

# kiana-mohammadinik-urgent-care-simulation

2026-02-09

## Instructions

Choose ONE of the three questions below and answer it using simulation.  
Your submission must include:

- Well-commented code (I should be able to follow your logic)
- A written analysis that answers the actual questions being asked (not just reporting numbers)
- At least one plot that directly supports a conclusion
- At least one sensitivity analysis (change an assumption and explain what changes)
- A clear recommendation in plain English, supported by your simulation results

You may make additional reasonable assumptions, but you must state and justify them.

Required (for any problem you choose):

Make one recommendation that optimizes something under a constraint.

- Queueing: minimize total cost *or* meet a service-level constraint
- Inventory: maximize profit while accounting for salvage/stockouts (or another realistic feature)
- Reliability: maximize uptime per dollar under a budget constraint

## Question 1: Urgent Care Center Queueing Systems

You operate an urgent care center that:

- Opens at 8am and closes at 5pm
- Customer arrivals follow a Poisson process
  - Rate = 4 per hour, except 12–1pm, when rate = 6 per hour
- Service times are i.i.d. random variables on the positive real line (*you must choose and justify a service-time distribution*)

You have 2 servers.

You are considering three queueing systems:

1. Single pooled line with parallel servers
2. Tandem service (In-N-Out style): customers must complete service at Server 1, then Server 2

3. Separate lines, one per server (grocery-store style)

#### Core Tasks

- Simulate 1 work week (5 days).
- Repeat the week simulation 7 times and average results.
- Estimate and report:
  - Mean waiting time
  - Distribution of waiting times (not just the mean)
- Compare the systems.

#### Decision Requirement (required)

Assume:

- Waiting cost = \$0.50 per customer-minute
- Server cost = \$30 per hour per server

Your task is to:

- Choose the “best” system and clearly define what “best” means
- Report the total expected daily cost
- Make a clear operational recommendation

Add ONE realism feature (choose one)

Pick one of the following additions and incorporate it in your model:

A. Staffing tradeoff: allow 2 vs 3 servers during 12–1pm and choose the best option under total cost

Hard

- In System 3 (separate lines), allow customers to switch lines
- Compare outcomes with and without switching

*Note:* If you want a concrete service-time option, you may use either:

- Lognormal service times (e.g., mean 10 min, sd 8 min), or
- A mixture model (e.g., 80% “quick” mean 6 min, 20% “long” mean 20 min)

**Extra Credit (+10%, choose ONE)**

- Event-driven simulation: implement an event-based simulator (priority queue) rather than time-stepping

Grading Emphasis You are graded on:

- Correct simulation logic
- Clear explanation and interpretation
- Thoughtful modeling assumptions
- Decision quality and justification
- Code readability and documentation

This is not about closed-form answers. It's about modeling, simulation, and decision-making under uncertainty.

---

## Event list helpers + arrivals + service distribution

```
# 1) GLOBAL TIME CONSTANTS

# minutes in workday (8am-5pm)
T_END    <- 540
# 12:00
T_LUNCH_START <- 240
# 1:00
T_LUNCH_END   <- 300

rate_per_min <- function(t){
  # piecewise arrival rate (per minute)
  if (t < T_LUNCH_START) return(4/60)
  if (t < T_LUNCH_END)   return(6/60)
  return(4/60)
}

# 2) ARRIVAL GENERATION (EVENT-DRIVEN)

# For piecewise-constant rates: generate next arrival time from current t
next_arrival_time <- function(t_now){
  t <- t_now

  while(TRUE){
    lam <- rate_per_min(t)
    if (lam <= 0) return(Inf)

    # boundary of current segment
    t_bound <- if (t < T_LUNCH_START) T_LUNCH_START else if (t < T_LUNCH_END) T_LUNCH_END else T_END

    # sample exponential interarrival with current lam
    dt <- rexp(1, rate = lam)
    t_candidate <- t + dt

    if (t_candidate < t_bound) {
      return(t_candidate)
    } else {
      # if we hit the boundary before the arrival, move to boundary and try again
      t <- t_bound
    }
  }
}
```

```

    if (t >= T_END) return(Inf)
  }
}
}

generate_arrivals_one_day <- function(){
  t <- 0
  arr <- c()
  while(TRUE){
    t <- next_arrival_time(t)
    if (!is.finite(t) || t >= T_END) break
    arr <- c(arr, t)
  }
  arr
}

# 3) SERVICE TIME DISTRIBUTION CHOICE

# Lognormal option: mean ~ 10, sd ~ 8 minutes
# Converting (mean, sd) to lognormal parameters:
lognormal_params <- function(mean, sd){
  var <- sd^2
  sigma2 <- log(1 + var/mean^2)
  sigma <- sqrt(sigma2)
  mu <- log(mean) - 0.5*sigma2
  list(mu = mu, sigma = sigma)
}

svc_params <- lognormal_params(mean = 10, sd = 8)

sample_service_time <- function(n = 1){
  # returns positive service times in minutes
  rlnorm(n, meanlog = svc_params$mu, sdlog = svc_params$sigma)
}

# 4) EVENT LIST (PRIORITY QUEUE)

# Implement as a data.frame and always pop the smallest time.
add_event <- function(event_list, time, type, server = NA_integer_, cust_id = NA_integer_){
  new <- data.frame(time = time, type = type, server = server, cust_id = cust_id)
  rbind(event_list, new)
}

pop_next_event <- function(event_list){
  idx <- which.min(event_list$time)
  ev <- event_list[idx, , drop = FALSE]
  event_list <- event_list[-idx, , drop = FALSE]
  list(event = ev, event_list = event_list)
}

```

This arrival method (used above) works because since arrivals are Poisson with constant rate within each segment, interarrivals are exponential within the segment. If a sampled interarrival would cross into the next

segment, we “jump to the boundary” and sample again using the new rate. That gives an exact NHPP for this piecewise-constant case.

Also, sorting the event list works because his simulation will have on the order of:

- $4/\text{hr} \times 9 \text{ hr} = 36$  customers/day (a bit more because lunch)
- plus service completion events

So the event list stays small. Sorting is fast and clean.

### System 1 (Pooled line, 2 servers) : Event-driven simulator in R

```
simulate_day_system1 <- function(n_servers = 2, seed = NULL){
  if (!is.null(seed)) set.seed(seed)

  # Customer storage
  # create customers on the fly as arrivals happen
  customers <- data.frame(
    cust_id = integer(),
    arrival_time = numeric(),
    service_start = numeric(),
    service_end = numeric(),
    wait_time = numeric()
  )

  # System state
  # pooled FIFO queue of customer IDs
  queue <- integer()
  # if Inf => idle
  busy_until <- rep(Inf, n_servers)
  server_job <- rep(NA_integer_, n_servers)

  # find an idle server index (or NA if none)
  idle_server <- function(){
    idx <- which(!is.finite(busy_until))[1]
    if (length(idx) == 0) return(NA_integer_)
    idx
  }

  idle_server <- function(){
    idx <- which(busy_until == Inf)[1]
    if (length(idx) == 0) return(NA_integer_)
    idx
  }

  # start service for next customer in queue if server is idle
  start_service_if_possible <- function(t_now, event_list){
    s <- idle_server()
    while (!is.na(s) && length(queue) > 0) {
      cid <- queue[1]
      queue <- queue[-1]
    }
  }
}
```

```

st <- t_now
svc <- sample_service_time(1)
et <- t_now + svc

# update customer record
customers$service_start[customers$cust_id == cid] <- st
customers$service_end[customers$cust_id == cid] <- et
customers$wait_time[customers$cust_id == cid] <- st - customers$arrival_time[customers$cust_id == cid]

# update server state
busy_until[s] <- et
server_job[s] <- cid

# schedule departure event for this server
event_list <- add_event(event_list, time = et, type = "depart", server = s, cust_id = cid)

# check for another idle server
s <- idle_server()
}
event_list
}

# Event list initialization
event_list <- data.frame(time = numeric(), type = character(), server = integer(), cust_id = integer())

# schedule first arrival
t_first <- next_arrival_time(0)
if (is.finite(t_first) && t_first < T_END) {
  event_list <- add_event(event_list, time = t_first, type = "arrival")
}

# customer id counter
next_id <- 1

# Main event loop
while (nrow(event_list) > 0) {
  popped <- pop_next_event(event_list)
  ev <- popped$event
  event_list <- popped$event_list

  t_now <- ev$time
  if (!is.finite(t_now)) break

  # If it is after closing, we stop new arrivals but let service finish. But, since we only schedule
  if (ev$type == "arrival") {
    # create new customer
    cid <- next_id
    next_id <- next_id + 1

    customers <- rbind(
      customers,
      data.frame(

```

```

    cust_id = cid,
    arrival_time = t_now,
    service_start = NA_real_,
    service_end = NA_real_,
    wait_time = NA_real_
  )
)

# join the pooled queue
queue <- c(queue, cid)

# schedule next arrival
t_next <- next_arrival_time(t_now)
if (is.finite(t_next) && t_next < T_END) {
  event_list <- add_event(event_list, time = t_next, type = "arrival")
}

# start service if any server idle
event_list <- start_service_if_possible(t_now, event_list)

} else if (ev$type == "depart") {
  s <- ev$server

  # free server
  busy_until[s] <- Inf
  server_job[s] <- NA_integer_

  # immediately start next customer if waiting
  event_list <- start_service_if_possible(t_now, event_list)
}
}

# Return only customers who actually started service (removes weird edge cases)
customers
}

# Quick summary helper
summarize_waits <- function(customers){
  waits <- customers$wait_time
  waits <- waits[is.finite(waits)]
  list(
    n_customers = length(waits),
    mean_wait = mean(waits),
    median_wait = median(waits),
    p90_wait = unname(quantile(waits, 0.90)),
    p95_wait = unname(quantile(waits, 0.95))
  )
}

```

Test (1 day)

```

set.seed(1)
cust1 <- simulate_day_system1(n_servers = 2)

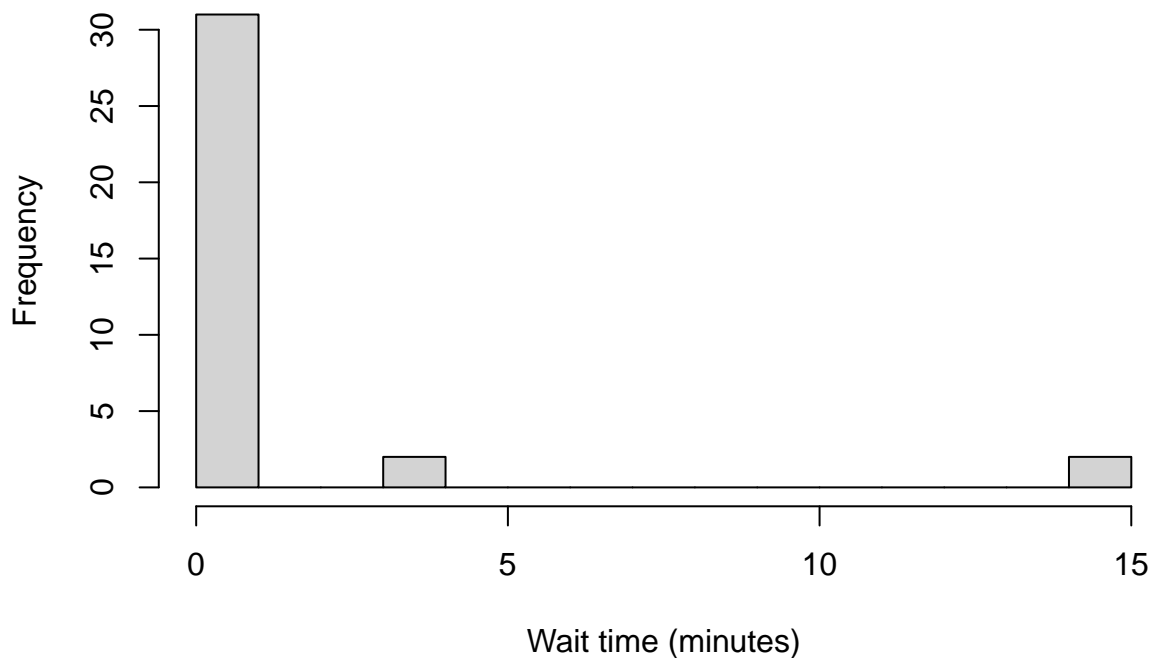
```

```
summarize_waits(cust1)
```

```
## $n_customers
## [1] 35
##
## $mean_wait
## [1] 1.05209
##
## $median_wait
## [1] 0
##
## $p90_wait
## [1] 2.249934
##
## $p95_wait
## [1] 6.892557
```

```
# Basic plot: waiting time distribution
hist(cust1$wait_time, breaks = 20,
     main = "System 1: Waiting Time Distribution (1 Day)",
     xlab = "Wait time (minutes)")
```

### System 1: Waiting Time Distribution (1 Day)



Simulating 1 work week (5 days), repeat 7 weeks, average results, and compute daily costs for System 1



```

simulate_week_system1 <- function(n_servers = 2, seed = NULL){
  if (!is.null(seed)) set.seed(seed)

  # simulate 5 independent days (fresh randomness each day)
  days <- vector("list", 5)
  for (d in 1:5){
    days[[d]] <- simulate_day_system1(n_servers = n_servers)
    days[[d]]$day <- d
  }
  do.call(rbind, days)
}

# Run 7 independent weeks and return a list:
# - all customer-level data
# - week-level summaries
# - overall averages
run_system1_experiment <- function(n_weeks = 7, n_servers = 2, seed = 123){
  set.seed(seed)

  all_weeks <- vector("list", n_weeks)
  week_summaries <- data.frame(
    week = integer(),
    mean_wait = numeric(),
    median_wait = numeric(),
    p90_wait = numeric(),
    avg_customers_per_day = numeric(),
    total_wait_minutes_per_day = numeric()
  )

  for (w in 1:n_weeks){
    cust <- simulate_week_system1(n_servers = n_servers)
    cust$week <- w
    all_weeks[[w]] <- cust

    # daily totals for that week
    daily <- aggregate(wait_time ~ day, data = cust, FUN = function(x) sum(x, na.rm = TRUE))
    mean_total_wait_per_day <- mean(daily$wait_time)

    waits <- cust$wait_time
    waits <- waits[is.finite(waits)]

    week_summaries <- rbind(
      week_summaries,
      data.frame(
        week = w,
        mean_wait = mean(waits),
        median_wait = median(waits),
        p90_wait = unname(quantile(waits, 0.90)),
        avg_customers_per_day = nrow(cust)/5,
        total_wait_minutes_per_day = mean_total_wait_per_day
      )
    )
  }
}

```

```

}

all_customer_data <- do.call(rbind, all_weeks)

# Overall averages across the 7 week summaries
overall <- data.frame(
  mean_wait = mean(week_summaries$mean_wait),
  median_wait = mean(week_summaries$median_wait),
  p90_wait = mean(week_summaries$p90_wait),
  avg_customers_per_day = mean(week_summaries$avg_customers_per_day),
  total_wait_minutes_per_day = mean(week_summaries$total_wait_minutes_per_day)
)

list(
  customers = all_customer_data,
  week_summaries = week_summaries,
  overall = overall
)
}

```

### 1) Week + multi-week wrapper for System 1

```

compute_daily_cost <- function(total_wait_minutes_per_day, n_servers, hours_open = 9,
                                wait_cost_per_min = 0.50, server_cost_per_hr = 30){

  waiting_cost <- wait_cost_per_min * total_wait_minutes_per_day
  staffing_cost <- server_cost_per_hr * hours_open * n_servers
  total_cost <- waiting_cost + staffing_cost

  data.frame(
    waiting_cost = waiting_cost,
    staffing_cost = staffing_cost,
    total_cost = total_cost
  )
}

```

### 2) Cost calculation (daily)

```

res1 <- run_system1_experiment(n_weeks = 7, n_servers = 2, seed = 1)

res1$overall

```

### 3) Run System 1 (2 servers) + plot

```

##   mean_wait median_wait p90_wait avg_customers_per_day
## 1    1.15835          0 4.342446          38.25714
##   total_wait_minutes_per_day
## 1                44.76268

```

```

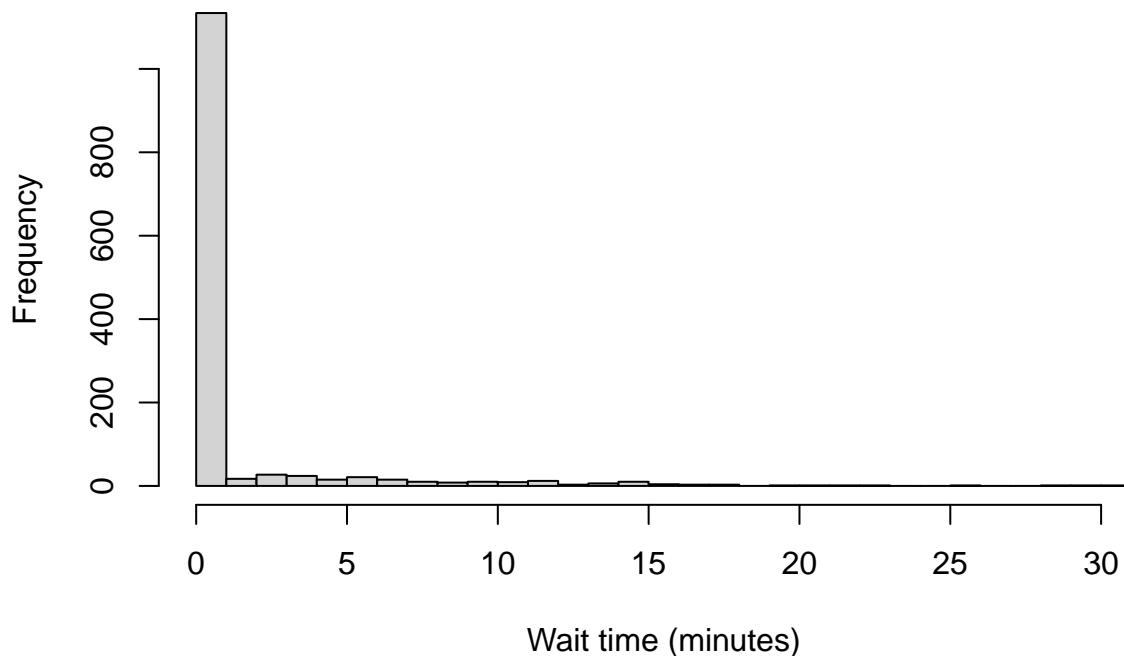
# Total expected daily cost (using the averaged daily total wait)
cost1 <- compute_daily_cost(
  total_wait_minutes_per_day = res1$overall$total_wait_minutes_per_day,
  n_servers = 2
)
cost1

##   waiting_cost staffing_cost total_cost
## 1      22.38134           540    562.3813

# Plot: waiting time distribution
hist(res1$customers$wait_time, breaks = 30,
     main = "System 1 (Pooled): Waiting Time Distribution\n(7 weeks of customer waits)",
     xlab = "Wait time (minutes)")

```

### System 1 (Pooled): Waiting Time Distribution (7 weeks of customer waits)



System 2:

```

simulate_day_system2 <- function(seed = NULL){
  if (!is.null(seed)) set.seed(seed)

  # Customer storage
  customers <- data.frame(
    cust_id = integer(),
    arrival_time = numeric(),

```

```

s1_start = numeric(), s1_end = numeric(), wait1 = numeric(),
s2_start = numeric(), s2_end = numeric(), wait2 = numeric(),
total_wait = numeric(),
total_time_in_system = numeric()
)

# Two FIFO queues
q1 <- integer()
q2 <- integer()

# Server states (single server at each stage)
s1_busy_until <- Inf
s1_job <- NA_integer_

s2_busy_until <- Inf
s2_job <- NA_integer_

s1_idle <- function() s1_busy_until == Inf
s2_idle <- function() s2_busy_until == Inf

# Event list
event_list <- data.frame(time = numeric(), type = character(), server = integer(), cust_id = integer())

# customer id counter
next_id <- 1

# helper: start service at server 1 if possible
start_s1_if_possible <- function(t_now, event_list){
  if (s1_idle() && length(q1) > 0) {
    cid <- q1[1]; q1 <- q1[-1]

    st <- t_now
    svc <- sample_service_time(1)
    et <- t_now + svc

    customers$s1_start[customers$cust_id == cid] <- st
    customers$s1_end[customers$cust_id == cid] <- et
    customers$wait1[customers$cust_id == cid] <- st - customers$arrival_time[customers$cust_id == cid]

    s1_busy_until <- et
    s1_job <- cid

    event_list <- add_event(event_list, time = et, type = "depart1", server = 1, cust_id = cid)
  }
  event_list
}

# helper: start service at server 2 if possible
start_s2_if_possible <- function(t_now, event_list){
  if (s2_idle() && length(q2) > 0) {
    cid <- q2[1]; q2 <- q2[-1]

    st <- t_now

```

```

    svc <- sample_service_time(1)
    et <- t_now + svc

    customers$s2_start[customers$cust_id == cid] <<- st
    customers$s2_end[customers$cust_id == cid] <<- et
    customers$wait2[customers$cust_id == cid] <<- st - customers$s1_end[customers$cust_id == cid]

    # total wait and total time in system now known
    customers$total_wait[customers$cust_id == cid] <<-
      customers$wait1[customers$cust_id == cid] + customers$wait2[customers$cust_id == cid]

    customers$total_time_in_system[customers$cust_id == cid] <<- et - customers$arrival_time[customers$cust_id == cid]

    s2_busy_until <<- et
    s2_job <<- cid

    event_list <- add_event(event_list, time = et, type = "depart2", server = 2, cust_id = cid)
  }
  event_list
}

# Schedule first arrival
t_first <- next_arrival_time(0)
if (is.finite(t_first) && t_first < T_END) {
  event_list <- add_event(event_list, time = t_first, type = "arrival")
}

# Main event loop
while (nrow(event_list) > 0) {
  popped <- pop_next_event(event_list)
  ev <- popped$event
  event_list <- popped$event_list
  t_now <- ev$time

  if (ev$type == "arrival") {
    cid <- next_id
    next_id <- next_id + 1

    customers <- rbind(
      customers,
      data.frame(
        cust_id = cid,
        arrival_time = t_now,
        s1_start = NA_real_, s1_end = NA_real_, wait1 = NA_real_,
        s2_start = NA_real_, s2_end = NA_real_, wait2 = NA_real_,
        total_wait = NA_real_,
        total_time_in_system = NA_real_
      )
    )

    # join queue 1
    q1 <- c(q1, cid)
  }
}

```

```

    # schedule next arrival
    t_next <- next_arrival_time(t_now)
    if (is.finite(t_next) && t_next < T_END) {
      event_list <- add_event(event_list, time = t_next, type = "arrival")
    }

    # start service at s1 if idle
    event_list <- start_s1_if_possible(t_now, event_list)

  } else if (ev$type == "depart1") {
    # server 1 finishes; customer moves to queue 2
    s1_busy_until <- Inf
    s1_job <- NA_integer_

    cid <- ev$cust_id
    q2 <- c(q2, cid)

    # immediately try starting s1 (next in q1) and s2 (from q2)
    event_list <- start_s1_if_possible(t_now, event_list)
    event_list <- start_s2_if_possible(t_now, event_list)

  } else if (ev$type == "depart2") {
    # server 2 finishes, customer exits system
    s2_busy_until <- Inf
    s2_job <- NA_integer_

    # start next at s2 if possible
    event_list <- start_s2_if_possible(t_now, event_list)
  }
}

customers
}

```

#### A) one-day event-driven simulator (tandem)

```

simulate_week_system2 <- function(seed = NULL){
  if (!is.null(seed)) set.seed(seed)

  days <- vector("list", 5)
  for (d in 1:5){
    days[[d]] <- simulate_day_system2()
    days[[d]]$day <- d
  }
  do.call(rbind, days)
}

run_system2_experiment <- function(n_weeks = 7, seed = 123){
  set.seed(seed)

  all_weeks <- vector("list", n_weeks)

```

```

week_summaries <- data.frame(
  week = integer(),
  mean_total_wait = numeric(),
  p90_total_wait = numeric(),
  mean_time_in_system = numeric(),
  avg_customers_per_day = numeric(),
  total_wait_minutes_per_day = numeric()
)

for (w in 1:n_weeks){
  cust <- simulate_week_system2()
  cust$week <- w
  all_weeks[[w]] <- cust

  # total waiting minutes per day (sum total_wait within each day)
  daily_totals <- aggregate(total_wait ~ day, data = cust, FUN = function(x) sum(x, na.rm = TRUE))
  mean_total_wait_per_day <- mean(daily_totals$total_wait)

  waits <- cust$total_wait
  waits <- waits[is.finite(waits)]

  tis <- cust$total_time_in_system
  tis <- tis[is.finite(tis)]

  week_summaries <- rbind(
    week_summaries,
    data.frame(
      week = w,
      mean_total_wait = mean(waits),
      p90_total_wait = unname(quantile(waits, 0.90)),
      mean_time_in_system = mean(tis),
      avg_customers_per_day = nrow(cust)/5,
      total_wait_minutes_per_day = mean_total_wait_per_day
    )
  )
}

all_customer_data <- do.call(rbind, all_weeks)

overall <- data.frame(
  mean_total_wait = mean(week_summaries$mean_total_wait),
  p90_total_wait = mean(week_summaries$p90_total_wait),
  mean_time_in_system = mean(week_summaries$mean_time_in_system),
  avg_customers_per_day = mean(week_summaries$avg_customers_per_day),
  total_wait_minutes_per_day = mean(week_summaries$total_wait_minutes_per_day)
)

list(
  customers = all_customer_data,
  week_summaries = week_summaries,
  overall = overall
)
}

```

## B) Week + 7-week experiment wrapper for System 2

```
res2 <- run_system2_experiment(n_weeks = 7, seed = 1)
res2$overall
```

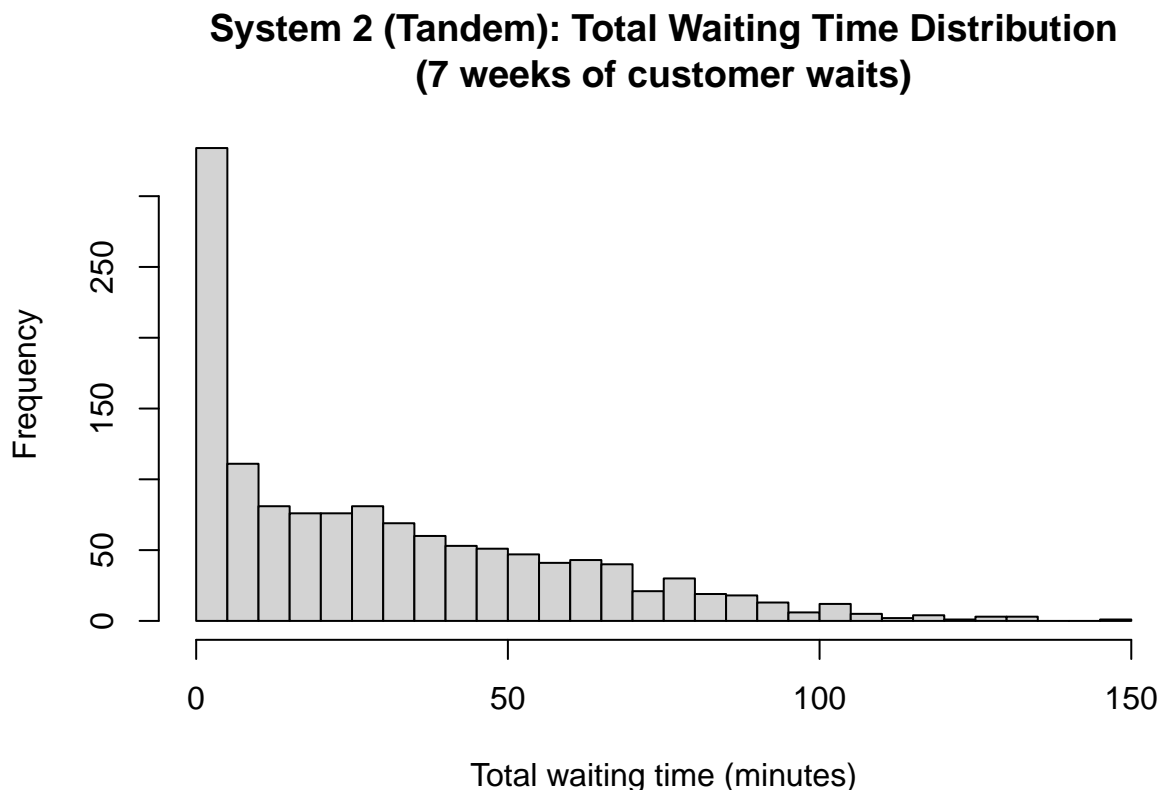
## C) Run System 2 + required plot + daily cost

```
##   mean_total_wait p90_total_wait mean_time_in_system avg_customers_per_day
## 1      29.18839      70.91036          49.46122          37.17143
##   total_wait_minutes_per_day
## 1                   1111.618
```

```
# Daily cost: still 2 servers total (one at each stage), 9 hours/day
cost2 <- compute_daily_cost(
  total_wait_minutes_per_day = res2$overall$total_wait_minutes_per_day,
  n_servers = 2
)
cost2
```

```
##   waiting_cost staffing_cost total_cost
## 1      555.8091          540    1095.809
```

```
# Plot: distribution of total waiting time
hist(res2$customers$total_wait, breaks = 30,
     main = "System 2 (Tandem): Total Waiting Time Distribution\n(7 weeks of customer waits)",
     xlab = "Total waiting time (minutes)")
```





### System 3: one-day simulator (separate lines, optional switching)

**Switching rule (From chosen difficulty level: Hard)** At each service completion, we allow switching if the difference in queue lengths is  $\geq 2$ . We move the last person in the longer line to the end of the shorter line. This would be helpful in scenarios where someone near the back decides to swap.

```
simulate_day_system3 <- function(switching = FALSE, switch_threshold = 2, seed = NULL){
  if (!is.null(seed)) set.seed(seed)

  customers <- data.frame(
    cust_id = integer(),
    arrival_time = numeric(),
    service_start = numeric(),
    service_end = numeric(),
    wait_time = numeric(),
    chosen_line = integer()
  )

  # Two separate FIFO queues (store customer IDs)
  q <- list(integer(), integer())

  # Two servers
  busy_until <- c(Inf, Inf)
  server_job <- c(NA_integer_, NA_integer_)

  is_idle <- function(s) busy_until[s] == Inf

  # Event list
  event_list <- data.frame(time = numeric(), type = character(), server = integer(), cust_id = integer())

  next_id <- 1

  # Choose a line at arrival: shorter queue (tie-break random)
  choose_line <- function(){
    len1 <- length(q[[1]])
    len2 <- length(q[[2]])
    if (len1 < len2) return(1L)
    if (len2 < len1) return(2L)
    sample(c(1L, 2L), 1) # tie
  }

  # Start service for a given server if idle and its queue nonempty
  start_service <- function(t_now, s, event_list){
    if (is_idle(s) && length(q[[s]]) > 0){
      cid <- q[[s]][1]
      q[[s]] <- q[[s]][-1]

      st <- t_now
      svc <- sample_service_time(1)
      et <- t_now + svc
    }
  }
}
```

```

customers$service_start[customers$cust_id == cid] <- st
customers$service_end[customers$cust_id == cid] <- et
customers$wait_time[customers$cust_id == cid] <- st - customers$arrival_time[customers$cust_id == cid]

busy_until[s] <- et
server_job[s] <- cid

event_list <- add_event(event_list, time = et, type = "depart", server = s, cust_id = cid)
}
event_list
}

# Hard feature: allow switching after a departure event
do_switching_if_needed <- function(){
  len1 <- length(q[[1]])
  len2 <- length(q[[2]])
  diff <- abs(len1 - len2)

  if (diff >= switch_threshold){
    longer <- if (len1 > len2) 1L else 2L
    shorter <- if (longer == 1L) 2L else 1L

    # move last person from longer to end of shorter
    mover <- tail(q[[longer]], 1)
    q[[longer]] <- q[[longer]][-length(q[[longer]])]
    q[[shorter]] <- c(q[[shorter]], mover)
  }
}

# schedule first arrival
t_first <- next_arrival_time(0)
if (is.finite(t_first) && t_first < T_END){
  event_list <- add_event(event_list, time = t_first, type = "arrival")
}

# main loop
while (nrow(event_list) > 0){
  popped <- pop_next_event(event_list)
  ev <- popped$event
  event_list <- popped$event_list
  t_now <- ev$time

  if (ev$type == "arrival"){
    cid <- next_id
    next_id <- next_id + 1

    line <- choose_line()

    customers <- rbind(
      customers,
      data.frame(
        cust_id = cid,
        arrival_time = t_now,

```

```

        service_start = NA_real_,
        service_end = NA_real_,
        wait_time = NA_real_,
        chosen_line = line
    )
)

# join that line
q[[line]] <- c(q[[line]], cid)

# schedule next arrival
t_next <- next_arrival_time(t_now)
if (is.finite(t_next) && t_next < T_END){
    event_list <- add_event(event_list, time = t_next, type = "arrival")
}

# try starting service on that line if its server is idle
event_list <- start_service(t_now, line, event_list)

# if the other server is idle, it can start too
other <- if (line == 1L) 2L else 1L
event_list <- start_service(t_now, other, event_list)

} else if (ev$type == "depart"){
    s <- ev$server

    # free server s
    busy_until[s] <- Inf
    server_job[s] <- NA_integer_

    # optional switching happens at service completions
    if (switching) do_switching_if_needed()

    # start service on both servers if possible
    event_list <- start_service(t_now, 1, event_list)
    event_list <- start_service(t_now, 2, event_list)
}
}

customers
}

```

A)

```

simulate_week_system3 <- function(switching = FALSE, switch_threshold = 2, seed = NULL){
    if (!is.null(seed)) set.seed(seed)

    days <- vector("list", 5)
    for (d in 1:5){
        days[[d]] <- simulate_day_system3(switching = switching, switch_threshold = switch_threshold)
        days[[d]]$day <- d
    }
}

```

```

}
do.call(rbind, days)
}

run_system3_experiment <- function(n_weeks = 7, switching = FALSE, switch_threshold = 2, seed = 123){
  set.seed(seed)

  all_weeks <- vector("list", n_weeks)
  week_summaries <- data.frame(
    week = integer(),
    mean_wait = numeric(),
    p90_wait = numeric(),
    avg_customers_per_day = numeric(),
    total_wait_minutes_per_day = numeric()
  )

  for (w in 1:n_weeks){
    cust <- simulate_week_system3(switching = switching, switch_threshold = switch_threshold)
    cust$week <- w
    all_weeks[[w]] <- cust

    daily_totals <- aggregate(wait_time ~ day, data = cust, FUN = function(x) sum(x, na.rm = TRUE))
    mean_total_wait_per_day <- mean(daily_totals$wait_time)

    waits <- cust$wait_time
    waits <- waits[is.finite(waits)]

    week_summaries <- rbind(
      week_summaries,
      data.frame(
        week = w,
        mean_wait = mean(waits),
        p90_wait = unname(quantile(waits, 0.90)),
        avg_customers_per_day = nrow(cust)/5,
        total_wait_minutes_per_day = mean_total_wait_per_day
      )
    )
  }

  all_customer_data <- do.call(rbind, all_weeks)

  overall <- data.frame(
    mean_wait = mean(week_summaries$mean_wait),
    p90_wait = mean(week_summaries$p90_wait),
    avg_customers_per_day = mean(week_summaries$avg_customers_per_day),
    total_wait_minutes_per_day = mean(week_summaries$total_wait_minutes_per_day)
  )

  list(customers = all_customer_data, week_summaries = week_summaries, overall = overall)
}

```

## B) Week + 7-week experiment wrappers (System 3)

```
# No switching
res3_no <- run_system3_experiment(n_weeks = 7, switching = FALSE, seed = 1)
res3_no$overall
```

### C) Run System 3: without switching vs with switching + plot (distribution comparison)

```
##   mean_wait p90_wait avg_customers_per_day total_wait_minutes_per_day
## 1  2.674667 9.156186          37.71429          100.7513
```

```
cost3_no <- compute_daily_cost(res3_no$overall$total_wait_minutes_per_day, n_servers = 2)
cost3_no
```

```
##   waiting_cost staffing_cost total_cost
## 1    50.37566          540    590.3757
```

```
# With switching
res3_sw <- run_system3_experiment(n_weeks = 7, switching = TRUE, switch_threshold = 2, seed = 1)
res3_sw$overall
```

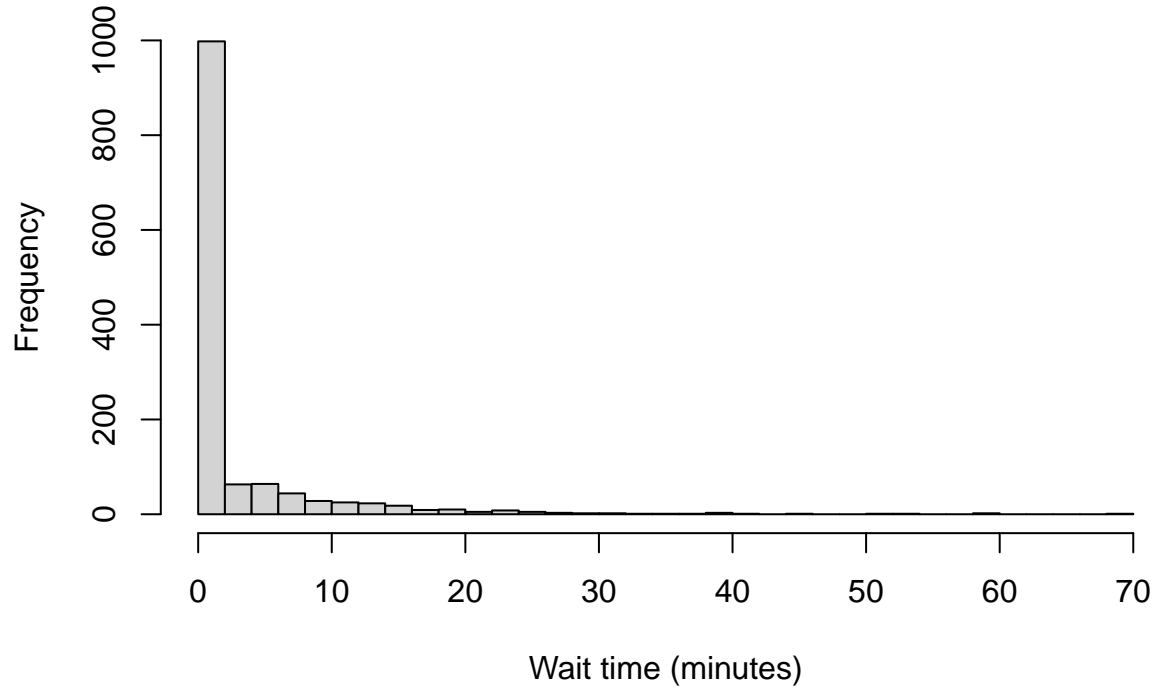
```
##   mean_wait p90_wait avg_customers_per_day total_wait_minutes_per_day
## 1  2.498084 8.233334          38          95.12854
```

```
cost3_sw <- compute_daily_cost(res3_sw$overall$total_wait_minutes_per_day, n_servers = 2)
cost3_sw
```

```
##   waiting_cost staffing_cost total_cost
## 1    47.56427          540    587.5643
```

```
# plot (distribution comparison)
hist(res3_no$customers$wait_time, breaks = 30,
     main = "System 3: Waiting Time Distribution (No Switching)",
     xlab = "Wait time (minutes)")
```

### System 3: Waiting Time Distribution (No Switching)



```
hist(res3_sw$customers$wait_time, breaks = 30,  
     main = "System 3: Waiting Time Distribution (With Switching)",  
     xlab = "Wait time (minutes)")
```

### System 3: Waiting Time Distribution (With Switching)

