

# Collective labeling-based argumentation using social choice methods

Reza Karimi<sup>a</sup>, Faria Nassiri-Mofakham<sup>\*,a</sup>

<sup>a</sup>Faculty of Computer Engineering, University of Isfahan, 81746-73441, Isfahan, Iran

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## ABSTRACT

Argumentation-based debates are mechanisms that a group can use to resolve conflicting opinions and as a result, reach a final agreement that. The purpose of argument-based discussions is to answer the question that how participants in a debate can reach agreement about the outcome of the debate, given all the arguments that have been made. Consolidating all arguments in a society of agents deals with complex challenges when there are multiple options and diverse opinions. To aggregate individual preferences and determine social preference, it is necessary to perform methods of aggregation functions. These methods are rooted in social choice literature. In the existing method, only one type of social choice methods (i.e. majority) has been discussed and other methods have not been considered. Considering the diversity of social choice methods, it is essential to pay attention to how these methods function and behave in argumentation-based debates in group decision-making. The purpose of this study is to pay attention to how social choice methods work in arguments based debates that use from the combination of a set of arguments, relations between them, and also a set of opinions about whether the them hold or not. This study presents collective decision-making by developing several social choice methods into an argumentation based debates to address these issues. Considering the different behaviors and outcome of the proposed functions, we present new concepts to define the behavior of aggregation methods based on collective arguments.

## 1. Introduction

Many studies examine how collective decisions are made in societies and analyze the processes human group decisions are developed to reach a final decision that is as favorable as possible for the group members [31]. Various aggregation rules such as voting, ranking, scoring, consensus, and argumentation have been used for collective decision making.


Social choice is a theory for analyzing and evaluating different methods that study how individual preferences, opinions, judgments, or values can be aggregated into a collective decision or social welfare that reflects the interests of the group or society. [This theory analyzes the challenges and complexities in different real-world decision-making scenarios in situations where there are multiple options and diverse opinions.](#)[30]


Computational social choice is an interdisciplinary field that studies the application of social choice theory to problems in artificial intelligence, computer science, multi-agent systems, economics and political science [25, 13].

On the other hand, collective argumentation is a field of study that is also listed among collective decision-making methods [5, 11, 24]. Argumentation is a rational and persuasive method, but it may be complex and subjective. The group members exchange arguments and counterarguments for or against the alternatives. To resolve conflicting opinions and reach an agreement, the members' opinions are consolidated. Argumentation-based debates address the need to consolidate all the arguments related to the decision to determine the final status of the arguments [24]. The arguments are evaluated based on their acceptability, persuasiveness, strength, and relevance [23, 8, 33, 2, 32]. The structure of arguments and the *relations* between them (such as "support", "attack", or "defeat") are represented by *Argumentation frameworks*.

Unlike structured argumentation (including logic-based argumentation, assumption-based argumentation, and other ones) [10, 15, 7, 21], there is no internal structure for the arguments in abstract argumentation frameworks (e.g., value-based frameworks [23, 4]). Evaluation of the arguments given an argumentation framework is based on extension, labeling, or ranking semantics families. In (3-valued) labeling-based semantics, when the argument is accepted ("in"), rejected ("out"), or unknown ("undec") to the individual [15, 16, 1], arguments are evaluated as accepted, attacked, or

\*Corresponding author

 [reza.karimi@eng.ui.ac.ir](mailto:reza.karimi@eng.ui.ac.ir) (R. Karimi); [fnasiri@eng.ui.ac.ir](mailto:fnasiri@eng.ui.ac.ir) (F. Nassiri-Mofakham)

 <http://eng.ui.ac.ir/~fnasiri> (F. Nassiri-Mofakham)

ORCID(s): 0000-0001-5147-9136 (F. Nassiri-Mofakham)

undecided, while ranking-based semantics determines the acceptability of the arguments by comparing them so that the most acceptable ones gains higher numerical degrees [22].

Labeling is the approach taken in this study. This technique is a way of expressing the preferences of an individual or each group member toward arguments, in which the arguments are accepted, rejected, or undecidable [24].

To guarantee a fair agreement, the preferences of group members concerning each argument should be taken into account. Moreover, to be contradiction-free in practice, this process should pay attention to the relationships between the arguments, where they attack or support each other. Not only in the group, but also in individual decision-making, it is important to apply arguments that survive attack relations. This would require appropriate procedures to combine the arguments considering both concerns.

This study stands over the two following pillars.

*Computational Social Choice* (COMSOC) is a bridge between social and technical sciences as well as classic and modern topics. Social choice is to combine individual preferences into a collective choice [19, 26]. That is, the group takes a vote to reach a consensus on the collective decision. People express their positions on important issues by voting for the preferred options. The voting process is an important and main process in COMSOC [21]. The act of voting is a person's preference for certain candidates, which can be expressed on an ordinal scale (candidates are ranked) or on a cardinal scale (a score is assigned to each candidate). In the simplest model, only the name of the candidate/alternative of interest can be mentioned.

There are many important situations where a group is asked to make a choice, namely: *Management*: to elect the chairman of the board of directors, if a management company is considering new board members, shareholders will receive proxy materials detailing the candidates and can cast their votes accordingly. The process involves various types of shares with differing voting rights, mechanisms for casting votes, and scenarios where shareholder votes are essential for major corporate actions; *Policy-making*: in policy debates, social choice theory aids in selecting the best policy option by considering the diverse preferences and opinions of stakeholders. This helps ensure that the chosen policy aligns with the collective goals of the society; *Resource Allocation*: social choice theory is used in resource allocation decisions, such as distributing resources among different groups or individuals. This helps ensure that resources are allocated fairly and in line with the collective preferences of the society; *Conflict Resolution*: in situations where there are conflicting opinions or values, social choice theory provides a framework for resolving these conflicts fairly and collectively. This helps ensure that the resolution reflects the general will of the society; *Decision-making in Committees*: social choice theory is applied in committee decision-making, where individual preferences are aggregated to determine the best course of action. This helps ensure that the collective decision reflects the diverse perspectives of the committee members; *Voting Systems*: social choice theory is used to design and evaluate voting systems, to ensure that they are fair and representative of the collective preferences of the voters.

Social choice can be implemented using several methods including 'voting rules,' 'raking rules,' 'scoring methods,' and 'consensus' [21, 28, 3]. Each of them has advantages and disadvantages. Voting is a simple and efficient method, but it may not reflect the intensity or diversity of preferences, and it may be subject to strategic manipulation or paradoxes. Ranking is a more expressive and informative method than voting, but it may be more complex and computationally demanding, and it may also face some impossibility results or inconsistency issues. Scoring is a flexible and intuitive method, but it may be affected by the scale or the distribution of the scores, and it may also be influenced by the subjective or relative judgments of the group members. Consensus is a participatory and democratic method, but it may be time-consuming and challenging, and it may also require compromise or concession from some group members [24, 11, 9, 27]. Due to the need for aggregating individual preferences into a collective decision fairly and consistently, given the divergent interests of group members, philosophers, and mathematicians have long grappled with how to best reflect the preferences of a society's members in its collective decisions [14, 20]. The ongoing effort to develop fair, rational, and effective ways to make collective decisions has brought various social choice methods [14, 20]. Social choice methods consolidate different kinds of opinions about each argument from the group members. Depending on the type of agents' voting, these methods can be classified into three categories, which are: labeling-based, Ordinal, and Cardinal. A few of these rules are well known such as majority, plurality, Borda count method, approval, Copeland, veto, runoff voting, and so on (cf. [13], for details).

*Collective argumentation* plays a crucial role in collective decision-making processes. It extends the notion of argumentation from an individual to a collective endeavor, where arguments emerge and evolve through the discourse and interactions of the participants; they are determined by the participants as they interact, rather than being predetermined. It refers to the co-construction of arguments through the interactions among multiple individuals within a group.

Collective argumentation facilitates the co-construction of arguments through group interactions, enabling the resolution of conflicting opinions and the emergence of coherent collective decisions by aggregating individual viewpoints using computational argumentation techniques. Computational argumentation formalizes this dialectical process of reasoning about arguments and counter-arguments, allowing the identification of coherent collective opinions from individual opinions. Argumentation-based debates are mechanisms that a group can use to resolve conflicting opinions and reach an agreement on a decision. Collective argumentation may depend on the representation and aggregation of the arguments. Aggregation functions combine individual opinions about arguments to generate a coherent collective labeling (opinion) on the decision topic, even when individual opinions are conflicting. Some aggregation functions guarantee coherent collective rationality, ensuring the collective opinion is coherent and reasonable [24, 12].

Several problems are considered in collective argumentation, namely:

- Aggregation problems: How to combine the individual arguments or opinions of the group members into a collective argument or decision that reflects the group's view [12, 17].
- Presentation problems: How to model and capture the structure and the content of the arguments or opinions of the group members, as well as the relations and interactions among them [12].
- Evaluation problems: How to measure and compare the quality or the performance of the arguments or opinions of the group members, as well as the outcomes or the results of the collective argumentation process [12, 18].

In this study, we address the problems in the first category mentioned above. In other words, we pay attention to incorporating preferences or values into the aggregation process, and how to elicit this from the input sources. In parallel to providing a fair joint agreement based on the group members' arguments, the relationships (attack or defense) among the arguments need to be taken into account to guarantee a logical outcome in practice, as well [24].

The scope of this study is in labeling-based argumentation frameworks, where the group members express their opinions about arguments over any collective decision or target. By considering the relationship between arguments, an argumentative discussion is achieved through the following steps [24]:

1. One of the agents expresses the purpose of the discussion (norm or goal),
2. Each agent is then allowed to present an argument for or against the target and/or any argument previously presented. This process continues until no agent has any more argument to provide,
3. Agents express their opinions about the presented arguments and goal by assigning labels “in” (accept or like), “out” (not accept or dislike), or “undec” (no decision about the argument or unknown),
4. Agents' opinions are then aggregated to form a consensus on the state of each argument and the state of the target.

### 1.1. State-of-the-art

The state-of-the-art method in collective argumentation in the context of labeling-based argumentation frameworks combines social choice theory and argumentation for computing the collective decision that emerges from a set of arguments and opinions. It is based on the following definitions.

**Definition 1 (Labeling profile [24]).** If  $AG = \{Ag_1, \dots, Ag_n\}$  is the set of agents taking part in a group, a labeling profile is a tuple  $\mathcal{L} = (L_1, \dots, L_n)$ , where  $L_i$  is the labeling encoding the opinion of agent  $Ag_i \in AG$ .

**Definition 2 (Agents ‘in’ [24]).**  $in_{\mathcal{L}}(a) = \left| \{Ag \in AG : L_{Ag}(a) = in\} \right|$  is the number of agents accepting argument  $a$ .

**Definition 3 (Agents ‘out’ [24]).**  $out_{\mathcal{L}}(a) = \left| \{Ag \in AG : L_{Ag}(a) = out\} \right|$  is the number of agents rejecting argument  $a$ .

**Definition 4 (Defending arguments [24]).**  $D(a) = \{b : b \Vdash a\}$  is the set of arguments defending argument  $a$ .

**Definition 5 (Attacking arguments [24]).**  $A(a) = \{b : b \dashv a\}$  is the set of arguments attacking argument  $a$ .

Assume argumentation framework  $AF = \langle \mathcal{A}, \dashv, \Vdash \rangle$ , where  $\dashv \subseteq \mathcal{A} \times \mathcal{A}$  and  $\Vdash \subseteq \mathcal{A} \times \mathcal{A}$  represent attack and defense relationships among arguments in the finite set  $\mathcal{A}$  of arguments. Given  $D(a)$ ,  $A(a)$ ,  $in_{\mathcal{L}}(a)$ , and  $out_{\mathcal{L}}(a)$ , the positive support  $Pro(a)$ , and the negative support  $Con(a)$  of argument  $a \in \mathcal{A}$  are defined as Eq. 1 and Eq. 2.

**Definition 6 (Positive support [24]).** The total number of agents accepting (i.e., labeled “in”) the arguments defending argument  $a$  and the total number of agents rejecting (i.e., labeled “out”) the arguments attacking argument  $a$  positively support argument  $a$ :

$$Pro_L(a) = in_L(D(a)) + out_L(A(a)). \quad (1)$$

**Definition 7 (Negative support [24]).** The total number of agents rejecting (i.e., labeled “out”) the arguments defending argument  $a$  and the total number of agents accepting (i.e., labeled “in”) the arguments attacking argument  $a$  negatively support argument  $a$ :

$$Con_L(a) = out_L(D(a)) + in_L(A(a)). \quad (2)$$

It formally defines aggregation functions to consider dependencies between arguments to ensure that the collective decision is coherent. It consolidates arguments and opinions using aggregation functions based on majority social choice [24]. Moreover, the opinions are considered in two types: *direct* and *indirect*.

**Definition 8 (Direct opinion [24]).** Direct opinion about an argument is the label of the argument.

**Definition 9 (Indirect opinion [24]).** Indirect opinions about an argument are labels of the arguments that attack or defend it.

An argument that attacks or defends an argument is considered the generation of that argument. These arguments are called descendants of the argument. Aggregation functions are divided into two main categories: *with* and *without* considering dependencies. Ignoring dependencies means that it is determined based on the majority of the labels (status) of each argument, which is defined as Eq. 3.

$$M_L(a) = \begin{cases} in, & in_L(a) > out_L(a) \\ out, & in_L(a) < out_L(a) \\ undec, & otherwise \end{cases} \quad (3)$$

Paying attention to dependencies means considering the label of attack and defense arguments, which includes the following three functions, namely:

**Opinion First Function (OF)** is a type of majority function that prioritizes direct opinions over indirect opinions. Therefore, the function first considers direct opinion to obtain a collective opinion about an argument. If using direct opinions results in a tie (equal number of acceptances and rejections), *OF* will use indirect opinions if possible to break the tie (cf. Eq. 4).

**Support First Function (SF)** prioritizes indirect opinions over direct opinions. In this way, it first considers indirect opinions to obtain a collective opinion about an argument. If using indirect opinions results in a tie, *SF* will use direct opinions to resolve the tie if possible, Eq. 5.

**Balanced Function (BF)** balances prioritizing direct opinions or indirect opinions. To this end, it equally combines direct and indirect opinions as Eq. 6. In this equation, *IO* represents an indirect opinion that is expressed about positive and negative supports (i.e., Pro and Con); *DO* is the direct opinion or label of each argument.

$$OF_L(a) = \begin{cases} in, & in_L(a) > out_L(a) \\ in, & in_L(a) = out_L(a) \text{ and } Pro_{OF_L}(a) > Con_{OF_L}(a) \\ out, & in_L(a) < out_L(a) \\ out, & in_L(a) = out_L(a) \text{ and } Pro_{OF_L}(a) < Con_{OF_L}(a) \\ undec, & otherwise \end{cases} \quad (4)$$

$$SF_L(a) = \begin{cases} in, & Pro_{SF_L}(a) > Con_{SF_L}(a) \\ in, & Pro_{SF_L}(a) = Con_{SF_L}(a) \text{ and } in_L(a) > out_L(a) \\ out, & Pro_{SF_L}(a) < Con_{SF_L}(a) \\ out, & Pro_{SF_L}(a) = Con_{SF_L}(a) \text{ and } in_L(a) < out_L(a) \\ undec, & otherwise \end{cases} \quad (5)$$

$$\begin{aligned}
BF_{\mathcal{L}}(a) &= \begin{cases} in, & IO_{\mathcal{L}}(a) + DO_{\mathcal{L}}(a) > 0 \\ out, & IO_{\mathcal{L}}(a) + DO_{\mathcal{L}}(a) < 0 \\ undec, & IO_{\mathcal{L}}(a) + DO_{\mathcal{L}}(a) = 0 \end{cases} \\
IO_{\mathcal{L}}(a) &= \begin{cases} 1, & Pro_{BF_{\mathcal{L}}}(a) > Con_{BF_{\mathcal{L}}}(a) \\ 0, & Pro_{BF_{\mathcal{L}}}(a) = Con_{BF_{\mathcal{L}}}(a) \\ -1, & Pro_{BF_{\mathcal{L}}}(a) < Con_{BF_{\mathcal{L}}}(a) \end{cases} \\
DO_{\mathcal{L}}(a) &= \begin{cases} 1, & in_{\mathcal{L}}(a) > out_{\mathcal{L}}(a) \\ 0, & in_{\mathcal{L}}(a) = out_{\mathcal{L}}(a) \\ -1, & in_{\mathcal{L}}(a) < out_{\mathcal{L}}(a) \end{cases}
\end{aligned} \tag{6}$$

The approach is explained in Example 1. This example assumes the following presuppositions and assumptions:

- The problem has a single norm (target),
- The target neither attacks nor defends the argument,
- In the argumentation graph, each argument attacks only one argument (other than the target),
- In the argumentation graph, each argument defends only one argument (other than the goal),
- All arguments are clear to everyone.

**Example 1 (marketing campaign).** *A company is planning a sponsored search marketing campaign [29] for a new line of eco-friendly home cleaning products. Finn, Sal, and Mark as the directors of the Finance, Sales, and Marketing departments of the company have different ideas for implementing this campaign with “budget allocation and performance tracking”, “promotional offers and landing pages”, and “keyword optimization and ad copy” perspectives, respectively.*

*The norm or rule is:*

$N =$  “The company should run a sponsored search marketing campaign, this year.”

*The three arguments are:*

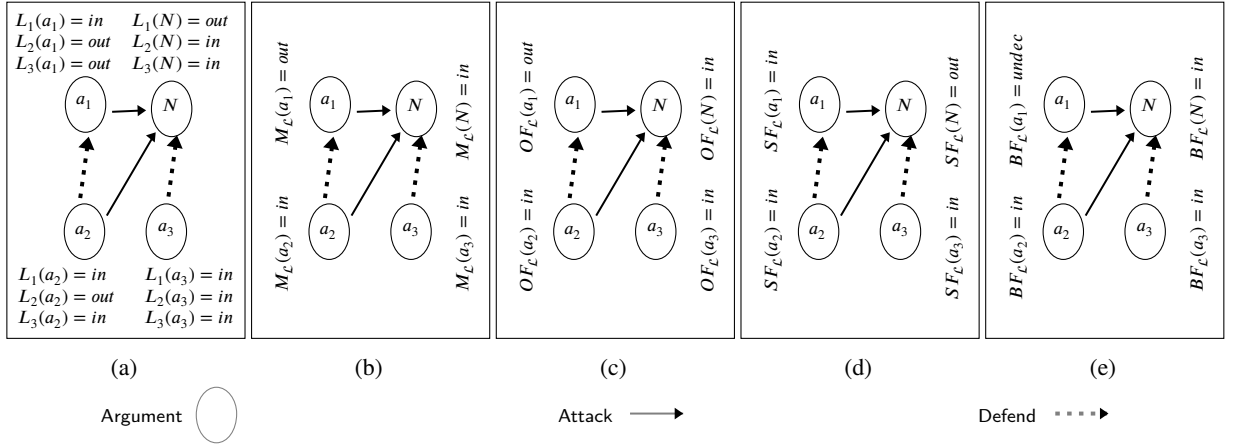
$a_1 =$  This campaign needs a large investment, but the Finance department cannot allocate a budget for this program, this year.

$a_2 =$  The Sales department does not have a team to implement promotional offers and optimized landing pages to convert traffic into sales through this campaign, this year.

$a_3 =$  The marketing department is ready for keyword research and compelling ad copy to attract clicks for eco-friendly home cleaning products.

*Considering this situation, the question that arises is whether the departments should accept this marketing norm or not. In other words, how should they gather their individual opinions into a consensus opinion and how will be the balance of the opinions?*

In this example, the persons’ labeling (direct opinions) creates the argumentation graph as depicted in Fig 1a. Arguments  $a_1$  and  $a_2$  are against  $N$ , while  $a_3$  and  $a_2$  respectively favor  $N$  and  $a_1$  because the department that does not have enough budget prefers to change the plan. Agents’ opinions about the goal and arguments are indicated in Table 1a. Finn ( $Ag_1$ ) cannot allocate a budget, so he rejects the target by assigning it an “out” label and accepts arguments  $a_1$  and  $a_2$  by labeling them “in”. However, he accepts  $a_3$  and labeled it by an “in”. On the other hand, Sal ( $Ag_2$ ) is ready to follow the campaign and is clearly in favor of norm  $N$ . As a result, she accepts both; accepts norm  $N$  and argument  $a_3$  and rejects arguments  $a_1$  and  $a_2$ , which are against  $N$ . Finally, Mark ( $Ag_3$ ) is interested in marketing campaigns and therefore accepts norm  $N$  and argument  $a_3$  and rejects argument  $a_1$ . However, he likes to receive more budgets, so he accepts  $a_2$ .



**Figure 1:** State-of-the-art collective labeling-based argumentation: (a) Graph representation of direct and indirect opinions of the agents in Example 1; and Aggregated labeling (and decision over target  $N$ ) computed by (b)  $M_L$ , (c)  $OF_L$ , (d)  $SF_L$ , and (e)  $BF_L$  (adapted from [24]).

**Table 1**

Collective argumentation in Example 1: (a) Opinions of three agents about a group schedule and their arguments [24], (b) Aggregated labeling computed by  $M_L$ ,  $OF_L$ ,  $SF_L$ , and  $BF_L$

Agents	Arguments				Argument	$out_L$	$in_L$	$M_L$	$OF_L$	$SF_L$	$BF_L$
	$N$	$a_1$	$a_2$	$a_3$							
$Ag_1 : Finn$	✗	✓	✓	✓	$N$	1	2	in	in	out	in
$Ag_2 : Sal$	✓	✗	✗	✓	$a_1$	2	1	out	out	in	undec
$Ag_3 : Mark$	✓	✗	✓	✓	$a_2$	1	2	in	in	in	in
					$a_3$	0	3	in	in	in	in

Employing Eq. 3 on Fig. 1a results in the collected label of each argument using the *Majority* aggregation function  $M_L$  as shown in Table 1b and Fig. 1b. In this example, the number of “in” or “out” labels is not equal. Hence, the first and the last condition in Eq. 4 cause  $OF_L$  results in the same outcome as  $M_L$  (Fig. 1c). The result of  $SF_L$  function for each argument in the marketing campaign example is determined in Fig. 1d.  $SF_L$  considers indirect opinions first. Since the arguments  $a_2, a_3$  have no generation, their collective labeling follows from the majority in the direct opinion, hence,  $SF_L(a_2) = SF_L(a_3) = in$ . In the case of argument  $a_1$ ,  $SF_L$  first considers the collective labeling of  $a_2$ , which is *in*, and so  $SF_L(a_1) = in$ . Target  $N$  is attacked by arguments  $a_1, a_2$  (both collectively labeled “in”), and  $a_3$  labeled “in”. Therefore, argument  $N$  has two attackers and one defender; that is, its negative support is more than its positive support. Thus,  $SF_L(N) = out$  according to Eq. 5.  $BF_L$  balances prioritizing between direct and indirect opinions by equally combining them using Eq. 6. Finally, the outcome of  $BF_L$  in the marketing campaign example is as illustrated in Fig. 1e.

## 1.2. Research objective

In argumentation-based debates in which a goal or a norm is discussed by group members, agents with different points of view present their arguments about the norm. In argumentation-based debates the agents in addition to expressing their arguments, express their opinions in the form of labelling framework about each argument and goal raised in the group discussion. The label for each argument can be “in” meaning the agent likes the argument, “out” the agent does not like the argument, or “undec” the agent is undecided about the argument. Determining the outcome as the final decision of the group in which the agents have different preferences is a important and challenge. In other words, the question that is raised is, what should be combined in the argument-based debates where the arguments and opinions of the agents are raised, so that all the preferences of the group members are reflected to a large extent and the final result is determined. In response to this question, the existing method deals with only one method from the set of social choice methods and has not paid attention to other social choice methods. The main objective of this paper is to

design and develop new aggregation functions with a view to other social choice methods. The outcome of each of these functions determines whether the group's goal should be accepted by the members, rejected, or the decision about it is undecided.

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### 1.3. Contribution of the study

The focus of this study is on collective labeling-based argumentation based on social choice. We consider the problems of *what to aggregate* and *how to aggregate*. We address how to combine the individual arguments or opinions of the group members into a collective argument or decision that reflects the group's view. The state-of-the-art approach to collective labeling-based argumentation brings graceful variations to majority rule. It also considers relations among arguments, which were naturally ignored in this social choice method. Our contribution follow:

1. We advance the state-of-the-art one step further by proposing promising functions based on dozens of social choice methods.
2. Our functions could potentially express and calculate the outcome based on the group members' arguments, and the relationships (attack or defense) among the arguments.
3. We present variations of these functions by taking into account direct and indirect opinions as well as constraints over incoming opinions to guarantee consistent and contradiction-free outcomes.
4. We present new measures which define the behavior of aggregation functions.
5. We discuss, prove, and compare the properties of our proposed functions.

To the best of our knowledge, this is the first study that proposes a collective labeling-based argumentation by extending multiple social choice methods and evaluating their collective behavior.

### 1.4. Organization of the paper

After preliminaries on computational social choice, argumentation, collective argumentation, and state-of-the-art collective labeling-based argumentation, we present a collection of group decision-making functions by proposing numerous collective labeling-based argumentation using several social choice rules in Section 2; we extend nine aggregation rules. To guarantee consistent and contradiction-free outcomes, they combine the arguments by considering relationships between arguments in addition to the preferences of group members concerning each argument. In Section 3, we analyze and discuss our proposed collective argumentation functions, where we present some new measures for describing the collective behavior of the aggregation functions. Section 4 concludes and opens some directions to future works.

## 2. Collective labeling-based argumentation

We extend the state-of-the-art collective aggregation method (cf. §1.1). We consider the following within the labeling-based argumentation framework:

1. Relationships between arguments (i.e., argumentation graph with labels, e.g., Fig. 1a),
2. Preferences of group members concerning each argument (i.e., both direct and indirect opinions, e.g., Table 1a).

The state-of-the-art labeling-based aggregation method, which is based on the *majority* aggregation rule, is not easily applicable to all social choice strategies. A few of these rules need to elicit the members' preferences, a few of them are repeated in multiple steps, and some others impose some constraints over incoming opinions. Table 2 demonstrates labeling-based social choice methods according to their characteristics and the challenges to adapt with labeling-based collective argumentation. We propose a plethora of collective argumentation functions based on dozens of labeling-based social choice methods addressing these challenges. As shown in Table 2, these social choice methods are labeling-based, hence, already are ready to extend into collective labeling-based argumentation.

In this section, we present related functions for this collective labeling-based argumentation based on inherent labeling-based social choice methods. As illustrated in Table 2, they are *Borda*, *Copeland*, *veto*, *cumulative*, *Kemeney*, *pairwise comparison*, *Hare*, *Simpson*, *majority runoff*, and *truncation*<sup>1</sup>. Our functions are based on direct opinions (cf.

<sup>1</sup>The *Majority* is the method developed in state-of-the-art (§1.1).

**Table 2**

Characteristics of labeling-based social choice methods regarding type of and relations among votes on arguments, requirement for preference elicitation, and one-shot or repeating implementation.

Method	Relation		Pref. Elicit.	Repeating	Constrained
	Direct	Indirect			
Borda	+	+	-	-	-
Copeland	+	+	-	-	-
Cumulative	+	+	-	-	+
Hare	+	+	+	-	-
Kemeny	+	+	+	-	-
Majority	+	-	-	-	-
Majoritarian run-off	+	-	-	+	-
Pairwise comparison	+	+	+	-	-
Simpson	+	+	+	-	-
Truncation	+	+	-	-	+
Veto	+	+	-	-	-

Definition 8). When the outcome of the function based on direct opinions does not result in a label, we consider indirect opinions using Definition 10 and Definition 11; otherwise, Definition 9 works out well.

**Definition 10 ( $Score_D$ ).** *It is the sum of all the scores of all defenders of the argument of  $a$ :*

$$Score_D(a) = \sum_{c \in D(b)} score_c(a)$$

**Definition 11 ( $Score_A$ ).** *It is the sum of all the scores of all attackers to the argument of  $a$ :*

$$Score_A(a) = \sum_{c \in A(b)} score_c(a)$$

## 2.1. Borda-based collective argumentation

Borda method has a high potential for aggregating arguments<sup>2</sup>. However, to make it applicable for aggregating labeling-based arguments we extend it by proposing the following approaches.

**ABORDA<sup>S</sup>.** The agents take turns based on their Seniority, hence the score of their arguments. ABORDA<sup>S</sup> function is presented as Eq. 8, where  $a$  is an argument and  $Score_{ABORDA^S}(a)$  is earned through Eq.7. This function only considers direct opinions and does not pay attention to indirect opinions. It is not very efficient because we have to consider the relationships between the arguments. Table 3 represents one sample situation, where  $a_1$ ,  $a_2$ , and  $a_3$  are in positions 1, 2, and 3, respectively, since Mark is the senior and Finn is the junior in the {Finn, Sal, and Mark} committee. The goal( $N$ ) is introduced first and is the basis of the group discussion, i.e., the most important argument, then it is assigned position 4. In this table, each group member expressed his/her opinion about the arguments by labeling ‘in’ (✓), ‘out’ (✗), and ‘undec’ (!).

$$Score_{ABORDA^S}(a) = \left\{ a : \sum_{Ag \in Ag} Rank_{Ag}(a), Opinion_{Ag}(a) = \checkmark \right\} \quad (7)$$

$$ABORDA^S(a) = \begin{cases} in, & Score_{ABORDA^S}(a) > threshold \\ out, & Score_{ABORDA^S}(a) < threshold \\ undec, & Score_{ABORDA^S}(a) = threshold \end{cases} \quad (8)$$

<sup>2</sup>For a description of this social choice rule, cf. [13].

**Table 3**Agents' labels in Example 1 using ABORDA<sup>S</sup> method

Argument Agents	Rank			
	1	2	3	4
	$a_1$	$a_2$	$a_3$	$N$
Finn	✓	✓	✓	✗
Sal	✗	✗	✓	✓
Mark	✗	✓	✓	✓
<b>Total Score</b>	1	4	9	8

The argument's rank could be determined based on the number of attacks and defenses from the argument, which of course may lead to multiple draws. In this case, it is reasonable to say that the argument that is more defended or less attacked should be ranked the highest,  $|A|$ , concerning the arguments that are more attacked and less defended, ranked as 1.

**ABORDA<sup>SDI</sup>**. In this version of Borda, in addition to direct opinion to argument  $a$ , indirect opinions are also considered by adding  $Score_D(a)$  and subtracting  $Score_A(a)$  (See Eq. 9):

$$ABORDA^{SDI}(a) = \begin{cases} in, & Score_{ABORDA^S}(a) + Score_D(a) - Score_A(a) > threshold \\ out, & Score_{ABORDA^S}(a) + Score_D(a) - Score_A(a) < threshold \\ undec, & Score_{ABORDA^S}(a) + Score_D(a) - Score_A(a) = threshold \end{cases} \quad (9)$$

**ABORDA<sup>P</sup>**. In this approach, agents express their opinions (labeling) on all permutations of the arguments' ranks. The opinions are then aggregated for each argument. Fig. 4 shows how this works for Example 1. In this figure,  $n$  is the number of arguments, which equals 4. In ABORDA<sup>P</sup>, the score of each argument is calculated per each permutation (i.e., any ranking of the arguments), and then the final score of each argument (from all permutations) is determined. This approach does not use indirect opinions.

$$ABORDA^P(a) = \begin{cases} in, & Score_{ABORDA^P}(a) > threshold \\ out, & Score_{ABORDA^P}(a) < threshold \\ undec, & Score_{ABORDA^P}(a) = threshold \end{cases} \quad (10)$$

**ABORDA<sup>PDI</sup>**. It is another version of ABORDA<sup>P</sup> which also takes into account the indirect opinions to an argument  $a$  by adding  $Score_D(a)$  and subtracting  $Score_A(a)$  (See Eq. 11):

$$ABORDA^{PDI}(a) = \begin{cases} in, & Score_{ABORDA^P}(a) + Score_D(a) - Score_A(a) > threshold \\ out, & Score_{ABORDA^P}(a) + Score_D(a) - Score_A(a) < threshold \\ undec, & Score_{ABORDA^P}(a) + Score_D(a) - Score_A(a) = threshold \end{cases} \quad (11)$$

## 2.2. Copeland-based collective argumentation

In this method, after presenting the agents' (direct) opinions about the arguments, they are compared pairwise from different aspects, presented below.

**ACOP<sup>D</sup>**. The arguments are compared pairwise according to the number of labels "in". This comparison is shown in the ACOP<sup>D</sup> matrix. In this matrix, the argument located in the column is compared with the argument located in the row. An argument  $a$  with more "in" labels will get +1 points, otherwise, it will get -1 points (if there is no label "in", it is considered the smallest "out"). Eq. 12 shows the ACOP<sup>D</sup> function, which only considers *Direct* opinions.

**Table 4**  
ABORDA<sup>P</sup> collective argumentation of Example 1 (among  $n = 3$  agents)

Argument	Rank				Rank	Rank	Rank	...	Rank
	1	2	3	4					
Agent	$N$	$a_1$	$a_2$	$a_3$	$N$	$a_3$	$a_1$	$a_2$	$N$
Finn	X	✓	✓	X	X	X	X	✓	✓
Sal	✓	X	X	✓	✓	✓	X	✓	X
Mark	✓	✓	X	✓	✓	X	✓	✓	✓
Total point	2	4	3	8	2	2	3	12	0

$n!$

**Table 5**  
ABORDA collective argumentation of Example 1

Argument	ABORDA <sup>S</sup>	ABORDA <sup>S</sup> Label	ABORDA <sup>SDI</sup>	ABORDA <sup>SDI</sup> Label	ABORDA <sup>P</sup>	ABORDA <sup>PDI</sup>
$N$	8	in	12	in		
$a_1$	1	undec	5	in	$\mathcal{O}(n!)$	$\mathcal{O}(n!)$
$a_2$	4	in	4	in		
$a_3$	9	in	9	in		

**Table 6**  
ACOP<sup>D</sup> & ACOP<sup>I</sup> matrices of Example 1

	$N$	$a_1$	$a_2$	$a_3$	Arguments	$N$	$a_1$	$a_2$	$a_3$	
$N$	0	0	0	1		0	0	0	0	$N$
$a_1$	1	0	1	1		-1	0	0	0	$a_1$
$a_2$	0	0	0	1		-1	1	0	0	$a_2$
$a_3$	0	0	0	0		1	0	0	0	$a_3$
ACOP <sup>D</sup> score	1	0	1	3		-1	1	0	0	ACOP <sup>I</sup> score

$$ACOP^D(a) = \begin{cases} in, & Score_{ACOP^D}(a) > threshold \\ out, & Score_{ACOP^D}(a) < threshold \\ undec, & Score_{ACOP^D}(a) = threshold \end{cases} \quad (12)$$

Table 6 shows the ACOP<sup>D</sup> matrix of Example 1. It represents the pairwise comparison of arguments based on the number of labels “in”. Summing up the scores in each column, the “ACOP<sup>D</sup> score” of every argument is obtained.

ACOP<sup>DI(ATT/DEF)</sup>. This method considers *indirect* opinions besides direct ones in two ways.

The first approach forms an ACOP<sup>I</sup> matrix by counting the number of attacks and defenses. In this way, if the argument is attacked, it gets -1 point and if it is defended, it gets +1 point, as formalized in Eq.13.

$$ACOP^{DI(ATT/DEF)}(a) = \begin{cases} in, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) > threshold \\ out, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) < threshold \\ undec, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) = threshold \end{cases} \quad (13)$$

Table 6 depicts the ACOP<sup>I</sup> matrix of Example 1. It illustrates the pairwise comparison of arguments based on the number of attacks and defenses. Summing up the scores in each column, the “ACOP<sup>I</sup> score” of the related argument is obtained. For example,  $a_1$  has attacked  $N$ , so  $N$  and  $a_1$  respectively get -1 and 0 in the corresponding matrix.

Since each argument must have a single label, the second approach uses the majority regarding the positive and negative support of the argument. If its positive support is greater than its negative support, the argument is labeled “in”, which corresponds to 1, and 0 otherwise. Eq. 14 represents this function for determining the label of each argument  $a$ .

**Table 7**Computing  $Pro_M$  &  $Con_M$  to get indirect opinions scores of Example 1

Argument	$Pro_M$	$Con_M$	Score $_{Pro/Con}$
$N$	1	0	1
$a_1$	0	0	0
$a_2$	0	0	0
$a_3$	2	1	1

**Table 8**

ACOP collective argumentation in Example 1

Argument	ACOP <sup>D</sup>	ACOP <sup>D</sup> Label	ACOP <sup>DI(ATT/DEF)</sup>	ACOP <sup>DI(ATT/DEF)</sup> Label	ACOP <sup>DI(PRO/CON)</sup>	ACOP <sup>DI(PRO/CON)</sup> Label
$N$	1	undec	0	out	2	in
$a_1$	0	out	1	undec	0	out
$a_2$	1	undec	1	undec	1	undec
$a_3$	3	in	3	in	4	in

$$ACOP^{DI(PRO/CON)}(a) = \begin{cases} in, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) > threshold \\ out, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) < threshold \\ undec, & Score_{ACOP^D}(a) + Score_{ACOP^I}(a) = threshold \end{cases} \quad (14)$$

Table 7 illustrates the Score $_{Pro/Con}$  of each argument by comparing its Pro and Con obtained by also considering the indirect opinions.

Table 8 shows all methods based on ACOP in Example 1.

### 2.3. Veto-based collective argumentation

We introduce this method in two versions, with and without considering indirect opinions about arguments.

**ARGVET<sup>D</sup>.** In veto voting (cf. [13]), it is enough to count the number of “out”s to each argument. The argument that has the least number of “out”s will win (i.e., its final label will be “in”). Actually, it looks only at the number of “out”s. In Example 1, argument  $a_3$  has no “out” label, so the final label is “in”, and the rest of the arguments take the “out” label (See Eq. 15).

$$ARGVET^D(a) = \begin{cases} in, & \text{if } a = \operatorname{argmin}_{b \in A} out(b) \\ out, & \text{if } a = \operatorname{argmax}_{b \in A} out(b) \end{cases} \quad (15)$$

**ARGVET<sup>DI</sup>.** By adding *indirect* opinions to ARGVET<sup>D</sup>, in addition to counting the number of “out” to each argument, the number of “out” attacking and defending arguments against/for each argument is also counted and added to it. Then, according to function Eq. 16, the label of each argument  $a$  is determined.

$$ARGVET^{DI}(a) = \begin{cases} in, & \text{if } a = \operatorname{argmin}_{b \in A} out(b) + \sum c \in A(b)out(c) - \sum c \in D(b)out(c) \\ out, & \text{if } a = \operatorname{argmax}_{b \in A} out(b) + \sum c \in A(b)out(c) - \sum c \in D(b)out(c) \end{cases} \quad (16)$$

All methods based on ARGVET on Example 1 are shown in Table 9.

### 2.4. Cumulative-based collective argumentation

This method can be defined by assigning a fixed value (coupon) for each agent. For example, we can assume a fixed number of four votes for each agent, including two “in” and two “out” votes that the agent can label. In cumulative voting, each agent divides a fixed number of points (i.e., a specified number of labels) among all arguments. Table 10 depicts the voting of agents in Example 1 in a cumulative way. The distribution of points between arguments can be

**Table 9**

ARGVET collective argumentation in Example 1

Argument	ARGVET <sup>D</sup>	ARGVET <sup>D</sup> Label	ARGVET <sup>DI</sup>	ARGVET <sup>DI</sup> Label
$N$	1	out	4	out
$a_1$	2	out	1	out
$a_2$	1	out	1	out
$a_3$	0	in	0	in

**Table 10**

ACUMUL collective argumentation in Example 1

Arguments Agents	$N$	$a_1$	$a_2$	$a_3$
<i>Finn</i>	X	✓	✓	X
<i>Sal</i>	X	X	!	✓✓
<i>Mark</i>	✓	✓	X	X

**Table 11**

AKEMEN collective argumentation in Example 1

(a) Counting the number of preferences

All possible pairs of arguments	Number of votes with indicated preferences		
	Prefer X over Y	Equal preference	Prefer Y over X
$X = a_1, Y = a_2$	0	2	1
$X = a_1, Y = a_3$	0	1	2
$X = a_1, Y = N$	1	0	2
$X = a_2, Y = a_3$	0	2	1
$X = a_2, Y = N$	0	0	0
$X = a_3, Y = N$	1	2	0

(b) Score of the sequence of preferences using the AKEMEN method

Preference	Score
$N > a_2$	1
$N > a_1$	2
$N > a_3$	0
$a_2 > a_1$	1
$a_2 > a_3$	0
$a_1 > a_3$	0
$N > a_1 > a_3$	2
...	-
$N > a_2 > a_1 > a_3$	4

considered a way of expressing the intensity of the vote because, in addition to voting for the favorite argument, the points are distributed among the arguments. Afterward, any other aggregation function can be applied to this table. For example, using the Majority method, the arguments that get the highest votes are accepted (i.e., get the label “in”).

## 2.5. Kemeny–Young based collective argumentation

In this method, there is a need for ranking preferences of the agents on the arguments. We extract all possible preferences from the agents’ opinions over the arguments (see Fig. 2) and apply the AKEMEN method to them.

In the AKEMEN method, counting all pairwise comparisons of the arguments are arranged in three columns according to their preferences (Prefer X over Y, Equal preference, and Prefer Y over X). Then, after finishing the counting, the score of all possible sequences is calculated, which shows how many times that sequence holds (how many times it is preferred). To calculate the score of a sequence, it is enough to calculate the pairwise preference scores of the arguments in that sequence according to Table 11a. Finally, after calculating all the pairwise preferences, all the scores are summed up (see Table 11b). The sequence with the highest point is the winner. In this sequence, the argument that is at the beginning of the sequence is labeled “in” and the rest is labeled “out”. For example, the score of the sequence of preferences  $a_1 > N > a_2 > a_3$  equals 3 and  $N > a_2 > a_1 > a_3$  equals 4 (see Table 11b). Table 12 shows all methods based on AKEMEN in Example 1. Since arguments  $a_2, a_3$  have no descendants, their collective labellings based on  $AKEMEN^{DI}(a)$  stem from the  $AKEMEN^D(a)$ , and hence,  $AKEMEN^{DI}(a_2) = \text{“out”}$  and  $AKEMEN^{DI}(a_3) = \text{“out”}$

$$AKEMEN^D(a) = \begin{cases} in, & \text{First argument at the beginning of the sequence} \\ out, & \text{Otherwise} \end{cases} \quad (17)$$

**Table 12**

AKEMEN collective argumentation in Example 1

Argument	AKEMEN <sup>D</sup>	AKEMEN <sup>DI</sup>
$N$	in	out
$a_1$	out	out
$a_2$	out	out
$a_3$	out	out

$$Ag_1 : a_1 \sim a_2 \sim a_3 > N$$

$$Ag_2 : N \sim a_3 > a_1 \sim a_2$$

$$Ag_3 : N \sim a_2 \sim a_3 > a_1$$

**Figure 2:** Preferences elicited from the agents' opinions in Table 1a.

$$AKEMEN^{DI}(a) = \begin{cases} in, & Pro_{AKEMEN^D} > Con_{AKEMEN^D} \\ out, & Pro_{AKEMEN^D} < Con_{AKEMEN^D} \\ undec, & Pro_{AKEMEN^D} = Con_{AKEMEN^D} \end{cases} \quad (18)$$

## 2.6. Pairwise-comparison/Hare-based collective argumentation

In these two methods, it is necessary to have a preference ordering of agents over arguments. Ordinal preferences of the agents are extracted from their opinions (i.e., “in” and “out” votes), so that any arguments  $a_1, \dots, a_m$  with similar opinions are considered indifferent ( $\sim$ ), and the arguments having “in” labels are preferred ( $>$ ) to those with labels “out”.

According to the opinions of the agents in Example 1 depicted in Table 1a, the preferences of the agents are extracted as illustrated in Fig. 2.

Every argument that is preferred to another argument is assigned +1, and -1 if not preferred. These numbers are summed up per each argument. That is, per each argument, the number of times it is preferred or not preferred to another argument is counted. The argument with the highest score is chosen. This can be done with/without a decision threshold for  $score_p$  of argument  $a$ , Eq. 19.

$$APREF^D(a) = \begin{cases} in, & Score_{APREF^p} > threshold \\ out, & Score_{APREF^p} < threshold \\ undec, & Score_{APREF^p} = threshold \end{cases} \quad (19)$$

In the next step, scoring can be done in three ways:  $APREF^{MLD}$  and  $APREF^{MD}$  when they consider only direct opinions in counting the most and least number of preferences regarding the argument, and  $APREF^{DI}$  which takes into account both direct and indirect opinions, where this can be achieved by either  $APREF^{MLD}$  or  $APREF^{MD}$ .

The  $APREF^{MLD}$  approach considers both the *Most preferred* and the *Least non-preferred* scores per each argument. For example, as shown in Fig. 2,  $N$  is placed once in the non-preferred part and twice in the preferred part, so  $Score(N) = 2(1) + 1(-1) = 1$ . Table 13 illustrates the score of the arguments.

$APREF^{MD}$  approach is similar to  $APREF^{MLD}$  but it ignores the non-preferred part of the preferences in counting the scores; that is, per each argument, the number of times it is preferred to another argument is not counted. In Table 1a which shows the opinions of the agents in Example 1, Eq. 19 and Fig. 2 results in the scores and labels depicted in Table 13. For example,  $N$  is preferred only twice –for the second and third agents– as we ignore its positions as the least preferred ones.

**Table 13**

APREF collective argumentation in Example 1

Argument	APREF <sup>MLD</sup>	APREF <sup>MLD</sup> Label	APREF <sup>MLD</sup> Label with threshold = 30%	APREF <sup>MD</sup>	APREF <sup>MD</sup> Label	APREF <sup>MD</sup> Label with threshold = 30%
$N$	1	out	undec	2	out	in
$a_1$	-1	out	out	1	out	undec
$a_2$	1	out	undec	2	out	in
$a_3$	3	in	in	3	in	in

Despite APREF<sup>MD</sup> and APREF<sup>MLD</sup>, indirect arguments (parent arguments) are also used by APREF<sup>DI</sup> to determine the final status of each argument  $a$  (Eq. 20). We add the preference score based on the attack and defense of each argument  $a$  (i.e., indirect opinions) to  $score_p$  of the argument gained by using only direct opinion through either APREF<sup>MD</sup> or APREF<sup>MLD</sup>, Eq. 20.

$$APREF^{DI}(a) = \begin{cases} in, & (Score_p(a) + Score_D(a) - Score_A(a)) > threshold \\ out, & (Score_p(a) + Score_D(a) - Score_A(a)) < threshold \\ undec, & (Score_p(a) + Score_D(a) - Score_A(a)) = threshold \end{cases} \quad (20)$$

## 2.7. Simpson-based collective argumentation

The agents' preferences over arguments are elicited (similar to APREF). In the Simpson voting method, the number of agents who prefer argument  $a$  to argument  $b$  is represented by  $C(a, b)$ . The "Maximin" score of argument  $a$  is denoted by  $S(a)$  and is calculated by Eq. 21, which is the lowest score it gets in each pairwise election. The score of a pairwise election is equal to the number of agents who prefer  $a$  (over  $b$ ). The bigger the "Maximin" score of an argument, the higher the argument rank.

$$S(a) = \min_{a \neq b} C(a, b) \quad (21)$$

Argument  $a$  gaining the maximum score is assigned the label "in", otherwise, "out", Eq. 22.

$$ASIMP^D(a) = \begin{cases} in, & \text{if } a = \operatorname{argmax}_b S(b) \\ out, & \text{otherwise} \end{cases} \quad (22)$$

$ASIMP^{DI}$ . To add indirect opinions, we use Pro and Con according to Eq. 23.

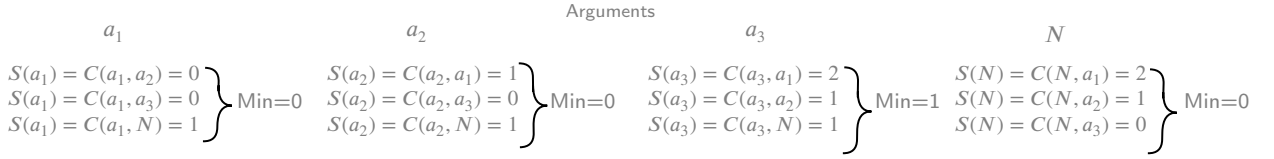
$$ASIMP^{DI}(a) = \begin{cases} in, & Pro_{ASIMP^D} > Con_{ASIMP^D} \\ out, & Pro_{ASIMP^D} < Con_{ASIMP^D} \\ undec, & Pro_{ASIMP^D} = Con_{ASIMP^D} \end{cases} \quad (23)$$

Given the preferences of the agents in Example 1 (Fig. 2), and according to Eq. 22 and Eq. 23, the label of each argument is determined using ASIMP as shown in Fig. 3 and Table 14.

Since arguments  $a_2, a_3$  have no descendants, their collective labellings based on  $ASIMP^{DI}(a)$  stem from the  $ASIMP^D(a)$ , and hence,  $ASIMP^{DI}(a_2) = \text{"out"}$  and  $ASIMP^{DI}(a_3) = \text{"in"}$

## 2.8. Majoritarian-runoff-based collective argumentation

In this method, agents declare their desired argument by presenting a label for each argument. If an argument gets the majority of votes, that argument is accepted for everyone and the rest of the arguments are assigned the "out" label as a sign of non-acceptance. Otherwise, the top two arguments (having the highest number of "in") will be placed in the secondary simple majority election. One of the two arguments that gets the most votes in the second election will be declared the winner. In Example 1, this situation does not occur because  $a_3$  wins by having three "in" labels (the majority is only congruent to one argument).



**Figure 3:** Comparing each argument with other arguments in Example 1 according to ASIMP method

**Table 14**

ASIMP collective argumentation in Example 1

Argument	$s(a)$	ASIMP <sup>D</sup> Label	ASIMP <sup>DI</sup> Label
$N$	0	out	out
$a_1$	0	out	out
$a_2$	0	out	out
$a_3$	1	in	in

### 2.9. Truncation-Borda-based collective argumentation

Truncation means that not all arguments need to be scored: only the top set of arguments is scored (e.g., the top three arguments), while the rest are given zero points. This method can be a suitable method for times when the number of arguments is large. Then, with each of the mentioned methods, we can continue the process of aggregating arguments.

## 3. Discussion

We proposed 20 group decision-making functions in Section 2. We discuss their properties in Section 3.2 based on the measures introduced in Section 3.1.

### 3.1. Measures

Appendix A explains the required properties for a social choice rule. They are Exhaustive Domain (ED), Anonymity (A), Non-Dictatorship (ND), Direct Unanimity (DU), Endorsed Unanimity (EU), Supportiveness (S), Monotonicity (M), Binary Monotonicity (BM), Familiar Monotonicity (FM), and Independence (I).

In addition to the above-mentioned properties, here we present new concepts to define the behavior of collective labeling-based aggregation methods. Assume  $\mathcal{F} = \{f : f \text{ is a collective labeling-based argumentation function}\}$ . For analyzing the behavior of collective labeling-based argumentation functions  $f \in \mathcal{F}$ , we present a higher level labeling-based aggregation over functions, which determines a single collective outcome. To this end, we adapt Definition 2 and Definition 3 for counting the number of ‘in’, ‘out’, and ‘undec’ votes of the functions  $f$  to a given target  $N$ .

**Definition 12 (Functions ‘in’).**  $in_{\mathcal{L}}^{\mathcal{F}}(N) = \left| \{f \in \mathcal{F} : L_f(N) = in\} \right|$  is the number of functions accepting target  $N$ .

**Definition 13 (Functions ‘out’).**  $out_{\mathcal{L}}^{\mathcal{F}}(N) = \left| \{f \in \mathcal{F} : L_f(N) = out\} \right|$  is the number of functions rejecting target  $N$ .

**Definition 14 (Functions ‘undec’).**  $undec_{\mathcal{L}}^{\mathcal{F}}(N) = \left| \{f \in \mathcal{F} : L_f(N) = undec\} \right|$  is the number of functions that are undecided about target  $N$ .

$$Collective\ Outcome(N) = \begin{cases} in, & (in_{\mathcal{L}}^{\mathcal{F}}(N) > out_{\mathcal{L}}^{\mathcal{F}}(N)) \text{ and } (undec_{\mathcal{L}}^{\mathcal{F}}(N) < \min \{in_{\mathcal{L}}^{\mathcal{F}}(N), out_{\mathcal{L}}^{\mathcal{F}}(N)\}) \\ out, & (out_{\mathcal{L}}^{\mathcal{F}}(N) > in_{\mathcal{L}}^{\mathcal{F}}(N)) \text{ and } (undec_{\mathcal{L}}^{\mathcal{F}}(N) < \min \{in_{\mathcal{L}}^{\mathcal{F}}(N), out_{\mathcal{L}}^{\mathcal{F}}(N)\}) \\ undec, & (undec_{\mathcal{L}}^{\mathcal{F}}(N) > \max \{in_{\mathcal{L}}^{\mathcal{F}}(N), out_{\mathcal{L}}^{\mathcal{F}}(N)\}) \text{ or } (in_{\mathcal{L}}^{\mathcal{F}}(N) = out_{\mathcal{L}}^{\mathcal{F}}(N)) \end{cases} \quad (24)$$

When  $Collective\ outcome(N) = \text{‘in’}$ , ‘out’, or ‘undec’, the agents have been respectively cooperative, competitive, or neutral regarding target  $N$  in the scenario. It is worth mentioning that this is without any respect to the good or bad nature of the target itself (i.e.,  $N$  might be a good habit, but the agents reject it, or vice versa). We define the behavior of a labeling-based aggregation function depending on the consistency of its outcome with the Collective outcome of labeling-based aggregation functions.

**Table 15**

Features of aggregation functions: + means fully satisfied; (+) means satisfied under some assumptions; and – stands for unsatisfied feature.

Method	ED	A	ND	DU	EU	S	M	FM	I
ABORDA <sup>S</sup>	+	+	+	(+) <sup>1</sup>	–	–	–	–	+
ABORDA <sup>P</sup>	+	+	+	(+) <sup>2</sup>	–	–	–	–	+
ABORDA <sup>SDI</sup>	+	+	+	–	–	–	–	–	–
ABORDA <sup>PDI</sup>	+	+	+	–	–	–	–	–	–
ACOP <sup>D</sup>	+	+	+	–	–	–	–	–	–
ACOP <sup>DI(Aff,Def)</sup>	+	+	+	–	–	–	–	–	–
ACOP <sup>DI(Pro,Com)</sup>	+	+	+	–	–	–	–	–	–
ACUMUL <sup>3</sup>	–	–	–	–	–	–	–	–	–
AKEMEN <sup>D</sup>	+	+	+	+	–	–	–	–	–
AKEMEN <sup>DI</sup>	+	+	+	–	–	–	–	–	–
AMRV	+	+	+	+	(+)	–	–	–	+
APREF <sup>MLD</sup>	+	+	+	+	–	–	–	–	–
APREF <sup>MD</sup>	+	+	+	+	–	–	–	–	–
APREF <sup>MLD(T)</sup>	+	+	+	–	–	–	–	–	–
APREF <sup>MD(T)</sup>	+	+	+	–	–	–	–	–	–
APREF <sup>DIIMLD</sup>	+	+	+	–	–	–	–	–	–
APREF <sup>DIIMD</sup>	+	+	+	–	–	–	–	–	–
ARGVET <sup>D</sup>	–	+	+	–	–	–	–	–	+
ARGVET <sup>DI</sup>	–	+	+	–	–	–	–	–	–
ASIMP <sup>D</sup>	+	+	+	+	–	–	–	–	–
ASIMP <sup>DI</sup>	+	+	+	–	–	–	–	–	–
Majority	+	+	+	+	(+)	–	–	–	+
OF	+	+	+	+	(+)	–	–	–	–
BF	+	+	+	+	(+)	–	–	–	–
SF	+	+	+	–	–	–	–	–	–

**ED:** Exhaustive Domain, **A:** Anonymity, **ND:** Non Dictatorship, **DU:** Direct Unanimity, **EU:** Endorsed Unanimity, **S:** Supportiveness, **M:** Monotonicity, **FM:** Familiar Monotonicity, **I:** Independence

<sup>1</sup> It depends on the threshold.

<sup>2</sup> This property is satisfied under certain conditions, so that an argument is placed in the best position in all modes (i.e., all orders and arrangement of arguments by agents).

<sup>3</sup> This method focuses on the number of labeling of each agent. After determining the quota of each agent in the number of labeling, voting can be continued with any other proposed function.

**Definition 15 (Mild aggregation).** When a function  $f \in \mathcal{F}$  results in the same outcome as the collective outcome, we call it Mild:

$$f \text{ is Mild, iff } \forall N \in \mathcal{A}, f(N) = \text{Collective outcome}(N). \quad (25)$$

**Definition 16 (Mild aggregation behavior).** We call a Mild function cooperative, antagonistic, or neutral regarding target  $N$ , if it results in an ‘in’, ‘undec’, and ‘out’ outcomes, respectively.

$$\text{Collective Behavior}(f, N) = \begin{cases} \text{Cooperative,} & \text{Collective outcome}(N) = \text{in} \\ \text{Neutral,} & \text{Collective outcome}(N) = \text{undec} \\ \text{Antagonistic,} & \text{Collective outcome}(N) = \text{out} \end{cases} \quad (26)$$

### 3.2. Findings

We classify our proposed collective labeling-based argumentation methods by considering the required properties for social choice rules (cf. Appendix A). Table 15 illustrates the properties of our proposed functions. The following propositions describe characteristics of these functions.

**Proposition 1.**  $ABORDA^S$  has ED, A, ND, DU, and I properties, but it does not satisfy EU, S, M, and FM.

**Proposition 2.**  $ABORDA^P$  has ED, A, ND, DU, and I properties, but it does not satisfy EU, S, M, and FM.

**Proposition 3.**  $ABORDA^{SDI}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 4.**  $ABORDA^{PDI}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 5.**  $ACOP^D$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 6.**  $ACOP^{DI(Att,Def)}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 7.**  $ACOP^{DI(Pro,Con)}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 8.**  $AKEMEN^D$  has ED, A, ND, and DU properties, but it does not satisfy EU, S, M, FM, and I.

**Proposition 9.**  $AKEMEN^{DI}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 10.**  $AMRV$  has ED, A, ND, DU, and  $EU^3$  and I properties, but it does not satisfy S, M, and FM.

**Proposition 11.**  $APREF^{MLD}$  has ED, A, ND, and DU properties, but it does not satisfy EU, S, M, FM, and I.

**Proposition 12.**  $APREF^{MD}$  has ED, A, ND, and DU properties, but it does not satisfy EU, S, M, FM, and I.

**Proposition 13.**  $APREF^{MLD(T)}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 14.**  $APREF^{MD(T)}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 15.**  $APREF^{DIMLD}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 16.**  $APREF^{DIMD}$  has ED, A, and ND, properties, but it does not satisfy DU, EU, S, M, FM, and I.

**Proposition 17.**  $ARGVET^D$  has A, ND, and I properties, but it does not satisfy ED, DU, EU, S, M, and FM.

**Proposition 18.**  $ARGVET^{DI}$  has A and, ND properties, but it does not satisfy ED, DU, EU, M, FM, and I.

**Proposition 19.**  $ASIMP^D$  has ED, A, ND, and DU properties, but it does not satisfy EU, S, M, FM, and I.

**Proposition 20.**  $ASIMP^{DI}$  has ED, A, and ND properties, but it does not satisfy DU, EU, S, M, FM, and I.

### 3.2.1. Justification

Here, we prove our Propositions (1 – 20) based on the required properties for social choice methods (cf. Appendix A). We prove the properties of Proposition 6 by showing how each feature of the aggregation function can be satisfied in the proposed Copeland method. The proofs for other propositions can be followed similarly.

*Exhaustive Domain.* ACOP's method uses all labels of all agents as input. The proposed function returns a final result using matrices that calculate different types of pairwise comparisons. Hence, whatever the labeling profiles are, these matrices can be calculated and thus the proposed method can calculate a result.

*Anonymity.* Considering that the proposed function is based on the comparison of the number of “in” labels and the number of attacks and defenses, we must prove that the permutation of agents (permutation of each agent with its label) does not affect these two comparisons. ACOP's method uses all the labels of all agents regardless of the position of each one for the input.

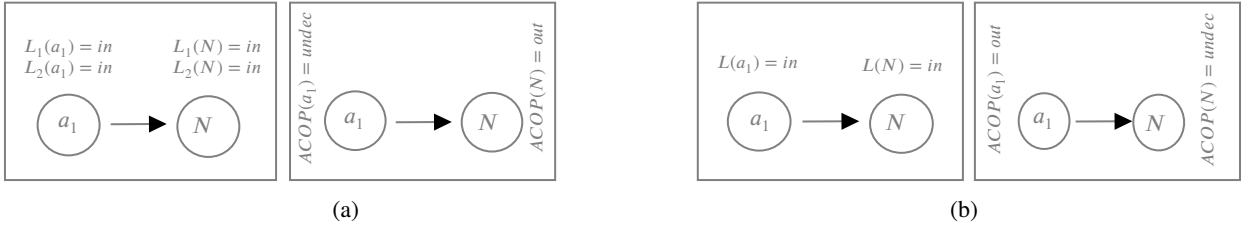
First, we show that permuting the agents does not effect the first comparison of arguments. Let  $\mathcal{AG} = \{Ag_1, \dots, Ag_n\}$  be the set of agents involved and  $\mathcal{L} = (L_1, \dots, L_n)$  be a labeling profile. Let  $\sigma$  be a permutation of the set of agents and  $\mathcal{L}' = (L'_{\sigma(1)}, \dots, L'_{\sigma(n)})$  the labeling profile resulting from applying the permutation  $\sigma$  over  $\mathcal{L}$ . Let  $F$  be an aggregation function based on ACOP. Let's consider any argument  $a$  and see that the following equations hold:

$$in_{\mathcal{L}}(a) = in_{\mathcal{L}'}(a) \text{ and } out_{\mathcal{L}}(a) = out_{\mathcal{L}'}(a).$$

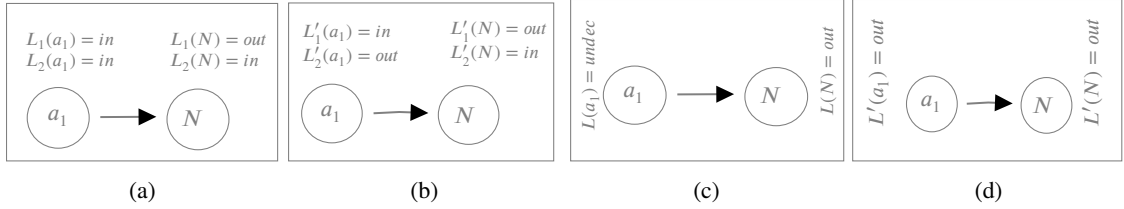
On the one hand  $in_{\mathcal{L}}(a) = |\{Ag_i \in \mathcal{AG} : L_i(a) = in\}| = |\{\sigma(Ag_i) \in \mathcal{AG} : L\sigma(i)(a) = in\}| = in_{\mathcal{L}'}(a)$ . On the other hand,  $out_{\mathcal{L}}(a) = |\{Ag_i \in \mathcal{AG} : L_i(a) = out\}| = |\{\sigma(Ag_i) \in \mathcal{AG} : L\sigma(i)(a) = out\}| = out_{\mathcal{L}'}(a)$ . Hence, the direct opinion of an argument is not affected by any permutation of the agents.

To prove the second comparison, it is clear that this comparison is between the number of attacks and defenses, so the permutation of agents does not affect it. Attacks and defenses are between arguments and have nothing to do with agents.

<sup>3</sup>under some assumptions



**Figure 4:** Properties of direct and endorsed unanimity for ACOP: (a) Counter example for direct unanimity: the argument labeling (left), the result of the ACOP function (right), (b) Counter example for endorsed unanimity: Labeling profile  $\mathcal{L}$  (left), ACOP on  $\mathcal{L}$  (right).



**Figure 5:** Counter example for monotonicity of ACOP: (a) Labeling profile  $\mathcal{L}$ , (b) Change opinion  $\mathcal{L}'$ , (c) ACOP on  $\mathcal{L}$ , (d) ACOP on  $\mathcal{L}'$ .

*Non-dictatorship.* According to the proof of the previous property that the proposed function is not affected by the permutation of agents, the non-dictatorship property is proved. By proving the previous feature, this feature is also proven in a way.

*Direct Unanimity.* In the ACOP's method, considering that the output of two matrices are added together (the number of labels "in"/ the number of attack and defense) and a threshold must be set, it is proved that direct unanimity is not obtained under any conditions. We prove this by the counter-example displayed in Fig. 4a.

According to ACOP and setting the threshold equal to 1, the situation obtained for each argument is contrary to the property of direct unanimity (because according to this property, every argument should be labeled "in", but the result is the opposite. Argument labels  $a_1$  and  $N_1$  were equal to "undec" and "out", respectively). With a similar counter-example, we also can discuss the endorsed unanimity property of ACOP.

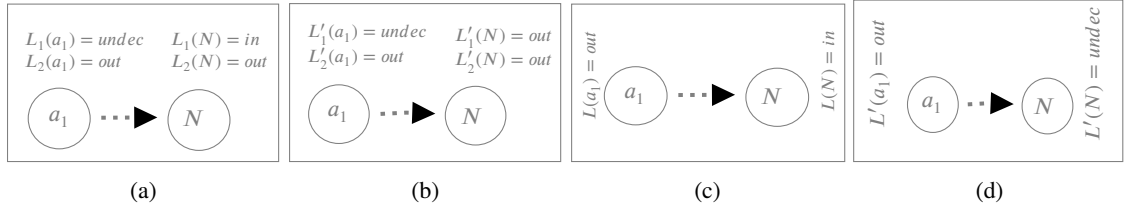
*Endorsed Unanimity.* The counter-example presented in Fig. 4b shows that the proposed method does not satisfy this property.

According to the definition of this property, considering that argument  $N$  is fully negative, its label should be "out", but according to the proposed method, the label "undec" is obtained, which shows that this property is not satisfied by the proposed function.

*Supportiveness.* The previous example is also a counter-example for supportiveness by only using  $L$  and its aggregation outcome  $F(L)$  (see Fig. 4a). Note that  $F(L)(a_1) = \text{"undec"}$  though no agent's opinion is "undec" on no argument  $a_1$ .

*Independence.* In the method based on ACOP, this property is not satisfied because, in one of the ACOP's matrices, the  $ACOP^I$  score of each argument is based on the number of attacks and defenses of that argument, which is against the essence of the independence property.

*Monotonicity.* We use the counter-example shown in Fig. 5 to show that this property is not satisfied in ACOP. First, by applying ACOPs, the label of argument  $a_1$  is calculated as "undec". By changing its opinion from "in" to "out" after calculations of the ACOP method, instead of being the same as "undec", it becomes equal to "out", which indicates the disproving of the characteristic of monotonicity. So by changing the opinion about  $a_1$ , the final result also changed, and this is against the property of monotonicity.



**Figure 6:** Counter example for familiar monotonicity: (a) Labeling profile  $\mathcal{L}$ , (b) Change opinion  $\mathcal{L}'$ , (c) ACOP on  $\mathcal{L}$ , (d) ACOP on  $\mathcal{L}'$ .

**Familiar Monotonicity.** As in the proof of the previous property, we use the counter-example shown in Fig. 6 to show that this property does not hold in ACOP.

If the opinion of descent of an argument remains the same, the direct opinion of the argument changes, and the outcome of the function is the same, according to the above definition of this property, the function satisfies Familiar Monotonicity. The labels of argument  $a_1$ , which is the descent of argument  $N$ , are fixed. By changing the direct opinion of argument  $a_1$  (i.e.,  $N$ ) from “in” to “out” after calculating the ACOP method, its label was also changed. Therefore, by changing the opinion about  $N$ , the result also changed, and this is against the Familiar Monotonicity property.

### 3.2.2. Summary result

Table 16 summarizes the collective choice for Example 1 using labeling-based collective argumentation functions. This table compares the outcomes of each proposed function regarding target  $N$  and the agents’ arguments  $a_1$ ,  $a_2$ , and  $a_3$ .

Among all the proposed functions, those based on the Veto social choice method do not represent Exhaustive Domain property. This values for the group decisions about social bad (e.g., establishing an industry which threatens the climate or human life); the others benefits decisions for social good. Moreover, the aggregation function based on Majoritarian Runoff does not present consistent and contradiction-free outcomes due to ignoring the relationships between arguments.

Following Eq. 24, Example 2 illustrates the collective outcome of employing all of the collective labeling-based argumentation functions regarding Example 1.

**Example 2.** Counting the number of ‘in’s, ‘out’s, and ‘undec’s raised by all the functions listed in Table 16 about target  $N$  in Example 1, we have  $\text{in}_L^F(N) = 12$ ,  $\text{out}_L^F(N) = 11$ , and  $\text{undec}_L^F(N) = 2$ . Since  $12 > 11$ , and  $2 < \min\{12, 11\}$ , Eq. 24 results in  $\text{Collective Outcome}(N) = \text{‘in’}$  as the collective decision raised by all kinds of aggregation functions.

According to Definition 16, as it is observed in Table 16, the behavior of our proposed collective labeling-based argumentation functions are determined, where ‘C’, ‘A’, and ‘N’ stand for Cooperative, Antagonistic, and Neutral, respectively. When Eq. 25 does not hold, ‘ $\sim M$ ’ shows that the function is not a *Mild* one. For example,  $ABORDA^{SDI}$ ,  $ACOP^D$ ,  $ACOP^{DI}$ ,  $ARGVET^D$ ,  $ACUMUL$ ,  $AKEMEN^{DI}$ ,  $ASIMP^D$ ,  $APREF^{MLD}$ ,  $APREF^{MD}$ ,  $APREF^{MLD(T)}$ , and  $SF$  are not *Mild*, but  $ABORDA^S$ ,  $ACOP^{DI(Pro/Con)}$ ,  $ARGVET^{DI}$ ,  $AKEMEN^D$ ,  $ASIMP^{DI}$ ,  $APREF^{MD(T)}$ ,  $APREF^{DIMLD}$ ,  $APREF^{DIMLD}$ ,  $M$ ,  $OF$ , and  $BF$  result in a cooperative outcome in the scenario described in Example 1.

## 4. Conclusions

Bridging between computational social choice and collective argumentation, this study presented a formal tool for evaluating and consolidating different and even contradicting opinions of individuals about their arguments into an aggregated decision for the group. We combine the individual arguments or opinions of the group members into a collective view by considering the direct and indirect relations between their arguments attacking or defending each other and the target decision to guarantee a logical outcome in practice. State-of-the-art achieved this by introducing varieties of majority rule functions into a labeling-based argumentation framework. The agents express their opinions about the presented arguments and goal by assigning labels “in” (accept or like), “out” (not accept or dislike), or “undec” (no decision about the argument or unknown). These problems address a real-world problem that has many potential applications in online communities and other open environments in multi-agent systems. We presented promising functions based on dozens of social choice methods. To the best of our knowledge, this is the first study that proposes a collective labeling-based argumentation by extending multiple social choice methods. And finally, we presented

**Table 16**

Comparing outcomes of the proposed collective labeling-based argumentation functions for aggregating agents' opinions about arguments  $N, a_1, a_2$  and  $a_3$  in Example 1

Argument			$N$	$a_1$	$a_2$	$a_3$	Behavior
Agent			Opinion				
Finn			in	out	out	in	
Sal			out	in	in	in	
Mark			in	out	in	in	
Function	Type	Eq.	Outcome				
ABORDA (§ 2.1)	S	8	in	undec	in	in	$C^2$
	SDI	9	in	in	in	in	C
	P	10	TCC <sup>1</sup>				-
	PDI	11	TCC				-
ACOP (§ 2.2)	D	12	undec	out	undec	in	$\sim M^3$
	DI <sub>Att,Def</sub>	13	out	undec	undec	in	$\sim M$
	DI <sub>Pro,Con</sub>	14	in	out	undec	in	C
ARGVET (§ 2.3)	D	15	out	out	out	in	$\sim M$
	DI	16	out	out	out	in	$\sim M$
ACUMUL <sub>Majority</sub> (§ 2.4)	D	-	out	in	out	in	$\sim M$
AKEMEN (§ 2.5)	D	17	in	out	out	out	C
	DI	18	out	out	out	out	$\sim M$
ASIMP (§ 2.7)	D	22	out	out	out	in	$\sim M$
	DI	23	out	out	out	in	$\sim M$
APREF (§ 2.6)	MLD	19	out	out	out	in	$\sim M$
	MD	20	out	out	out	in	$\sim M$
	MLD(T)	-	undec	out	undec	in	$\sim M$
	MD(T)	-	in	undec	in	in	C
	DIMLD	-	in	out	undec	in	C
	DIMD	-	in	in	in	in	C
D/I Majority [24] (§ 1.1)	M	3	in	out	in	in	C
	OF	4	in	out	in	in	C
	SF	5	out	in	in	in	$\sim M$
	BF	6	in	undec	in	in	C
Collective Outcome							in

<sup>1</sup> TCC: Time consuming calculations. Time complexity is  $\mathcal{O}(n!)$ , where  $n$  is the number of agents.

<sup>2</sup> C: Cooperative

<sup>3</sup>  $\sim M$ : Not Mild

*cooperative*, *neutral*, and *antagonistic* concepts for the behavior of the collective outcome of all aggregation functions based on the number of “in”, “out”, and “undec” resulting from the functions. We proved the properties of these functions based on different criteria regarding appropriate group decision characteristics.

In future work, we will address social choice methods that are based on cardinal or ordinal opinions. We also will investigate our proposed functions in the group-decision scenarios where the individuals express their opinions via CP-Nets or lexicographic preferences over the arguments.

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## CRedit authorship contribution statement

**Reza Karimi:** Conceptualization, Methodology, Formal analysis, Resources, Visualization Preparation, Writing - Original Draft Preparation. **Faria Nassiri-Mofakham:** Conceptualization, Methodology, Formal analysis, Writing - Original Draft Preparation, Writing - Review & Editing, Visualization Preparation, Supervision.

## A. Properties of aggregation rules

Formal properties desirable for aggregation functions and their outcomes are adapted from the social choice theory and in terms of argumentation frameworks and collective decision-making [24, 6], as follows:

- *Exhaustive Domain (ED)* [6]. This property defines aggregation functions based on the labeling they can accept as input..
- *Anonymity (A)* [6]. In argumentation, all agents' opinions must be equally valued. Anonymity ensures this by requiring that if a permutation is applied to the agents' labeling, the aggregation function's outcome remains unchanged. Let  $\mathcal{L} = (L_1, \dots, L_n)$  be a labeling profile  $\sigma$  a permutation over  $Ag$ , and  $\mathcal{L}' = (L\sigma(a), \dots, L\sigma(n))$  the labeling profile resulting from applying  $\sigma$  over  $L$ . An aggregation function  $F$  satisfies anonymity if  $F(L) = F(L')$ .
- *Non-Dictatorship (ND)* [6]. This property ensures that no single agent's opinion can dominate the others. It is a weaker form of anonymity, requiring that no agent's label is always the aggregation function's result for every profile. An aggregation function  $F$  satisfies ND if no agent  $ag_i \in Ag$  satisfies that  $F(L) = L_i$  for every labeling profile  $L \in D$ .
- *Direct Unanimity (DU)* [24]. Let  $\mathcal{L} = (L_1, \dots, L_n)$  be a labeling profile, where  $\mathcal{L} \in D$ . An aggregation function  $F$  satisfies DU if, for any  $a \in A$  such that  $L_i(a) = l$  for all  $L_i \in \mathcal{L}$ , where  $l \in \{in, out, undec\}$ , then  $F(L)(a) = l$  holds. That is, If all agents have the same direct opinion on an argument, the collective opinion should match this unanimous view.
- *Endorsed Unanimity (EU)* [24]. This property extends unanimity to indirect opinions. If all agents unanimously support or oppose an argument indirectly, the collective opinion should reflect this consensus. Let  $\mathcal{L} = (L_1, \dots, L_n)$  be a labeling profile such that  $\mathcal{L} \in D$ . An aggregation function  $F$  satisfies EU if:
  1. For any  $a \in A$  such that  $a$  counts on full positive support for all  $L_i \in \mathcal{L}$ , then  $F(\mathcal{L})(a) = in$ ;
  2. For any  $a \in A$  such that  $a$  counts on full negative support for all  $L_i \in \mathcal{L}$ , then  $F(\mathcal{L})(a) = out$ .
- *Supportiveness (S)* [6]. An aggregation function should not assign a label to an argument that no agent has used. An aggregation function  $F$  satisfies S if for every argument  $a \in A$  and for all labeling profile  $\mathcal{L} = (L_1, \dots, L_n)$ ,  $\mathcal{L} \in D$ , we can find some agent  $Ag_i \in AG$  for which  $F(\mathcal{L})(a) = L_i(a)$  holds.
- *Monotonicity (M)* [6]. If some agents' direct opinions on an argument change to match the collective label, the collective label should remain unchanged. Let  $l \in \{in, out, undec\}$  be a label,  $a \in A$  an argument, and  $\mathcal{L} = (L_1, \dots, L_i, \dots, L_{i+k}, \dots, L_n)$ ,  $\mathcal{L}' = (L'_1, \dots, L'_i, \dots, L'_{i+k}, \dots, L'_n)$ ,  $\mathcal{L}, \mathcal{L}' \in D$ , two profiles that only differ on the labelings of agents  $i, \dots, i+k$ . An aggregation function  $F$  is monotonic if  $L_j(a) \neq l$  while  $L'_j(a) = l$  for all  $j \in i, \dots, i+k$ , then  $F(L)(a) = l$  implies that  $F(L')(a) = l$ .
- *Binary Monotonicity (BM)* [6]. It is similar to monotonicity, but specifically for binary labels (in or out). Let  $l \in \{in, out\}$  be a label,  $a \in A$  an argument, and  $\mathcal{L} = (L_1, \dots, L_i, \dots, L_{i+k}, \dots, L_n)$ ,  $\mathcal{L}' = (L'_1, \dots, L'_i, \dots, L'_{i+k}, \dots, L'_n)$ ,  $\mathcal{L}, \mathcal{L}' \in D$ , two profiles that only differ on the labelings of agents  $i, \dots, i+k$ . An aggregation function  $F$  is monotonic if  $L_j(a) \neq l$  while  $L'_j(a) = l$  for all  $j \in i, \dots, i+k$ , then  $F(L)(a) = l$  implies that  $F(L')(a) = l$ .
- *Familiar Monotonicity (FM)* [24]. This property considers the opinions of an argument's descendants. If the direct opinions of agents on an argument and its descendants change to match the collective label, the collective label should remain unchanged. Let  $l \in \{in, out, undec\}$  be a label,  $a \in A$  an argument, and  $\mathcal{L} = (L_1, \dots, L_i, \dots, L_{i+k}, \dots, L_n)$ ,  $\mathcal{L}' = (L'_1, \dots, L'_i, \dots, L'_{i+k}, \dots, L'_n)$ ,  $\mathcal{L}, \mathcal{L}' \in D$ , two profiles that only

differ on the labelings of agents  $i, \dots, i + k$ . An aggregation function  $F$  satisfies FM if  $L_j(a) \neq l$  while  $L'_j(a) = l$  and  $L_j(b) = L'_j(b)$  for all  $j \in \{i, \dots, i + k\}$  and argument  $b$  descendant of  $a$ , then  $F(L)(a) = l$  implies that  $F(L')(a) = l$ .

- **Independence (I)** [6]. The aggregated label for an argument should depend only on the labels assigned to that argument by different agents, not on the labels for other arguments. Let be two profiles  $\mathcal{L} = (L_1, \dots, L_n)$  and  $\mathcal{L}' = (L'_1, \dots, L'_n)$ , such that  $L, L' \in D$ ; and  $a \in A$  an argument, such that for all agents  $i \in \{1, \dots, n\}$   $L_i(a) = L'_i(a)$ . An aggregation function  $F$  satisfies I if  $F(\mathcal{L})(a) = F(\mathcal{L}')(a)$ .

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