Inflation theory

EWS - exam paper

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ABSTRACT: By looking at the isotropy of the cosmic microwave background and by calculating the flatness of our universe, some problems arise. These problems can be solved by introducing a period of exponential increase in the size of the universe called inflation. The underlying physics are given by the inflaton field. This paper first poses the problems, then describes the inflation theory in order to solve the given problems and at last a single experiment is discussed.

Friedmann equations

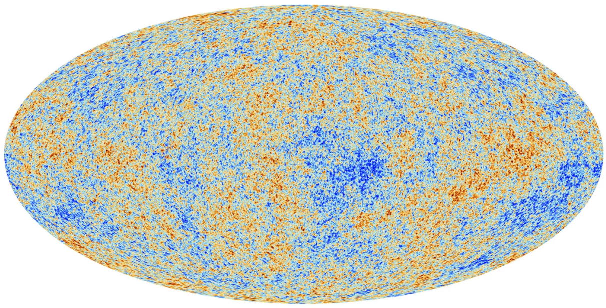
First, I give the Friedmann equations in this section because I will use them for some simple calculations in this paper [18]:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3} \tag{2}$$

1 Horizon problem

A very useful tool to look back in time in the universe is the Cosmic Microwave Background (CMB). This consists of photons at different energies, coming from the decoupling of matter and radiation when the universe was around 300 000 years old.



Figuur 1: Map of the CMB with fluctuations of the order of 10^{-5}K [1]

In this figure, the CMB does not look isotropic. This is due to the scale of the fluctuations that is being used. The CMB is actually very isotropic with a temperature of 2.73K, but there are some temperature fluctuations of the order of 10^{-5}K which can be observed in this figure. This is thus very small so we can conclude that the CMB has an overall temperature of 2.73K with very small temperature fluctuations.
This isotropic property of the CMB imposes a problem called: The horizon problem. We first need to introduce the concept of a horizon.
A horizon is the distance over which one can observe a particle by exchange of photons. In a static universe this can be easily calculated:

$$D_{static} = c \cdot t \tag{3}$$

With t the time the photon has traveled. Our universe is not static but expanding. We define the comoving distance r(z) [3]:

$$r(z) = c\int_0^z \frac{dz'}{H(z')} \tag{4}$$

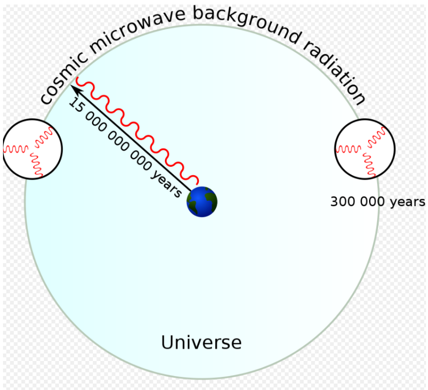
H(z) = $H_0[\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^3]^{\frac{1}{2}}$ [6]
We are living in a matter dominated universe, so in order to solve this problem analytically, we assume

the matter term to be dominant: $H(z) \approx H_0\sqrt{\Omega_m}(1+z)^{\frac{3}{2}}$.

$$\begin{aligned} r(z) &= \frac{c}{H_0\sqrt{\Omega_m}}\int_0^z \frac{dz'}{(1+z')^{3/2}} && (5) \\ &= \frac{2c}{H_0\sqrt{\Omega_m}}\left[-(1+z')^{-1/2}\right]_0^z && (6) \\ &= \frac{2c}{H_0\sqrt{\Omega_m}}\left(1-\frac{1}{\sqrt{1+z}}\right) && (7) \\ & && (8) \end{aligned}$$

With this formula we can calculate the distance from us to the CMB by assuming z very large (z=1100 for the CMB).
This means that $r(z) \approx \frac{2c}{H_0\sqrt{\Omega_m}}$
The particle horizon of the CMB can be calculated in a same way:

$$\begin{aligned} d_{horizon}(z) &= c\int_z^\infty \frac{dz'}{H(z')} && (9) \\ &= \frac{2c}{H_0\sqrt{\Omega_m}}\frac{1}{\sqrt{1+z}} && (10) \end{aligned}$$



Figuur 2: Visual representation of the trigonometry problem

The size of the CMB horizon from our point of view now comes down to a simple trigonometry problem. In a small angle approximation we get:

$$\theta_{horizon} = \frac{d_{horizon}(z)}{r(z)} = \frac{1}{\sqrt{1+z}} = \frac{1}{\sqrt{1+1100}} \approx 1.7° \tag{11}$$

This means that the patches in the sky that are causally connected could only be around the size of 1.7°. But the whole sky is in thermal equilibrium instead of different 1.7° patches. How is it possible that the photons have travelled for a far more distance than they could have? This is called **The Horizon Problem**.

2 Flatness problem

In the derivation in the previous section, we assumed the universe to be flat and expanding. This resulted in a horizon of 1.7° for the CMB, which we actually measure today by looking at the well known CMB fluctuations plot in terms of its multipole moments. The problem here is that the universe kept expanding after the matter-radiation decoupling. The expanding of the universe induces a curvature in space-time. This means that in order to have the correct $\approx 1°$ measurement we have today, the universe had to keep flat from the time of decoupling until the present day. This means that the parameters which describe the flatness of the universe had to be incredibly fine tuned. This mystery of where this very fine tuned flatness comes from is called: **The flatness problem**.

3 Magnetic monopole problem

Some Grand Unification Theories describe the production of magnetic monopoles in a vast majority as topological defects of these theories [2],[4]. But if the density of those monopoles is really high as predicted, why don't we observe them today? This is called: **The magnetic monopole problem.**

4 Inflation

In 1981, Alan Guth came up with the theory of inflation [12]. A very rapid, exponential expansion in the early universe (around 60 e-folds [17]). This theory is able to explain the problems encountered before. A straightforward way to ensure an exponential growth is to assume that during the inflation period, the energy density is constant and very large compared to the other terms in the Friedmann equations. We can rewrite them as:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} = \zeta^2 \tag{12}$$

With ζ a constant.

$$\int_{t_1}^{t_2} \left(\frac{\dot{a}}{a}\right) dt = \int_{t_1}^{t_2} \zeta dt \tag{13}$$

$$\tag{14}$$

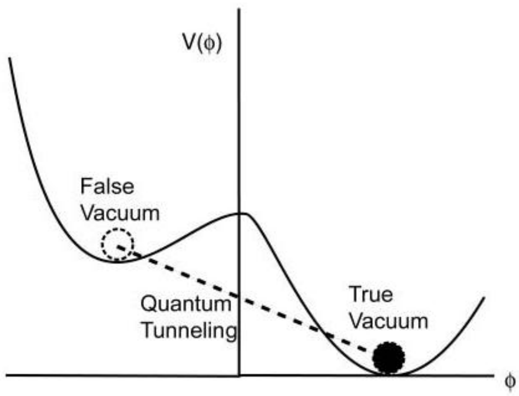
$$ln(a_2) - ln(a_1) = \zeta(t_2 - t_1) \tag{15}$$

$$\tag{16}$$

$$\frac{a_2}{a_1} = e^{\zeta(t_2 - t_1)} \tag{17}$$

5 Inflaton field

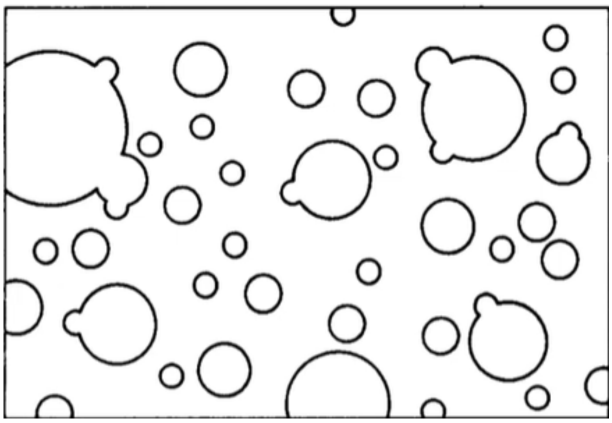
By introducing a new scalar field, the inflaton field, one can explain the constant non-zero energy density that was used in the Friedmann equations. The form of this field can be described using the following potential energy curve:



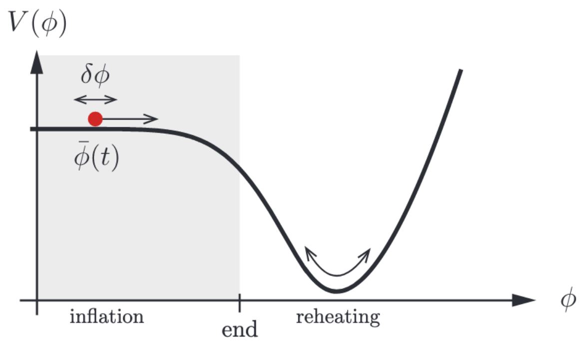
Figuur 3: Alan Guth's original model for the inflaton field [16]

This was Alan Guth's first proposal for the potential energy curve for the inflaton field. We see that the field is stuck for a long time in a state with non-zero potential energy. This state is called, the false vacuum.

The long period of non-zero potential energy is needed to explain the exponential growth as calculated in the previous section. The problem with this concept is that the only way for the field to go to the true vacuum state is trough quantum mechanical tunneling. The problem here is that tunneling is a process with a small probability. Not every part of the inflaton field will tunnel at the exact same time. Some regions will have gone to the true vacuum state, where radiation and matter can exist, and other regions will still be stuck in an inflationary phase. This makes the universe inhomogeneous and we don't observe this today. This is called: the graceful exit problem.



Figuur 4: Artistic impression of the universe bubbles that are formed by a non graceful exit



Figuur 5: Form of the potential energy curve in the new slow roll inflation theory [15]

A solution to this problem is given by using a different potential energy curve. The field seems to be stuck for a long time in a state we still call the false vacuum state. This state has an energy density that only slowly varies (how slow depends on the steepness of the potential curve as seen in the figure above).

If we now use the second Friedmann equation and use the fact that $\dot{\rho} = 0$ we get:

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + \frac{P}{c^2}) = 0 \tag{18}$$
$$P = -\rho c^2 = -u \tag{19}$$

Where u is the energy density. By now using another form of the Friedmann equations:

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3\frac{P}{c^2}) \tag{20}$$
$$\ddot{a} = -\frac{4\pi Ga}{3}(\rho - 3\frac{\rho c^2}{c^2}) \tag{21}$$
$$\ddot{a} = \frac{4\pi Ga}{3}(2\rho) > 0 \tag{22}$$

We have now proven that the universe is positively expanding during this false vacuum state.

6 Dynamics of the inflaton field

The energy density of this inflaton field can be described by [13]:

$$\rho_\phi c^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{23}$$

In our inflation model, we assumed a slow rolling potential $V(\phi)$.
This also means that during the inflation period, we can neglect the kinetic energy term in comparison to the potential.

$$\rho_\phi c^2 \approx V(\phi) = \text{constant} \tag{24}$$

Here we can see again that ρ is a constant value. Thus as calculated in section 4, this leads to an exponential expansion of the universe.

Of course,inflation has to stop at some point. This exponential growth stops when the potential V(ϕ) is not a constant anymore and the kinetic energy term can't be neglected. This phase is called: the reheating phase.
We want the inflaton field to reach its true vacuum state. By looking at the potential energy curve, we can state that the reheating phase can be described by a damped harmonic oscillator:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{dV(\phi)}{dt} = 0 \tag{25}$$

Where H(t) is the hubble constant and works as a friction term.
The potential energy gets converted in kinetic energy by the production of relativistic particles (matter, dark matter, radiation, etc). These are the particles we know exist in the universe today

7 Solutions to problems

7.1 Horizon problem

If the universe has indeed undergone an exponential expansion, we can easily see that even a very tiny universe can grow into something macroscopic. The most general explanation to this problem is stating that the universe before inflation was so small, that it was in complete thermal equilibrium/causal contact. By inflating the universe, most regions get causally disconnected from each other, they freeze in. But the temperature is more or less the same everywhere due to the causal contact before inflation.

7.2 Flatness problem

To solve this problem, we will first rewrite the Friedmann equation:

$$H^2 + \frac{kc^2}{a(t)^2} = \frac{8\pi G}{3}\rho \tag{26}$$

Define $\Omega = \frac{\rho}{\rho_c}$ with $\rho_c = \frac{3H^2}{8\pi G}$

$$\Omega = \frac{8\pi G\rho}{3H^2} \tag{27}$$

The Friedmann equation now becomes:

$$1 + \frac{kc^2}{H^2 a(t)^2} = \Omega \tag{28}$$

$$|\Omega - 1| = \frac{|k|c^2}{a(t)^2 H^2} \tag{29}$$

Imagine that before inflation, the universe is very curved so that $|\Omega - 1|$ is equal to 1.
If we now compare the curvature before and after inflation we get:

$$\frac{\frac{kc^2}{a(t2)^2 H^2}}{\frac{kc^2}{a(t1)^2 H^2}} = \left(\frac{1/a(t2)}{1/a(t1)}\right)^2 \tag{30}$$

We proved before that a(t) = $e^{\zeta t}$ and stated that a(t) $\approx e^{60}$ during inflation, so we get:

$$\left(\frac{1/a(t2)}{1/a(t1)}\right)^2 = e^{-2\zeta(t_2-t_1)} = e^{-2\cdot 60} \approx 10^{-52} \tag{31}$$

This means that after inflation we have:

$$\Omega = 1 \pm 10^{-52} \tag{32}$$

Inflation thus flattens out the universe and causes the incredibly finetuning needed for the universe to stay flat for a very long time.

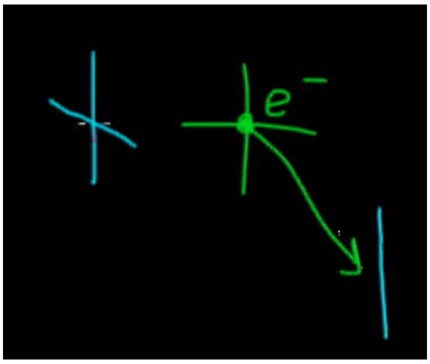
7.3 Magnetic monopoles problem

Because I have only briefly discussed this problem and the existence of magnetic monopoles stays hypothetical, I will not go in depth here. The monopoles could have formed before the inflation period happened when the universe was still hot enough for the unification of the strong,weak and electromagnetic forces. Some believe that the high density of monopoles has been diluted in great extent due to the exponential growth of the universe. So the monopoles would still be present, but the density is too low to observe them (yet?).

8 Search for new evidence

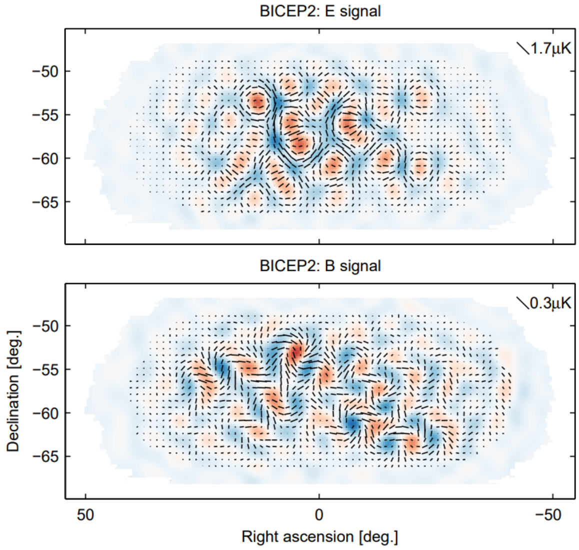
In 2014, BICEP2 made the announcement that they have found evidence for primordial gravitational waves by looking at the polarisation of the CMB [7].

The CMB light got polarised due to the scattering on free electrons before the time of matter-radiation decoupling. If unpolarised light (this means light that is for example equally polarised in the vertical direction and horizontal direction as is represented by the left blue cross on the figure) reaches a free electron, the electron starts moving back and forth (no light emitted in the direction of the green arrow. But the electron also moves up and down and thus emits vertical polarised light represented by the single blue line.
If we apply the same reasoning for light that comes from above the electron, we have only horizontal polarisation in the direction of the green arrow. If the polarisation strength is equal from both CMB photons, the light stays unpolarised.
As we know, there are tiny temperature fluctuations in the CMB. This means that the polarisation strength from the light that comes from the left could be stronger if the plasma is hotter than the plasma that emits the CMB photons from above. This would result in a net vertical polarisation.



Figuur 6: Sketch of CMB light polarisation on a free electron [10]

BICEP2 measured these polarisations and came up with following map of the CMB polarisation:



Figuur 7: CMB polarisation modes measured by BICEP2 [7]

They measured 2 types of polarisations: E-mode and B-mode. The E-mode pattern seems to flow around the hotter patches of space, while the B-mode is more twisted/curled.
The E-mode pattern is believed to originate from the previously discussed temperature anisotropy in the CMB.
The B-mode pattern could not be produced by this temperature anisotropy. A good alternative could be the stretching of space-time due to primordial gravitational waves produced by the inflation period that happened earlier.

Not only are the B-modes an indication that inflation happened, but the strength of these modes can tell us something about the real model for inflation. In this paper, I discussed only two models (the original model proposed by Alan Guth and the slow roll approximation), but there are more. Each model describes the production of primordial gravitational waves in their own way and with different strengths. Measuring the B-modes can thus help us identify what inflationary model is the correct one.

Unfortunately, this was disproven by new measurements of the galactic dust [8]. Scattering of the CMB light on 0.1μm dust particles proved to be the main source of the B-modes measured by BICEP2.
Does this mean that inflation did not happen? No, maybe the inflation models are wrong and are there no primordial gravitational waves. Or we didn't measure them yet.

9 Conclusion

When studying early universe cosmology, some problems arise: the horizon problem, flatness problem and magnetic monopole problem. These three problems can all be solved by stating that a period of exponential growth ($\approx e^{60}$) did occur in the early universe. The inflation theory has been introduced in order to solve these problems. One way to directly detect the inflation is by searching for the primordial gravitational waves it could have produced. This was done by BICEP2 in 2014, but quickly disproven. The search still goes on.

10 Sources

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