Lab4

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```
library(ISLR)
library(leaps)

library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --
```

```
## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.7
## v tidyr   1.1.4     v stringr 1.4.0
## v readr   2.1.1     v forcats 0.5.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(caret)
```

```
## Loading required package: lattice
```

```
##
## Attaching package: 'caret'
```

```
## The following object is masked from 'package:purrr':
##
##     lift
```

Textbook Example

```
data(Hitters)
summary(Hitters)
```

```
##      AtBat            Hits         HmRun            Runs
##  Min.   : 16.0   Min.   :  1   Min.   : 0.00   Min.   :  0.00
##  1st Qu.:255.2   1st Qu.: 64   1st Qu.: 4.00   1st Qu.: 30.25
##  Median :379.5   Median : 96   Median : 8.00   Median : 48.00
##  Mean   :380.9   Mean   :101   Mean   :10.77   Mean   : 50.91
##  3rd Qu.:512.0   3rd Qu.:137   3rd Qu.:16.00   3rd Qu.: 69.00
##  Max.   :687.0   Max.   :238   Max.   :40.00   Max.   :130.00
##
##       RBI             Walks            Years            CAtBat
##  Min.   :  0.00   Min.   :  0.00   Min.   : 1.000   Min.   :   19.0
##  1st Qu.: 28.00   1st Qu.: 22.00   1st Qu.: 4.000   1st Qu.:  816.8
##  Median : 44.00   Median : 35.00   Median : 6.000   Median : 1928.0
##  Mean   : 48.03   Mean   : 38.74   Mean   : 7.444   Mean   : 2648.7
##  3rd Qu.: 64.75   3rd Qu.: 53.00   3rd Qu.:11.000   3rd Qu.: 3924.2
##  Max.   :121.00   Max.   :105.00   Max.   :24.000   Max.   :14053.0
```

```
##
##      CHits            CHmRun           CRuns             CRBI
##  Min.   :   4.0   Min.   :  0.00   Min.   :   1.0   Min.   :   0.00
##  1st Qu.: 209.0   1st Qu.: 14.00   1st Qu.: 100.2   1st Qu.:  88.75
##  Median : 508.0   Median : 37.50   Median : 247.0   Median : 220.50
##  Mean   : 717.6   Mean   : 69.49   Mean   : 358.8   Mean   : 330.12
##  3rd Qu.:1059.2   3rd Qu.: 90.00   3rd Qu.: 526.2   3rd Qu.: 426.25
##  Max.   :4256.0   Max.   :548.00   Max.   :2165.0   Max.   :1659.00
##
##      CWalks        League  Division    PutOuts          Assists
##  Min.   :   0.00   A:175   E:157    Min.   :   0.0   Min.   :  0.0
##  1st Qu.:  67.25   N:147   W:165    1st Qu.: 109.2   1st Qu.:  7.0
##  Median : 170.50                    Median : 212.0   Median : 39.5
##  Mean   : 260.24                    Mean   : 288.9   Mean   :106.9
##  3rd Qu.: 339.25                    3rd Qu.: 325.0   3rd Qu.:166.0
##  Max.   :1566.00                    Max.   :1378.0   Max.   :492.0
##
##      Errors           Salary       NewLeague
##  Min.   : 0.00   Min.   :  67.5   A:176
##  1st Qu.: 3.00   1st Qu.: 190.0   N:146
##  Median : 6.00   Median : 425.0
##  Mean   : 8.04   Mean   : 535.9
##  3rd Qu.:11.00   3rd Qu.: 750.0
##  Max.   :32.00   Max.   :2460.0
##                  NA's   :59
```

Need to remove missing values.

```
# remove rows with missing data
Hitters=na.omit(Hitters)
with(Hitters, sum(is.na(Hitters$Salary)))
```

```
## [1] 0
```

Best subset

```
regfit.full=regsubsets(Salary~.,Hitters)
summary(regfit.full)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., Hitters)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
```

```
## CWalks          FALSE      FALSE
## LeagueN         FALSE      FALSE
## DivisionW       FALSE      FALSE
## PutOuts         FALSE      FALSE
## Assists         FALSE      FALSE
## Errors          FALSE      FALSE
## NewLeagueN      FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##          AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 ) " "   " "  " "   " "  " " " " " "   " "    " "   " "    " "   "*"
## 2  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "    " "   " "    " "   "*"
## 3  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "    " "   " "    " "   "*"
## 4  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "    " "   " "    " "   "*"
## 5  ( 1 ) "*"   "*"  " "   " "  " " " " " "   " "    " "   " "    " "   "*"
## 6  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "    " "   " "    " "   "*"
## 7  ( 1 ) " "   "*"  " "   " "  " " " " "*"   " "    "*"   "*"    "*"   " "
## 8  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "    " "   " "    "*"   " "
##          CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 ) " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 ) "*"    " "     "*"       "*"     " "     " "    " "
```

looks lie it only goes upto 8 best variables.

Let's try all 19 variables.

```
regfit.full=regsubsets(Salary~.,data=Hitters,nvmax=19)
(reg.summary=summary(regfit.full))
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
```

```
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  " "   "*"  " "   " "  " " " " "*"   " "   "*"    "*"   "*"    " "   " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "    " "   "*"    "*"   " "
## 9  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
names(reg.summary)
```

```
## [1] "which"  "rsq"    "rss"    "adjr2"  "cp"     "bic"    "outmat" "obj"
```

```
reg.summary$rsq
```

```
##  [1] 0.3214501 0.4252237 0.4514294 0.4754067 0.4908036 0.5087146 0.5141227
##  [8] 0.5285569 0.5346124 0.5404950 0.5426153 0.5436302 0.5444570 0.5452164
## [15] 0.5454692 0.5457656 0.5459518 0.5460945 0.5461159
```

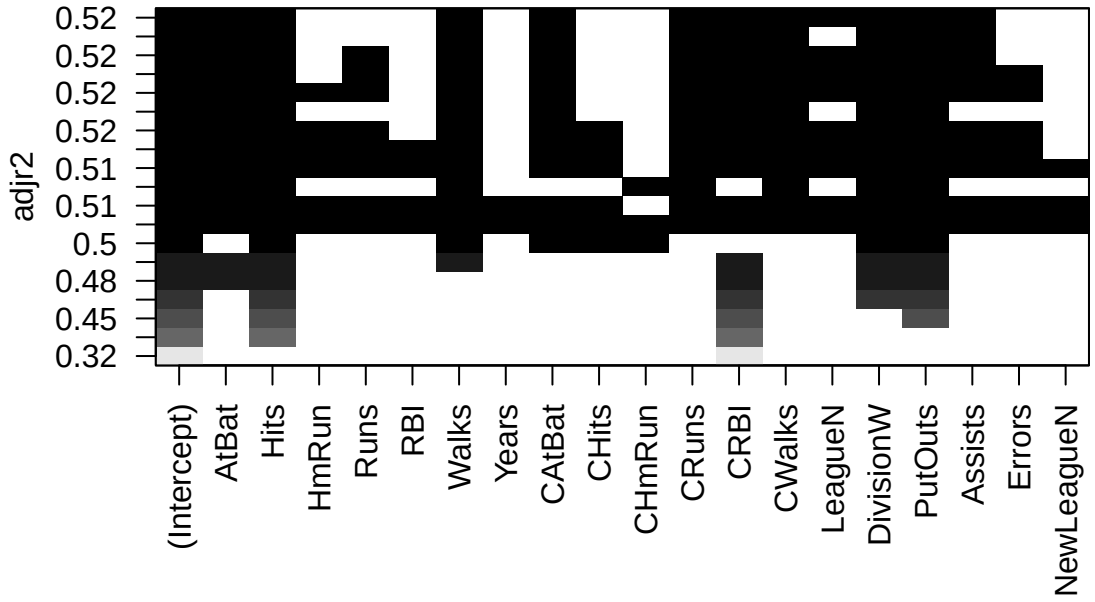
```
summary(regfit.full)$adjr2
```

```
##  [1] 0.3188503 0.4208024 0.4450753 0.4672734 0.4808971 0.4972001 0.5007849
##  [8] 0.5137083 0.5180572 0.5222606 0.5225706 0.5217245 0.5206736 0.5195431
## [15] 0.5178661 0.5162219 0.5144464 0.5126097 0.5106270
```

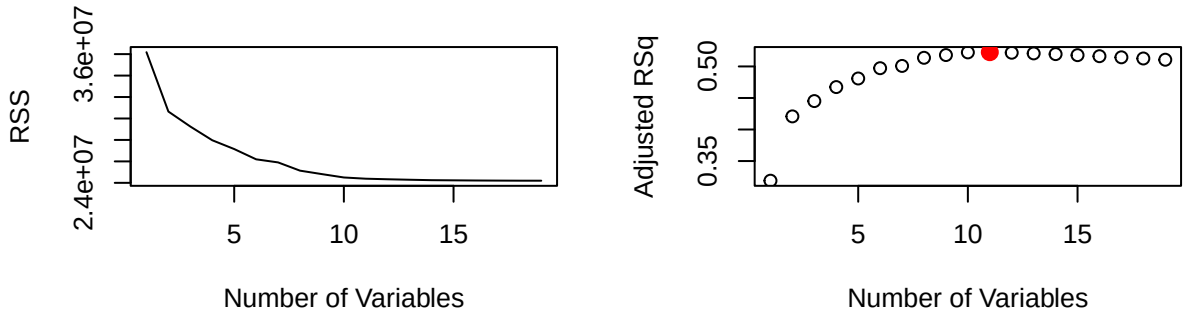
```
which.max(reg.summary$adjr2)
```

```
## [1] 11
```

```
plot(regfit.full,scale="adjr2")
```

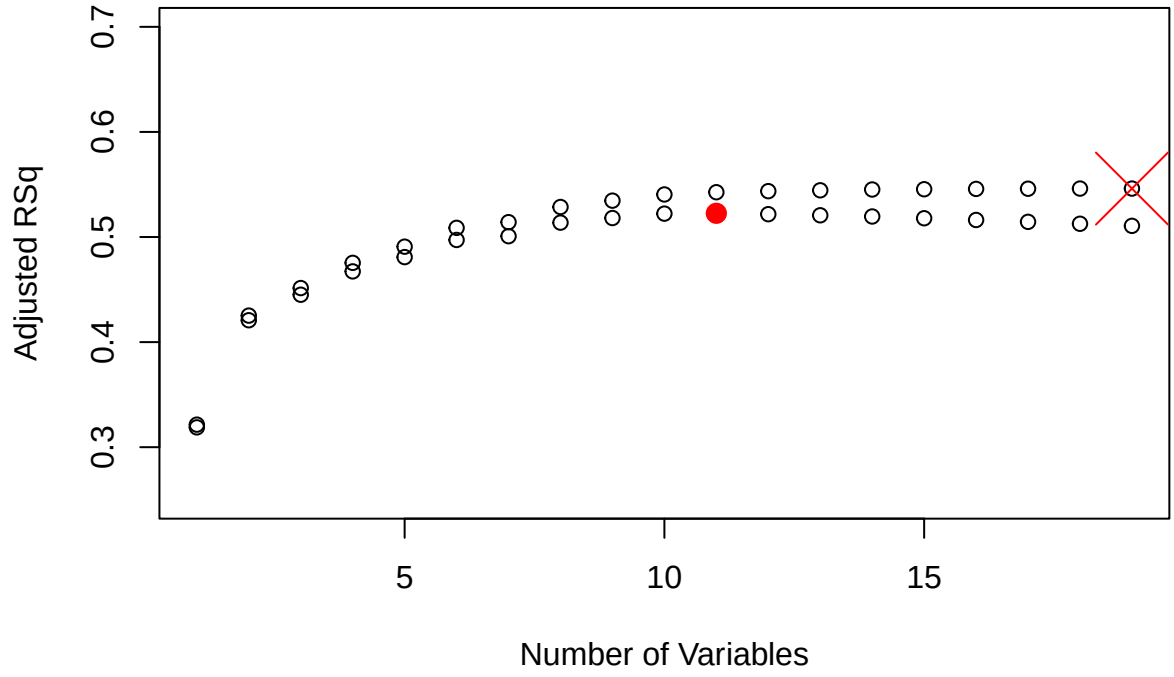



```
par(mfrow=c(2,2))
plot(reg.summary$rss,xlab="Number of Variables",ylab="RSS",type="l")


plot(reg.summary$adjr2,xlab="Number of Variables",ylab="Adjusted RSq")

m=which.max(reg.summary$adjr2)
points(m,reg.summary$adjr2[m], col="red",cex=2,pch=20)
```

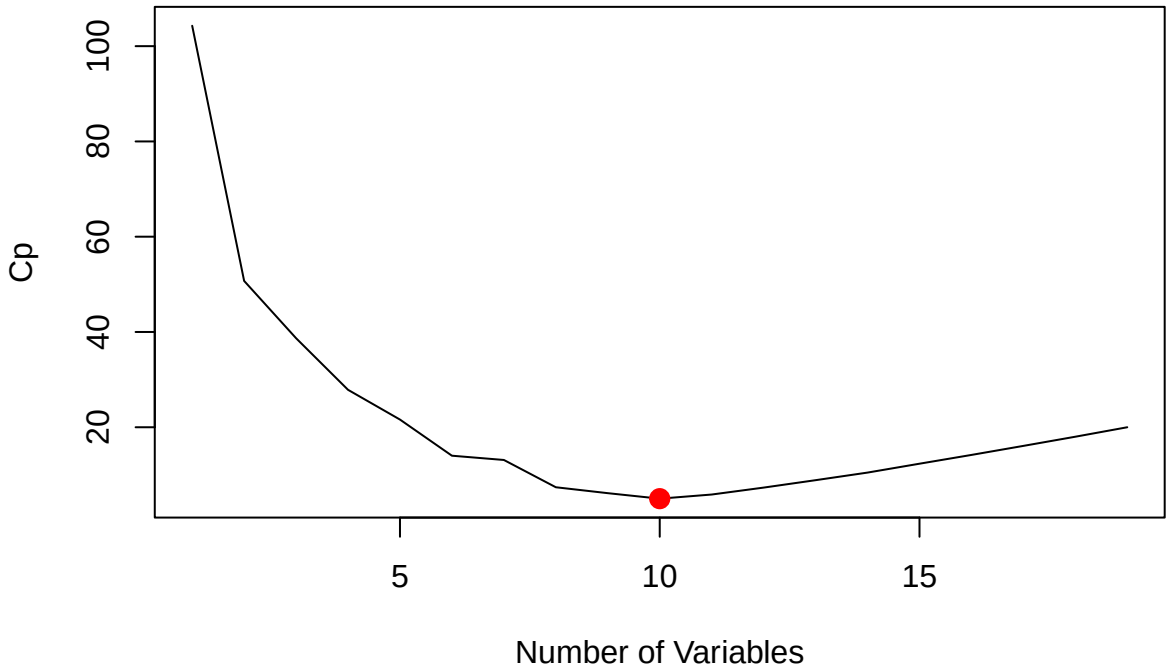



```
#compare Rsq and adjusted srq

plot(reg.summary$adjr2,xlab="Number of Variables",ylab="Adjusted RSq", ylim=c(0.25,0.7))

m=which.max(reg.summary$adjr2)
```

```
n=which.max(reg.summary$rsq)
points(m,reg.summary$adjr2[m], col="red",cex=2,pch=20)
points(reg.summary$rsq)
points(n,reg.summary$rsq[n], col="red",cex=5,pch=4)
```

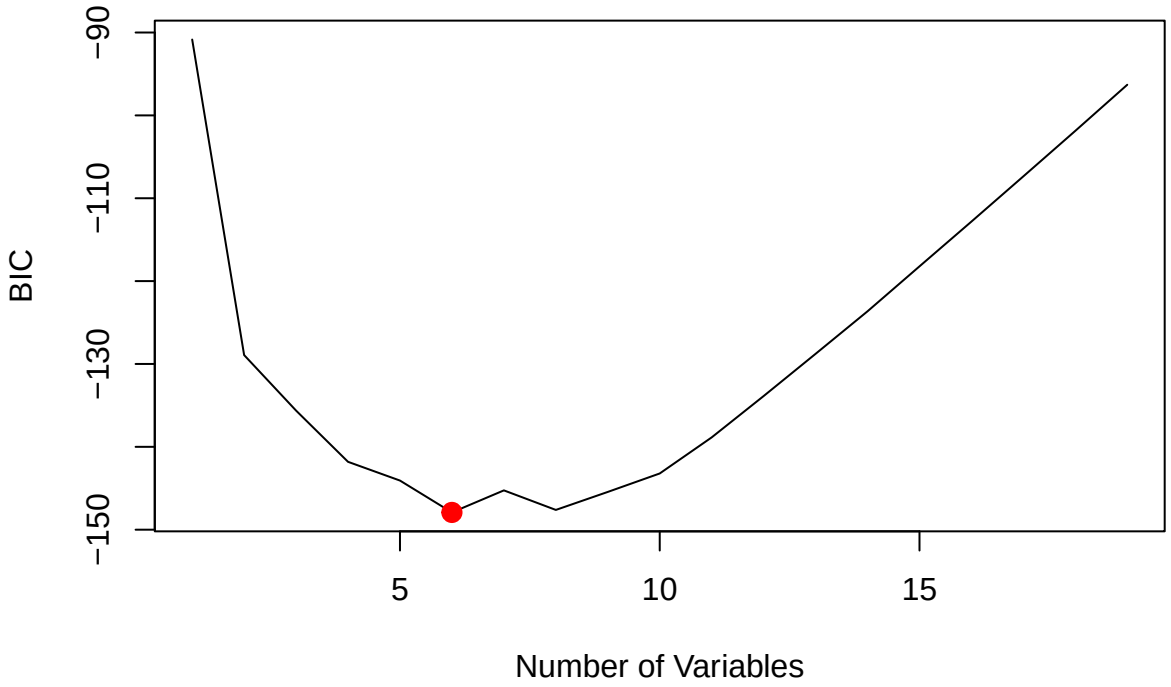



```
plot(reg.summary$cp,xlab="Number of Variables",ylab="Cp",type='l')
m=which.min(reg.summary$cp)
points(m,reg.summary$cp[m],col="red",cex=2,pch=20)
```

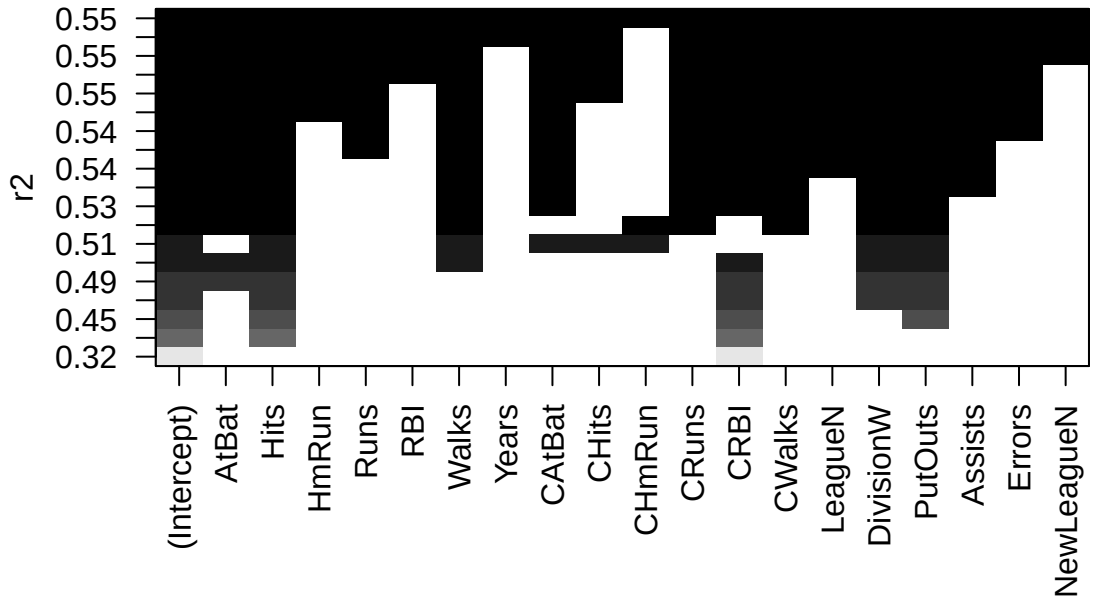



```
m=which.min(reg.summary$bic)
plot(reg.summary$bic,xlab="Number of Variables",ylab="BIC",type='l')
```

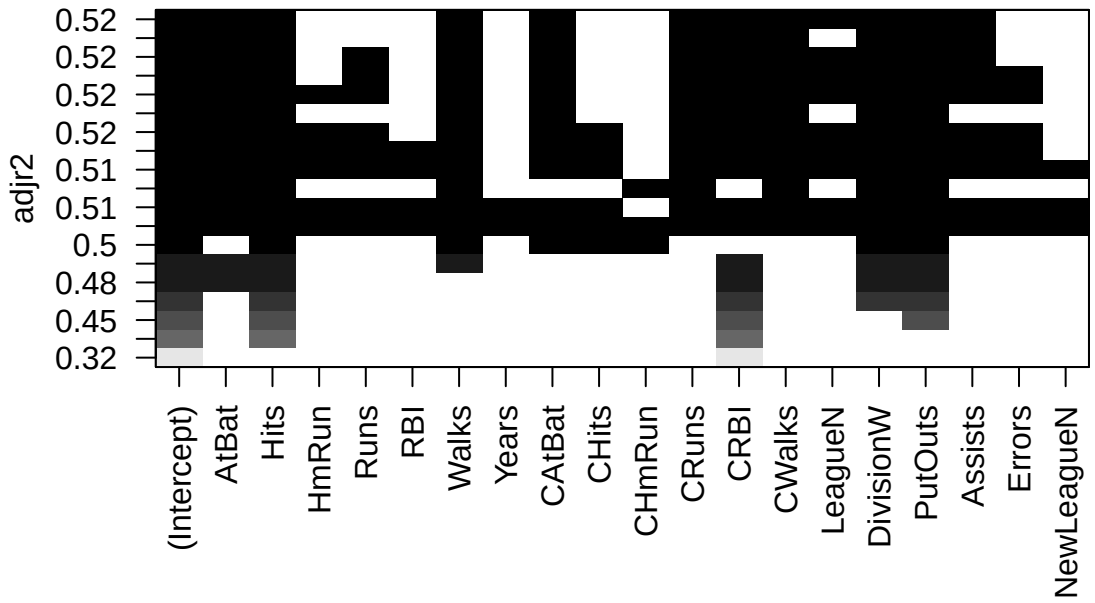
```
points(m,reg.summary$bic[6],col="red",cex=2,pch=20)
```



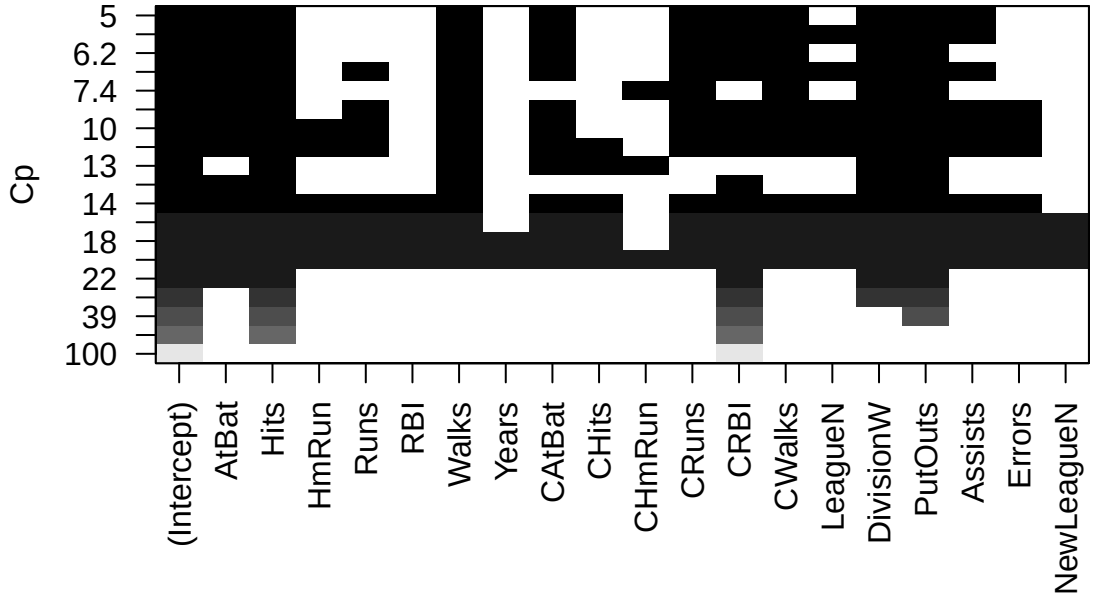

```
plot(regfit.full,scale="r2")
```



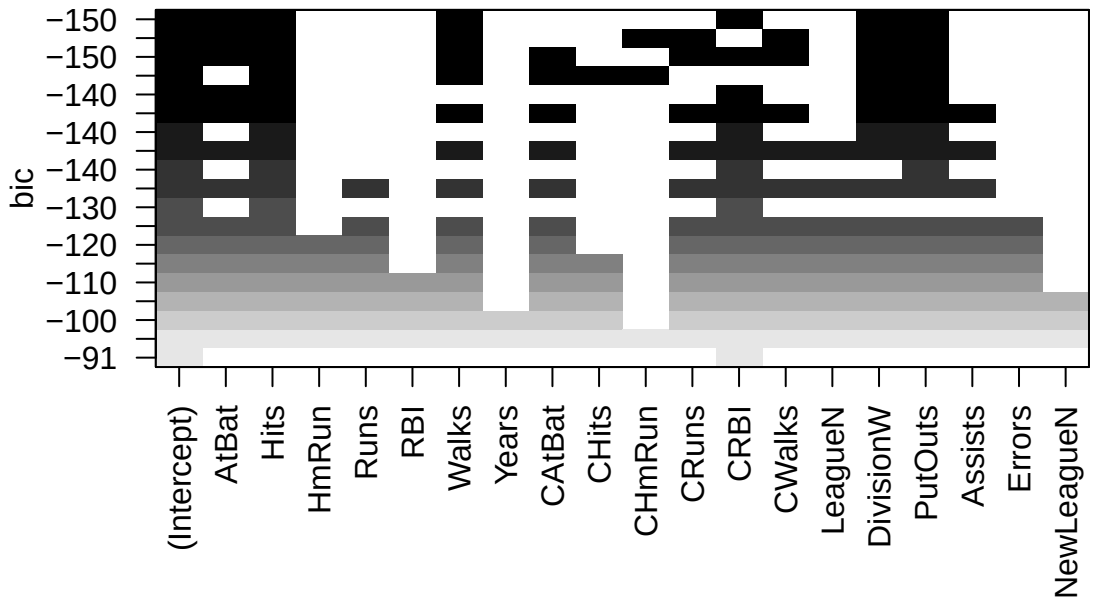

```
plot(regfit.full,scale="adjr2")
```




```
plot(regfit.full,scale="Cp")
```




```
plot(regfit.full,scale="bic")
```




```
coef(regfit.full,6)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI    DivisionW
##   91.5117981   -1.8685892    7.6043976    3.6976468    0.6430169 -122.9515338
##      PutOuts
##    0.2643076
```

Forward and Backward Stepwise Selection

```
regfit.fwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="forward")

summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables  (and intercept)
##             Forced in Forced out
## AtBat           FALSE      FALSE
## Hits            FALSE      FALSE
## HmRun           FALSE      FALSE
## Runs            FALSE      FALSE
## RBI             FALSE      FALSE
## Walks           FALSE      FALSE
## Years           FALSE      FALSE
## CAtBat          FALSE      FALSE
## CHits           FALSE      FALSE
## CHmRun          FALSE      FALSE
## CRuns           FALSE      FALSE
## CRBI            FALSE      FALSE
## CWalks          FALSE      FALSE
## LeagueN         FALSE      FALSE
## DivisionW       FALSE      FALSE
## PutOuts         FALSE      FALSE
## Assists         FALSE      FALSE
## Errors          FALSE      FALSE
```

```
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

variables are nested.

```
regfit.bwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="backward")
summary(regfit.bwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "backward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
```

```
## HmRun           FALSE      FALSE
## Runs            FALSE      FALSE
## RBI             FALSE      FALSE
## Walks           FALSE      FALSE
## Years           FALSE      FALSE
## CAtBat          FALSE      FALSE
## CHits           FALSE      FALSE
## CHmRun          FALSE      FALSE
## CRuns           FALSE      FALSE
## CRBI            FALSE      FALSE
## CWalks          FALSE      FALSE
## LeagueN         FALSE      FALSE
## DivisionW       FALSE      FALSE
## PutOuts         FALSE      FALSE
## Assists         FALSE      FALSE
## Errors          FALSE      FALSE
## NewLeagueN      FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: backward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " " " "   " "    " "   " "    "*"   " "
## 2  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "    " "   " "    "*"   " "
## 3  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "    " "   " "    "*"   " "
## 4  ( 1 )  "*"   "*"  " "   " "  " " " " " "   " "    " "   " "    "*"   " "
## 5  ( 1 )  "*"   "*"  " "   " "  " " " "*"   " "    " "   " "    "*"   " "
## 6  ( 1 )  "*"   "*"  " "   " "  " " " "*"   " "    " "   " "    "*"   " "
## 7  ( 1 )  "*"   "*"  " "   " "  " " " "*"   " "    " "   " "    "*"   " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " " "*"   " "    " "   " "    "*"   "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " " "*"   " "    "*"   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " " "*"   " "    "*"   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " " "*"   " "    "*"   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " " "*"   " "    "*"   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " " "*"   " "    "*"   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " " "*"   " "    "*"   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " " "*"   " "    "*"   "*"    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "    "*"   "*"    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "    "*"   "*"    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"    "*"   "*"    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
```

```
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
coef(regfit.full,7)
```

```
##  (Intercept)         Hits        Walks       CAtBat        CHits       CHmRun
##   79.4509472    1.2833513    3.2274264   -0.3752350    1.4957073    1.4420538
##    DivisionW      PutOuts
## -129.9866432    0.2366813
```

```
coef(regfit.fwd,7)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI       CWalks
##  109.7873062   -1.9588851    7.4498772    4.9131401    0.8537622   -0.3053070
##    DivisionW      PutOuts
## -127.1223928    0.2533404
```

```
coef(regfit.bwd,7)
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns       CWalks
##  105.6487488   -1.9762838    6.7574914    6.0558691    1.1293095   -0.7163346
##    DivisionW      PutOuts
## -116.1692169    0.3028847
```

Hybrid method

```
regfit.bwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="seqrep")
summary(regfit.bwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "seqrep")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
```

```
## Selection Algorithm: 'sequential replacement'
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  "*"   "*"  "*" " " " "   " "   " "    " "   " "    " "   " "
## 6  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "    " "   " "    "*"   " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   " "    " "   "*"    "*"   " "
## 9  ( 1 )  "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

Choosing Among Models

In order to use the validation set approach, we begin by splitting the observations into a training set and a test set.

We do this by creating a random vector, train, of elements equal to TRUE if the corresponding observation is in the training set, and FALSE otherwise.

The vector test has a TRUE if the observation is in the test set, and a FALSE otherwise.

Note the ! in the command to create test causes TRUEs to be switched to FALSEs and vice versa.

We also set a random seed so that the user will obtain the same training set/test set split.

```
set.seed(1)
train=sample(c(TRUE,FALSE), nrow(Hitters),rep=TRUE)
test=(!train)
```

Now, we apply regsubsets() to the training set in order to perform best subset selection.

Notice that we subset the Hitters data frame directly in the call in order to access only the training subset of the data, using the expression Hitters[train,].

```
regfit.best=regsubsets(Salary~.,data=Hitters[train,],nvmax=19)
```

We now compute the validation set error for the best model of each model size.

We first make a model matrix from the test data.

The model.matrix() function is used in many regression packages for building an "X" matrix from data.

```
test.mat=model.matrix(Salary~.,data=Hitters[test,])
```

Now we run a loop, and for each size i, we extract the coefficients from regfit.best for the best model of that size, multiply them into the appropriate columns of the test model matrix to form the predictions, and compute the test MSE.

```
val.errors=rep(NA,19)
for(i in 1:19){
  coefi=coef(regfit.best,id=i)
  pred=test.mat[,names(coefi)]%*%coefi
  val.errors[i]=mean((Hitters$Salary[test]-pred)^2)
}
```

We find that the best model is the one that contains ten variables.

```
val.errors
```

```
##  [1] 164377.3 144405.5 152175.7 145198.4 137902.1 139175.7 126849.0 136191.4
##  [9] 132889.6 135434.9 136963.3 140694.9 140690.9 141951.2 141508.2 142164.4
## [17] 141767.4 142339.6 142238.2
```

```
m=which.min(val.errors) #m=10
coef(regfit.best,m)
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns       CWalks
##   67.1085369   -2.1462987    7.0149547    8.0716640    1.2425113   -0.8337844
##    DivisionW      PutOuts
## -118.4364998    0.2526925
```

This was a little tedious, partly because there is no predict() method for regsubsets().

Since we will be using this function again, we can capture our steps above and write our own predict method.

```
predict.regsubsets=function(object,newdata,id,...){
  form=as.formula(object$call[[2]])
  mat=model.matrix(form,newdata)
  coefi=coef(object,id=id)
  xvars=names(coefi)
  mat[,xvars]%*%coefi
}
```

Our function pretty much mimics what we did above.

Finally, we perform best subset selection on the full data set, and select the best ten-variable model.

```
regfit.best=regsubsets(Salary~.,data=Hitters,nvmax=19)
coef(regfit.best,10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798    1.4082490
##         CRBI       CWalks    DivisionW      PutOuts      Assists
##    0.7743122   -0.8308264 -112.3800575    0.2973726    0.2831680
```

In fact, we see that the best ten-variable model on the full data set has a different set of variables than the best ten-variable model on the training set.

We now try to choose among the models of different sizes using cross- validation.

we must perform best subset selection within each of the k training sets.

Despite this, we see that with its clever subsetting syntax, R makes this job quite easy.

First, we create a vector that allocates each observation to one of k = 10 folds, and we create a matrix in which we will store the results.

```
k=10
set.seed(1)
folds=sample(1:k,nrow(Hitters),replace=TRUE)
cv.errors=matrix(NA,k,19, dimnames=list(NULL, paste(1:19)))
```

Now we write a for loop that performs cross-validation.

In the jth fold, the elements of folds that equal j are in the test set, and the remainder are in the training set.

We make our predictions for each model size (using our new predict() method), compute the test errors on the appropriate subset, and store them in the appropriate slot in the matrix cv.errors.

```
for(j in 1:k){
  best.fit=regsubsets(Salary~.,data=Hitters[folds!=j,], nvmax=19)
  for(i in 1:19){
    pred=predict(best.fit,Hitters[folds==j,],id=i)
    cv.errors[j,i]=mean( (Hitters$Salary[folds==j]-pred)^2)
  }
}
```

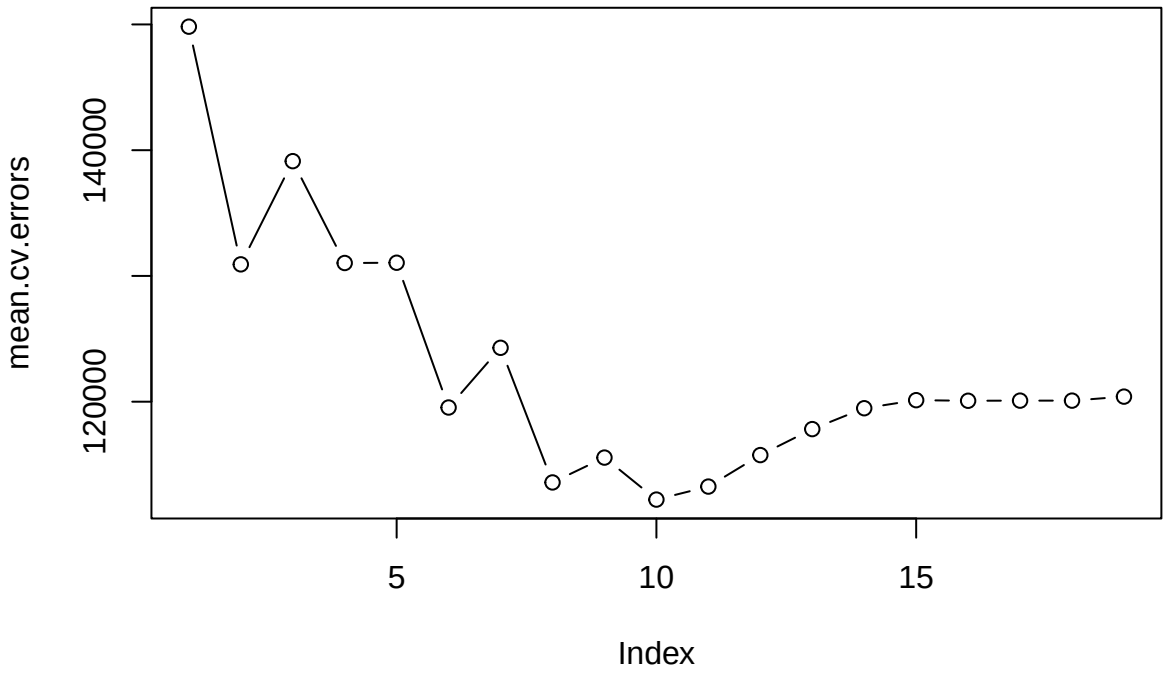
This has given us a 10×19 matrix, of which the (i, j)th element corresponds to the test MSE for the ith cross-validation fold for the best j-variable model.

We use the apply() function to average over the columns of this matrix in order to obtain a vector for which the jth element is the cross- validation error for the j-variable model.

```
mean.cv.errors=apply(cv.errors,2,mean)
mean.cv.errors
```

```
##        1        2        3        4        5        6        7        8
## 149821.1 130922.0 139127.0 131028.8 131050.2 119538.6 124286.1 113580.0
##        9       10       11       12       13       14       15       16
## 115556.5 112216.7 113251.2 115755.9 117820.8 119481.2 120121.6 120074.3
##       17       18       19
## 120084.8 120085.8 120403.5
```

```
par(mfrow=c(1,1))
plot(mean.cv.errors,type='b')
```



We see that cross-validation selects an 11-variable model.

We now perform best subset selection on the full data set in order to obtain the 11-variable model.

```
reg.best=regsubsets(Salary~.,data=Hitters, nvmax=19)
coef(reg.best,11)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns
##  135.7512195   -2.1277482    6.9236994    5.6202755   -0.1389914    1.4553310
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##    0.7852528   -0.8228559   43.1116152 -111.1460252    0.2894087    0.2688277
```

Example

Predicting fertility score on the basis of socio-economic indicators.

```
# Load the data
data("swiss")
# Inspect the data
sample_n(swiss, 3)
```

```
##           Fertility Agriculture Examination Education Catholic Infant.Mortality
## Nyone          56.6        50.9          22        12    15.14             16.7
## Moutier        85.8        36.5          12         7    33.77             20.3
## Echallens      68.3        72.6          18         2    24.20             21.2
```

(a). Computing best subsets regression.

In our example, we have only 5 predictor variables in the data. So, we'll use nvmax = 5.

```
models <- regsubsets(Fertility~., data = swiss, nvmax = 5)
summary(models)
```

```
## Subset selection object
## Call: regsubsets.formula(Fertility ~ ., data = swiss, nvmax = 5)
## 5 Variables  (and intercept)
```

```
##                   Forced in Forced out
## Agriculture           FALSE      FALSE
## Examination           FALSE      FALSE
## Education             FALSE      FALSE
## Catholic              FALSE      FALSE
## Infant.Mortality      FALSE      FALSE
## 1 subsets of each size up to 5
## Selection Algorithm: exhaustive
##          Agriculture Examination Education Catholic Infant.Mortality
## 1  ( 1 ) " "         " "         "*"       " "      " "
## 2  ( 1 ) " "         " "         "*"       "*"      " "
## 3  ( 1 ) " "         " "         "*"       "*"      "*"
## 4  ( 1 ) "*"         " "         "*"       "*"      "*"
## 5  ( 1 ) "*"         "*"         "*"       "*"      "*"
```

It can be seen that the best 2-variables model contains only Education and Catholic variables

($Fertility \sim Education + Catholic$).

The best three-variable model is

($Fertility \sim Education + Catholic + Infant.mortality$), and so forth.

A natural question is: which of these best models should we finally choose for our predictive analytics?

(c) Choosing the optimal model.

```
res.sum <- summary(models)

data.frame(
  Adj.R2 = which.max(res.sum$adjr2),
  CP = which.min(res.sum$cp),
  BIC = which.min(res.sum$bic)
)
```

```
##   Adj.R2 CP BIC
## 1      5  4   4
```

Here, adjusted R2 tells us that the best model is the one with all the 5 predictor variables. However, using the BIC and Cp criteria, we should go for the model with 4 variables.

Note also that the adjusted R2, BIC and Cp are calculated on the training data that have been used to fit the model. This means that, the model selection, using these metrics, is possibly subject to overfitting and may not perform as well when applied to new data.

A more rigorous approach is to select a models based on the prediction error computed on a new test data using k-fold cross-validation

(d). K-fold cross-validation

(i) get_model_formula(), allowing to access easily the formula of the models returned by the function regsubsets().

```
# id: model id
# object: regsubsets object
# data: data used to fit regsubsets
# outcome: outcome variable
get_model_formula <- function(id, object, outcome){
  # get models data
  models <- summary(object)$which[id,-1]
  # Get outcome variable
```

```
  #form <- as.formula(object$call[[2]])
  #outcome <- all.vars(form)[1]
  # Get model predictors
  predictors <- names(which(models == TRUE))
  predictors <- paste(predictors, collapse = "+")
  # Build model formula
  as.formula(paste0(outcome, "~", predictors))
}
```

For example to have the best 3-variable model formula;

```
get_model_formula(3, models, "Fertility")
```

```
## Fertility ~ Education + Catholic + Infant.Mortality
## <environment: 0x7fd8080cbbd8>
```

(ii) get_cv_error(), to get the cross-validation (CV) error for a given model:

```
get_cv_error <- function(model.formula, data){
  set.seed(1)
  train.control <- trainControl(method = "cv", number = 5)
  cv <- train(model.formula, data = data, method = "lm",
              trControl = train.control)
  cv$results$RMSE
}
```

Finally, use the above defined helper functions to compute the prediction error of the different best models returned by the regsubsets() function:

```
# Compute cross-validation error
model.ids <- 1:5
cv.errors <-  map(model.ids, get_model_formula, models, "Fertility") %>%
  map(get_cv_error, data = swiss) %>%
  unlist()
cv.errors
```

```
## [1] 9.464156 8.517433 7.855267 7.601072 7.736328
```

```
# Select the model that minimize the CV error
which.min(cv.errors)
```

```
## [1] 4
```

It can be seen that the model with 4 variables is the best model. It has the lower prediction error.

The regression coefficients of this model can be extracted as follow:

```
coef(models, 4)
```

```
##      (Intercept)      Agriculture        Education         Catholic
##       62.1013116       -0.1546175       -0.9802638        0.1246664
## Infant.Mortality
##        1.0784422
```

Read more:

http://www.sthda.com/english/articles/37-model-selection-essentials-in-r/155-best-subsets-regression-essentials-in-r/

http://www.sthda.com/english/articles/37-model-selection-essentials-in-r/154-stepwise-regression-essentials-in-r/

Lab Assignment

Explore the a different dataset from the ISLR2 package (ISLR2::) and use Best Subset selection with the regsubsets() function. From the outputs of regsubsets(), we can obtain metrics like BIC for each sub-model with K predictors.

1. Make a few plots showing BIC, and Mallow's Cp, and 1-AdjustedRsquared.

Are the different metrics in rough agreement?

Try to answer these questions using different plots

a. compare RSS and R^2
b. Compare R^2 with adjusted R^2
c. Compare adjusted R^2, BIC, cp with Cross validation results
d. Also get the feature plots for Adj. R^2 , BIC, cp; What is the number of predictors to include in the model for each different metric and what are these predictors?

2. These metrics are evaluted with the full training set. Another way we can approach the problem of model selection is to use cross validation. What are the pros and cons of each approach?