

Tube-Load Modeling of Aorta with Abdominal Aortic Aneurysm

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1. Motivation and Background

- ❖ Motivation:
 - To investigate the feasibility of tube-load modeling in tracking the progression of abdominal aortic aneurysm (AAA) in a patient pre- and post-endovascular repair (EVAR).
 - Non-imaging-based monitoring of AA is practically non-existent.
 - Most AAA are asymptomatic and ruptured aorta has a mortality rate of >80%.
- ❖ Background:
 - Aortic aneurysm is a balloon-like bulge in the main artery which supplies blood to the body.
 - Weakened aortic walls and distending blood pressure causes progressive vessel expansion and potential rupture.
 - Conventional methods for AAA diagnosis are:
 - Imaging-based techniques → underutilized due to inconvenience.
 - AAA alters pulse wave propagation and reflection characteristics of the arteries → morphological changes of arterial blood pressure (ABP) waveforms.
 - Non-invasive measurements of arterial blood pressure, like tonometry waveform measurements reflects the changes that AAA causes.
 - Thus, more convenient and affordable ways to monitor AAA will enhance patient safety.

2. Methods

- ❖ Experimental Data:
 - We collected carotid and femoral tonometry waveforms from 43 AAA patients before and after EVAR.
 - We preprocessed the artery tonometry waveforms and derived a representative carotid and femoral waveform beat.
 - We calibrate the tonometry waveforms to the arm cuff mean and diastolic blood pressure levels.
 - We excluded patients that had PAD or endoleaks post-EVAR since both conditions may affect the shape of arterial blood pressure waveforms.
 - A quantitative measure of waveform quality was used to exclude subjects with poor waveform quality, followed by manual inspection.

2.1 Tube-Load Model

- ❖ A Tube-Load (TL) model represents the arterial system via uniform, loss-less tubes with parametric loads.
 - Tube has characteristic impedance, Z_c , and additional components may be used to better represent physiology:
 - Arterial compliance C_T and arterial resistance R_T
- ❖ Eq. (1) TL model is characterized by two parameters:
 - Pulse Transit Time (PTT) denoted by τ
 - Reflection coefficient denoted by Γ
- ❖ Eq. (2) TL model is characterized by three parameters:
 - Pulse Transit Time (PTT) denoted by τ
 - Reflection coefficient as lumped parameters η_2/η_1

- ❖ Mathematical model of 2-parameter TL model:

$$P_F(s) = \frac{1 + \Gamma}{e^{\tau s} + \Gamma e^{-\tau s}} \cdot P_C(s) \quad (1)$$

- Reflection coefficient, Γ , is given by $\Gamma = \frac{R_T - Z_c}{R_T + Z_c}$

- ❖ Mathematical model of three-parameter TL model:

$$P_F(s) = \frac{s + \eta_1 + \eta_2}{(s + \eta_1)e^{\tau s} + \eta_2 e^{-\tau s}} \cdot P_C(s) \quad (2)$$

- $\frac{\eta_2}{\eta_1}$ is the reflection coefficient in the 3-parameter model.

- $\eta_1 = \frac{2Z_c + R_T}{2Z_c R_T C_T}$ and $\eta_2 = \frac{R_T}{2Z_c R_T C_T}$

- ❖ We fit the tube-load models to the carotid and femoral waveform pertaining to the patient and inferred patient-specific model parameters based on a nonlinear optimization procedure.

Visuals

- ❖ Arterial waveforms are the sum of the forward (P_f) and backward (P_b) traveling waves.
- ❖ Forward and backward waves have a time delay τ (i.e., PTT) and wave reflection phenomena is accounted for by impedance mismatch (i.e., $Z_c \neq Z_L$).

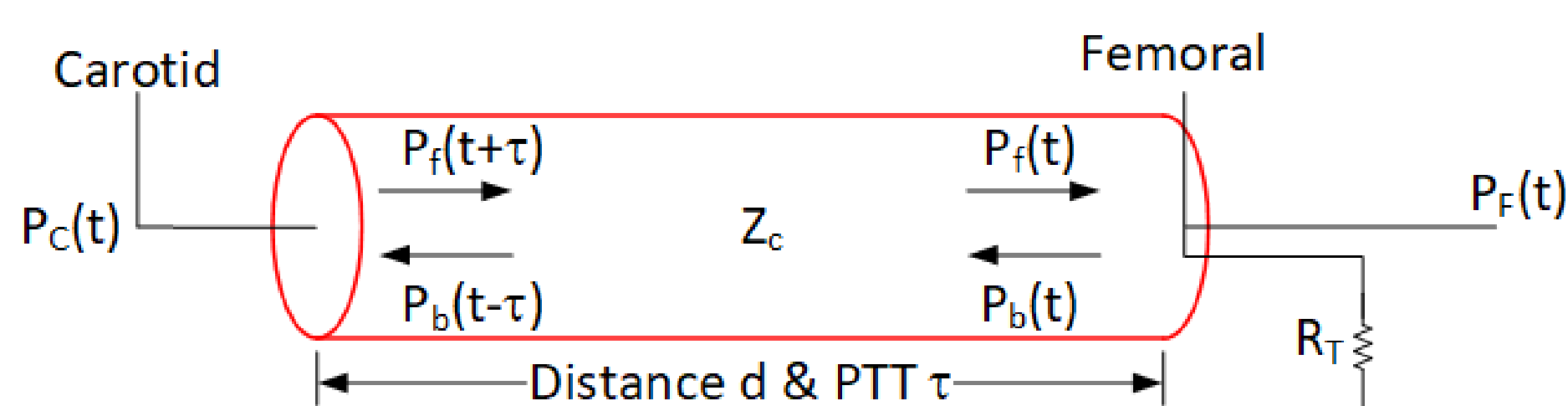


Fig. 1: Diagram of 2-parameter tube-load model characterized by τ and Γ .

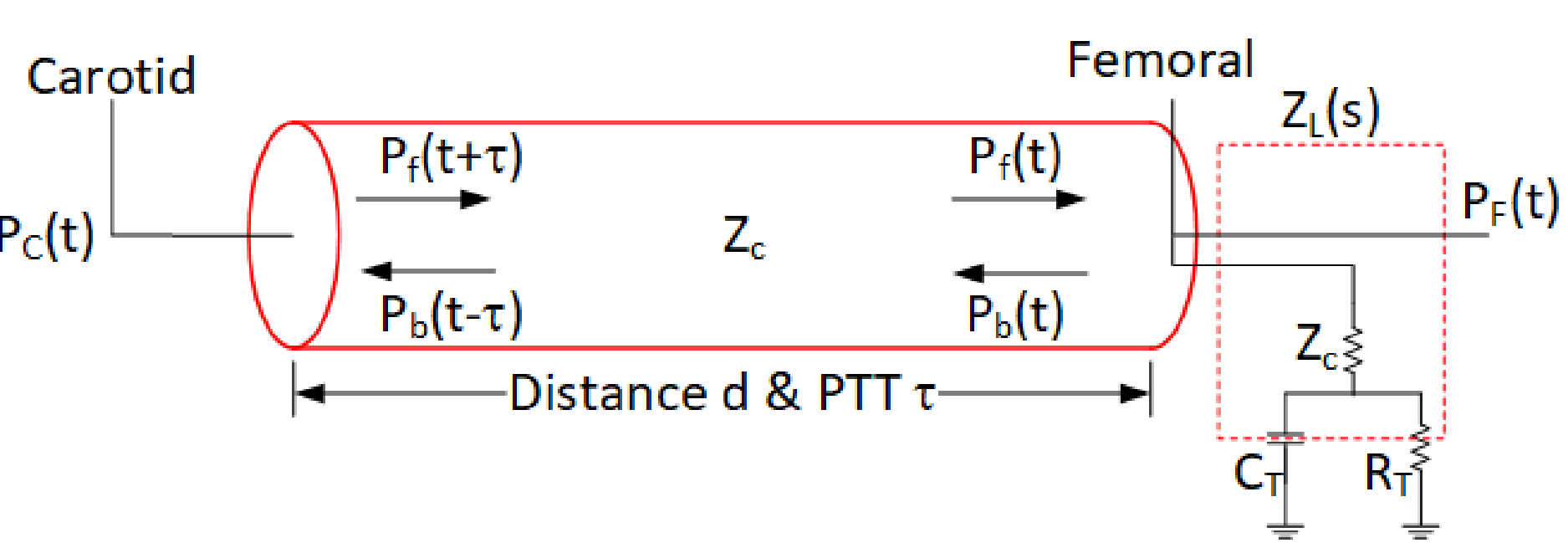


Fig. 2: Diagram of 3-parameter tube-load model characterized by τ , η_1 , and η_2 .

3. Model Fitting

- ❖ Carotid-femoral PTT (cf-PTT) was calculated via the intersecting tangents method which is a robust method for estimating PTT.
- ❖ We also obtain the model parameter estimates for cf-PTT and the reflection coefficient from both the 2-parameter and 3-parameter models.
- ❖ The TL model uses the femoral waveform as the input signal and fits to the carotid waveform by adjusting its estimate for PTT and reflection coefficient parameters.

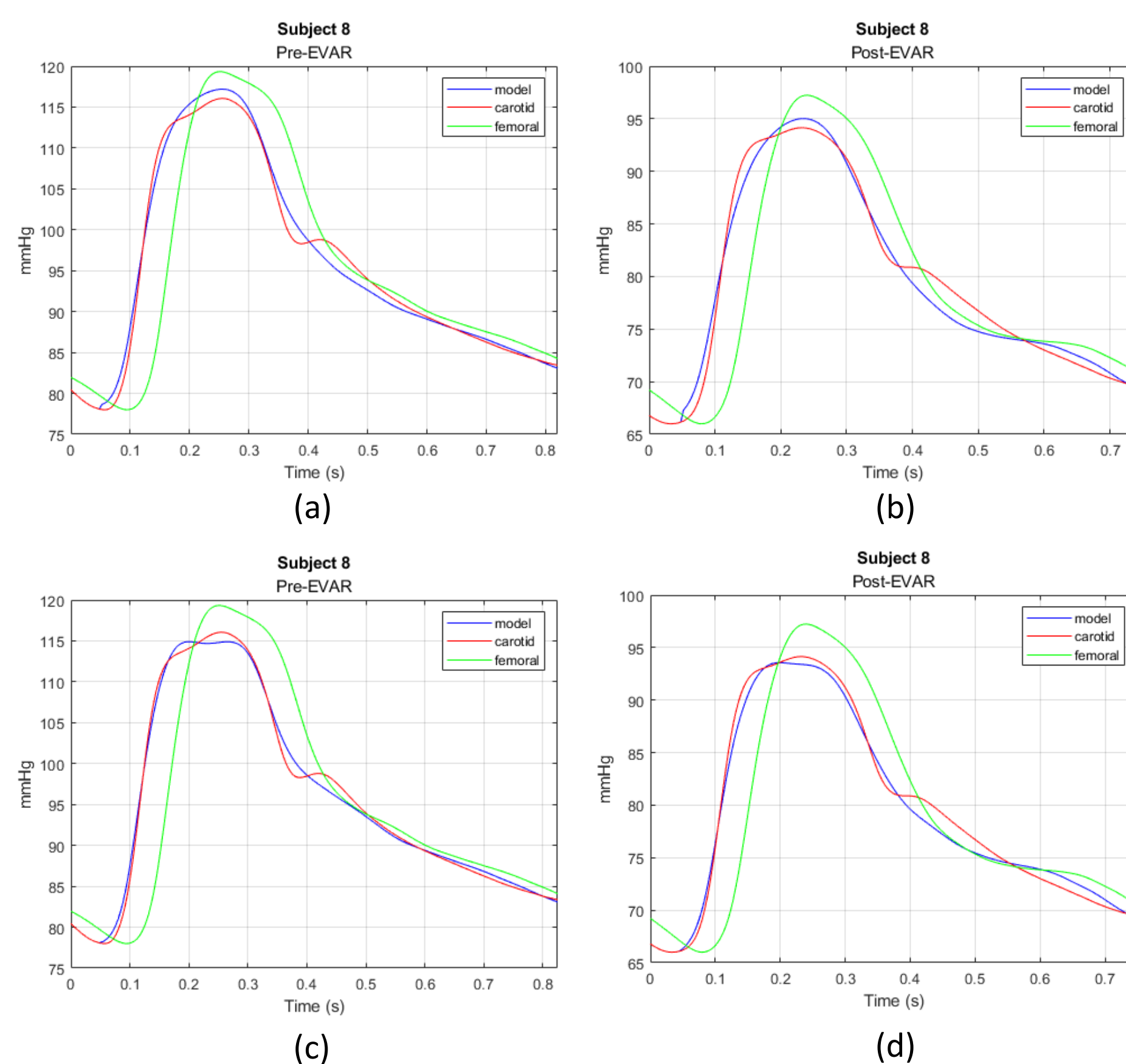


Fig 3: 2-parameter model fitting results for (a) pre-EVAR and (b) post-EVAR. 3-parameter model fitting results for (c) pre-EVAR and (d) post-EVAR.

- ❖ The 3-parameter model was found to have a lower average RMSE compared to the 2-parameter model.

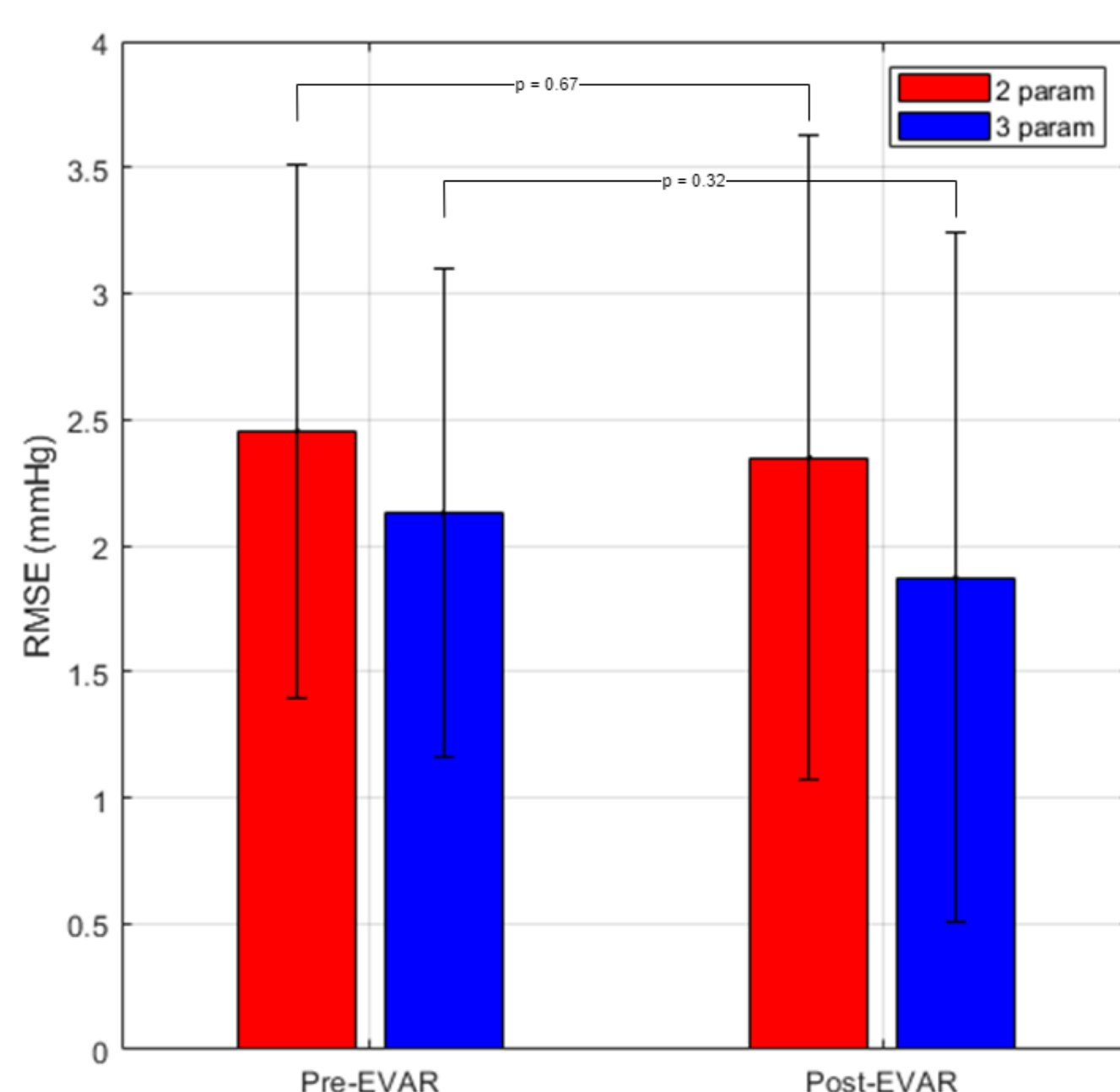


Fig. 4: Average RMSE of the model between pre- and post-EVAR subjects. No significant difference in RMSE was observed between the 2-parameter and 3-parameter models.

4. Data Analysis

- ❖ We compared the cf-PTT and reflection coefficient values in pre-EVAR vs. post-EVAR subjects. We normalized the cf-PTT for age, height, mean blood pressure, and pulse rate, and normalized the reflection coefficient for age and mean blood pressure.
- These normalizations account for the fact that both PTT and reflection coefficient are likely affected by blood pressure and age, and PTT is affected by height and pulse rate.

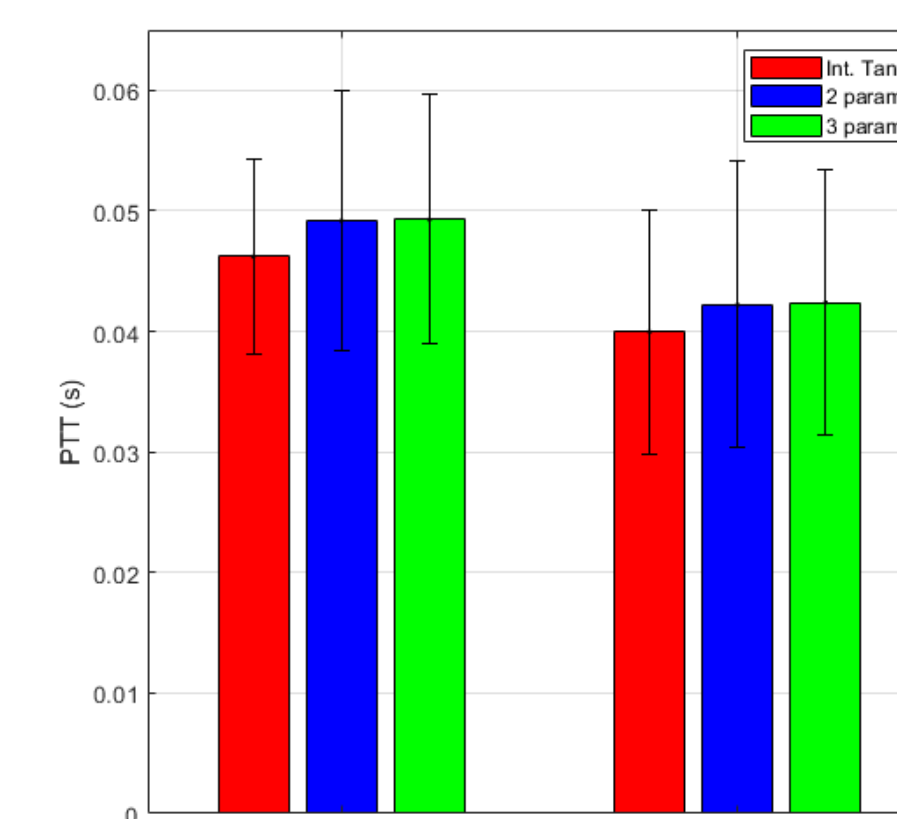


Fig 5: Unnormalized cf-PTT estimated from intersecting tangents and TL-models in pre-EVAR vs. post-EVAR subjects

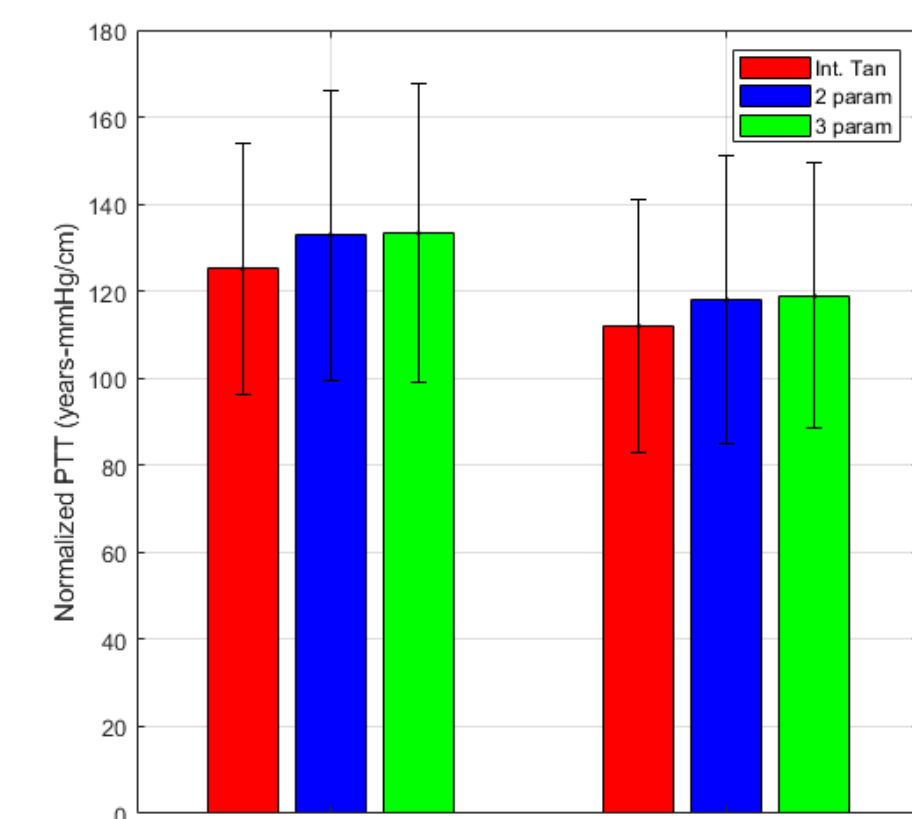


Fig. 6: Normalized cf-PTT estimated from intersecting tangents and TL-models in pre-EVAR vs. post-EVAR subjects. Normalization formula given by $NPTT = \frac{MBP \cdot Age \cdot PR}{Height} \cdot PTT$

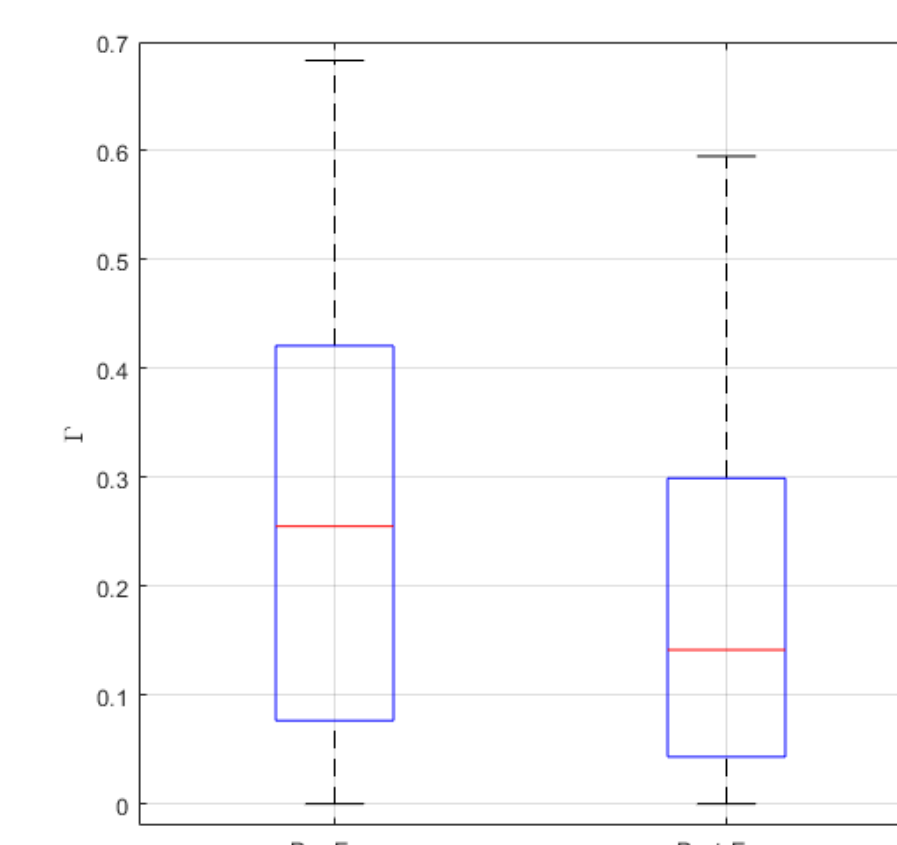


Fig 7: Unnormalized reflection coefficient from 2-parameter model.

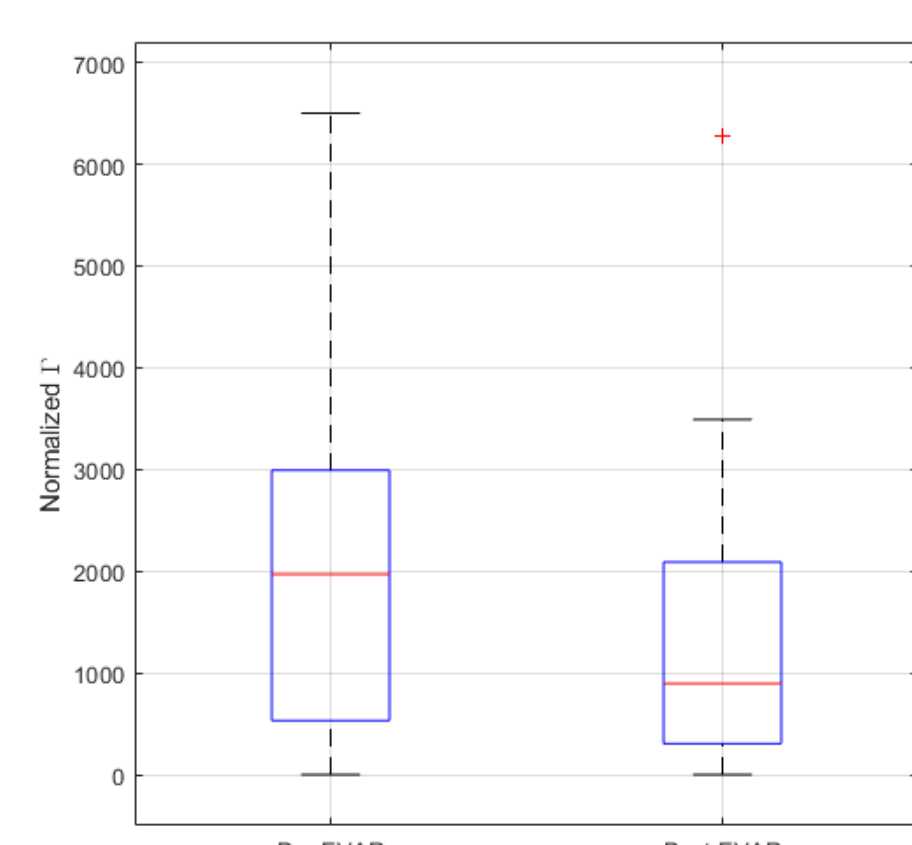


Fig 8: Normalized reflection coefficient from 2-parameter model.

5. Results

- ❖ Our results suggests that TL model may have the potential to track AAA progression within an individual without the use of conventional imaging techniques.
- ❖ Significant differences in PTT and reflection coefficient, and their normalizations, were found between pre-EVAR and post-EVAR subjects.
- ❖ The trends in the changes of TL model parameters were consistent with physiological expectations.
 1. PTT has been shown to decrease after EVAR.
 2. Reflection coefficient is expected decrease following EVAR due to a decrease in arterial compliance.

Table 1: Normalized PTT and reflection coefficient values of 43 pre-EVAR and post-EVAR subjects (mean \pm SD). *:p < 0.05.

	Pre-EVAR	Post-EVAR	Pre-EVAR	Post-EVAR
	Normalized PTT (mmHg-years/cm)		Normalized Reflection Coefficient	
2-parameter Model	132.96 \pm 33.30	118.01 \pm 32.96*	2076.0 \pm 1732.0	1259.7 \pm 1300.8*
3-parameter Model	133.37 \pm 34.25	118.87 \pm 30.47*	2526.2 \pm 2270.5	2343.3 \pm 2319.0

Table 2: PTT and reflection coefficient values of 43 pre-EVAR and post-EVAR subjects (mean \pm SD). *:p < 0.05.

	Pre-EVAR	Post-EVAR	Pre-EVAR	Post-EVAR
	PTT (s)		Reflection Coefficient	
2-parameter Model	0.0492 \pm 0.0108	0.0422 \pm 0.0119*	0.271 \pm 0.213	0.184 \pm 0.175*
3-parameter Model	0.0493 \pm 0.0104	0.0424 \pm 0.0110*	0.330 \pm 0.276	0.343 \pm 0.322

Acknowledgment

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