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Assingment - Parameter Estimation

Ans1.

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sample size 'n' is taken

($x_1, x_2 \ldots x_n$)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Here, $\mu = \theta_1$, $\sigma^2 = \theta_2$

$$f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}, \quad i = 1, 2 \ldots n$$

Joint density function

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking ln on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{1}{2} \ln(2\pi\theta_2) - \left(\frac{(x_i-\theta_1)^2}{2\theta_2} \right) \right)$$

For θ_1

$$\frac{\partial \ln(L(\theta_1, \theta_2))}{\partial \theta_1} = \frac{\partial}{\partial \theta_1} \left(\sum_{i=1}^{n} \left(-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right) \right) = 0$$

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$$\Rightarrow \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$\sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$n\theta_1 = \sum_{i=1}^{n} x_i$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

MLE of θ_1 = sample Mean

for θ_2,

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \frac{\partial}{\partial \theta_2} \left(\sum_{i=1}^{n} \left(-\frac{1}{2} \ln(2\pi\theta_2) - \left(\frac{(x_i - \theta_1)^2}{2\theta_2} \right) \right) \right) = 0$$

$$= \sum_{i=1}^{n} \left(\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2(\theta_2)^2} \right) = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$n\theta_2 = \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$\Rightarrow$ MLE of θ_2 is sample variance.

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Q2. Sample = $(X_{-1}, X_{-2}, \dots, X_{-n})$

$$f(x; M, \theta) = {}^{M}C_{x}\, \theta^{x} (1-\theta)^{M-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{M}C_{x_i}\, \theta^{x_i} (1-\theta)^{M-x_i}$$

Taking ln on both sides

$$\ln L(\theta) = \ln \prod_{i=1}^{n} {}^{M}C_{x_i}\, \theta^{x_i} (1-\theta)^{M-x_i}$$

$$= \sum_{i=1}^{n} \left(\ln {}^{M}C_{x_i} + x_i \ln\theta + \left((M-x_i)\ln(1-\theta)\right)\right)$$

For θ

$$\frac{\partial}{\partial\theta} \ln L(\theta) = \frac{\partial}{\partial\theta}\left(\sum_{i=1}^{n} \left(\ln \left[{}^{M}C_{x_i} + x_i \ln\theta + (M-x_i)\ln(1-\theta)\right)\right] \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{M-x_i}{1-\theta} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i - \theta.M}{\theta(1-\theta)} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \theta.M) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\theta.M = 0$$

$$\theta = \frac{1}{nM} - \sum_{i=1}^{n} x_i$$

$\Rightarrow$ MLE of θ = $\dfrac{\text{sample Mean}}{M}$