Neutrino Physics
Graduate Lectures (Jan-Feb 2005)

Lecture 2: 27 January 2005

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Lecture 2

2. Neutrino interactions (cont.)
    - 2.2 Neutrino-electron scattering
    - 2.3 Neutrino-nucleon quasi-elastic scattering
    - 2.4 Neutrino-nucleon deep inelastic scattering
        - Variables
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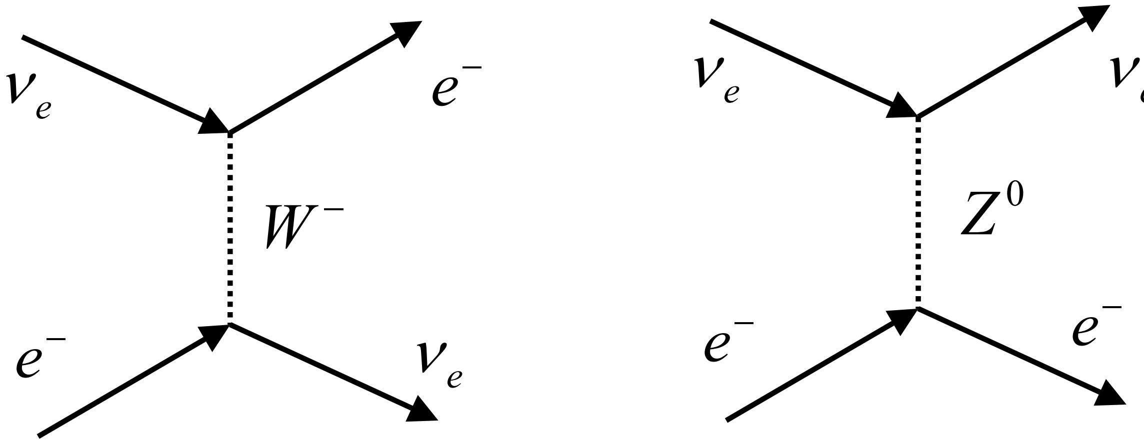
2.2 Neutrino-electron scattering

- Tree level Feynman diagrams: $\nu_e + e^- \rightarrow \nu_e + e^-$



- Effective Hamiltonian:

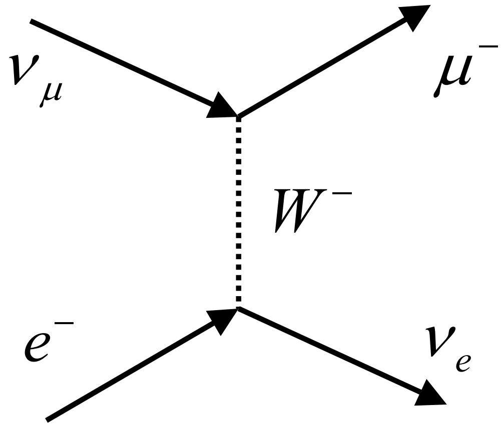
$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right]\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]+\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\right]\left[\bar{e}\gamma_\mu(g_V-g_A\gamma_5)e\right]\right\}$$

$$= \frac{G_F}{\sqrt{2}}\left\{\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\right]\left[\bar{e}\gamma_\mu(1+g_V-(1+g_A)\gamma_5)e\right]\right\}$$

(through a Fierz transformation)

Neutrino-electron scattering (cont)

- Only charged current: $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$



$$s = \left(p(\nu_\mu) + p(e)\right)^2 = 2m_e E(\nu_\mu)\ (in\ LAB)$$

$$t = q^2 = \left(p(\nu_\mu) - p(\mu)\right)^2$$

$$y = \frac{p(e)\cdot\left(p(\nu_\mu) - p(\mu)\right)}{p(e)\cdot p(\nu_\mu)} = \frac{E(\nu_\mu) - E(\mu)}{E(\nu_\mu)}\ (in\ LAB)$$

Inelasticity variable (0<y<1)

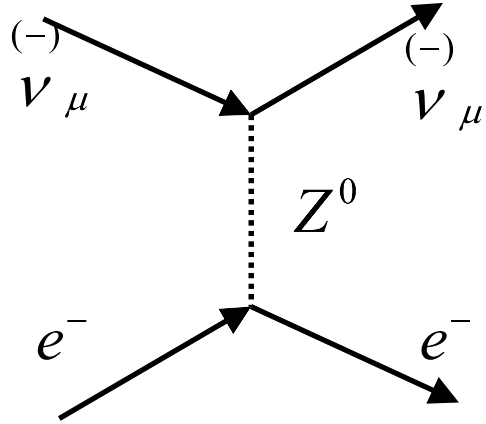
$$\frac{d\sigma_{CC}(\nu_\mu e^-)}{dy} = \frac{G_F{}^2 s}{\pi}\frac{m_W{}^2}{q^2 - m_W{}^2} \approx \frac{2G_F{}^2 m_e}{\pi}E(\nu_\mu)\ (in\ LAB)$$

Total cross-section: $\sigma_{CC}(\nu_\mu e^-) = \frac{G_F{}^2 s}{\pi} = 0.4\times 10^{-43}\left(\frac{E}{10\ MeV}\right)cm^2$

(cross-section proportional to energy!)

Neutrino-electron scattering (cont)

- Only neutral current: $\overset{(-)}{\nu}_{\mu} + e^- \rightarrow \overset{(-)}{\nu}_{\mu} + e^-$



$$\bar{e}\gamma_{\mu}(g_V - g_A\gamma_5)e = g_L\bar{e}\gamma_{\mu}(1-\gamma_5)e + g_R\bar{e}\gamma_{\mu}(1+\gamma_5)e$$

$$g_L = \frac{1}{2}(g_V + g_A) = -\frac{1}{2} + \sin^2\theta_W$$

$$g_R = \frac{1}{2}(g_V - g_A) = \sin^2\theta_W$$

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F{}^2 s}{\pi}\frac{m_Z{}^2}{q^2 - m_Z{}^2}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2 + \sin^4\theta_W(1-y)^2\right]$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu e^-)}{dy} = \frac{G_F{}^2 s}{\pi}\frac{m_Z{}^2}{q^2 - m_Z{}^2}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2(1-y)^2 + \sin^4\theta_W\right]$$

Neutrino-electron scattering (cont)

- Only neutral current (total cross-section): $\overset{(-)}{\nu}_\mu + e^- \rightarrow \overset{(-)}{\nu}_\mu + e^-$

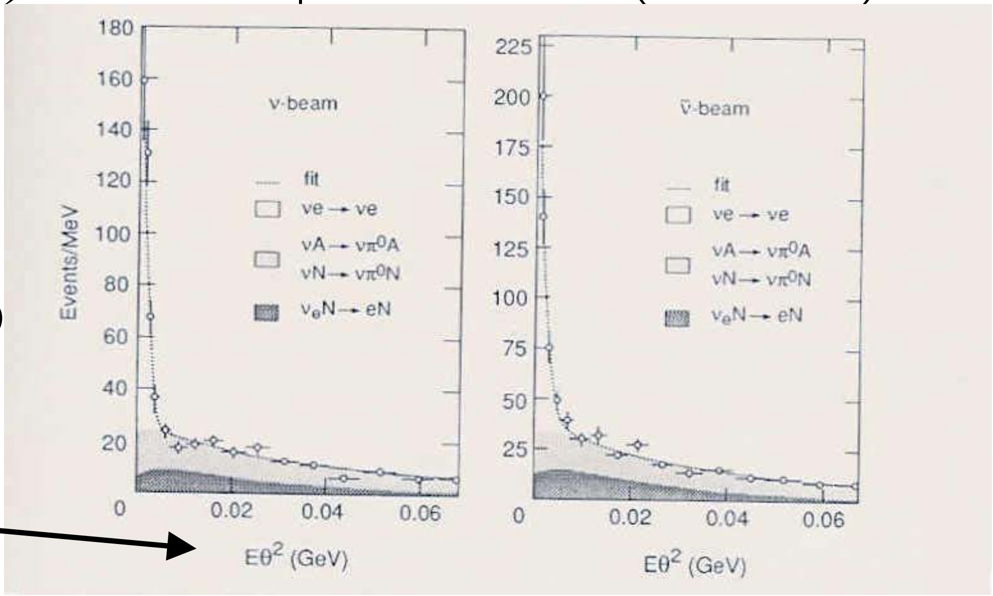
$$\sigma_{NC}(\nu_\mu e^-) = \frac{G_F^{\,2} s}{\pi}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)^2 + \frac{1}{3}\sin^4\theta_W\right] = 0.15\times 10^{-43}\left(\frac{E_\nu}{10\ MeV}\right) cm^2$$

$$\sigma_{NC}(\bar{\nu}_\mu e^-) = \frac{G_F^{\,2} s}{\pi}\left[\frac{1}{3}\left(-\frac{1}{2}+\sin^2\theta_W\right)^2 + \sin^4\theta_W\right] = 0.14\times 10^{-43}\left(\frac{E_\nu}{10\ MeV}\right) cm^2$$

- Can obtain value of $\sin^2\theta_W$ from neutrino electron scattering (CHARM II):

$$\sin^2\theta_W = 0.2324 \pm 0.0058 \pm 0.0059$$

$$E_e\Theta^2 = 2m_e(1-y)$$



Neutrino-electron scattering (cont)

- Back to $\nu_e + e^- \rightarrow \nu_e + e^-$ (charged and neutral currents)

$$g_L = \frac{1}{2}(1 + g_V + 1 + g_A) = -\frac{1}{2} + \sin^2\theta_W + 1 = \frac{1}{2} + \sin^2\theta_W$$

$$g_R = \frac{1}{2}(1 + g_V - (1 + g_A)) = \sin^2\theta_W$$

Then: $$\frac{d\sigma(\nu_e e^-)}{dy} = \frac{G_F^{\ 2} s}{\pi}\left[\left(\frac{1}{2} + \sin^2\theta_W\right)^2 + \sin^4\theta_W (1-y)^2\right]$$

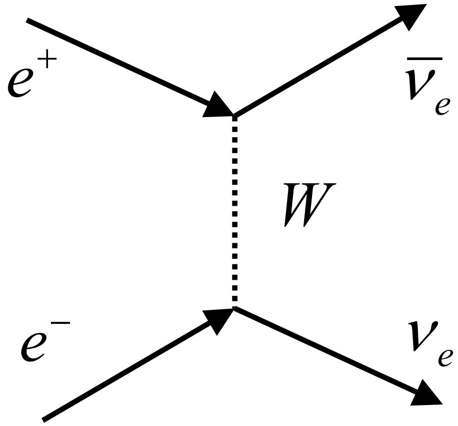
$$\Rightarrow \sigma(\nu_e e^-) = \frac{G_F^{\ 2} s}{\pi}\left[\left(\frac{1}{2} + \sin^2\theta_W\right)^2 + \frac{1}{3}\sin^4\theta_W\right] = 0.9 \times 10^{-43}\left(\frac{E_\nu}{10\ MeV}\right) cm^2$$

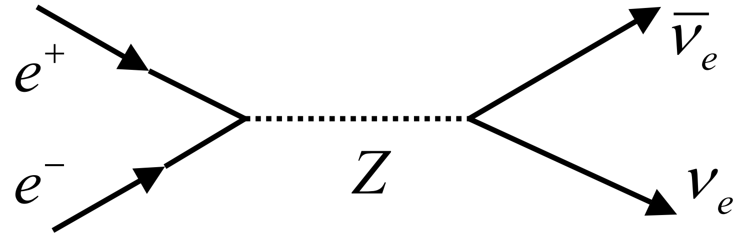
This cross-section is a consequence of the interference of the charged and neutral current diagrams.

Neutrino-electron scattering (cont)

- Neutrino pair production: $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$

Contribution from both W and Z graphs.





Then:

$$\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_e) = \frac{G_F{}^2 s}{12\pi}\left[\left(\frac{1}{2} + 2\sin^2\theta_W\right)^2 + \frac{1}{4}\right]$$

- Only neutral current contribution to: $e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$

$$\sigma(e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu) = \frac{G_F{}^2 s}{12\pi}\left[\left(\frac{1}{2} - 2\sin^2\theta_W\right)^2 + \frac{1}{4}\right]$$

Neutrino-electron scattering (cont)

- Summary neutrino electron scattering processes:

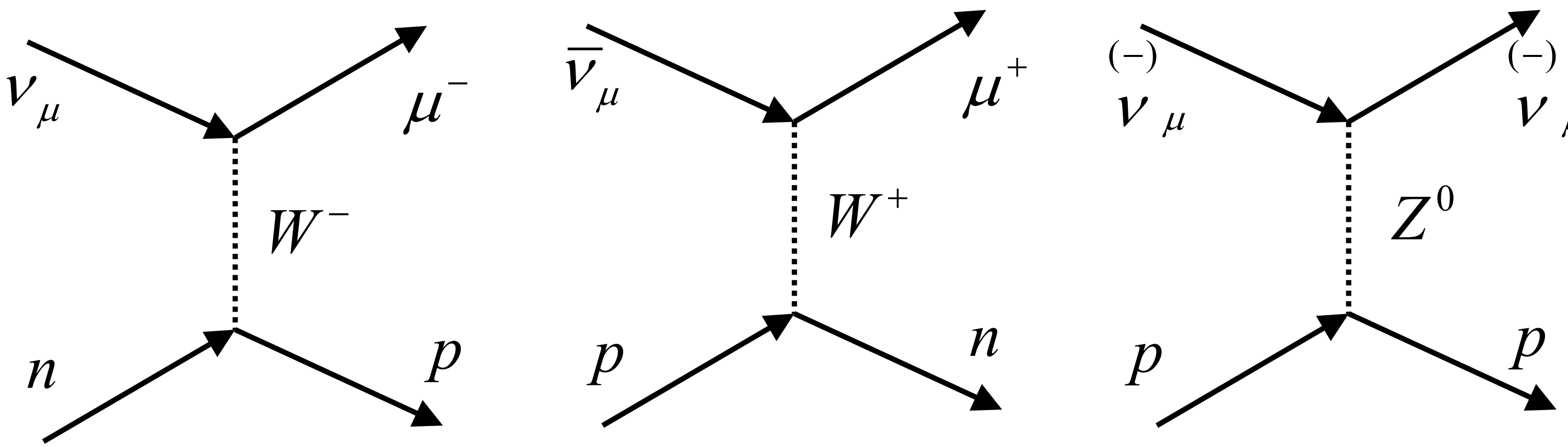
Process	Total cross-section
$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$	$\frac{G_F{}^2 s}{\pi}$
$\nu_e + e^- \rightarrow \nu_e + e^-$	$\frac{G_F{}^2 s}{4\pi}\left[\left(2\sin^2\theta_W - 1\right)^2 + \frac{4}{3}\sin^4\theta_W\right]$
$\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$	$\frac{G_F{}^2 s}{4\pi}\left[\frac{1}{3}\left(2\sin^2\theta_W + 1\right)^2 + 4\sin^4\theta_W\right]$
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$	$\frac{G_F{}^2 s}{4\pi}\left[\left(2\sin^2\theta_W - 1\right)^2 + \frac{4}{3}\sin^4\theta_W\right]$
$\overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$	$\frac{G_F{}^2 s}{4\pi}\left[\frac{1}{3}\left(2\sin^2\theta_W - 1\right)^2 + 4\sin^4\theta_W\right]$
$e^+ + e^- \rightarrow \nu_e + \overline{\nu}_e$	$\frac{G_F{}^2 s}{12\pi}\left[\frac{1}{2} + 2\sin^2\theta_W + 4\sin^4\theta_W\right]$
$e^+ + e^- \rightarrow \nu_\mu + \overline{\nu}_\mu$	$\frac{G_F{}^2 s}{12\pi}\left[\frac{1}{2} - 2\sin^2\theta_W + 4\sin^4\theta_W\right]$

$$s = 2m_e E(\nu_\mu)\ (in\ the\ LAB\ \ frame)$$

2.3 Neutrino-nucleon quasi-elastic scattering

- Quasi-elastic neutrino-nucleon scattering reactions (small q^2):

$$\nu_\mu + n \rightarrow \mu^- + p \qquad \bar{\nu}_\mu + p \rightarrow \mu^+ + n \qquad \overset{(-)}{\nu}_\mu + p \rightarrow \overset{(-)}{\nu}_\mu + p$$



$$M = \left\langle \mu^-, p \middle| H_{eff} \middle| \nu_\mu, n \right\rangle =$$

$$\frac{G_F \cos\theta_c}{\sqrt{2}} \left[\bar{\mu} \gamma^\mu (1-\gamma_5) \nu_\mu \right] \left[\bar{p} \gamma_\mu \left(F_V(q^2) + F_A(q^2)\gamma_5 \right) n \right]$$

$F_V(q^2) = vector\ form\ factor$ $\qquad \cos\theta_C = 0.975\ (Cabbibo\ angle)$

$F_A(q^2) = axial-vector\ form\ factor$

Neutrino-nucleon quasi-elastic scattering (cont)

- Form factors introduced since proton, neutron not elementary.
- Depend on vector and axial weak charges of the proton and neutron.
- Two hypotheses:
  - Conservation of Vector Current (CVC):
  - Partial conservation of Axial Current (PCAC):

$$F_V(q^2) = \frac{F_V(0)}{\left(1 - q^2/0.71\right)^2} \qquad F_V(0) = 1$$

$$F_A(q^2) = \frac{F_A(0)}{\left(1 - q^2/1.065\right)^2} \qquad F_A(0) = g_A = -1.2573 \pm 0.028$$

- For low energy neutrinos ($E_\nu << m_N$):

$$\sigma(\nu_e n) = \sigma(\bar{\nu}_e p) = \frac{\left(G_F \cos\theta_C\right)^2 E_\nu^{\ 2}}{\pi}\left[F_V(0)^2 + 3F_A(0)^2\right]$$

$$\approx 9.75 \times 10^{-42}\left(\frac{E_\nu}{10\ MeV}\right)^2 cm^2$$

2.4 Neutrino-nucleon deep inelastic scattering

- Deep inelastic neutrino-nucleon scattering reactions have large q^2 ($q^2 \gg m_N^2, E_\nu \gg m_N$): $\quad \nu_l(p) + N \to l^-(p') + X$
- Quark-parton model valid due to asymptotic freedom of QCD, which makes quarks behave as free point-like particles.
- Infinite momentum frame: a parton takes a fraction *x (0<x<1)*, of momentum when struck by a neutrino. Final quark state:

$$(xp_N + q)^2 = m_q^2 \Rightarrow x \approx -\frac{q^2}{2p_N \cdot q} \quad if\ q^2 \gg m_q^2$$

- Variables in DIS:

$$s = (p + p_N)^2 \approx 2ME_\nu = 2ME$$

$$Q^2 = -q^2 = -(p + p')^2 = 4EE' \sin^2\frac{\theta}{2}$$

$$W^2 = E_X^2 - p_X^2 = -Q^2 + 2M\nu + M^2$$

$$\nu = \frac{q \cdot p_N}{M} = E - E'$$

Bjorken Variables (*0<x<1, 0<y<1*):

$$x = \frac{-q^2}{2q \cdot p_N} = \frac{Q^2}{2M\nu}$$

$$y = \frac{q \cdot p_N}{p \cdot p_N} = \frac{\nu}{E} = \frac{Q^2}{2MEx}$$

Neutrino-nucleon deep inelastic scattering (cont)

- Neutrino proton CC scattering: $\nu_\mu(p) + p \to \mu^-(p') + X$

  $u(x)dx$ =number of u-quarks in proton between x and x+dx

  $u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$

  In the sea: $u_S(x) = \bar{u}(x) \quad d_S(x) = \bar{d}(x)$

  For proton (uud):
  $$\int_0^1 u_V(x)dx = \int_0^1 \left[u(x) - \bar{u}(x)\right]dx = 2$$
  $$\int_0^1 d_V(x)dx = \int_0^1 \left[d(x) - \bar{d}(x)\right]dx = 1$$

- Scattering off quarks:

$$\frac{d\sigma_{CC}(\nu_\mu q)}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu \bar{q})}{dy} = \frac{2G_F{}^2 m_q E}{\pi} \quad with \ \ y = 1 - \frac{E}{E'} = \frac{1}{2}(1 - \cos\theta)$$

$$\frac{d\sigma_{CC}(\nu_\mu \bar{q})}{dy} = \frac{d\sigma_{CC}(\bar{\nu}_\mu q)}{dy} = \frac{2G_F{}^2 m_q E}{\pi}(1-y)^2$$

Neutrino-nucleon deep inelastic scattering (cont)

- Scattering off proton:

$$\frac{d\sigma_{CC}(\nu_\mu p)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} 2x\left\{\left[d(x)+s(x)\right]+\left[\overline{u}(x)+\overline{c}(x)\right](1-y)^2\right\}$$

$$\frac{d\sigma_{CC}(\overline{\nu}_\mu p)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} 2x\left\{\left[u(x)+c(x)\right](1-y)^2+\left[\overline{d}(x)+\overline{s}(x)\right]\right\}$$

- Structure functions:

Callan-Gross relationship: $2xF_1(x) = F_2(x)$

$$F_2^{\nu p}(x) = 2x\left[d(x)+\overline{u}(x)+s(x)+\overline{c}(x)\right]$$
$$xF_3^{\nu p}(x) = 2x\left[d(x)-\overline{u}(x)+s(x)-\overline{c}(x)\right]$$
$$F_2^{\overline{\nu} p}(x) = 2x\left[u(x)+c(x)+\overline{d}(x)+\overline{s}(x)\right]$$
$$xF_3^{\overline{\nu} p}(x) = 2x\left[u(x)+c(x)-\overline{d}(x)-\overline{s}(x)\right]$$

- Neutron (isospin symmetry):

$$F_2^{\nu n}(x) = 2x\left[u(x)+\overline{d}(x)+s(x)+\overline{c}(x)\right]$$
$$xF_3^{\nu n}(x) = 2x\left[u(x)-\overline{d}(x)+s(x)-\overline{c}(x)\right]$$

Neutrino-nucleon deep inelastic scattering (cont)

- Scattering off isoscalar target (equal number neutrons and protons):

$$q \equiv u + d + s + c \qquad \overline{q} \equiv \overline{u} + \overline{d} + \overline{s} + \overline{c}$$

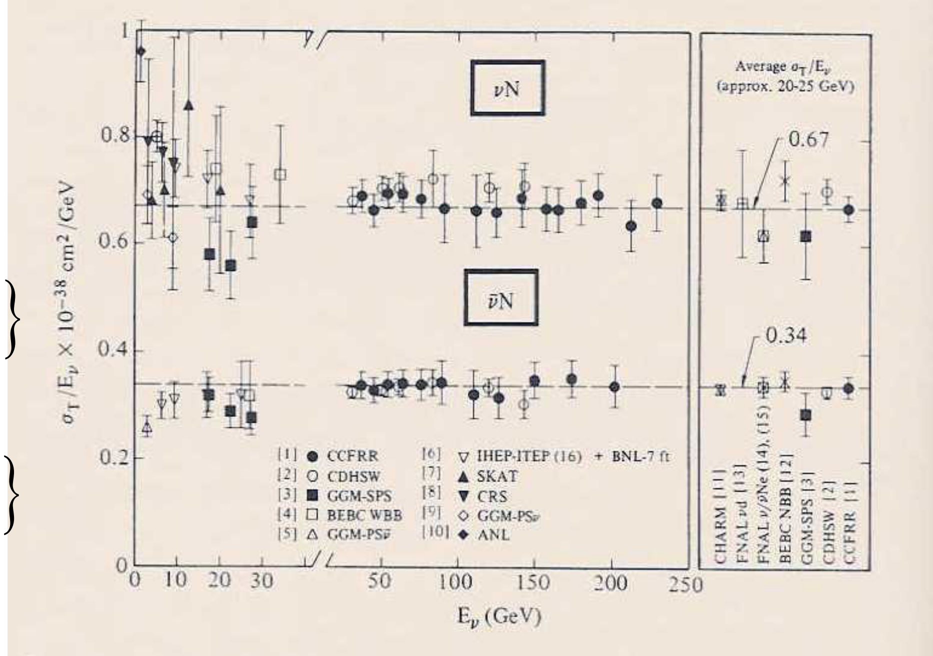
$$F_2^{\nu N}(x) = x\left[q(x) + \overline{q}(x)\right]$$

$$xF_3^{\nu N}(x) = x\left[q(x) - \overline{q}(x) + 2\left(s(x) - c(x)\right)\right]$$

$$xF_3^{\overline{\nu} N}(x) = x\left[q(x) - \overline{q}(x) - 2\left(s(x) - c(x)\right)\right]$$

$$\frac{d\sigma_{CC}(\nu_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{ q(x) + \overline{q}(x)\,(1-y)^2 \right\}$$

$$\frac{d\sigma_{CC}(\overline{\nu}_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{ q(x)(1-y)^2 + \overline{q}(x) \right\}$$



- Total cross-section:

$$\sigma_{CC}(\nu_\mu N) = \frac{G_F^{\,2} s}{2\pi}\left[\langle Q\rangle + \frac{1}{3}\langle\overline{Q}\rangle\right] = 0.67\times 10^{-38}\,cm^2/GeV \times E(GeV)$$

$$\sigma_{CC}(\overline{\nu}_\mu N) = \frac{G_F^{\,2} s}{2\pi}\left[\frac{1}{3}\langle Q\rangle + \langle\overline{Q}\rangle\right] = 0.34\times 10^{-38}\,cm^2/GeV \times E(GeV)$$

Neutrino-nucleon deep inelastic scattering (cont)

- Quark content of nucleons from CC cross-sections
- Define:

$$U = \int_0^1 xu(x)dx \text{ , } etc.$$

- Experimental values from y distribution of cross-sections yields:

$$\frac{\overline{Q}}{Q+\overline{Q}} = 0.15 \pm 0.03 \qquad \frac{S}{Q+\overline{Q}} = 0.00 \pm 0.03 \qquad \frac{\overline{Q}+S}{Q+\overline{Q}} = 0.16 \pm 0.01$$

- If $\quad r \equiv \dfrac{\sigma_{CC}(\overline{\nu}N)}{\sigma_{CC}(\nu N)} = 0.495\,(measured) \quad \Rightarrow \dfrac{\overline{Q}}{Q} = \dfrac{3r-1}{3-r} \approx 0.19$

$$Q_V = Q - \overline{Q} \approx 0.33 \qquad Q_S = \overline{Q}_S = \overline{Q} \approx 0.08$$

$$\int_0^1 F_2^{\nu N}(x)dx = Q + \overline{Q} \approx 0.49$$

- Quarks and antiquarks carry 49% of proton momentum, valence quarks only 33% and sea quarks only 16%.

Neutrino-nucleon deep inelastic scattering (cont)

- Sum rules:
  - Gross-Llewellyn Smith: $S_{GLS}=\frac{1}{2}\int_0^1 (F_3^{\nu}(x)+F_3^{\bar{\nu}}(x))dx$

$$S_{GLS}=\int_0^1 (q(x)-\bar{q}(x))dx = 3\left[1-\frac{\alpha_s}{\pi}-a\left(\frac{\alpha_s}{\pi}\right)^2-b\left(\frac{\alpha_s}{\pi}\right)^3\right]=2.64\pm 0.06$$

  - Adler:

$$S_A=\frac{1}{2}\int_0^1 \frac{1}{x}(F_2^{\nu n}(x)+F_2^{\nu p}(x))dx=\int_0^1 (u_V(x)-d_V(x))dx=1$$

  - Gottfried:

$$S_G=\frac{1}{2}\int_0^1 \frac{1}{x}(F_2^{\mu n}(x)+F_2^{\mu p}(x))dx=\frac{1}{3}\int_0^1 (u(x)+\bar{u}(x)-d(x)-\bar{d}(x))dx=\frac{1}{3}$$

$S_G=0.235\pm 0.026$ Maybe isospin asymmetry: $\bar{u}(x)\neq\bar{d}(x)$

  - Bjorken:

$$S_B=\int_0^1 (F_1^{\bar{\nu}p}(x)+F_1^{\nu p}(x))dx=1-\frac{2\alpha_s(Q^2)}{3\pi}$$

Neutrino-nucleon deep inelastic scattering (cont)

- Neutral currents: $\overset{(-)}{\nu}_{\mu} + p \rightarrow \overset{(-)}{\nu}_{\mu} + X$

$$\frac{d\sigma_{NC}(\nu_\mu q)}{dxdy} = \frac{d\sigma_{NC}(\bar{\nu}_\mu \bar{q})}{dxdy} =$$

$$\frac{G_F^{\ 2} m_q E_\nu}{2\pi} x \left\{ (g_V + g_A)^2 + (g_V - g_A)^2 (1-y)^2 + \frac{m_q}{E_\nu}(g_A^{\ 2} - g_V^{\ 2})y \right\}$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu q)}{dxdy} = \frac{d\sigma_{NC}(\nu_\mu \bar{q})}{dxdy} =$$

$$\frac{G_F^{\ 2} m_q E_\nu}{2\pi} x \left\{ (g_V - g_A)^2 + (g_V + g_A)^2 (1-y)^2 + \frac{m_q}{E_\nu}(g_A^{\ 2} - g_V^{\ 2})y \right\}$$

- Coupling constants:

$$g_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \qquad g_V = \frac{1}{2} \qquad \text{for q=u,c}$$

$$g'_V = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \qquad g'_V = -\frac{1}{2} \qquad \text{for q=d,s}$$

$$\begin{cases} g_L = \frac{1}{2}(g_V + g_A) \\ g_R = \frac{1}{2}(g_V - g_A) \end{cases}$$

Neutrino-nucleon deep inelastic scattering (cont)

- Neutral currents off nucleons (neglecting c and s quark contributions):

$$\overset{(-)}{\nu}_\mu + N \to \overset{(-)}{\nu}_\mu + X$$

$$\frac{d\sigma_{NC}(\nu_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{ (g_L^{\,2} + g_L'^{\,2})^2 \left[q + \bar{q}(1-y)^2 \right] + (g_R^{\,2} + g_R'^{\,2})^2 \left[\bar{q} + q(1-y)^2 \right] \right\}$$

$$\frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{dxdy} = \frac{G_F^{\,2} ME}{\pi} x\left\{ (g_R^{\,2} + g_R'^{\,2})^2 \left[q + \bar{q}(1-y)^2 \right] + (g_L^{\,2} + g_L'^{\,2})^2 \left[\bar{q} + q(1-y)^2 \right] \right\}$$

- Defining: $R_\nu \equiv \dfrac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)}$ $\quad R_{\bar{\nu}} \equiv \dfrac{\sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\bar{\nu} N)}$ $\quad r \equiv \dfrac{\sigma_{CC}(\bar{\nu} N)}{\sigma_{CC}(\nu N)}$

yields: $g_L^{\,2} + g_L'^{\,2} = \dfrac{R_\nu - r^2 R_{\bar{\nu}}}{1-r^2}$ $\quad g_R^{\,2} + g_R'^{\,2} = \dfrac{r(R_{\bar{\nu}} - R_\nu)}{1-r^2}$

$$R_\nu = (g_L^{\,2} + g_L'^{\,2}) + r(g_R^{\,2} + g_R'^{\,2}) = \frac{1}{2} - \sin^2\theta_W + (1+r)\frac{5}{9}\sin^4\theta_W$$

$$R_{\bar{\nu}} = (g_L^{\,2} + g_L'^{\,2}) + \frac{1}{r}(g_R^{\,2} + g_R'^{\,2}) = \frac{1}{2} - \sin^2\theta_W + \left(1 + \frac{1}{r}\right)\frac{5}{9}\sin^4\theta_W$$

(Llewelyn-Smith relationships)

Neutrino-nucleon deep inelastic scattering (cont)

- More relationships:

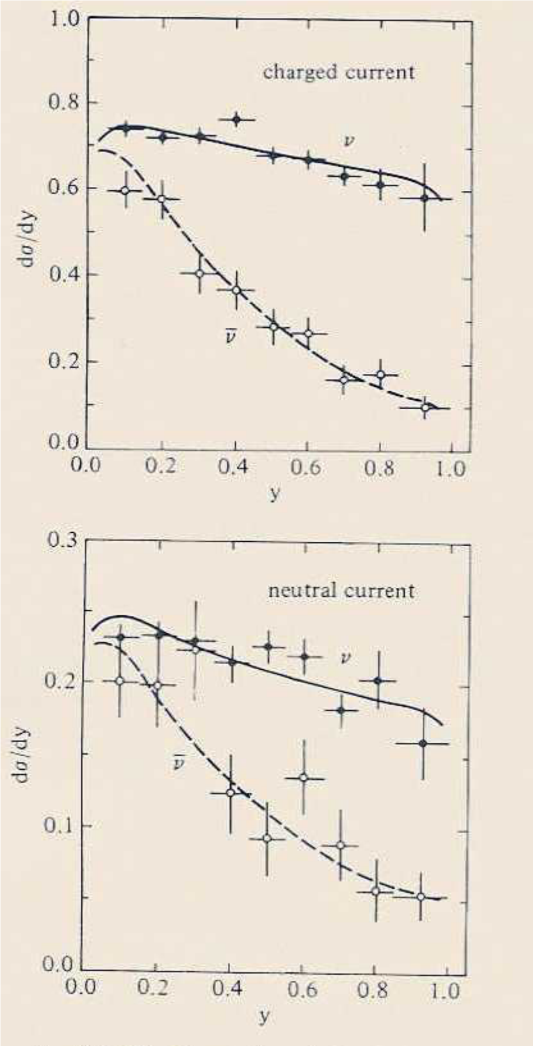
$$\frac{\frac{d\sigma_{NC}(\nu_\mu N)}{dy}+\frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{dy}}{\frac{d\sigma_{CC}(\nu_\mu N)}{dy}+\frac{d\sigma_{CC}(\bar{\nu}_\mu N)}{dy}}=\frac{1}{2}-\sin^2\theta_W+\frac{10}{9}\sin^4\theta_W$$

$$\frac{\frac{d\sigma_{NC}(\nu_\mu N)}{dy}-\frac{d\sigma_{NC}(\bar{\nu}_\mu N)}{dy}}{\frac{d\sigma_{CC}(\nu_\mu N)}{dy}-\frac{d\sigma_{CC}(\bar{\nu}_\mu N)}{dy}}=\frac{1}{2}-\sin^2\theta_W$$

(Paschos-Wolfenstein relationship)

- Llewellyn-Smith relationship used to measure $\sin^2\theta_W$ by performing ratios of charged current to neutral current of neutrino nucleon scattering. CHARM, CDHS and CCFR have consistent results:

$$\sin^2\theta_W=0.233\pm0.003\pm0.005$$



CHARM data

Neutrino-nucleon deep inelastic scattering (cont)

- NuTeV experiment at Fermilab uses Paschos-Wolfenstein relationship and obtains reduced systematic errors but their result is >3σ away from world average:

$$NUTEV:\ \sin^2\theta_W = 0.2277 \pm 0.0013 \pm 0.0009$$

$$World\ average:\ \sin^2\theta_W = 0.2227 \pm 0.00037$$

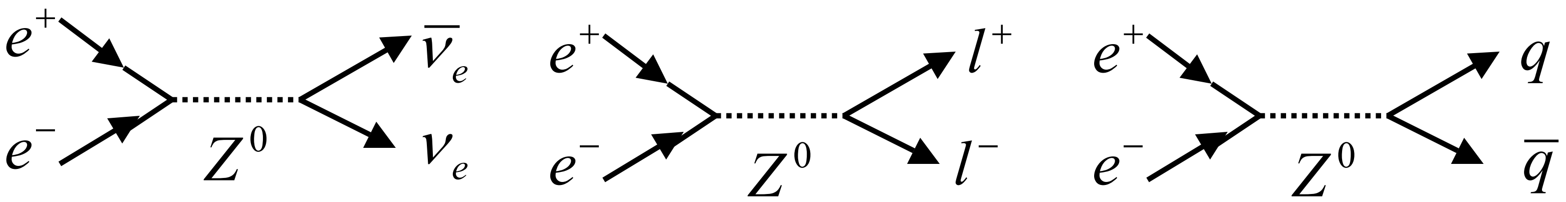
- NuTeV is 600 tonne calorimeter that ran at Fermilab with a sign-selected neutrino beam, so it was able to distinguish neutrino from antineutrino interactions (from decay of π+ and π-). This allows one to use the Paschos-Wolfenstein formula and reduce systematics. Charged current events had a muon (μ^- from neutrinos and μ^+ from antineutrinos) and neutral current events were "short" events.

2.5 Number of neutrinos

- Width of the Z-pole resonance: Breit-Wigner distribution

$$\sigma(e^+e^- \to f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{s\Gamma_e\Gamma_f}{(s - M_Z{}^2)^2 + s^2\Gamma_Z^2 / M_Z}$$

$$\sigma_{peak}(e^+e^- \to f) = \frac{12\pi(\hbar c)^2}{M_Z} \frac{\Gamma_e\Gamma_f}{\Gamma_Z^2} = \frac{12\pi(\hbar c)^2}{M_Z} B(Z^0 \to e^+e^-) B(Z^0 \to f\bar{f})$$



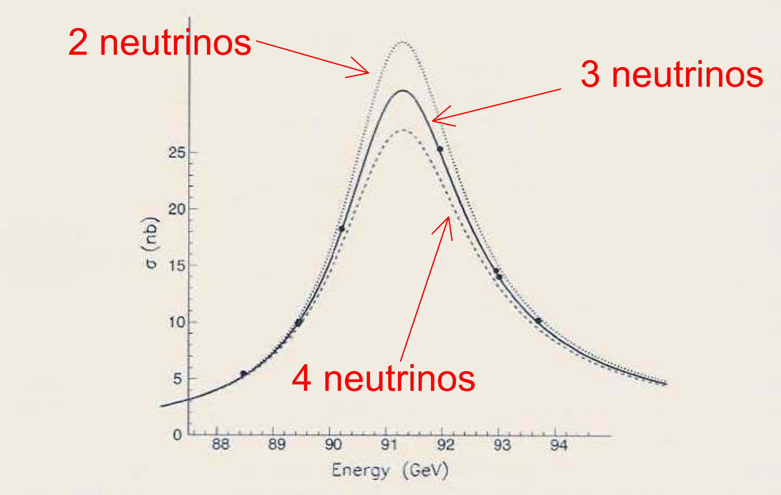
$$\Gamma_Z = \Gamma_{had} + 3\Gamma_{l^+l^-} + N_\nu \Gamma_{\nu\bar{\nu}} = 2490 Mev$$

$$\Gamma_{had} = \Gamma_u + \Gamma_d + \Gamma_c + \Gamma_s + \Gamma_b = 1741 MeV$$

$$\Gamma_{l^+l^-} = 83.9 MeV$$

$$\Gamma_{\nu\bar{\nu}} = 167.1 MeV$$

$$\Rightarrow N_\nu = 2.993 \pm 0.011$$



- Only 3 neutrinos with mass less than the Z mass

3.1 Dirac mass

- In the Standard Model neutrinos are assumed to be massless
- If neutrinos are not massless, then an assumption has to be made about the neutrino mass: Dirac neutrino (like electron mass)

$$L = \bar{\nu}\left(i\gamma \cdot \partial - m\right)\nu \Rightarrow \left(\gamma \cdot p - m\right)\nu = 0 \qquad \nu(x) = \int \frac{d\vec{p}}{(2\pi)^{3/2}} \sum_s \left[u(\vec{p},s)a(\vec{p},s)e^{-ip\cdot x} + v(\vec{p},s)b^{+}(\vec{p},s)e^{ip\cdot x}\right]$$

$a(\vec{p},s) =$ neutrino annihilation $\qquad b(\vec{p},s) =$ antineutrino creation

$$\left(\gamma \cdot p - m\right)u(\vec{p},s) = 0 \qquad \left(\gamma \cdot p + m\right)v(\vec{p},s) = 0$$

$$u(\vec{p},s) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} 1 \\ \vec{\sigma}\cdot\vec{p}/(E+m) \end{pmatrix}\chi_s \qquad v(\vec{p},s) = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} \vec{\sigma}\cdot\vec{p}/(E+m) \\ 1 \end{pmatrix}\bar{\chi}_s$$

$$\chi_s = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad s = +\frac{1}{2}, -\frac{1}{2} \qquad \bar{\chi}_s = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad s = +\frac{1}{2}, -\frac{1}{2}$$

- Chirality operator: for free neutrino, Dirac spinors are eigenstates

$$(\vec{\sigma}\cdot\vec{p})u(\vec{p},s) = 2su(\vec{p},s) \qquad \nu_L \equiv \frac{1-\gamma_5}{2}\nu \qquad \gamma_5\nu_L = -\nu_L$$

$$(-\vec{\sigma}\cdot\vec{p})v(\vec{p},s) = 2su(\vec{p},s) \qquad \nu_R \equiv \frac{1+\gamma_5}{2}\nu \qquad \gamma_5\nu_R = \nu_R$$

$$\nu = \nu_L + \nu_R$$

$$\vec{\sigma}\cdot\hat{p}\left\{\frac{(1\mp\gamma_5)}{2}\nu\right\} = \mp\left\{\frac{(1\mp\gamma_5)}{2}\nu\right\}$$

Dirac mass (cont)

- Charge conjugation:

$$C|\nu(\vec{p},s)\rangle = \eta_C|\bar{\nu}(\vec{p},s)\rangle \qquad \Rightarrow \nu^C \equiv C\nu C^{-1} = -\eta_C^* i\gamma^2\nu^* \quad |\eta_C| = 1$$

- CP operator:

$$CP|\nu(\vec{p},s)\rangle = \eta_{CP}|\bar{\nu}(-\vec{p},s)\rangle \qquad \Rightarrow \nu^{CP} \equiv CP\nu(t,\vec{x})(CP)^{-1} = -\eta_{CP}^* i\gamma^0\gamma^2\nu^*(t,-\vec{x}) \quad |\eta_{CP}| = 1$$

- Dirac Lagrangian: need 4 spinor fields for Dirac neutrino $\nu_L, \nu_R, (\nu_L)^C, (\nu_R)^C$

$$L_D = -m\bar{\nu}M\nu = -m(\bar{\nu}_L M\nu_R + \bar{\nu}_R M\nu_L) \qquad \nu_L = \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_L \qquad \nu_R = \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_R$$

$$MM^+ = Um_D^2U^+ \qquad \Rightarrow m_D = \begin{pmatrix} m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}$$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_L = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_L \qquad \Rightarrow U = \begin{pmatrix} c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & e^{-i\delta}\end{pmatrix}\cdot\begin{pmatrix} c_{13} & 0 & s_{13}\\ 0 & 1 & 0\\ -s_{13} & 0 & c_{13}\end{pmatrix}$$

$$where\ c_{ij} = \cos\theta_{ij},\ and\ s_{ij} = \sin\theta_{ij}$$

Weak eigenstates

Mass eigenstates

Neutrino PMNS (Pontecorvo, Maki, Nakagawa, Sakata) mixing matrix

3.2 Majorana mass

- Only 2 spinor fields observed $\nu_L, (\nu_L)^C$
- Majorana neutrino: the neutrino is its own antiparticle

$$\nu_M = \nu_M{}^C \equiv C\nu_M C^{-1}$$

- CP operator:

$$CP\left|\nu_M(\vec{p},s)\right\rangle = \eta_{CP}^M \left|\bar{\nu}_M(-\vec{p},s)\right\rangle = \eta_{CP}^M \left|\nu_M(-\vec{p},s)\right\rangle \quad \eta_{CP}^M = -(\eta_{CP}^M)^*$$ (imaginary)

- Majorana Lagrangian:

$$L_M = \frac{1}{2}\bar{\nu}_M(i\gamma\cdot\partial - m)\nu_M \qquad L_{mass}^M = -\frac{1}{2}\left[\bar{\nu}_L^C M\nu_L + \bar{\nu}_L M\nu_L^C\right] \quad M = M^T \quad U^T MU = m_D$$

$$\nu_{L,R} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{L,R} \quad N_{L,R} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{L,R} \qquad \nu_L = UN_L \Rightarrow L_{mass}^M = -\frac{1}{2}\left[\bar{N}_L^C m_D N_L + \bar{N}_L m_D N_L^C\right] = -\frac{1}{2}\bar{\nu}_M m_D \nu_M$$

$$if \quad \nu_M \equiv N_L + N_L^C$$

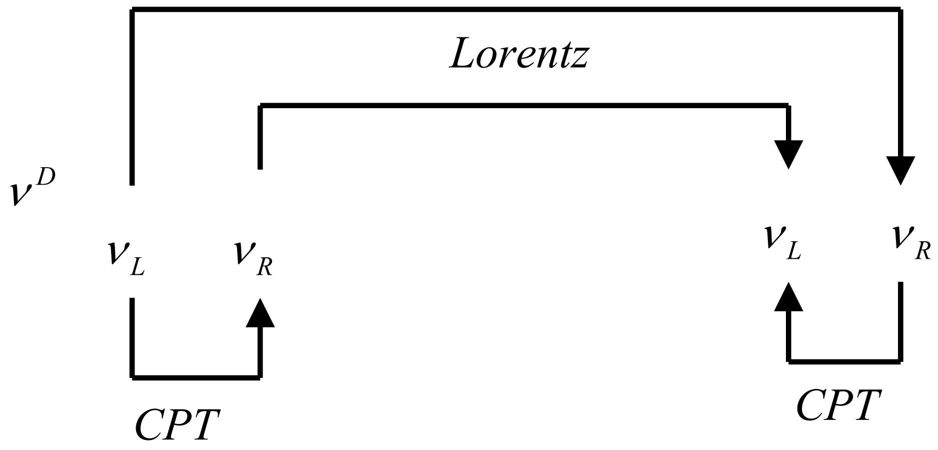
$$\Rightarrow U = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3e^{-i\rho} \\ -s_1c_2e^{i\gamma} & (c_1c_2c_3 + e^{i\delta}s_2s_3)e^{i\gamma} & (c_1c_2s_3 - e^{i\delta}s_2c_3)e^{i(\gamma-\rho)} \\ s_1c_2e^{i(\gamma-\rho)} & (c_1s_2c_3 - e^{i\delta}c_2s_3)e^{i(\gamma-\rho)} & (c_1s_2s_3 + e^{i\delta}c_2c_3)e^{i(\gamma-2\rho)} \end{pmatrix}$$

Majorana mixing matrix (3 phases)

Majorana mass (cont)

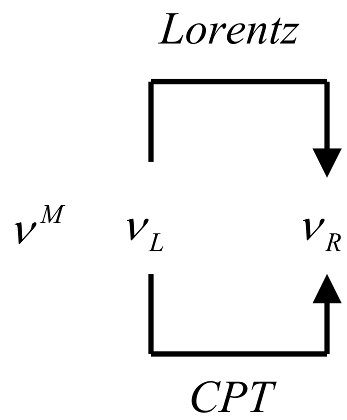
- Dirac model:

  (Lepton flavour conserving)



- Majorana model:

  (Lepton flavour violating: ΔL=+-2)



3.3 See-saw mechanism

- Need right handed neutrinos with mass scale much larger than 250 GeV (for example GUTs). Dirac-Majorana mass term for one generation:

$$L_{mass}^{D-M} = -m_D \overline{\nu}_L \nu_R - \frac{1}{2}\left[m_L \overline{\nu}_L^C \nu_L + m_R \overline{\nu}_R^C \nu_R\right] + h.c.$$

$$if \quad \nu \equiv \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} \quad \Rightarrow L_{mass}^{D-M} = -\frac{1}{2}\left[\overline{\nu}^C M \nu + \overline{\nu} M \nu^C\right] \quad with\ M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

- After diagonalisation: $m_1 \cong \frac{m_D{}^2}{m_R}, m_2 \cong m_R \quad if\ m_R >> m_D, m_L$

- For three generations: $m_\nu = m_D \frac{1}{M} m_D^T \qquad with\ m_\nu, m_D, M = 3 \times 3\ matrices$

$$L_{mass}^{D-M} = -\frac{1}{2}\left[\overline{\nu}^C M(6\times 6)\nu + h.c.\right] \quad with\ M(6\times 6) = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \quad if\ \left|M_{ij}\right| >> \left|(m_D)_{ij}\right|$$

Therefore: $m(\nu_i) \cong \frac{m_{f,i}{}^2}{m_R} \quad with\ m_{f,i} = (m_u, m_c, m_t)\ or\ (m_e, m_\mu, m_\tau)$

(depending on the model)