Homework 4

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Problem 1

Suppose $X = (X_1, X_2, X_3)$ has a multinomial distribution with size $n = 10$ and probabilities $p_1 = .2, p_2 = .5, p_3 = .3$. Use a simulation with **rmultinom** to show that $P(X_1 = 2, X_2 = 4, X_3 = 4) \approx 0.0638$. Confirm your results using **dmultinom**.

Problem 2

Suppose $X = (X_1, X_2, X_3)$ has a multinomial distribution with size $n = 10$ and probabilities $p_1 = .2, p_2 = .5, p_3 = .3$. Use a simulation with **sample (not rmultinom)** to show that $P(X_1 = 2, X_2 = 4, X_3 = 4) \approx 0.0638$. Confirm your results using **dmultinom**.

Problem 3

Let $X_1, \ldots, X_{12}$ be a random sample of size 12 from the $U(0, 1)$ distribution. Explain why $Z = X_1 + X_2 + \cdots + X_{12} - 6$ has an approximate standard normal distribution. You can either proove this theorotically by using CLT, or can use a simulation. *You will have to find or look up the variance of a single X_i.*

Problem 4

Problem 4.4 #14 a and b in Chihara/Hesterberg.

14. Let $X_1, X_2, \ldots, X_9 \overset{i.i.d.}{\sim} N(7, 3^2)$ and $Y_1, Y_2, \ldots, Y_{12} \overset{i.i.d.}{\sim} N(10, 5^2)$. Let $W = \bar{X} - \bar{Y}$.
    (a) Give the exact sampling distribution of W.
    (b) Simulate the sampling distribution of W in `R` and plot your results (adapt code from the previous exercise). Check that the simulated mean and the standard error are close to the theoretical mean and the standard error.
    (c) Use your simulation to find $P(W < -1.5)$. Calculate an exact answer and compare.

Hint:

Corollary A.2 *Let $X_1, X_2, \ldots, X_n$ be independent normal random variables with common mean μ and common variance σ^2. Let $\bar{X}$ denote the sample mean. Then $\bar{X}$ is normally distributed with mean μ and variance (σ^2/n).*

Theorem A.10 *Let X be a normal random variable with mean μ_1 and variance σ_1^2, and let Y be a normal random variable with mean μ_2, and variance σ_2^2. Assume that X and Y are independent. Then $X \pm Y$ is a normal random variable with mean $\mu_1 \pm \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.*

Problem 5

Problem 4.4 #18 in Chihara/Hesterberg.

18. Let $X_1, X_2, \ldots, X_{30} \overset{i.i.d.}{\sim} \text{Exp}(1/3)$ and let $\bar{X}$ denote the sample mean.
    (a) Simulate the sampling distribution of $\bar{X}$ in `R`.
    (b) Find the mean and standard error of the sampling distribution and compare to the theoretical results.
    (c) From your simulation, find $P(\bar{X} \leq 3.5)$.
    (d) Estimate $P(\bar{X} \leq 3.5)$ by assuming that the CLT approximation holds. Compare this result with the one in part (c).

Problem 6 (Bonus)

Let $X_1, \ldots, X_n$ be a random sample of size n from a $U(0, a)$ distribution, where $a > 0$. Find $E(X_1 + X_2 + \cdots + X_n)$. and find the approximate distribution of the sample mean, if n is large.