CAS 760

Simple Type Theory

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12 Software Support and Final Remarks

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Outline

1. Software support.
2. Final remarks.

1. Software Support

Basic Support

The most basic support is software for processing Alonzo types and expressions. There are three main approaches:

1. Build LaTeX commands that represent Alonzo types and expressions.
2. Write a language for representing Alonzo types and expressions as strings plus a parser for translating strings into data structures.
3. Produce an interactive development environment (IDE) for Alonzo types and expressions.

Advanced Support

Basic software support for Alonzo provides a foundation for advanced support for Alonzo in the five areas corresponding to the five aspects of the tetrapod model of mathematical knowledge:

1. Organization: Software (e.g., theory assistants) for building theory graphs.
2. Inference: Software for the three approaches to solving problems of the fundamental form $T \models^{\mathsf{s}} \mathbf{A}_o$.
3. Computation: Software for computing internal functions represented by constants and external SBMAs.
4. Concretization: Software for building and managing databases of Alonzo objects.
5. Narration: Software for communicating theory narratives and theory graph narratives.

Fully Integrated Support

- The most effective software system for Alonzo would be an interactive mathematical laboratory (IML) in which the kinds of support mentioned above are fully integrated.
- An IML of this kind is needed for large-scale mathematics research.
- A comprehensive IML for Alonzo would have the potential to transform how people learn and practice mathematics.

2. Final Remarks

Course Objective Revisited

- The overarching objective of Simple Type Theory is to transform how logic is taught in university to students who will actually need to use logic in a practical way.
- This course and the textbook seek to achieve this objective by:
  1. Offering Alonzo, a practical, general-purpose predicate logic for expressing and reasoning about mathematical ideas.
  2. Introducing the key concepts of predicate logic and type theory using Alonzo.
  3. Showing that mathematical ideas can be expressed in Alonzo in a manner that is very close to how they are usually expressed in mathematical practice.
  4. Showing that Alonzo is ideally suited for reasoning about mathematical structures and constructing libraries of mathematical knowledge.
  5. Focusing on the model theory instead of on the proof theory of Alonzo.
  6. Providing a foundation for the study of higher-order logic and type theory.

What is Formal Mathematics? [1/2]

- Formal mathematics is mathematics done with the aid of a formal logic.
- A formal logic (logic for short) is a family of languages such that:
  1. The languages of the logic have a common precise syntax.
  2. The languages of the logic have a common precise semantics with a notion of logical consequence
  3. There is a sound formal proof system for the logic in which proofs can be syntactically constructed..
- This is a very general definition of a logic that covers the various versions of first-order logic, simple type theory, and dependent type theory as well as the various versions of set theory.

What is Formal Mathematics? [2/2]

- Formal mathematics is done by:
  1. Choosing a formal logic.
  2. Constructing theories of the logic.
  3. Developing the theories by defining concepts and stating and proving facts.
  4. Interconnecting the theories with morphisms.
  5. Transporting definitions and theorems from one theory to another.

Five Big Benefits of Formal Mathematics

1. Mathematics can be done with greater rigor.
2. Conceptual errors can be systematically discovered.
3. Mathematics can be done with software support.
4. Results can be mechanically checked.
5. We can regard mathematical knowledge as a formal structure consisting of a network of interconnected theories.

Standard Approach to Formal Mathematics

- The standard approach to formal mathematics is to do mathematics with the help of a proof assistant and have all details formally proved and mechanically checked.
- It has three major strengths:
  1. It achieves all five benefits of formal mathematics.
  2. All theorems are verified by machine-checked formal proofs, and thus there is a very high level of assurance that the results produced are correct.
  3. There are several powerful proof assistants available that support the approach.
- It has two important weaknesses:
  1. It prioritizes certification over communication.
  2. It is not accessible to the great majority of mathematics practitioners.

Campaign for Formal Mathematics

- For at least 60 years, there has been a campaign to transform traditional mathematical practice into a formal discipline using the standard approach.
- The campaign appears to have been a great success:
  - Several powerful proof assistants have been developed.
  - Much mathematical knowledge has been formalized.
  - Researchers are now using proof assistants to check the proofs of the theorems they prove.
- However, the campaign has been a failure if one considers that no more than 1% of mathematics practitioners have ever used a formal logic or a proof assistant in their work.
- It has failed since the standard approach focuses on certification instead of on communication and accessibility.

An Alternative Approach

- I have proposed an alternative approach to formal mathematics focusing on communication and accessibility.
  1. The underlying logic is fully formal and supports standard mathematical practice.
  2. Proofs can be traditional, formal, or a mix of the two.
  3. Mathematical knowledge is organized as a theory graph using the little theories method.
  4. There are several levels of software support.
- Certification is still important, but it is can be done when appropriate via traditional proof and cross checks.
- In summary:
  1. Everything is done within a formal logic except proofs.
  2. Communication and accessibility are key.

Formal Mathematics for the Masses

- I am pursuing a project called Formal Mathematics for the Masses.
- Its objective is to make formal mathematics more useful, accessible, and natural to a wider range of mathematics practitioners by implementing the alternative approach.
- The goals of the project are to:
  1. Design a suitable practice-oriented logic with a formal proof system and a module system.
  2. Develop several levels of software support.
  3. Demonstrate the effectiveness of the alternative approach by formalizing a large body of mathematical knowledge.

Results

- Alonzo, a practice-oriented version of simple type theory.
- A formal proof system for Alonzo.
- A module system for Alonzo.
- A graduate-level textbook that presents simple type theory using Alonzo: W. Farmer, *Simple Type Theory, Second Edition*, Birkhäuser/Springer, 2025.
- A set of LaTeX macros and environments for Alonzo.
- A prototype expression editor for Alonzo (joint work with Andrew Thompson).
- A formalization of monoid theory in Alonzo using the little theories method and the alternative approach: W. M. Farmer and D. Y. Zvigelsky, "Monoid Theory in Alonzo", Journal of Applied Logics, 12:1853–1939, 2025.

Future Work

- A paper that presents the alternative approach to formal mathematics to the mathematics community.
- A fully implemented expression editor for Alonzo.
- A theory graph assistant for Alonzo.
- A formalization of a large body of mathematical knowledge in Alonzo.

The End.