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CSCI 5521

HW1 Q3 Written Responses

1) The discriminant function for: "Multivariate Gaussian classifier assuming class-dependent covariance" is given by the following:

$$g_i(x) = -\frac{1}{2}\log|S_i| - \frac{1}{2}(x-m_i)^T S_i^{-1}(x-m_i) + \log \hat{P}(C_i)$$

How I coded this equation in my program:

```
m = self.mean[c].reshape(8,1)
dot = np.float(np.dot(np.dot((x - m).T, inv(self.S[c])),(x - m)))
val = -0.5 * math.log(det(self.S[c])) - 0.5 * dot + math.log(self.p[c])
```

The discriminant function for: "Multivariate Gaussian classifier assuming class-independent covariance" is given by the following:

$$g_i(x) = -\frac{1}{2}(x-m_i)^T S^{-1}(x-m_i) + \log \hat{P}(C_i)$$

How I coded this equation in my program:

```
m = self.mean[c].reshape(8,1)
dot = np.float(np.dot(np.dot((x - m).T, inv(self.S)),(x - m)))
val = - 0.5 * dot + math.log(self.p[c])
```

The discriminant function takes in the sample mean, $\mathbf{m_i}$, covariance matrix, $\mathbf{S_i}$, priors, $\mathbf{p(C_i)}$, and sample test, $\mathbf{x}$, to calculate a score value. This value is used to determine which class the sample test, $\mathbf{x}$, belongs to. The main difference between the two discriminant functions: the second equation is a reduced version of the first equation. The reason for this reduction: the second equation utilizes a common covariance matrix whereas the first equation has separate $\mathbf{S_i}$ for each class, C_1 and C_2.

2) Confusion Matrix on the test set for each assumption:

```
In [1]: runfile('/Users/kiavang/CSCI5521/hw1_programming/hw1.py', wdir='/Users/kiavang/CSCI5521/
hw1_programming')
Confusion Matrix for Gaussian Discriminant with class-dependent covariance
[[20  7]
 [10 63]]
Confusion Matrix for Gaussian Discriminant with class-independent covariance
[[25  5]
 [ 5 65]]
```

The Confusion Matrix is visual representation (of TP, FP, FN, and TN) that shows the prediction performance of the algorithm.

Confusion Matrix		Actual Class	
		P	N
Predicted Class	P	TP	FP
	N	FN	TN

Class-dependent Covariance Case:

Sensitivity (True Positive Rate) = $\frac{TP}{TP+FN} = \frac{20}{20+10} = \frac{2}{3} \approx 0.667$

- → 66.7%, proportion of positives that were correctly identified

Specificity (True Negative Rate) = $\frac{TN}{TN+FP} = \frac{63}{63+7} = \frac{63}{70} \approx 0.90$

- → 90%, proportion of negatives that were correctly identified

Class-independent Covariance Case:

Sensitivity (True Positive Rate) = $\frac{TP}{TP+FN} = \frac{25}{25+5} = \frac{5}{6} \approx 0.833$

- → 83.3%, proportion of positives that were correctly identified

Specificity (True Negative Rate) = $\frac{TN}{TN+FP} = \frac{65}{65+5} = \frac{65}{70} \approx 0.928$

- → 92.8%, proportion of negatives that were correctly identified