THE UNIVERSITY OF SYDNEY

COMP2129 Assignment 4

Due: 11:59pm Sunday, 26th May 2013

Task description

In this assignment we will implement the basic PageRank algorithm in the C programming language using a variety of parallel programming techniques to ensure peak performance is achieved.

The PageRank algorithm was developed in 1996 by Larry Page and Sergey Brin when they were graduate students at Stanford University. Google and other search engines compare words in search phrases to words in web pages and use ranking algorithms to determine the most relevant results.

PageRank assigns a score to a set of web pages that indicates their importance. The underlying idea behind PageRank is to model a user who is clicking on web pages and following links from one page to another. In this framework, important pages are those which:

- have incoming links from many other pages; and/or
- have incoming links from other pages with a high PageRank score

The PageRank algorithm

PageRank is an iterative algorithm that is repeated until a stopping criteria is met. The last iteration gives us the result of the search, which is a score per web page. A high score indicates a very relevant web page whereas a low score indicates a not so relevant web page for a search. Sorting the web pages by their scores in descending order gives us the order for the result list of a search query.

For describing the PageRank algorithm we introduce the following symbols:

- S is the set of all web pages that we are computing the PageRank scores for
- $N = |S|$ is the total number of web pages
- $\mathbf{P} = [P_1^t, P_2^t, ..., P_N^t]$ is the vector of PageRank scores
- d is a dampening factor for the probability that the user continues clicking on web pages
- ϵ is the convergence threshold
- $\text{IN}(p)$ is the set of all pages in S which link to page p
- $\text{OUT}(p)$ is the set of all pages in S which page p links to

For the PageRank vector, we use the notation $\mathbf{P}_p^{(t)}$ to represent the PageRank score for page p at iteration t. We initialise the scores for all pages to an initial value of $\frac{1}{N}$ so that the sum equals 1.

$$\mathbf{P}_u^{(0)} = \frac{1}{N} \tag{1}$$

During each iteration of the algorithm, the value of $\mathbf{P}$ is updated as follows:

$$\mathbf{P}_u^{(t+1)} = \frac{1-d}{N} + d \sum_{v \in \text{IN}(u)} \frac{\mathbf{P}_v^{(t)}}{|\text{OUT}(v)|} \tag{2}$$

The algorithm continues to iterate until the convergence threshold is reached; that is, the algorithm terminates when PageRank scores stop varying between iterations. The PageRank scores have converged when the following condition is met:

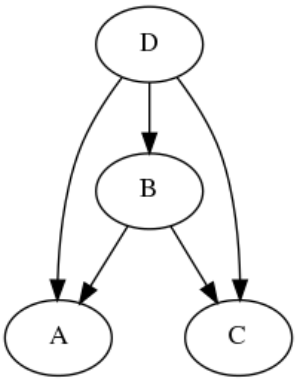
$$||\mathbf{P}^{(t+1)} - \mathbf{P}^{(t)}|| \leq \epsilon \tag{3}$$

The vector norm is defined to be the standard Euclidean vector norm; that is, for some vector $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$:

$$||\mathbf{x}|| = \sqrt{\sum_i x_i^2} \tag{4}$$

Example

Our example has four web pages: $S = \{A, B, C, D\}$. In this example, $d = 0.85$ and $\epsilon = 0.005$. The referencing structure of the web pages A, B, C, and D is given in the graph below.



$$\begin{aligned}
\text{IN}(A) &= \{B, D\} & \text{OUT}(A) &= \emptyset \\
\text{IN}(B) &= \{D\} & \text{OUT}(B) &= \{A, C\} \\
\text{IN}(C) &= \{B, D\} & \text{OUT}(C) &= \emptyset \\
\text{IN}(D) &= \emptyset & \text{OUT}(D) &= \{A, B, C\}
\end{aligned}$$

where a node represents a web page and an edge in the graph indicates that the source of the edge is linking to the destination of the edge.

Initialise $\mathbf{P}^{(0)} = \frac{1}{N}$. Then perform the first iteration for each page.

$$\mathbf{P}_A^{(1)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(0)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(0)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.25}{2} + \frac{0.25}{3}\right) \approx 0.214$$

$$\mathbf{P}_B^{(1)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_D^{(0)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.25}{3}\right) \approx 0.108$$

$$\mathbf{P}_C^{(1)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(0)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(0)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.25}{2} + \frac{0.25}{3}\right) \approx 0.214$$

$$\mathbf{P}_D^{(1)} = \frac{1-0.85}{4} + 0.85\,(0) = \frac{0.15}{4} + 0 \approx 0.038$$

The initialisation and first iteration result in the following values for $\mathbf{P}$:

t	A	B	C	D
0	0.250	0.250	0.250	0.250
1	0.214	0.108	0.214	0.038

Next, we check whether or not the algorithm has converged.

$$\begin{aligned} ||\mathbf{P}^{(1)} - \mathbf{P}^{(0)}|| &= ||\{-0.036, -0.142, -0.036, -0.215\}|| \\ &= \sqrt{0.0677} \\ &\approx 0.260 \end{aligned}$$

Since $0.260 \not\leq 0.005$ (ϵ), we perform another iteration.

$$\mathbf{P}_A^{(2)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(1)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(1)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.108}{2} + \frac{0.038}{3}\right) \approx 0.094$$

$$\mathbf{P}_B^{(2)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_D^{(1)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.038}{3}\right) \approx 0.048$$

$$\mathbf{P}_C^{(2)} = \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(1)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(1)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.108}{2} + \frac{0.038}{3}\right) \approx 0.094$$

$$\mathbf{P}_D^{(2)} = \frac{1-0.85}{4} + 0.85\,(0) = \frac{0.15}{4} + 0 \approx 0.038$$

Which leaves us with the following three iterations of $\mathbf{P}$:

t	A	B	C	D
0	0.250	0.250	0.250	0.250
1	0.214	0.108	0.214	0.038
2	0.094	0.048	0.094	0.038

Again, we check whether or not the algorithm has converged:

$$||\mathbf{P}^{(2)} - \mathbf{P}^{(1)}|| \approx 0.180 \not\leq \epsilon$$

Since convergence has not been reached, we perform another iteration.

$$\begin{aligned}
\mathbf{P}_A^{(3)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(2)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(2)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.048}{2} + \frac{0.038}{3}\right) \approx 0.069 \\
\mathbf{P}_B^{(3)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_D^{(2)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.038}{3}\right) \approx 0.048 \\
\mathbf{P}_C^{(3)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(2)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(2)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.048}{2} + \frac{0.038}{3}\right) \approx 0.069 \\
\mathbf{P}_D^{(3)} &= \frac{1-0.85}{4} + 0.85\,(0) = \frac{0.15}{4} + 0 \approx 0.038
\end{aligned}$$

Again, we check whether or not the algorithm has converged:

$$||\mathbf{P}^{(3)} - \mathbf{P}^{(2)}|| \approx 0.040 \not\leq \epsilon$$

Since convergence has not been reached, we perform another iteration.

$$\begin{aligned}
\mathbf{P}_A^{(4)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(3)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(3)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.048}{2} + \frac{0.038}{3}\right) \approx 0.069 \\
\mathbf{P}_B^{(4)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_D^{(3)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.038}{3}\right) \approx 0.048 \\
\mathbf{P}_C^{(4)} &= \frac{1-0.85}{4} + 0.85\left(\frac{\mathbf{P}_B^{(3)}}{|\{A,C\}|} + \frac{\mathbf{P}_D^{(3)}}{|\{A,B,C\}|}\right) = \frac{0.15}{4} + 0.85\left(\frac{0.048}{2} + \frac{0.038}{3}\right) \approx 0.069 \\
\mathbf{P}_D^{(4)} &= \frac{1-0.85}{4} + 0.85\,(0) = \frac{0.15}{4} + 0 \approx 0.038
\end{aligned}$$

Again, we check whether or not the algorithm has converged:

$$||\mathbf{P}^{(4)} - \mathbf{P}^{(3)}|| = 0 \leq \epsilon$$

Convergence has now been reached, so the algorithm terminates and the values of $\mathbf{P}^{(4)}$ are the final PageRank scores for each of the pages. If this were a query, the resulting ranks would be A, C, B, D, where we arbitrarily rank page A before page C since they have the same score.

Input and Output

In the header file **pagerank.h**, we have provided the function **read_input** that will process the input file for you. **read_input** prints **error** if the input is malformed in any way, and frees the memory that has been allocated if this occurs. We have also provided various structs in the header file that may be useful, including a struct for holding pages and a struct that is a linked list of pages. **read_input** returns the input data in these structs, which have the following form:

```
1  /* singly linked list to store all pages */
2  struct list {
3    node* head; /* pointer to the head of the list */
4    node* tail; /* pointer to the tail of the list */
5    int length; /* length of the entire list */
6  };
7
8  /* struct to hold a linked list of pages */
9  struct node {
10   page* page; /* pointer to page data structure */
11   node* next; /* pointer to next page in list */
12 };
13
14 /* struct to hold a page */
15 struct page {
16   char name[21]; /* page name */
17   int index;     /* index of the page */
18   int noutlinks; /* number of outlinks from this page */
19   list* inlinks; /* pointer to linked list of pages with inlinks to this page */
20 };
```

Warning: Do not change the header file or the **main** function.
We will replace the header file with our own copy when marking your code.

Try compiling the provided skeleton and running it in **gdb**. Set a breakpoint after **read_input** has been called, and run the program on a sample input to examine what the data structures look like. You may also find the funtion **page_list_destroy** to be useful. Given a pointer to a **struct list**, such as the one created by **read_input**, it will free all of the memory that is occupied by the list and the pages it contains. The provided **pagerank.c** file shows you how to use these functions.

Input format

```
<number of cores>
<dampening factors>
<number of pages>
<name of web page>
...
<number of edges>
<source page> <destination page>
...
```

The first line specifies how many cores are available on the computer. You need to take the number of cores into account when you are optimizing the performance of the program.

The next input lines specify the dampening factor used in the algorithm and the total number of web pages. The names of the web pages follow, one per line , where each name may be a maximum of 20 characters long. After declaring the names of the web pages, the number of edges in the graph is given, followed by each edge in the graph specified by its source and destination page.

`read_input` prints **`error`** and terminates if:

- there are memory allocation errors
- any page name exceeds its maximum length
- the dampening factor is less than 0 or greater than 1
- there is extraneous input or the input is otherwise malformed
- any page is declared twice or an edge is defined to a nonexistent page
- the number of cores or pages is not $>$ than 0 or the number of edges is $<$ than 0

In the example, the input is as follows assuming that the number of cores available is 2.

```
2
0.85
4
A
B
C
D
5
D A
D B
D C
B A
B C
```

Our example has four web pages with names A, B, C, and D.
There are five edges: $D \rightarrow A$, $D \rightarrow B$, $D \rightarrow C$, $B \rightarrow A$, and $B \rightarrow C$.

The output is the list of scores per web page, i.e.,

```
A 0.0686
B 0.0481
C 0.0686
D 0.0375
```

The scores are ordered in the same order they were defined in the input.

For printing the score, use the format string **`"%s %.4lf\n"`**.
For all test cases set $\epsilon = 0.005$ (**`pagerank.h`** defines this constant EPSILON)

The Makefile

We have provided you with a Makefile that can be used to compile and test your code using the test cases you define. The included Makefile will only run correctly on the **`ucpu0`** and **`ucpu1`** machines.

- **`$ make`**
  will compile your **`pagerank.c`** file into an executable file **`pagerank`**
- **`$ make test`**
  will compile your program and then check the correctness of your program using the test cases that you define in your **`tests/`** directory and highlight any differences between the **`expected`** output and the **`observed`** output of your program
- **`$ make rain`**
  will compile your program and then check whether your program has any memory leaks using the test cases that you define in your **`tests/`** directory. If any test cases require further investigation, you can run valgrind manually e.g. **`$ valgrind ./pagerank < sample.in`**
- **`$ make submission`**
  will submit your program to the benchmarking queue and also for marking

Marking

We have provided a set of test cases in the included **`tests/`** directory, none of these cases will test for any erroneous inputs as we have provided you with a standardised input reading function. In this assignment only **valid inputs** are tested, and your program will be checked for memory leaks. If your program leaks memory you will not pass the test case even if the output is correct. In addition to the provided cases we will run your program against a substantial collection of hidden cases.

For this assignment correctness and performance are equally weighted. However, your submission will only receive marks for the performance component if it passes all of the correctness test cases.

Before you attempt to write optimised code, you should first complete a working unoptimised version. Once you have the basic algorithm correct you can then begin to implement various optimisations.

5 **marks** are assigned based on automatic tests for the *correctness* of your program.
This component will use our own hidden test cases that cover every aspect of the specification, including memory leaks. Note that your test cases may also be used in this component.

5 **marks** are assigned based on the *performance* of your code relative to other students, and is automatically tested on a separate machine in a consistent manner. The fastest submission will receive 5 marks, with successively slower solutions receiving lower marks. Any implementation that is faster than our sequential reference implementation will receive at least 2.5 marks.

Warning: Any attempts to deceive the auto marker will result in an immediate 0 for the assignment.