KITUR CHELIMO MERCY

SCT221-0840/2022

DESIGN AND ANALYSIS OF ALGORITHM

ASSIGNMENT 1

05/04/2024

Q1) Write an efficient recursive algorithm that takes a sentence, starting index, and ending index. The algorithm should then return a sentence that contains words between the starting and ending indices. Write the recurrence relation of your algorithm and find time complexity using the tracing tree method.

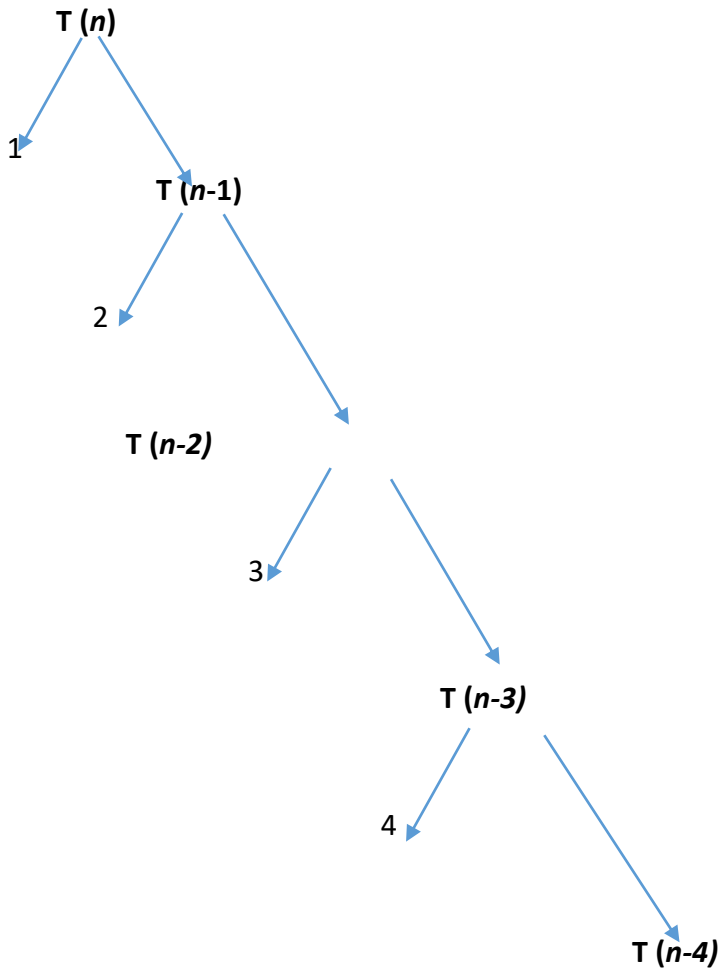
Recurrence relation of the algorithm:

Let **T (*n*)** be the time complexity of the algorithm, where n represents the number of characters in the sentence.

$$T(n) = T(n-1) + O(1)$$

The time complexity of the algorithm calculated using the tracing tree method is as follows:

Tracing Tree



From the tracing tree, we can observe that each level of the tree adds a constant time operation ***(O (1)).*** The height of the tree is ***n***. Therefore, the time complexity is the sum of the operations at each level, which is ***O*** (*n*).

So, the time complexity of the algorithm using the tracing tree method is ***O*** (*n*).

```
#include <iostream>
#include <string>

using namespace std;

// Recursive function to extract words between starting and ending indices
string extractWords (const string& sentence, int start, int end) {
   // if starting index exceeds ending index or sentence is empty
   if (start > end || start >= sentence.length()) {
      return "";
   }

   string result = ""; // Store the result

   // If the current character is not a space and falls within the range,
   // add it to the result string
   if (sentence[start] != ' ' && start <= end) {
      result += sentence[start];
   }

   // If the current character is a space and the previous character was part of a word,
   // add a space to separate words
```

```
    if (sentence[start] == ' ' && start <= end && start > 0 && sentence[start - 1] != ' ') {
        result += ' ';
    }

    // Recurr with the next index
    return result + extractWords(sentence, start + 1, end);
}

int main() {
    string sentence;
    int start, end;

    // Input sentence, starting index, and ending index
    cout << "Enter a sentence: ";
    getline(cin, sentence);
    cout << "Enter starting index: ";
    cin >> start;
    cout << "Enter ending index: ";
    cin >> end;
    // Extract words between starting and ending indices
    string result = extractWords(sentence, start, end);
    // Output the extracted words
    cout << "Words between indices " << start << " and " << end << ": " << result << endl;
    return 0;
}
```

Q2) Write an efficient algorithm that takes an array A [$n_1 ... n_n$) of sorted integers and returns an array with elements that have been circularly shifted k positions to the right. For example, a sorted array A = [5, 15, 29, 35, 42] is converted to A [35, 42, 5, 15, 27, 29] after circularly shifted 2 positions, while the same array A = [5, 15, 29, 35, 42] is converted to A [27, 29, 35, 42, 5, 15] after circularly shifted 4 positions. Write the recurrence relation of your solution and find the time complexity of your algorithm using an iterative method.

```
#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

void reverseArray(vector<int>& arr, int start, int end) {
    while (start < end) {
        swap(arr[start], arr[end]);
        start++;
        end--;
    }
}

vector<int> circularShift(vector<int>& arr, int n1, int n2, int k) {
    int n = n2 - n1 + 1;
    k %= n; // to handle cases where k is larger than n

    // Step 1: Reverse the entire range
    reverseArray(arr, n1, n2);
```

```
    // Step 2: Reverse the first k elements within the range
    reverseArray(arr, n1, n1 + k - 1);

    // Step 3: Reverse the remaining elements within the range
    reverseArray(arr, n1 + k, n2);

    return arr;
}

int main() {
    vector<int> A = {5, 15, 29, 35, 42};
    int n1 = 0; // Start index
    int n2 = A.size() - 1; // End index
    int k1 = 2;
    int k2 = 4;

    cout << "Original Array: ";
    for (int i = n1; i <= n2; ++i) {
        cout << A[i] << " ";
    }
    cout << endl;

    vector<int> result1 = circularShift(A, n1, n2, k1);
    cout << "Circularly shifted by " << k1 << " positions: ";
    for (int i = n1; i <= n2; ++i) {
        cout << result1[i] << " ";
    }
```

```
    cout << endl;


    vector<int> result2 = circularShift(A, n1, n2, k2);
    cout << "Circularly shifted by " << k2 << " positions: ";
    for (int i = n1; i <= n2; ++i) {
        cout << result2[i] << " ";
    }
    cout << endl;
    return 0;
}
```

SOLVING THE RECURRENCE RELATIONS

Given: T(n) = 3*T(n/2) + n

Let's expand:

T(n) = 3T(n/2) + n = 3(3T(n/4) + n/2) + n = 3^2 * T(n/4) + 3n/2 + n = 3^2 * (3T(n/8) + n/4) + 3n/2 + n = 3^3 * T(n/8) + 3^2 * n/4 + 3*n/2 + n ...

After k steps, we have: T(n) = 3^k * T(n/2^k) + n * (1 + 3/2 + (3/2)^2 + ... + (3/2)^(k-1))

We stop when n/2^k = 1, which happens when k = $\log_2 n$.

So, T(n) = 3^$\log_2 n$ * T(1) + n * (1 + 3/2 + (3/2)^2 + ... + (3/2)^($\log_2 n$-1))

Since T(1) is a constant, and the sum in the second term is a geometric series, we can solve it:

T(n) = O(3^$\log_2 n$) + O(n * (1 + (3/2) + (3/2)^2 + ... + (3/2)^($\log_2 n$-1)))

Using the formula for the sum of a finite geometric series:

T(n) = O(3^$\log_2 n$) + O(n * ((1 - (3/2)^$\log_2 n$) / (1 - 3/2)))

Since (3/2)^$\log_2 n$ = n^($\log_2 3$ - 1) and (1 - (3/2)^$\log_2 n$) / (1 - 3/2) is a constant, we have:

T(n) = O(3^$\log_2 n$) + O(n * (1 - n^($\log_2 3$ - 1)) / (1 - 3/2)) = O(n^$\log_2 3$) + O(n) = O(n^$\log_2 3$)

So, the time complexity of the given recurrence relation T(n) = 3*T(n/2) + n is O(n^$\log_2 3$).