0.18x_{1}+0.23x_{2}+0.05x_{3}

Constraint:
* $72x_1 + 121x_2 + 65x_3 \geq 2000$ * $72x_1 + 121x_2 + 65x_3 \leq 2250$ * $107x_1 + 500x_2 \geq 5000$ * $107x_1 + 500x_2 \leq 50000$
* $x_1 \leq 10$ * $x_2 \leq 10$ * $x_3 \leq 10$
* x_1, x_2, x_3

```r
objective.in <- c(0.18, 0.23, 0.05)
const.mat <- matrix(c(72, 121, 65,
                      72, 121, 65,
                      107, 500, 0,
                      107, 500, 0,
                      1, 0, 0,
                      0, 1, 0,
                      0, 0, 1), nrow = 7, ncol = 3, byrow = TRUE)
const.rhs <- c(2000, 2250, 5000, 50000, 10, 10, 10)
const.dir <- c(">=", "<=", ">=", "<=", "<=", "<=", "<=")
optimum <- lp(direction = "min", objective.in, const.mat, const.dir,
              const.rhs, all.int = TRUE) # with integer condition
optimum$solution
```

```
## [1]  2 10 10
```

```r
optimum$objval
```

```
## [1] 3.16
```

Solution: 1. 2 corns 2. 10 milk 3. 10 wheat bread

Question 5

Paper and wood products companies need to define cutting schedules that will maximize the total wood yield of their forests over some planning period. Suppose that a firm with control of 2 forest units wants to identify the best cutting schedule over a planning horizon of 3 years. Forest unit 1 has a total acreage of 2 and unit 2 has a total of 3. The studies that the company has undertaken predict that each acre in unit 1(2) will have 1, 1.3, 1.4 (1, 1.2, 1.6) tons of woods available for harvesting in year 1, 2, 3 respectively. Based on its prediction of economic conditions, the company believes that it should harvest at least 1.2, 1.5, 2 tons of wood in year 1, 2, 3 separately. Due to the availability of equipment and personnel, the company can harvest at most 2, 2, 3 tons of wood in year 1, 2, 3. What is this company's best cutting strategy that maximizes the total weights of wood? Here discounting of the time value should not be considered.

Variabless:
* x_i : unit 1 in year for wood i
* y_i : unit 2 in year for wood i

Constraints:
* $x_1 + y_1 \geq 1.2$
* $x_2 + y_2 \geq 1.5$
* $x_3 + y_3 \geq 2$
* $x_1 + y_1 \leq 2$
* $x_2 + y_2 \leq 2$
* $x_3 + y_3 \leq 3$
* $x_2 \leq (2 - x_1) * 1.3$
* $x_3 \leq [(2 - x_1)1.3 - x_2] * 1.4/1.3$
* $y_2 \leq (3 - y_1) * 1.2$
* $y_3 \leq [(3 - y_1)1.2 - y_2] * 1.6/1.2$
###

```
objective.in <- c(1,1,1,1,1,1)
const.mat <- matrix(c(1,0,0,1,0,0,
                      0,1,0,0,1,0,
                      0,0,1,0,0,1,
                      1,0,0,1,0,0,
                      0,1,0,0,1,0,
                      0,0,1,0,0,1,
                      1.3,1,0,0,0,0,
                      1.4,1.4/1.3,1,0,0,0,
                      0,0,0,1.2,1,0,
                      0,0,0,1.6,4/3,1), nrow=10, ncol=6, byrow = TRUE)
const.rhs <- c(1.2,1.5,2,2,2,3,2.6,2.8,3.6,4.8)
const.dir <- c(">=",">=",">=","<=","<=","<=","<=","<=","<=","<=")
optimum <- lp(direction = "max", objective.in, const.mat, const.dir,
              const.rhs)
optimum$solution
```

```
## [1] 0.4615385 2.0000000 0.0000000 1.1250000 0.0000000 3.0000000
```

```
optimum$objval
```

```
## [1] 6.586538
```

As a result, the amounts of wood for harvesting in unit1 are 0.4615385, 2, 0 tons in year 1, 2, 3. And the amounts of wood for harvesting in unit2 are 1.125, 0, 6.586538 tons in year 1, 2, 3. Solution: 1. unit 1 - 0.4615385, 2, 0 tons for 1, 2, 3 years cons. 2. unit 2 - 1.125, 0, 6.586538 tons for 1, 2, 3 years cons.