Stable Marriage Theorem

Group 30

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Storyboard

Problem Introduction (Time: 0 - 3)

The history of the topic:

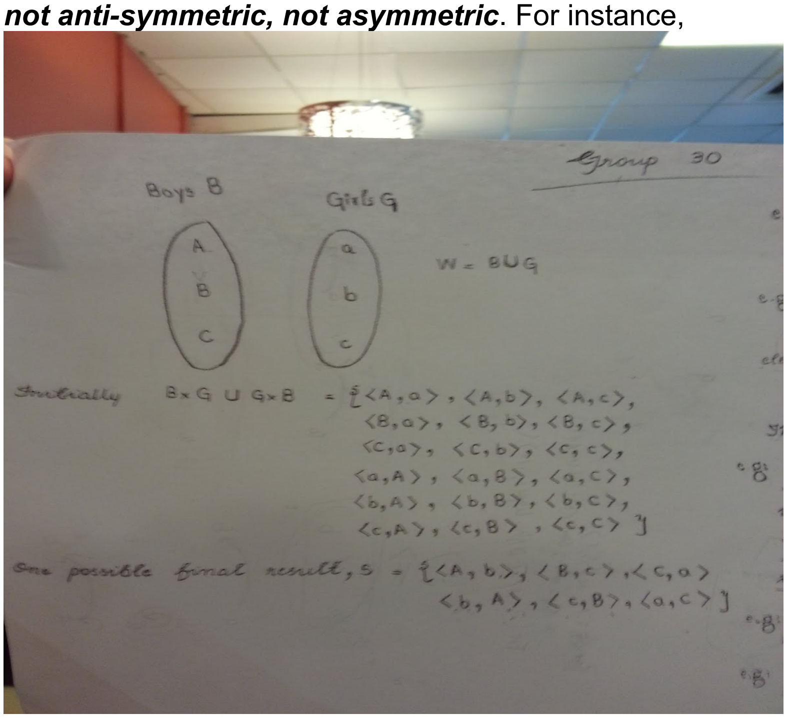
- In 1962, two mathematicians proved that it is always possible to solve the SMP (for any equal number of men and women)

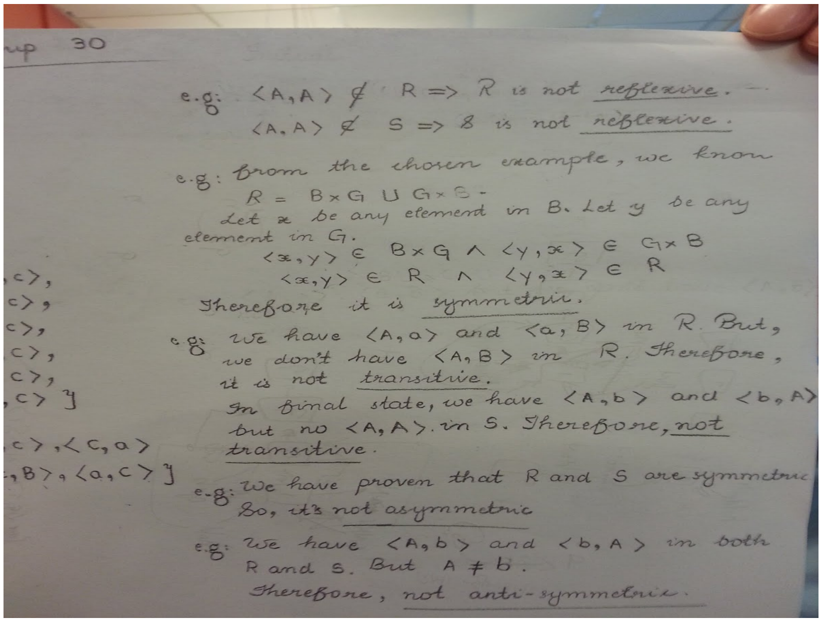
David Gale	Lloyd Shapley
 (December 13, 1921 – March 7, 2008) A distinguished American mathematician and economist. He was a Professor Emeritus at UC Berkeley, affiliated with departments of Mathematics, Economics, and Industrial Engineering and Operations Research. He has contributed to the fields of mathematical economics, game theory, and convex analysis.	 (born June 2, 1923) A Professor Emeritus at UCLA, affiliated with departments of Mathematics and Economics. He has contributed to the fields of mathematical economics and game theory.

Problem Definition:

- Original:

- ○ Given n men and n women, where each person has ranked all members of the opposite sex with a unique number between 1 and n in order of preference, marry the men and women together such that there are no two people of opposite sex who would both rather have each other than their current partners. If there are no such people, all the marriages are "stable".
- General:
  - ○ If two groups (A,B) with same number of objects, for every object in A has a unique ranking of objects in B and same for every object in B, each object can and must only link with one of the object in other group, then there must be a “stable” state for the two groups.
- Link with Relation/Function:
  - ○ Before pairing them up, the initial state is a relation. After pairing them up, we end up with a bijective function.
  - ○ Let B represents the set which contains all the boys and G be the representation of the set which contains all girls. The universe of discourse is A U B, we call it **W**. We know that initially, the matching relation **R** on W is BxG U GxB. After applying the algorithm, the matching relation **S** on W is a subset of R. S and R share the same properties namely, ***symmetric, not reflexive, not transitive, not anti-symmetric, not asymmetric***. For instance,





Format - Recorded slides
- It's not the main focus of the video, and it's clear enough to show the main points using slides

Solution Algorithm Explanation (Time: 3 - 8)

Gale-Shapley algorithm:
- unengaged man propose to most-prefered woman to whom he has not yet proposed
- every woman engaged with suitor whom she prefers most and rejects the rest
- repeat the process until everyone is engaged then we find the stable marriage
  - pseudocode:

```
function stableMatching {
  Initialize all m ∈ M and w ∈ W to free
  while ∃ free man m who still has a woman w to propose to {
    w = m's highest ranked such woman to whom he has not yet proposed
    if w is free
      (m, w) become engaged
    else some pair (m', w) already exists
      if w prefers m to m'
        (m, w) become engaged
        m' becomes free
      else
        (m', w) remain engaged
  }
}
```

This algorithm assumes that given m boys and n girls (m can be equal to n), but for each boy b, each b has n number of preferences and for each girl g, g has m number of preferences.

Theorem behind algorithm:
- The algorithm will end (Hint: when the unengaged man proposed to the woman who receives no other proposals)
  - *Once a woman becomes engaged, she is always engaged to someone. So, at the end, there cannot be a man and a woman both unengaged, as he must have proposed to her at some point*

(since a man will eventually propose to everyone, if necessary) and, being unengaged, she would have had to have said yes.

- The marriage is stable (Hint: if we change the marriage pair, either man or woman will be unsatisfied with his/her new partner)
    - *Let Alice be a woman and Bob be a man who are both engaged, but not to each other. Upon completion of the algorithm, it is not possible for both Alice and Bob to prefer each other over their current partners. If Bob prefers Alice to his current partner, he must have proposed to Alice before he proposed to his current partner. If Alice accepted his proposal, yet is not married to him at the end, she must have dumped him for someone she likes more, and therefore doesn't like Bob more than her current partner. If Alice rejected his proposal, she was already with someone she liked more than Bob.*

Algorithm Demonstration:

- step to step virtual demonstration of the algorithm
- generate n pairs of man & woman
- assign random preference on man & woman
- showing proposal process and outcome for each round
- showing final stable matching state

Algorithm Optimization:

- don't let a woman's choice depends on the man that proposes to her (resolve the woman's dependency problem,**Amnesiac Algorithm**)
- pick out best matching pairs first ('cheating' in the proposal, pair up the man and woman who both have 1st rank of each other)

Format - Recorded GUI animation (e.g. html5/flash/canvas website) demonstration

- We are computer science students, it's better to use program to explain everything

Similar Problem & Real World Application (Time: 8-9)

Variations of the problem:

- Stable roommates problem
- Hospital/residents problem
    - Difference in that "women" can accept "proposals" from more than one "man" (A hospital can accept more than one candidate, likewise for a college school)
- Hospital/residents problem with couples

Applications of the topic (i.e. in science, engineering, etc.)

- Chemistry: model solves the zero clustering problem of chemical graph polynomials successfully
- College application in US
- Graduating medical students intern assignment

Format - Recorded slides

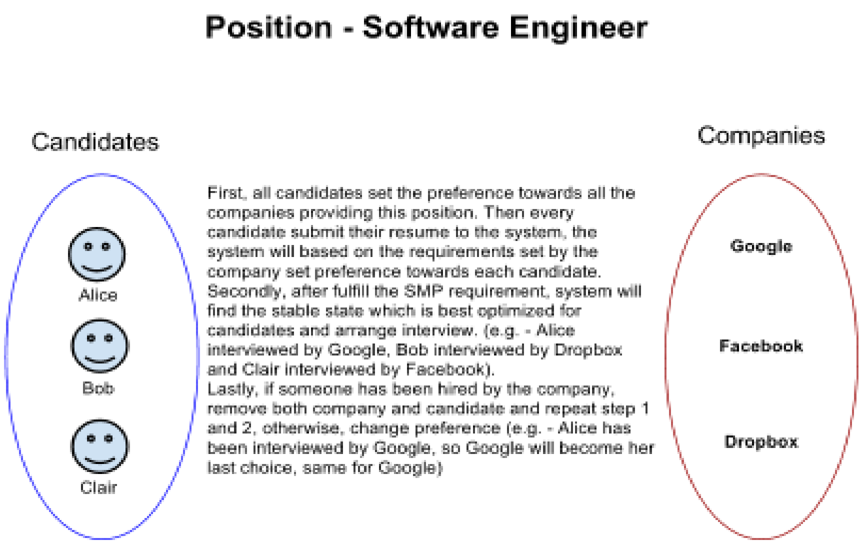
- It's not the main focus of the video, and it's clear enough to show the main points using slides

Our Application & Thought (Time: 9 - 14.5)

Application of SMP in recruitment industry:

- Why
    - help candidates to have a higher chance of getting an interview chance from his/ her prefered company

    ◦ help company to skip tedious interview selection (automate company ranking of candidates)
    ◦ make both candidates and company experience a better and easier recruitment process
- Reasoning
    ◦ Each candidate has his own preference towards the company
    ◦ Each company has a selection criteria for candidates
    ◦ For all the candidates applying same position from different companies, they will form up a SMP
- How it works



    ◦ JobStreet website has linkage with both candidate & companies
    ◦ JobStreet has a very good job position category system
    ◦ JobStreet will collect Resume and candidates preference towards companies (e.g. candidate A's ranking for Software Engineer [Google, Dropbox, Facebook …], candidate B's ranking for bank teller [OCBC, UOB, DBS …])
    ◦ We develop an automatic preference assignment tool for companies, the selection criteria is also based on resume (e.g. Google's criteria for software engineer [special contribution to open source project like android, educational background, programming skills …], OCBC's criteria for bank teller [educational background, work attitude, mathematical skills …])
    ◦ For every candidate applies same position and every company provide that position, they form up SMP
    ◦ After find out the 'stable' state, company can arrange interviews, if candidate has been selected for the position, then system remove both company and candidate to continue to find new stable state. If candidate is not successful in the interview, his ranking towards the company will become last, and same for the company's preference towards him then system will continue to find new stable state.

Format - Recorded 'handmade' animation or GUI animation
- It's always easier using pictures and flow diagram to explain a system, if we have enough time, we will use GUI animation

References & Credit (Time: 14.5 - 15)

[Wikipedia]
[The stable marriage, William Hunt]
[The stable marriage problem structure and algorithm, Dan Gusfield & Robert W. Irving]
[Relation types]