Lab 4 Assignment

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Problem 1:

Use simulation to answer the follwing questions.

Given independent uniform random variables $X_1 \sim U(a,b)$ and $X_2 \sim U(a,b)$. (You can use parameters of your choice)

a. Is the distribution of $X_1 + X_2$ uniform? Please Justify your answer.

b. Is the distribution of $X_1.X_2$ uniform? Please Justify your answer.

c. Is the distribution of $max(X_1, X_2)$ or $min(X_1, X_2)$ uniform? Please Justify your answer.

d. Is the distribution of $1 - X_1$ uniform? Please Justify your answer.

Problem 2:

Prove the following theorems using simulation. (You can use parameters of your choice)

a. Linear Transformations

Theorem 5.6.4 If X has the normal distribution with mean μ and variance σ^2 and if $Y = aX + b$, where a and b are given constants and $a \neq 0$, then Y has the normal distribution with mean $a\mu + b$ and variance $a^2\sigma^2$.

b. Linear Combinations of Normally Distributed Variables

Theorem 5.6.7 If the random variables $X_1, \ldots, X_k$ are independent and if X_i has the normal distribution with mean μ_i and variance σ_i^2 $(i = 1, \ldots, k)$, then the sum $X_1 + \cdots + X_k$ has the normal distribution with mean $\mu_1 + \cdots + \mu_k$ and variance $\sigma_1^2 + \cdots + \sigma_k^2$.

Problem 3:

Use the iris data set to check the normality of Sepal width of setosa using the following normality tests (Repeat the steps in the example 3 in lab 4, comment on your results)

a. Anderson Darling Test
b. Kolmogorov-Smirnov Test (Comapre to a standard normal distribution)

Problem 4 (BONUS): Exponential Distribution

Given exponentially distributed random variables $X_1, ..., X_k$. Think of waiting times for independent random alarm clocks $1, ..., k$ to go off.

Which of these are again exponentially distributed? Explore with a simulation. (Hint: you can use many methods here to compare the distributions, for example; using Kolmogorov-Smirnov Test, plotting cdfs or ecdfs(as we did in lab 3),..etc)

a. Distribution of $min(X_1, \dots, X_k)$? Waiting time for the first alarm to go off.
b. Distribution of $max(X_1, \dots, X_k)$? Waiting time for the last alarm to go off.
c. Distribution of $X_1 + \cdots + X_k$? Waiting time until Start of the next clock when the previous alarm/s has gone off.