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The Operation Research II Courses

Title
Modeling Integer Programming in the Real World: A Case Study of Knapsack Problems for a Day of Climbing

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Abstract

The purpose of this article is "To determine the optimal number of units of food items and high-energy snacks to bring on a 1-day mountain climbing trip, in order to maximize the total food value while satisfying weight and other constraints."

Constraints:

- The weight of the knapsack and its contents must be less than or equal to a specific limit (e.g. 25-30% of body weight).
- The knapsack and its contents must be within a specific budget.
- The knapsack contents must meet specific dietary and nutritional needs (e.g. sufficient calories per day).
- The decision variables representing the number of units of each food item and high-energy snack must be binary (0 or 1).

This model could be helpful in optimizing food choices for the specific activity, taking into account the nutritional value, weight, and other necessary factors that are important to consider when planning an outdoor activity. Also, it would be helpful to have an efficient use of the budget.

Keywords: Modelling, Integer Programming, Knapsack Problem

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Chapter 1

Introduction

1.1 BACKGROUND

The knapsack problem is a classic combinatorial optimization problem that has been widely studied in the fields of operations research and computer science. The problem can be formulated in several ways, but a common version is the so-called 0-1 knapsack problem, which involves deciding which items to include in a knapsack, with the goal of maximizing the total value of the items while staying within a certain weight limit.

In the context of a 1-day mountain climbing, the knapsack problem can be formulated as a 0-1 knapsack problem, where the items are the food items and high-energy snacks that you plan to bring, and the knapsack capacity is the weight limit that your knapsack can carry. The objective is to determine the optimal number of units of each food item and high-energy snack to bring, so that the total value of the food is maximized while staying within the weight limit.

The knapsack problem is NP-hard, meaning that no algorithm is known that can find an exact solution in polynomial time for all instances of the problem. However, there are many approximate and exact algorithms that have been proposed to solve the problem, such as dynamic programming, branch and bound, and linear programming. Additionally, there are many variations of the problem that have been studied such as the multi-dimensional knapsack problem, continuous knapsack problem and bounded knapsack problem.

In the recent years, the knapsack problem has been applied in different fields like logistics, production planning, and transportation. And it's also been used to optimize the food selection in outdoor activities or for people who have specific dietary needs such as knapsacker or athletes.

In a nutshell, the knapsack problem is a classic and important problem in the field of optimization, and has been widely studied in various applications. Its a way of planning the food choices for a specific activity and make an efficient use of the budget for the activity.

It is worth mentioning that the

Chapter 2

Define the Problem

2.1 Essential items

When planning a 1-day mountain climbing trip, it's important to pack carefully and only bring the essentials. The assumption of the problem is a normal middle-aged person who is capable of carrying a 20-pound knapsack. Here is a list of items, that should be carried in a knapsack:

1. Water: 2-3 liters (depending on the size of the water bottles) can weight around 2-3 pounds when full.
2. Food: High-energy snacks such as nuts, seeds, dried fruit, energy bars and sandwiches. The weight will vary depending on the amount of food you bring, but it's usually around 1-2 pounds.
3. Extra clothing: Insulated jacket or fleece (1-1.5 pounds); hat, gloves and an extra pair of socks (0.5-1 pound); and a rain jacket (1 pound)
4. Headlamp or flashlight: 0.25-0.5 pounds
5. First aid kit: 0.5-1 pound
6. Maps and compass: 0.25-0.5 pounds
7. Sun protection: Sunglasses, sunscreen, and lip balm (0.5 pound)
8. Personal items such as insect repellent, a personal hygiene kit, and a camera (0.5-1 pound)
9. Climbing gear (harness, helmet, carabiners, belay device, etc.) : 3-5 pounds (if necessary)
10. A small towel and a change of clothes for after the climb: 1-2 pounds
11. Trash bags and personal hygiene products: 1-2 pounds

2.1 Food items

Given a set of food items and high-energy snacks, with each item having a weight and a food value (in calories), and a weight limit for the knapsack, the objective is to select a subset of items that maximizes the total food value while staying within the weight limit. The decision variables are binary (0 or 1) indicating whether or not each food item or high-energy snack should be included in the knapsack, and the problem is subject to additional constraints such as budget, volume, and nutritional needs.

In summary, the problem that you want to model is the optimization of food selection for a 1-day mountain climbing knapsack, with the goal of maximizing the total food value while staying within the weight, budget and other necessary constraints, in order to plan an efficient and healthy outdoor activity.

For a one-day mountain climbing trip, you may not need to bring as much food as you would for a longer trip, but it's still important to bring snacks that will give you the energy you need to complete your climb. Here are some high-protein foods that are lightweight and easy to pack:

1. Peanut butter or almond butter: These spreads are high in protein and healthy fats. Two tablespoons typically contain around 8-10 grams of protein, and usually weigh around 2-3 ounces per serving. (Weight: 0.12-0.18 pounds; Caloric Value: around 190 calories per 2 tablespoons, 180,000 IRR)
2. Jerky: Beef, turkey, or venison jerky is a great source of protein. Depending on the brand and type, it can have around 10-15 grams of protein per ounce, and usually weigh around 1-2 ounces per serving. (Weight: 0.06-0.12 pounds; Caloric Value: 80-150 calories per ounce, 140,000 IRR)
3. Dried legumes like lentils, peas, and beans: They are lightweight and easy to pack, and they provide around 7-10 grams of protein per quarter-cup serving. (Weight:0.06-0.12 pounds; Caloric Value: around 150 calories per quarter cup serving, 50,000 IRR)
4. Protein bars: A lot of protein bars available in the market with high protein content, around 10-20 grams of protein per bar, and usually weigh around 1-2 ounces per bar. (Weight: 0.06-0.12 pounds; Caloric Value: around 200-400 calories per bar, 250,000 IRR)
5. Crackers or whole-grain biscuits: a serving of around 15 crackers can provide around 150 calories. (Weight: 0.09 pounds, Caloric Value: 150 calories per 15 crackers, 300,000 IRR)

6. Yogurt: yogurt is a good source of protein, typically with around 12-15 grams of protein per 6-ounce container. (Weight: 0.37-0.45 pounds; Caloric Value: around 100 calories per ounce, 300,000 IRR)
7. Canned tuna or salmon: Both of these fish are high in protein, with around 20-25 grams of protein per can (5-6 oz). They also pack healthy omega-3 fatty acids. (Weight: 0.3-0.4 pounds; Caloric Value: around 100 calories per ounce, 400,000 IRR)
8. Cheese: Hard cheeses like cheddar, parmesan, and gouda have around 7-10 grams of protein per ounce. And usually weigh around 1-2 ounces per serving. (Weight: 0.06-0.12 pounds; Caloric Value: 110-140 calories per ounce, 500,000 IRR)
9. Nuts: almonds, walnuts, cashews, etc are great sources of protein and healthy fats, a quarter-cup serving can provide around 150-200 calories. (Weight: 0.12-0.18 pounds, Caloric Value: 150-200 calories per quarter cup, 400,000 IRR)
10. Dark chocolate: High in antioxidants and a good source of healthy fats and carbohydrates, a 1 oz serving can provide around 170 calories (Weight: 0.06 pounds, Caloric Value: 170 calories per 1 oz, 600,000 IRR)
11. Dried fruits: raisins, apricots, cranberries, etc are great sources of natural sugar and fiber, a quarter cup serving can provide around 120-150 calories. (Weight: 0.06-0.09 pounds, Caloric Value: 120-150 calories per quarter cup, 500,00 IRR)

It's important to note that the weight and caloric value of each food item will depend on the brand and packaging, also, always bear in mind that individual caloric and nutritional needs can vary and it's always important to consult a nutritionist before planning any long-term activities, specially if you have any dietary.

Chapter 3
Modelling

3.1 The Objective Function

To determine the optimal number of units of food items and high-energy snacks to bring on a 1-day mountain climbing trip, in order to maximize the total food value while satisfying weight and other constraints.

$$Max\ Z = 570X_1 + 300X_2 + 150X_3 + 400X_4 + 150X_5 + 719X_6 + 639X_7 + 280X_8 + 200X_9 + 150X_{10} + 170X_{11}$$

3.2 The Constraints

- The weight of the knapsack and its contents must be less than or equal to a specific limit (By designing a questionnaire from people active in this field, we considered that the average weight of the knapsack and its contents should be less than or equal to 20 pounds for a middle-aged person, and the weight that we considered for food is 1 pound).

$$18X_1 + 12X_2 + 12X_3 + 12X_4 + 9X_5 + 45X_6 + 40X_7 + 12X_8 + 18X_9 + 9X_{10} + 6X_{11} \leq 100$$

Numbers are multiples of 0.01.

- The knapsack and its contents must be within a specific budget. (By designing a questionnaire from people active in this field, the average budget allocated to food for one day of climbing has been found to be 1,000,000 IRR.)

$$18X_1 + 14X_2 + 5X_3 + 25X_4 + 30X_5 + 30X_6 + 40X_7 + 50X_8 + 40X_9 + 50X_{10} + 60X_{11} \leq 100$$

Numbers are multiples of 10,000.

- The knapsack contents must meet specific dietary and nutritional needs (e.g. sufficient calories per day for a middle-aged person is 2800-3000 calories, we consider 2800 in this modeling).

$$570X_1 + 300X_2 + 150X_3 + 400X_4 + 150X_5 + 719X_6 + 639X_7 + 280X_8 + 200X_9 + 150X_{10} + 170X_{11} \leq 2800$$

- The decision variables representing the number of units of each food item and high-energy snack must be binary (0 or 1).

$$X_j = \begin{cases} 1 & if\ the\ type\ j\ food\ is\ with\ the\ climber \\ 0 & Otherwise \end{cases}$$

Chapter 4

Results

4.1 Solved by LINDO LINGO

4.1.1 Binary

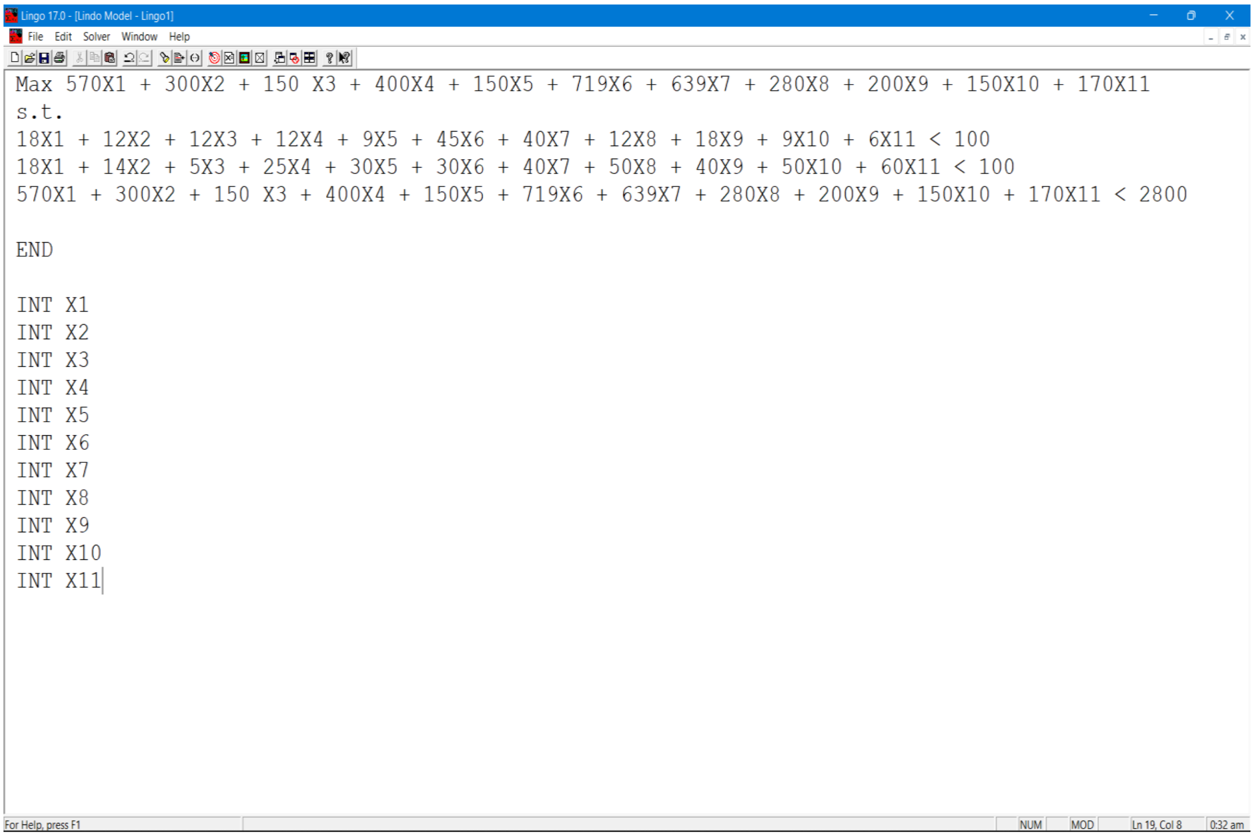
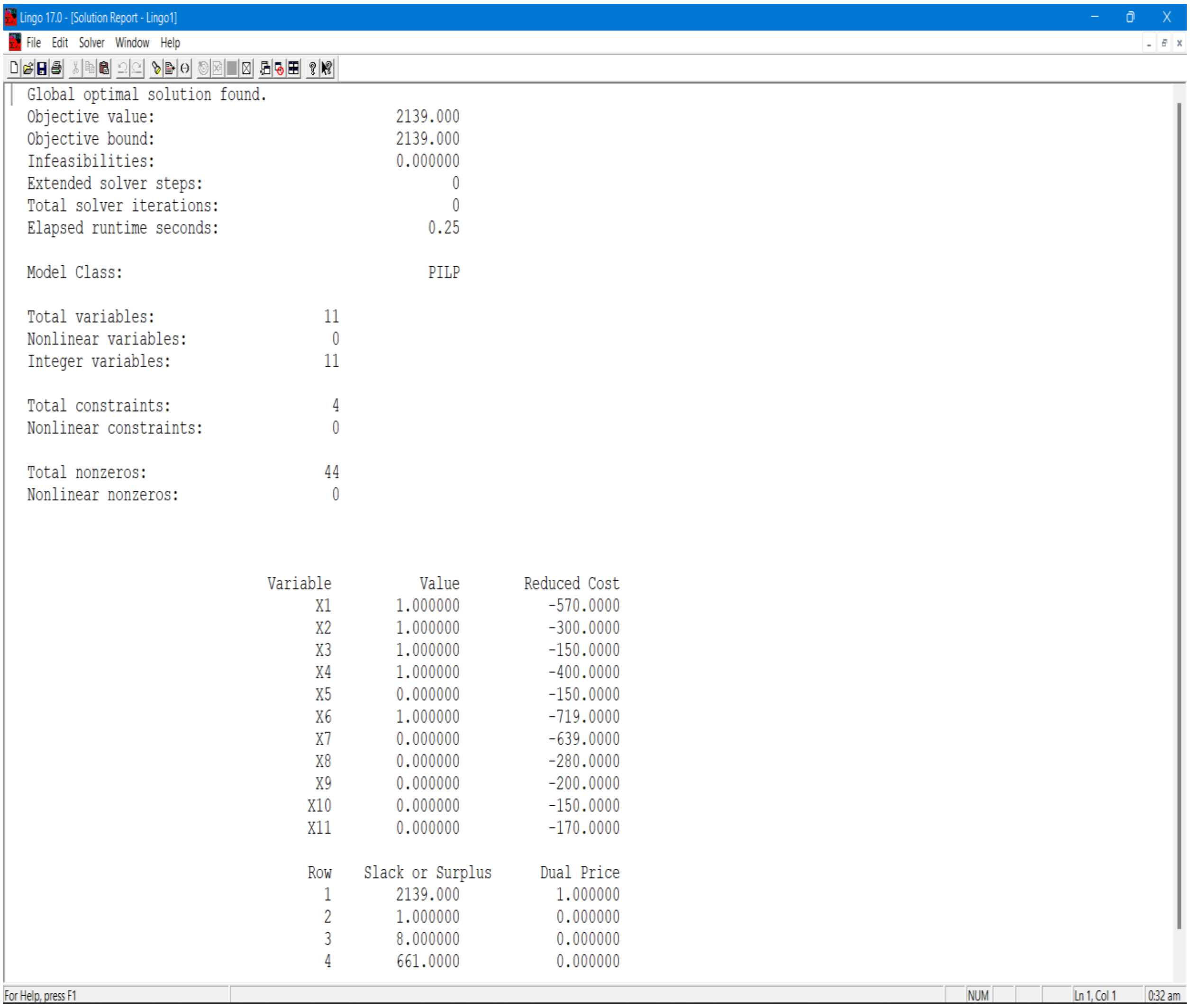


Fig. 1. Input variables of the Problem (binary)

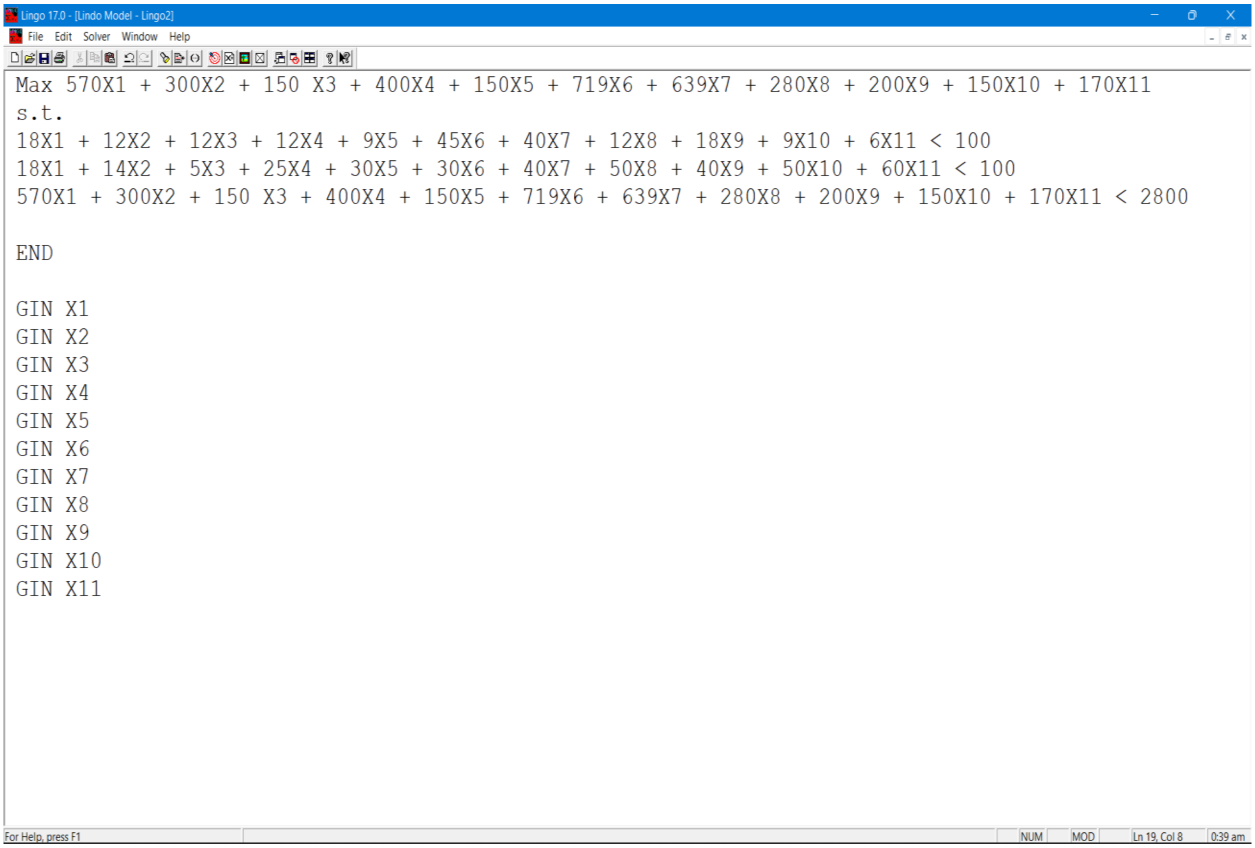


```
Global optimal solution found.
Objective value:                              2139.000
Objective bound:                              2139.000
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0
Elapsed runtime seconds:                          0.25

Model Class:                                      PILP

Total variables:                     11
Nonlinear variables:                  0
Integer variables:                   11

Total constraints:                    4
Nonlinear constraints:                0

Total nonzeros:                      44
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                    X1        1.000000           -570.0000
                                    X2        1.000000           -300.0000
                                    X3        1.000000           -150.0000
                                    X4        1.000000           -400.0000
                                    X5        0.000000           -150.0000
                                    X6        1.000000           -719.0000
                                    X7        0.000000           -639.0000
                                    X8        0.000000           -280.0000
                                    X9        0.000000           -200.0000
                                   X10        0.000000           -150.0000
                                   X11        0.000000           -170.0000

                                   Row    Slack or Surplus      Dual Price
                                     1        2139.000            1.000000
                                     2        1.000000            0.000000
                                     3        8.000000            0.000000
                                     4        661.0000            0.000000
```

Fig. 2. Problem-solving and optimal answer

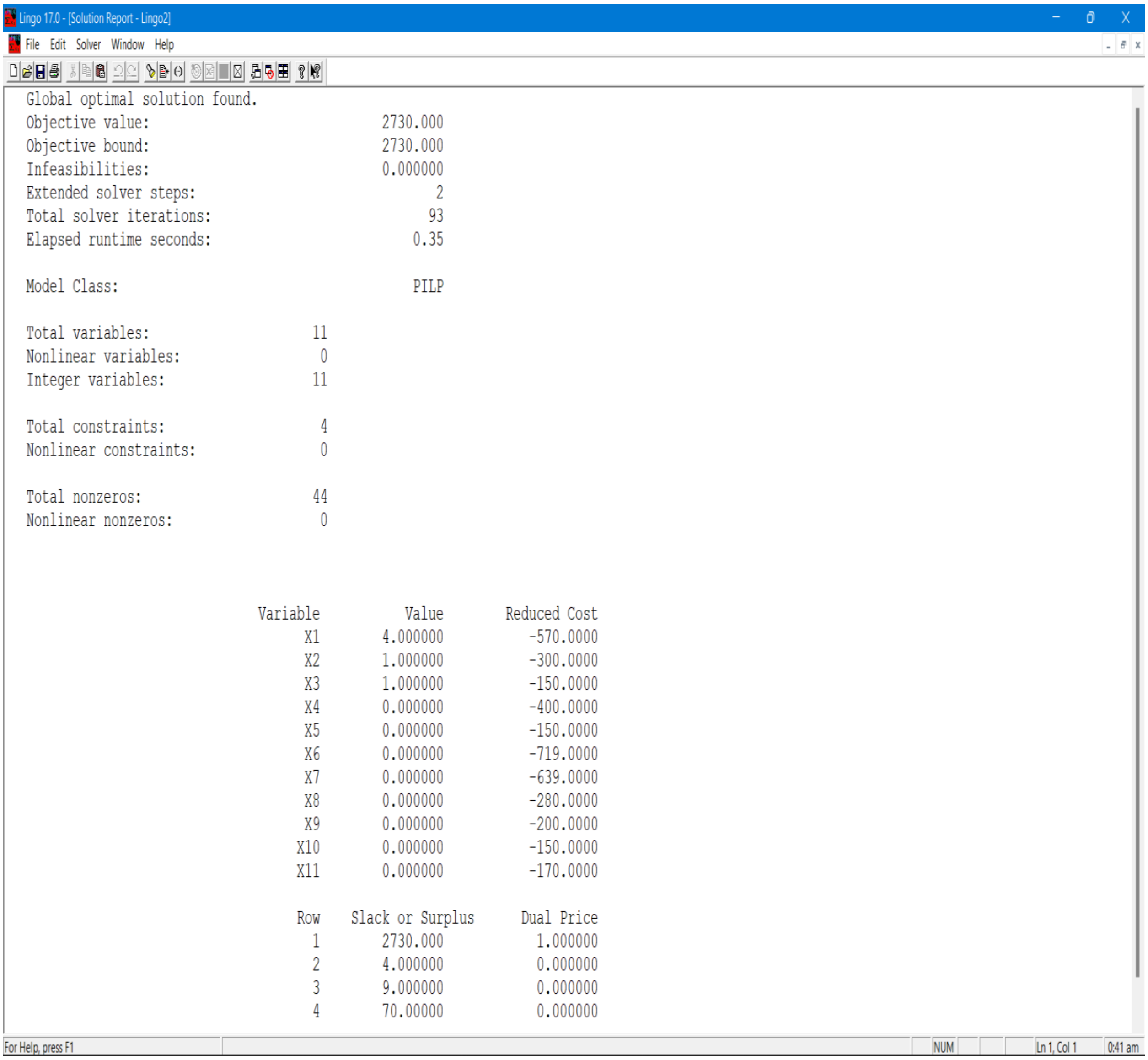
Results: $Z^* = 2139, \quad X_1^* = X_2^* = X_3^* = X_4^* = X_6^* = 1, \quad X_5^* = X_7^* = X_8^* = X_9^* = X_{10}^* = X_{11}^* = 0$

4.1.2 Integer



```
Max 570X1 + 300X2 + 150 X3 + 400X4 + 150X5 + 719X6 + 639X7 + 280X8 + 200X9 + 150X10 + 170X11
s.t.
18X1 + 12X2 + 12X3 + 12X4 + 9X5 + 45X6 + 40X7 + 12X8 + 18X9 + 9X10 + 6X11 < 100
18X1 + 14X2 + 5X3 + 25X4 + 30X5 + 30X6 + 40X7 + 50X8 + 40X9 + 50X10 + 60X11 < 100
570X1 + 300X2 + 150 X3 + 400X4 + 150X5 + 719X6 + 639X7 + 280X8 + 200X9 + 150X10 + 170X11 < 2800

END

GIN X1
GIN X2
GIN X3
GIN X4
GIN X5
GIN X6
GIN X7
GIN X8
GIN X9
GIN X10
GIN X11
```

Fig. 3. Input variables of the Problem (Integer)



Global optimal solution found.

Objective value:	2730.000
Objective bound:	2730.000
Infeasibilities:	0.000000
Extended solver steps:	2
Total solver iterations:	93
Elapsed runtime seconds:	0.35
Model Class:	PILP
Total variables:	11
Nonlinear variables:	0
Integer variables:	11
Total constraints:	4
Nonlinear constraints:	0
Total nonzeros:	44
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X1	4.000000	-570.0000
X2	1.000000	-300.0000
X3	1.000000	-150.0000
X4	0.000000	-400.0000
X5	0.000000	-150.0000
X6	0.000000	-719.0000
X7	0.000000	-639.0000
X8	0.000000	-280.0000
X9	0.000000	-200.0000
X10	0.000000	-150.0000
X11	0.000000	-170.0000

Row	Slack or Surplus	Dual Price
1	2730.000	1.000000
2	4.000000	0.000000
3	9.000000	0.000000
4	70.00000	0.000000

Fig. 4. Problem-solving and optimal answer (Integer)

Results:

$$Z^* = 2730, \quad X_2^* = X_3^* = 1, \quad X_4^* = X_5^* = X_6^* = X_7^* = X_8^* = X_9^* = X_{10}^* = X_{11}^* = 0, \quad X_1^* = 4$$

4.2 Solved by Dynamic Programming

$$\boldsymbol{Max\ Z} = 570X_1 + 300X_2 + 150X_3 + 400X_4 + 150X_5 + 719X_6 + 639X_7 + 280X_8 + 200X_9 + 150X_{10} + 170X_{11}$$

s.t.

$$18X_1 + 12X_2 + 12X_3 + 12X_4 + 9X_5 + 45X_6 + 40X_7 + 12X_8 + 18X_9 + 9X_{10} + 6X_{11} \leq 100$$

$$18X_1 + 14X_2 + 5X_3 + 25X_4 + 30X_5 + 30X_6 + 40X_7 + 50X_8 + 40X_9 + 50X_{10} + 60X_{11} \leq 100$$

$$570X_1 + 300X_2 + 150X_3 + 400X_4 + 150X_5 + 719X_6 + 639X_7 + 280X_8 + 200X_9 + 150X_{10} + 170X_{11} \leq 2800$$

$$X_j \geq 0 \ and\ integer$$

Forward Method

$T = 11$

t: *Step number*

t, t+1, …, T

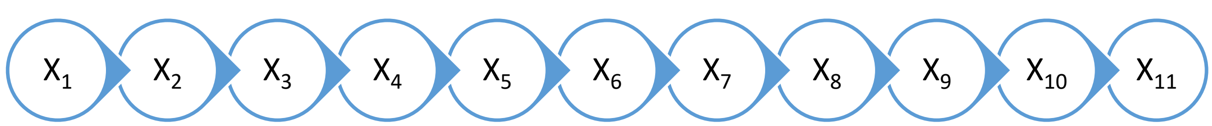


Fig. 6. Forward method

t = 1

$$0 \le X_1 \le 4, \qquad W_1 = 18 \qquad V_1 = 18, \qquad L_1 = 570, \qquad R_1 = 570$$

X_1	$a_1 = X_1 . W_1$	$b_1 = X_1 . V_1$	$c_1 = X_1 . L_1$	$f_1 = X_1 . R_1$
0	0	0	0	0 *
1	18	18	570	570 *
2	36	36	1140	1140 *
3	54	54	1710	1710 *
4	72	72	2280	2280 *

Table. 1. Step 1

t = 2

$$0 \le X_2 \le 7, \qquad W_2 = 12, \qquad V_2 = 14, \qquad L_2 = 300, \qquad R_2 = 300$$

X_2	X_1	$a_2 = a_1 + X_2 . W_2$	$b_2 = b_1 + X_2 . V_2$	$c_2 = c_1 + X_2 . L_2$	$f_2 = f_1 + X_2 . R_2$
0	0	0	0	0	0
0	1	18	18	570	570
0	2	36	36	1140	1140
0	3	54	54	1710	1710
0	4	72	72	2280	2280
1	0	12	14	300	300
1	1	30	32	870	870
1	2	48	50	1440	1440
1	3	66	68	2010	2010
1	4	84	86	2580	2580 *
2	0	24	28	600	600
2	1	42	46	1170	1170
2	2	60	64	1740	1740
2	3	78	82	2310	2310
2	4	0	0	0	0
3	0	36	42	900	900
3	1	54	60	1470	1470
3	2	72	78	2040	2040
3	3	90	96	2610	2610
3	4	0	0	0	0
4	0	48	56	1200	1200
4	1	66	74	1770	1770

4	2	84	92	2340	2340
4	3	0	0	0	0
4	4	0	0	0	0
5	0	60	70	1500	1500
5	1	78	88	2070	2070
5	2	0	0	0	0
5	3	0	0	0	0
5	4	0	0	0	0
6	0	72	84	1800	1800
6	1	0	0	0	0
6	2	0	0	0	0
6	3	0	0	0	0
6	4	0	0	0	0
7	0	84	98	2100	2100
7	1	0	0	0	0
7	2	0	0	0	0
7	3	0	0	0	0
7	4	0	0	0	0

Table. 2. Step 2

t = 3

$$0 \leq X_7 \leq 8, \qquad W_7 = 12, \qquad V_7 = 5, \qquad L_7 = 150, \qquad R_7 = 150$$

X_3	X_2	$a_3 = a_2 + X_3 \, . \, W_3$	$b_3 = b_2 + X_3 \, . \, V_3$	$c_3 = c_2 + X_3 \, . \, L_3$	$f_3 = f_2 + X_3 \, . \, R_3$
0	0	72	72	2280	2280
0	1	84	86	2580	2580
0	2	78	82	2310	2310
0	3	90	96	2610	2610
0	4	84	92	2340	2340
0	5	78	88	2070	2070
0	6	72	84	1800	1800
0	7	84	98	2100	2100
1	0	84	77	2430	2430
1	1	96	91	2730	2730 *
1	2	90	87	2460	2460
1	3	-	-	-	-
1	4	96	97	2490	2490
1	5	90	93	2220	2220
1	6	84	89	1950	1950
1	7	-	-	-	-
2	0	96	82	2580	2580
0	0	-	-	-	-
0	0	-	-	-	-
0	0	-	-	-	-
0	0	-	-	-	-
0	0	-	-	-	-
2	6	96	94	2100	2100
2	7	-	-	-	-
3	0	-	-	-	-
3	1	-	-	-	-
3	2	-	-	-	-
3	3	-	-	-	-
3	4	-	-	-	-
3	5	-	-	-	-
3	6	-	-	-	-
3	7	-	-	-	-
4	0	-	-	-	-

4	1	-	-	-	-
4	2	-	-	-	-
4	3	-	-	-	-
4	4	-	-	-	-
4	5	-	-	-	-
4	6	-	-	-	-
4	7	-	-	-	-
5	0	-	-	-	-
5	1	-	-	-	-
5	2	-	-	-	-
5	3	-	-	-	-
5	4	-	-	-	-
5	5	-	-	-	-
5	6	-	-	-	-
5	7	-	-	-	-
6	0	-	-	-	-
6	1	-	-	-	-
6	2	-	-	-	-
6	3	-	-	-	-
6	4	-	-	-	-
6	5	-	-	-	-
6	6	-	-	-	-
6	7	-	-	-	-
7	0	-	-	-	-
7	1	-	-	-	-
7	2	-	-	-	-
7	3	-	-	-	-
7	4	-	-	-	-
7	5	-	-	-	-
7	6	-	-	-	-
7	7	-	-	-	-
8	0	-	-	-	-
8	1	-	-	-	-
8	2	-	-	-	-
8	3	-	-	-	-
8	4	-	-	-	-
8	5	-	-	-	-
8	6	-	-	-	-

8	7	-	-	-	-

Table. 3. Step 3

t = 4

$$0 \leq X_4 \leq 4, \qquad W_4 = 12, \qquad V_4 = 25, \qquad L_4 = 400, \qquad R_4 = 400$$

X_4	X_3	$a_4 = a_3 + X_4 . W_4$	$b_4 = b_3 + X_4 . V_4$	$c_4 = c_3 + X_4 . L_4$	$f_4 = f_3 + X_4 . R_4$
0	0	90	96	2610	2610
0	1	96	91	2730	2730 *
0	2	96	94	2100	2100
1	0	-	-	-	-
1	1	-	-	-	-
1	2	-	-	-	-
2	0	-	-	-	-
2	1	-	-	-	-
2	2	-	-	-	-
3	0	-	-	-	-
3	1		-	-	-
3	2	-	-	-	-
4	0	-	-	-	-
4	1	-	-	-	-
4	2	-	-	-	-

Table. 4. Step 4

It can be clearly seen that $Vi > (100 - 96)$, $i \geq 4$, So $X_i = 0$, $i \geq 4$.

Results:

$Z^* = 2730, \quad X_{11}^* = X_{10}^* = X_9^* = X_8^* = X_7^* = X_6^* = X_5^* = X_4^* = 0, \quad X_2^* = X_3^* = 1, \quad X_1^* = 4$

Chapter 5
Conclusion

As it can be clearly seen, the optimal weight of units of food items and high-energy snacks to bring on a 1-day mountain climbing trip, in order to maximize the total food value while satisfying weight and other constraints are:

- Peanut butter or almond butter: $4 \times 18 = 72\ pounds$
- Jerky: $1 \times 12 = 12\ pounds$
- Dried legumes like lentils, peas, and beans: $1 \times 12 = 12\ pounds$
- and the others: 0 pounds

$$\big((18 \times 4) + (12 \times 1) + (12 \times 1)\big) \times 0.01 = 0.96 \text{ pounds}$$

The amount of money that should be allocated to this 1-day trip is equal to:

$$\big((18 \times 4) + (14 \times 1) + (5 \times 1)\big) \times 10{,}000 = 910{,}000\ IRR$$

Also, the food value of these, are:

$$(570 \times 4) + (300 \times 1) + (150 \times 1) = 2730$$