

Lab 6 Assignment

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Problem 1

Let $Y \sim N(0, 1)$ have a standard normal distribution. Set $X = \lfloor Y \rfloor$ (floor function).

- Find $median(Y|X = x)$ from simulations and plot it as a function of x .
- Find $sd(Y|X = x)$ from simulations and plot it as a function of x .

R Notebook

- a. Do this with a data frame. Two columns, one for X and one for Y. use *floor()* function.
floor function: https://en.wikipedia.org/wiki/Floor_and_ceiling_functions
- b. Find the median $Y|X$ and plot. (You can use the *aggregate()* function.). Comment.
- c. Find the standard deviation of $Y|X$ and plot. (You can use the *aggregate()* function.). Comment.

Problem 2:

$X \sim U(0, 1)$ and $Y \sim U(0, 1)$.

Plot the joint distribution of X & Y such that $X + Y < 1$

- a. First make a large number of samples from $X \sim U(0, 1)$ and $Y \sim U(0, 1)$ and plot X & Y and then plot the samples below the line $x + y = 1$.
- b. By looking at the plot, what is the joint p.d.f $f(X, Y)$?
- c. Find the marginal distribution of X by hand and making a probability histogram. Compare whether these are the same. (Hint: think about the limits of X and Y according to $0 < X + Y < 1$)
- d. Find the marginal distribution of Y by hand and making a probability histogram. Compare whether these are the same. (Hint: think about the limits of X and Y according to $0 < X + Y < 1$)

Problem 3 : Joint ditributions and conditional expectations

A pdf is defined by

$$f(x, y) = \begin{cases} C(x + 2y) & (0 < y < 1, 0 < x < 2) \\ 0 & (otherwise) \end{cases}$$

- a. Find the value of C .
- b. Find the Marginal density of X

- c. (BONUS) Find the Marginal density of Y
- d. Find the conditional density function of $Y|X = x$
- e. (BONUS) Find the Conditional expectation $E(Y|X)$