Arbitrage Opportunities and Overnight Returns in Leveraged and Inverse ETFs

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Abstract

We investigate how accurate leveraged and inverse ETFs are in delivering their intended single-day returns and add much-needed context to the analysis of leveraged ETF performance. We demonstrate that these funds exhibit predictable deviations in returns relative to their target and design a market-neutral portfolio to capture them. We then demonstrate the viability of leveraging the portfolio's returns through short selling and investments in option derivatives. Additionally, the study documents a case of asymmetric returns in overnight and intraday sessions in both stocks and options in leveraged and inverse ETFs.

I. Introduction

ETFs have become an everyday tool for investors and a major source of income for their issuers. As of Q1 2023, approximately 7 trillion dollars were invested in equity and fixed-income ETFs in the United States market[1], leading to a complicated network of interactions spanning all market sectors. We will concentrate on ETFs designed to provide a daily return multiple of a specific index, in particular the popular Nasdaq-100 and S&P 500 families offered by ProShares. These ETFs offer varying levels of leverage, including -1x, -2x, -3x, 2x, and 3x against the respective index. These funds have gained enormous popularity, with the 3x and -3x Nasdaq-100 ETFs posting an average daily volume of 170 million and 140 million shares respectively. Leveraged ETFs exhibit complex structures and significant expense ratios, ranging from 0.86% to 0.95%, which can result in minor deviations from their intended daily return, despite their high liquidity. Though one never expects these instruments to move in perfect synchrony, it is a natural assumption that any single day deviation, beyond decay caused by the expense ratio, is due to simple randomness. However, we will establish that these deviations are far from random and explore the internal holdings of the ETFs to understand the source of these divergences.

The performance of leveraged ETFs has been a source of confusion for many investors, primarily because these funds aim to generate accurate returns only for a single trading day. Existing literature has overwhelmingly focused on modeling the long-term returns of leveraged ETFs relative to their target indices. Papers such as Lu et al. (2009), Zhang and Judge (2016), and Peterburgsky (2018) all put emphasis on how market volatility and compounding of returns lead to long-term underperformance in leveraged ETFs. Yet we only

[1] Statistics provided by BlackRock: https://www.ishares.com/us/insights/global-etf-facts

found an attempt to quantify and explain the effects of single-day deviations in Avellaneda and Dobi (2012). Avellaneda and Dobi (2012) conclude that leveraged funds consistently underperform due to internal rebalancing and slippage. However, because their paper analyzes data in an exclusively low interest rate environment, they do not observe fundamental trends that contradict their assumptions and conclusions. Additionally, they overestimate the magnitude of returns that can be captured, as explained in Section IV. Our paper aims to thoroughly describe these single-day deviations so that returns of leveraged ETFs could be interpreted in the proper context. While we demonstrate that there is nothing inherently mysterious about these discrepancies, we want to emphasize that the market appears to fail at pricing this phenomenon.

This paper is organized as follows: Section II provides a brief overview of data collection methods. Section III considers the performance of popular ETFs relative to their target index and investigates their holdings to understand the source of discrepancies in returns. In Section IV, we will introduce capital constraints and consider optimal portfolio construction for high and low interest rate environments. Section V will demonstrate ways to leverage our returns through short selling and option investments. We will also address market-maker trading practices in the context of this strategy. Section VI is dedicated to observations regarding portfolio returns of overnight and intraday sessions. In Section VII, we will summarize our findings and address any remaining questions.

II. Data

Dividends and price data for equities and indices were provided by *Yahoo Finance*. Index price data was cross-referenced with the information provided by *MarketWatch* and the *BarChart for Excel* service. In addition, historical NAV data was provided by *YCharts.*

Current short-selling fees and share availability data were provided by *Interactive Brokers*, while historical records of this data were accessed through *IBorrowDesk*.

Historical intraday returns were accessed through *BarChart for Excel* and *TD Ameritrade's ThinkOrSwim On Demand* tool.

Historical end-of-day data for options contracts was provided by *MarketData.app*. The data required moderate cleaning and is likely less reliable than more reputable and expensive vendors. Values were manually cross-referenced with prices provided by *TD Ameritrade's ThinkOrSwim On Demand* tool.

Hypothetical margin requirements were calculated using the margin calculators provided by the *Options Clearing Corporation* and *Interactive Brokers*.

III. Initial Observation

To begin gauging the accuracy of index ETFs, we will compare the well-known S&P 500 Index (SPX) to its corresponding ETF replica SPY. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ will contain the daily percentage changes of SPX and SPY respectively. Vector $\boldsymbol{c}$ is defined as the cumulative product of the sum of vector $-1 * \boldsymbol{a}$ and vector $\boldsymbol{b}$, which is graphed against time and presented in Fig. 1. The periodic spikes in Fig. 1 illustrate that SPY is a dividend paying equity that experiences a negative price correction on the ex-dividend date. By adding dividends to vector $\boldsymbol{b}$ to reflect the total returns of SPY, we get Fig. 2. which illustrates the flow of money into SPY strictly for dividend purposes.

SH is the ticker symbol for a -1x multiple of the S&P 500 Index. By repeating the process described in the paragraph above (keeping vector $\boldsymbol{a}$ positive now), we isolate the return deviations of SH. A peculiar trend appears, one that can be replicated with SDS and SPXU, the -2x and -3x ETF multiples of SPX (taking care to scale vector $\boldsymbol{a}$ by factors of 2 and 3). The returns across portfolios constructed using SPX and SH/SDS/SPXU are presented in Fig. 3. The reverse trend can be observed in portfolios constructed using SPX and SSO/UPRO, the 2x and 3x ETF multiples of SPX, which are presented in Fig. 4.

As observed in Fig. 2, one cause of these divergences can be attributed to dividend expectations. Additionally, the holdings of inverse and leveraged funds primarily consist of index swaps, which are private contracts where one party agrees to pay a negotiated interest rate in exchange for the return of a target index. Inverse ETFs, which hold short positions in swaps, are on the receiving end of interest rate payments. Leveraged long ETFs, which hold long positions in swaps, pay out this interest rate. Fig. 5 displays the returns of SPXU and UPRO in the context of the federal funds rate and suggests that a high-rate environment increases returns for swap sellers and increases losses for swap buyers. Finally, expense ratios and transaction costs associated with the fund (not covered by an expense ratio), place downward pressure on the returns of the fund. We cannot fully explain the general opposing trends in long and inverse funds as presented in Fig. 3 and 4. However, we theorize that short swaps, much like option put contracts, behave as a form of insurance with associated premiums. This would explain why an inverse fund generally underperforms its target, whereas long funds generally outperform their target.

To be sure that these returns are completely associated with the structure of the fund, we substitute historical market prices with NAV data published for the ETF SH. Fig. 6 displays the same trends as analyzed previously, and we conclude that these trends are tied to the internals of the fund.

IV. Capital Constraints and Efficient Combinations

To build a real portfolio, we must move away from theoretical price indices. This paper will not analyze the viability of futures contracts for recreating returns discussed in Section III, we leave this for future exploration. We will continue by analyzing portfolios consisting of shares and leveraged with option derivatives.

First, we need to introduce capital constraints on the portfolios analyzed in section III. Consider the SH/SPX combination illustrated in Fig. 3. Our goal was to analyze the return deviations offered by the SH ETF relative to its target. We simulated a 100% investment in SH with a theoretical hedge in SPX purely for visualization purposes. This means that the hedge we construct using the underlying index is not feasible in practice without diluting our position in SH or borrowing additional capital. Going forward, vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ will be scaled to maintain a proper hedge such that their absolute sum will be 1. For example, a portfolio hedging SPX against SPXU will hold a 75% weight in SPX and a 25% weight in SPXU. An x dollar investment in SPXU demands a $3x$ dollar investment in SPX, a total capital requirement of $4x$. Assuming limited capital of value x, we allocate $\frac{1}{4}x$ to SPXU and $\frac{3}{4}x$ to SPX. This adjustment will have a serious impact on the magnitude of our returns.

Understanding which portfolio combination is the most efficient requires a concise summary of daily returns as a function of interest rates. Table 1 provides a linear approximation of this relationship for every ETF in question.

To rank the portfolio combinations, we employ a brute force approach. We sort our output by the slope and constant components of the linear approximation, which describes our portfolio's behavior as interest rates grow sufficiently large or as interest rates approach 0. The rankings are presented in Tables 2 and 3 and indicate that a hedge between SPY and SPXU is optimal for high interest rate environments, while a hedge between SPXU and UPRO is optimal for low interest rate environments. Moving forward, we will substitute SPXU and UPRO with the Nasdaq-100 equivalents, SQQQ and TQQQ, to take advantage of their superior liquidity.

V. Options Leverage

Before attempting to work with options, we briefly considered short selling as a form of leverage. Given that our portfolio is market neutral with 0 expected returns, margin calculators predict a near 0 collateral requirement for both short stock and option portfolios. Considering that borrowing fees constitute our largest expense, we question whether institutional market makers might achieve artificial short exposure by accruing liabilities through delayed share delivery, thereby circumventing the costs associated with borrowing. Evans et al. (2017) provide this statistic: "ETFs constitute 10% of U.S. equity market capitalization but over 20% of short interest and 78% of failures-to-deliver." Additionally, Baltussen et al. (2020) explain that "Market makers in products with gamma exposure, such as options and leveraged ETFs, are commonly net short these products." This suggests that our speculations may have grounds, and we welcome any insights from those more familiar with the business practices and regulations pertaining to these institutions.

The Put-Call parity relates the price of the underlying security to the price of its put and call (of equal strike and expiration) through equation 1:

$$Price_{Call} - Price_{Put} = Price_{Underlying} - Discount * Strike \tag{1}$$

where *Discount* is the price of a 1 Dollar treasury bill maturing at option expiration. We calculate its change with respect to time in equation 2:

$$\Delta(Price_{Call} - Price_{Put}) = \Delta(Price_{Underlying}) - \Delta(Discount) * Strike \tag{2}$$

Treasuries have linear growth with respect to time, therefore the daily change in the $Call - Put$ component reflects the change in the price of the underlying, less some constant given by $\Delta(Discount) * Strike$. The return on a synthetic long position lies below the return of long shares, and vice versa for a short position.

With this in mind, we presume that a double synthetic long position in our ETFs will produce insignificant returns, as it will be muted by the discount factor. Theoretically, a portfolio in which one ETF is held short and the other is held long will negate the discount factor and isolate the portfolio returns. Yet the strongest returns will lie where both ETFs are held short, pocketing both the portfolio returns and the discount rate, possibly enough to overcome transaction costs and price imperfections.

To illustrate the divergence between options and shares, consider the assets SPY and SH graphed in Fig. 7. Fig. 8 shows the effect of trying to replicate a profitable long position in SPY and SH in the options market. As predicted, the underperformance of options negates the positive returns the portfolio was designed to capture.

We now consider an options portfolio that maintains a hedge through a synthetic short position in SQQQ and TQQQ. The simulation holds contracts until expiration and reinvests funds into at-the-money contracts with the second nearest expiration, creating a bi-weekly rotation. For illustration purposes, we assume a frictionless environment with no commissions. Similar to Fig. 7, Fig. 9 illustrates the long-term performance of a synthetic short position in options relative to a short position in shares for the assets SQQQ and TQQQ. Fig. 10 shows the net effect of replicating a short position in SQQQ and TQQQ in the options market. The magnitude of returns for our portfolio is dependent largely on our margin requirement. Once again, FINRA regulation[2] points to the stress tests conducted by the *Options Clearing Corporation,* which have computed a near zero requirement for our market neutral portfolio. The returns in Fig. 11 are therefore calculated relative to the price of the options contracts, plus an arbitrary collateral of 10% of the underlying for each contract sold.

VI. Intraday and Overnight Sessions

The returns of the portfolios in the previous sections can further be broken down into intraday and overnight sessions[3]. In this section, we substitute the index SPX with the ETF SPY, as the index does not accurately reflect the value of its constituents overnight. In practice, this discrepancy is resolved immediately at the market open, with SPX making a move toward its fair value.

Additionally, Fig. 12 shows that portfolios carrying an index ETF and a leveraged inverse ETF have strong positive overnight returns that revert in the day session. We presume this stems from a temporary premium placed on instruments used for hedging in a riskier overnight environment. A more extreme example can be observed in Fig. 13, illustrating the

[2] FINRA's regulatory notice on margin requirements for leveraged ETFs: https://www.finra.org/sites/default/files/NoticeDocument/p119906.pdf
[3] An introduction to overnight returns: Haghani et al. (2022): https://ssrn.com/abstract=4139328

ETF pair SOXX and SOXS, tracking 1x and -3x multiples of the ICE Semiconductor Index respectively.

Finally, we consider overnight and intraday returns in the options market. To provide an example, we will compare two hedged portfolios: one consisting of investments in SPY and SH shares, and the other comprised of investments in synthetic long options for SPY and SH. Although the synthetic long portfolio mostly accurately tracks the share returns across an entire trading day, Figs. 14 and 15 illustrate that the options and stock portfolios behave very differently in overnight and intraday sessions. The culprit behind the magnitude of returns in the options portfolio are the puts and calls for SH. Whereas a synthetic long position in SPY corresponds with stock returns in both the intraday and overnight session, a synthetic long position in leveraged and inverse ETFs shows staggering returns overnight with an asymmetrical move back to equilibrium in the day session. We do not yet know the cause of these returns.

VII. Conclusion

While current literature focuses almost exclusively on the long-term performance of leveraged ETFs, we explore single-day return deviations to add important context to the performance of the funds. Leveraged and inverse ETFs produce predictable return deviations relative to their target, which we believe get carried over from the fund’s investment in swap derivatives. We tested market-neutral portfolios to isolate and capture these returns and calculated the optimal approach for high and low interest rate environments. We provided clear reasoning and evidence to demonstrate that these returns could be leveraged in the options market, although we limited our observations to a frictionless environment. In addition, upon analyzing the intraday and overnight trading sessions, we suggested that inverse ETFs trade at a premium at night, and self-correct in the day session. We also observed that a synthetic long position in an inverse or leveraged ETF produces a staggering return in the overnight and intraday session, far from the return predicted by the put-call parity. We conclude that borrowing rates for short positions must remain high to ensure that the observed returns cannot be meaningfully leveraged through short selling and investments in options derivatives. Additionally, we point out that this strategy could be a convenient method of generating returns for market-making institutions who could use their privileges to establish artificial short positions at lower costs.

Further research is needed to reach a definite answer regarding the profitability of this strategy in the options market. This would require more robust analysis of liquidity and spreads in the options market and a definite measurement of margin requirements for our positions. Additionally, we remain interested in whether our strategy could be executed by market makers. We seek a detailed breakdown of the regulations and potential workarounds for setting up a net short position in leveraged and inverse ETFs through naked short selling. Finally, we would like to continue exploring the significance of overnight options returns in inverse and leveraged ETFs.

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Figure I: Short SPX and Long SPY, Equal Weight Portfolio, Dividends Excluded

Figure I hedges the price movements featured in SPX and SPY to illustrate the excess value that SPY gains in anticipation of its dividend, along with the value it loses following a payout. We have intentionally excluded dividend cash flow from our returns vector for illustration purposes.

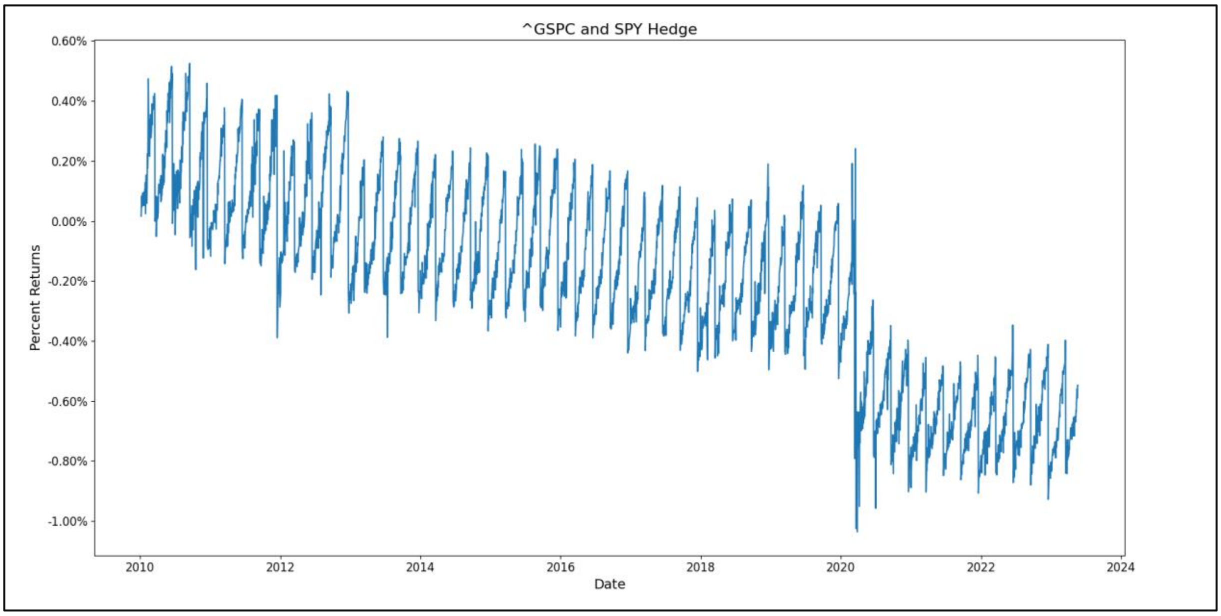

Figure II: Short SPX and Long SPY, Equal Weight Portfolio, Dividends Included

Figure II properly adds dividend payments to reflect the cash flow received on the day of a negative price correction. It demonstrates that SPY's outperformance relative to SPX stems from its behavior as a total return index rather than a price index.

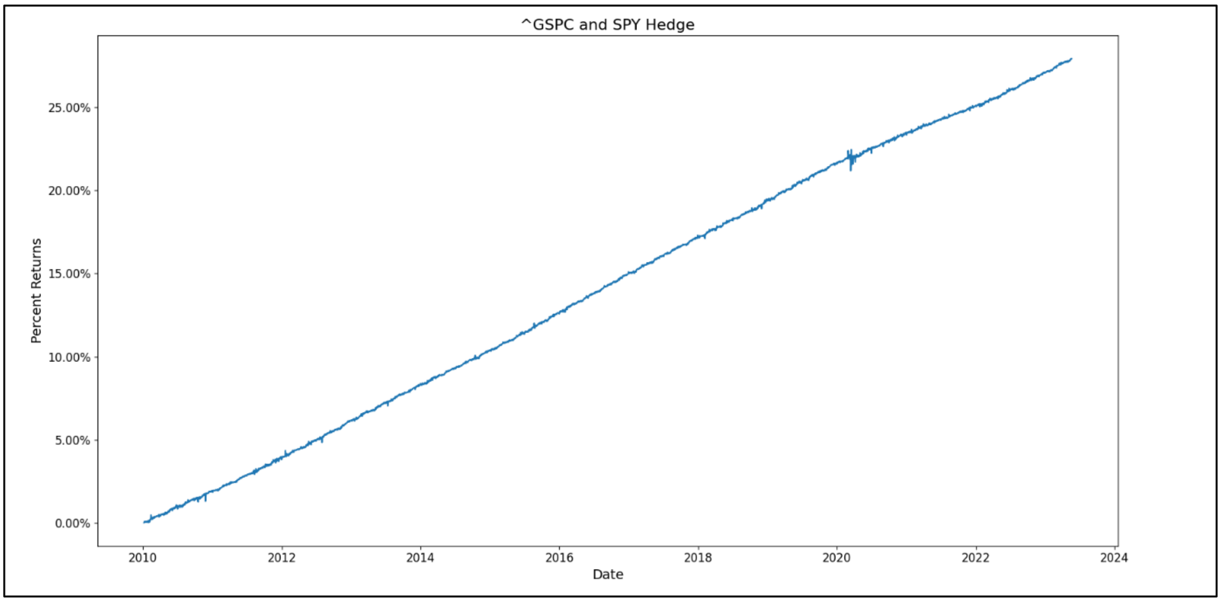

Figure III: Long SPX and Long SH/SDS/SPXU Equal Weight Portfolios

Figure III illustrates periods of excess and deficit returns across SH, SDS, and SPXU by hedging their price movements against their target index SPX. We notice that higher leverage translates to more negative returns.

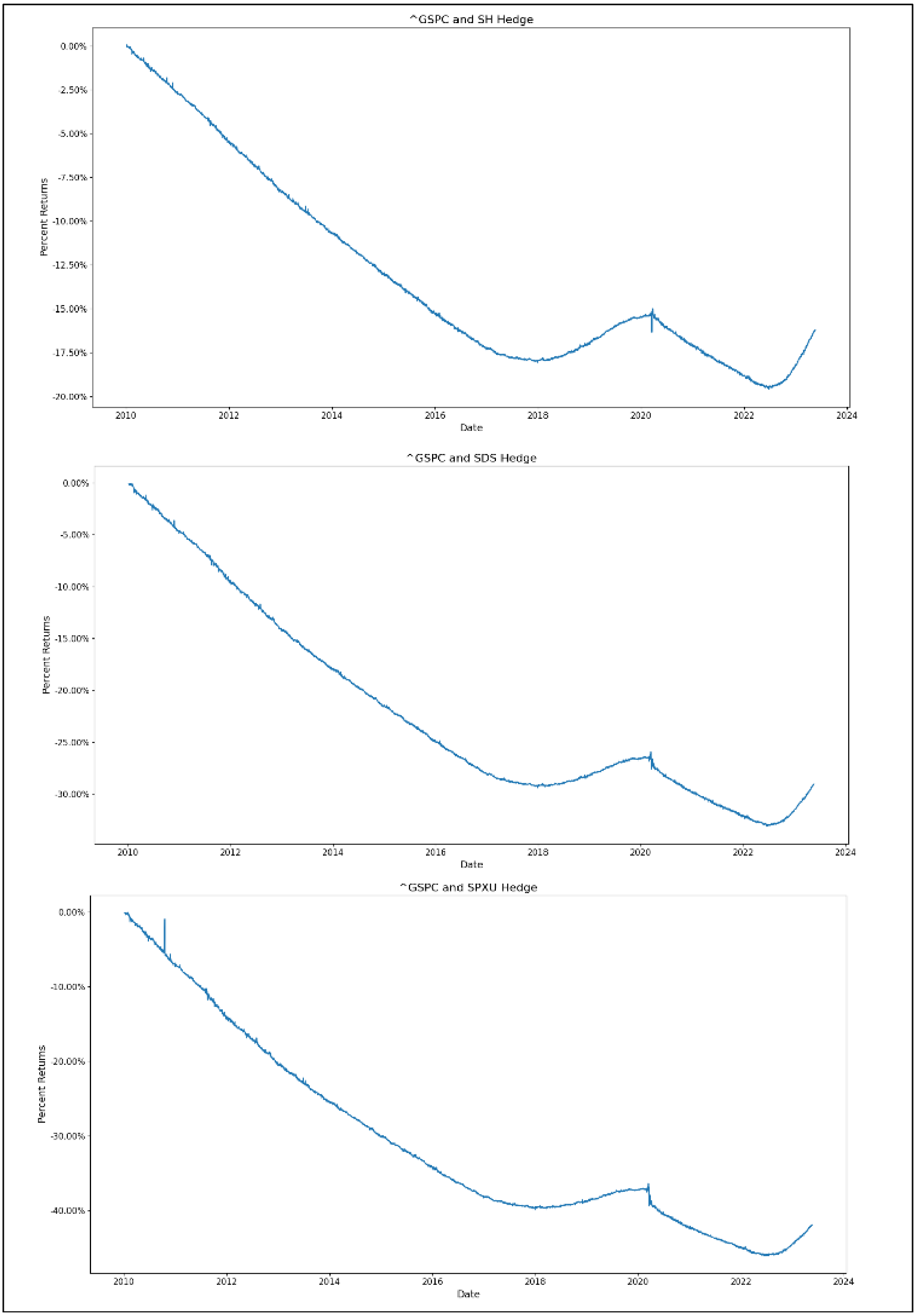

Figure IV: Short SPX and Long SSO/UPRO Equal Weight Portfolios

Figure IV illustrates periods of excess and deficit returns across SSO and UPRO by hedging their price movements against their target index SPX. We notice that higher leverage translates to more positive returns.

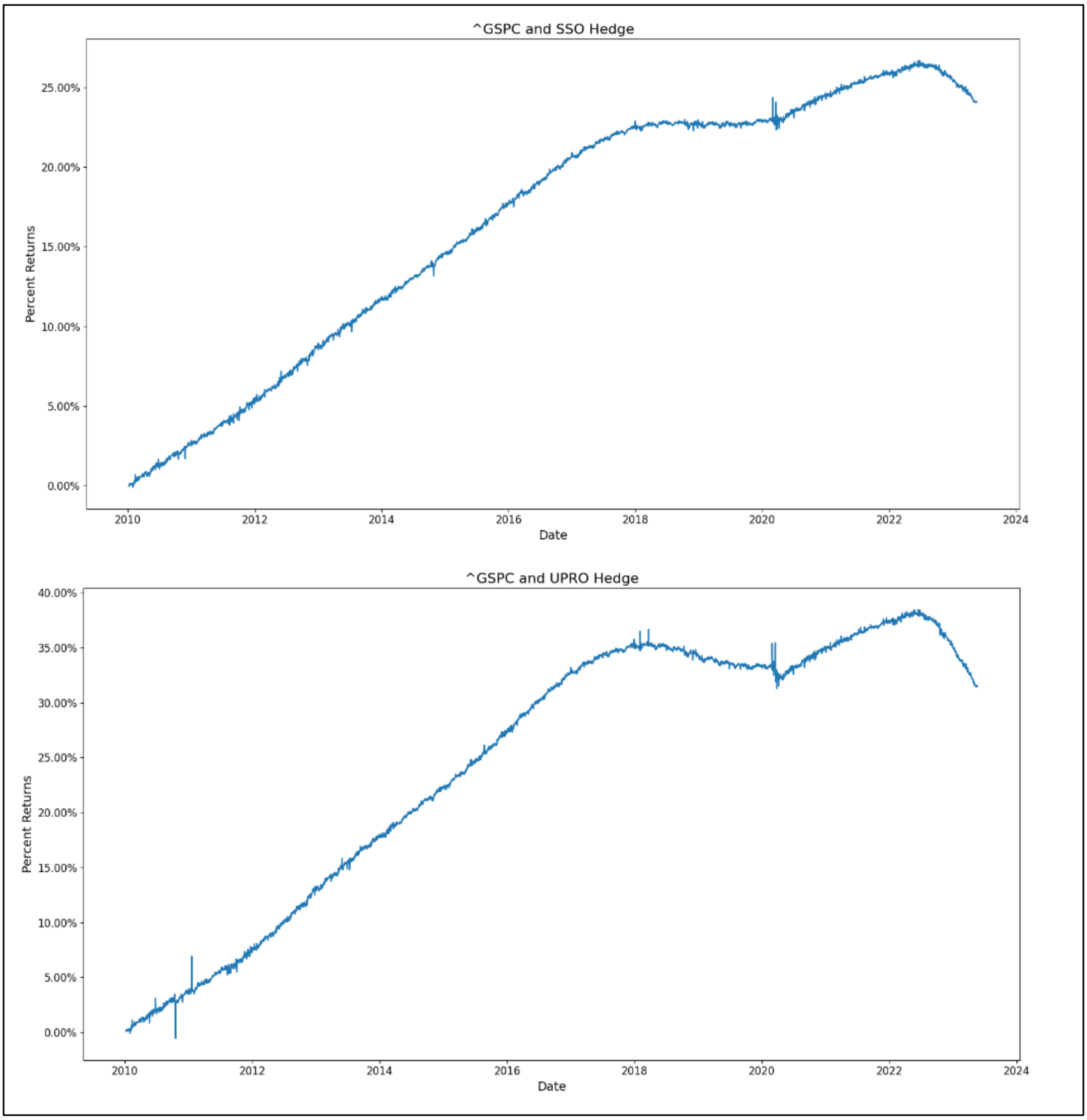

Figure V: Portfolio Returns in Context of Interest Rates

Figure V illustrates the relationship between portfolio returns and interest rates.

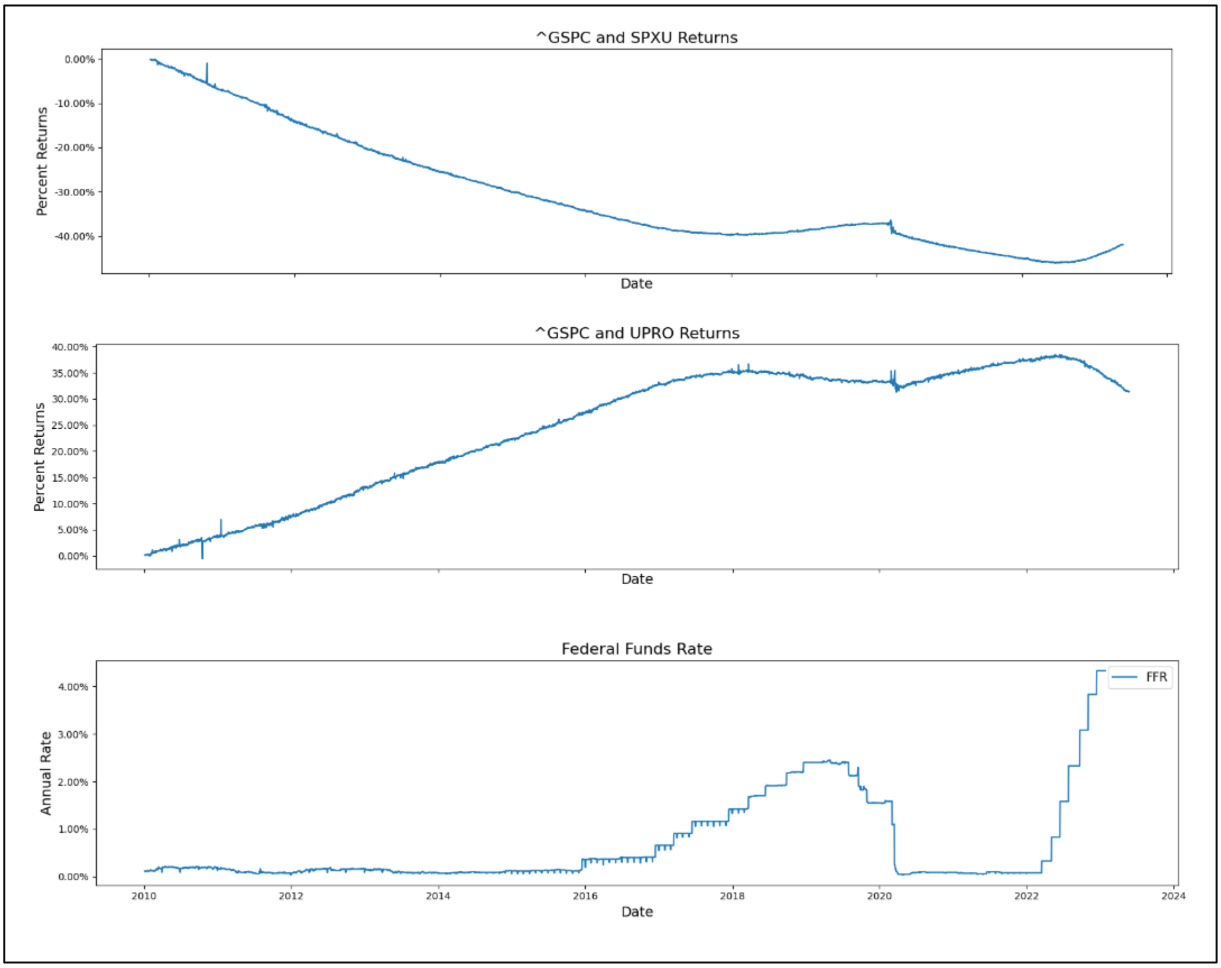

Figure VI: Long SPX and Long SH, NAV, Equal Weight Portfolios

Figure VI replicates the returns achieved through an equal-weight long position in SPX and SH, but substitutes end-of-day NAV data for the market price.

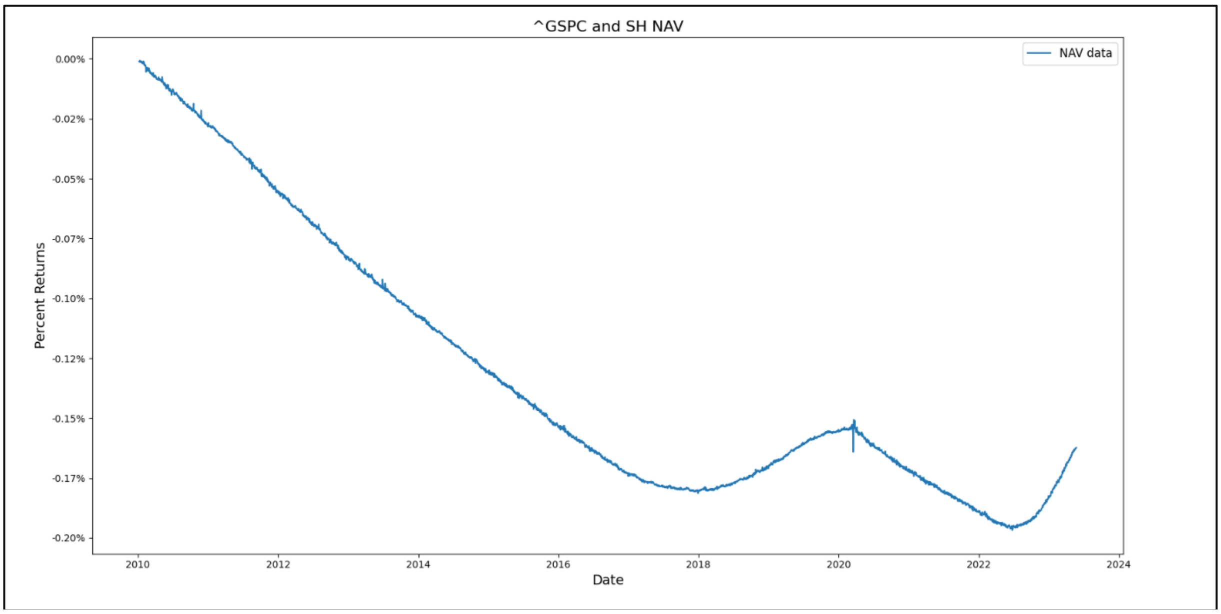

Table 1: Portfolio Sensitivity to Interest Rates

Portfolio	Multiple	Slope	Constant	Std Err	Significance
50% SPY - 50% SPX	1x	-0.000158	0.003770	0.00056	0.77909
33% SSO - 66% SPX	2x	-0.001678	0.003397	0.00073	0.02236
25% UPRO - 75% SPX	3x	-0.002500	0.003928	0.00100	0.01279
50% SH + 50% SPX	-1x	0.004310	-0.005806	0.00068	3.0654e-10
33% SDS + 66% SPX	-2x	0.004426	-0.006653	0.00075	4.2810e-9
25% SPXU + 75% SPX	-3x	0.004645	-0.007388	0.00115	5.622e-5

Daily percentage portfolio returns as a linear function of interest rates between January 2010 and January 2023 (3287 days). Each portfolio assumes a perfect hedge subject to capital constraints. Significance is measured as a two-sided test against the hypothesis that interest rates have no effect on returns.

Table 2: Optimal Portfolio Combination for Low Interest Rate Environments

Portfolio	Slope
0.75 SPY + 0.25 SPXU	0.0046
0.66 SPY + 0.33 SDS	0.0044
0.6 SSO + 0.4 SPXU	0.0043
0.5 SPY + 0.5 SH	0.0043
0.5 UPRO + 0.5 SPXU	0.0042
0.5 SSO + 0.5 SDS	0.0041
0.33 SSO + 0.66 SH	0.0040
0.4 UPRO + 0.6 SDS	0.0040
0.25 UPRO + 0.75 SH	0.0039
0.75 SPY - 0.25 UPRO	0.0025
0.75 SH - 0.25 SPXU	0.0018
0.66 SPY - 0.33 SSO	0.0017
0.66 SH - 0.33SDS	0.0013
0.6 SSO - 0.4 UPRO	0.0010
0.6 SDS - 0.4 SPXU	0.0006

Table 2 ranks the portfolios in order of descending slope, which represents the effect that interest rates have on daily percent portfolio returns, as predicted by our linear model.

Table 3: Optimal Portfolio Combination for High Interest Rate Environments

Portfolio	Constant
-0.5 UPRO - 0.5 SPXU	0.0068
-0.4 UPRO - 0.6 SDS	0.0057
-0.6 SSO - 0.4 SPXU	0.0056
-0.5 SSO - 0.5 SDS	0.0049
-0.25 UPRO - 0.75 SH	0.0047
-0.33 SSO - 0.66 SH	0.0043
-0.5 SPY - 0.5 SH	0.0021
-0.75 SPY - 0.25 SPXU	0.0018
-0.66 SPY - 0.33 SDS	0.0017
0.75 SPY - 0.25 UPRO	0.0017
0.66 SPY - 0.33 SSO	0.0016
-0.75 SH + 0.25 SPXU	0.0013
-0.66 SH + 0.33 SDS	0.0010
-0.6 SDS + 0.4 SPXU	0.0002
-0.6 SSO + 0.4 UPRO	0.0002

Table 3 ranks the portfolios in order of descending constants, which represents the daily portfolio returns in the absence of interest rates, as predicted by our linear model.

Figure VII: SH, SPY: Long Shares vs Synthetic Long Options

Figure VII demonstrates the growing divergence between a long position in shares and a synthetic long position in options, as predicted by the discount component of the Put-Call Parity.

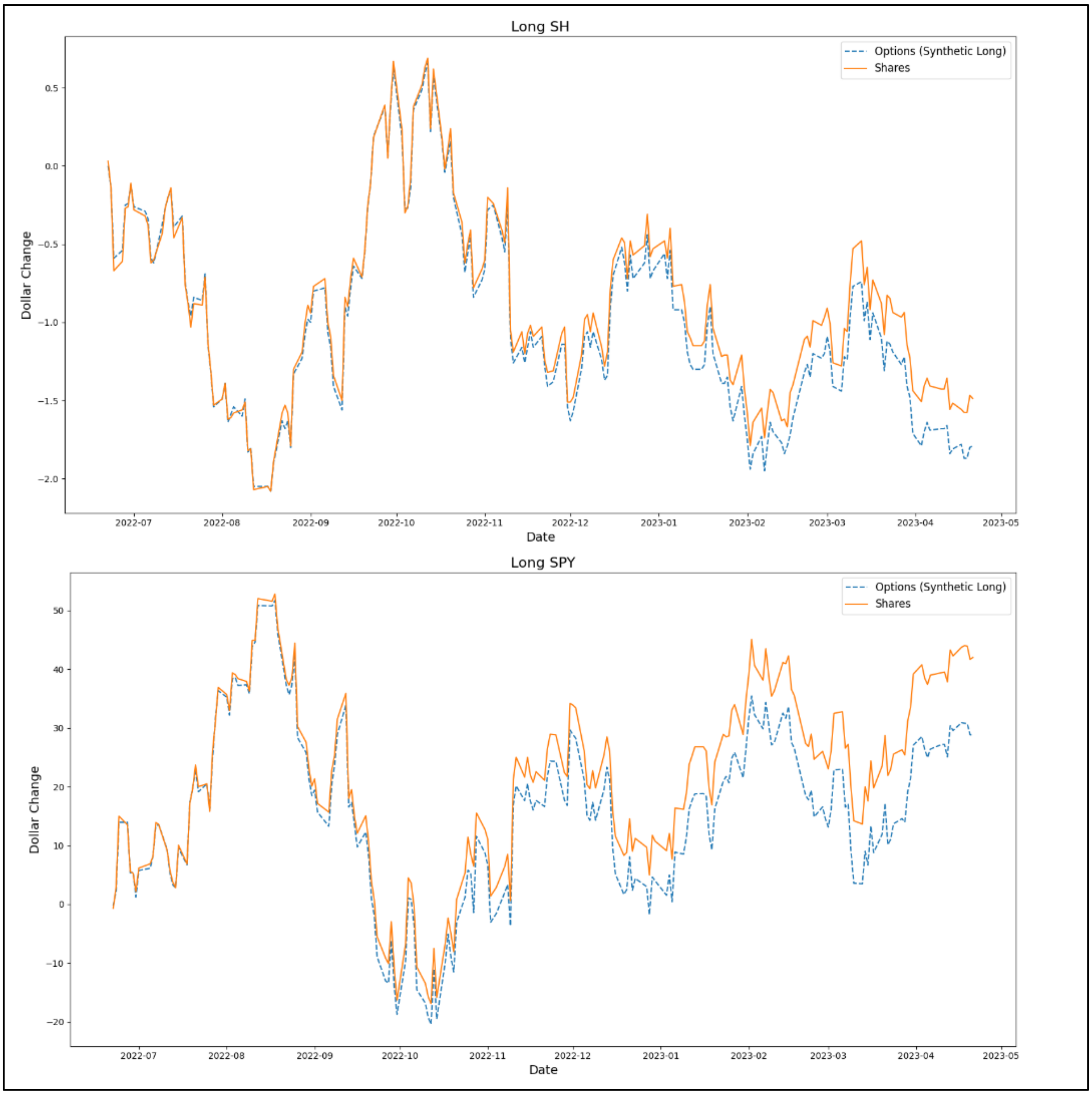

Figure VIII: Synthetic Long Position in SH and SPY

Figure VIII demonstrates how the divergence in Fig. VII creates an offset against the positive portfolio returns we attempt to capture.

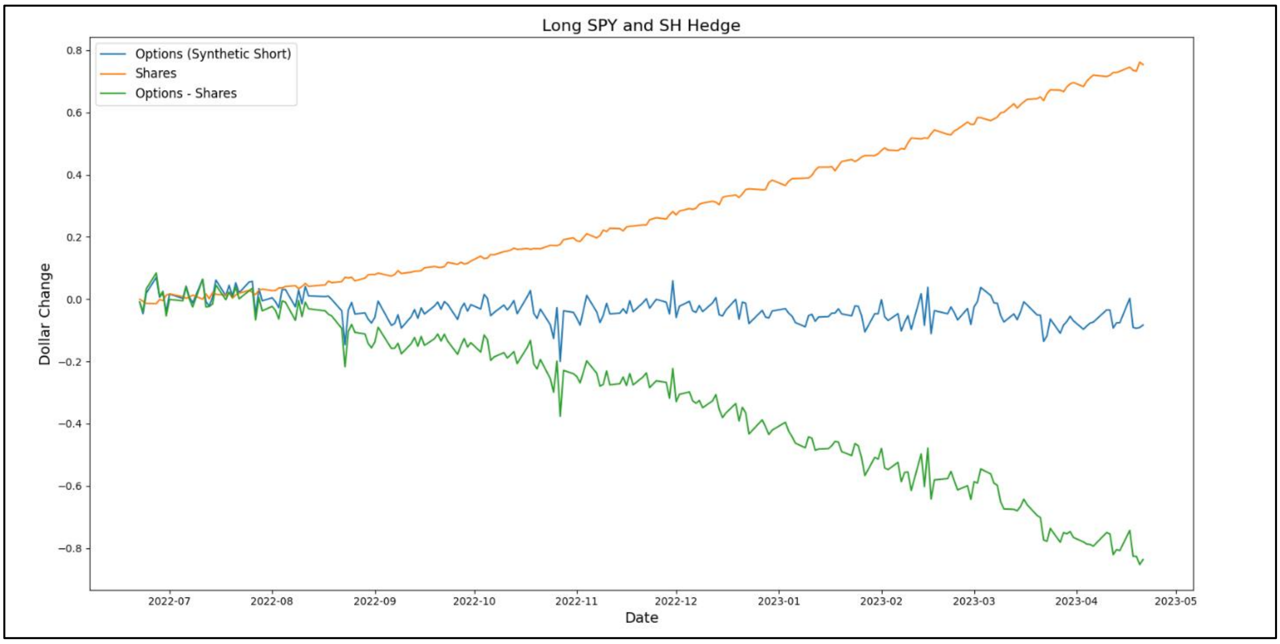

Figure IX: TQQQ, SQQQ: Short Shares vs Synthetic Short Options

Figure IX demonstrates the growing divergence between a short position in shares and a synthetic short position in options, as predicted by the discount component of the Put-Call Parity. Note that options now outperform.

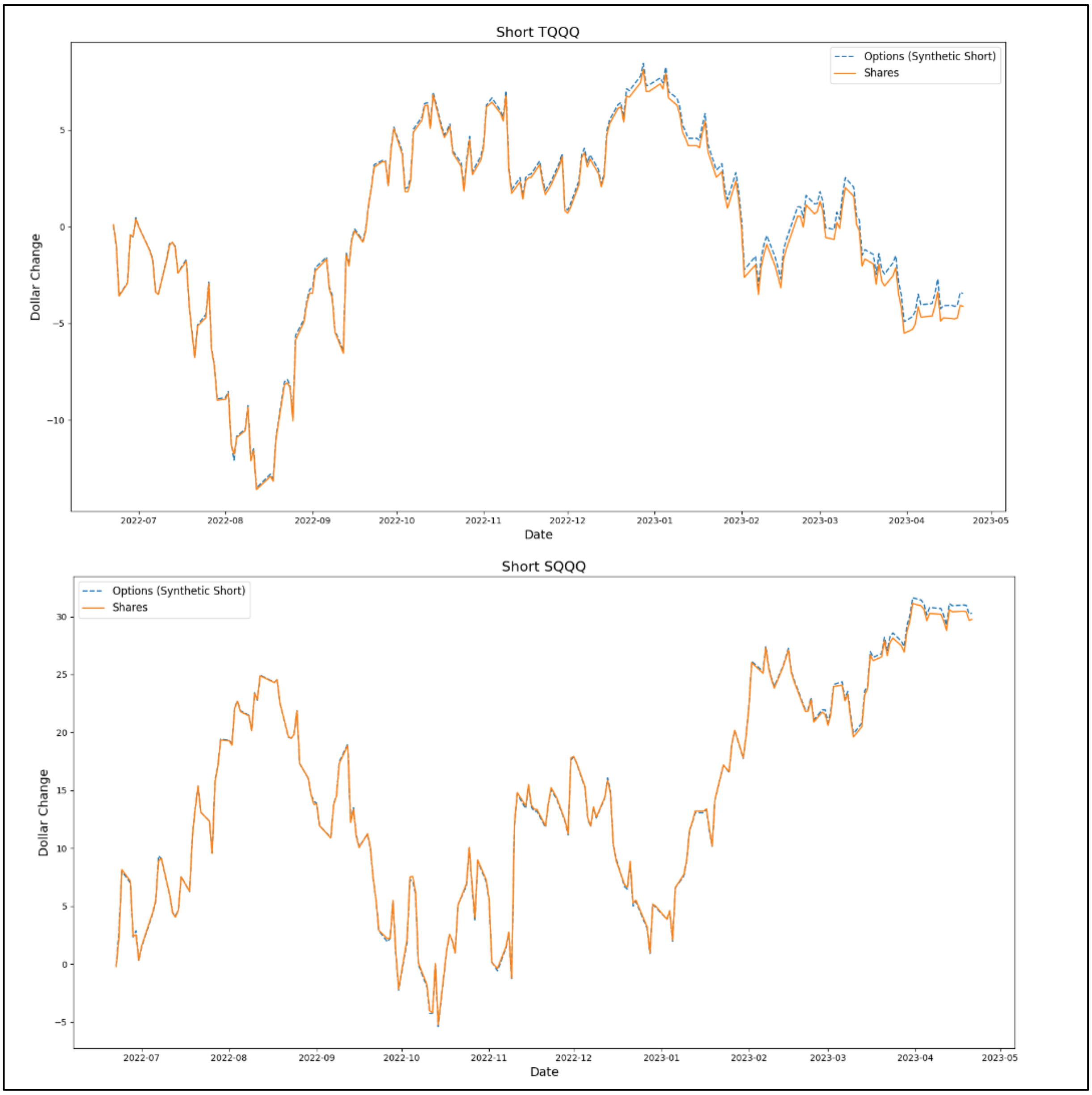

Figure X: Dollar Returns: Synthetic Short Position in SQQQ and TQQQ

Figure X graphs the returns of a hedged synthetic short position in SQQQ and TQQQ between July 2022 and May 2023. The effect of the divergence observed in Fig. IX continues to fuel positive returns even as the strategy loses profitability in a high interest rate environment.

Figure XI: Percent Returns: Synthetic Short Position in SQQQ and TQQQ

Figure XI graphs the returns of a hedged synthetic short position in SQQQ and TQQQ between July 2022 and May 2023, relative to the cost of the portfolio.

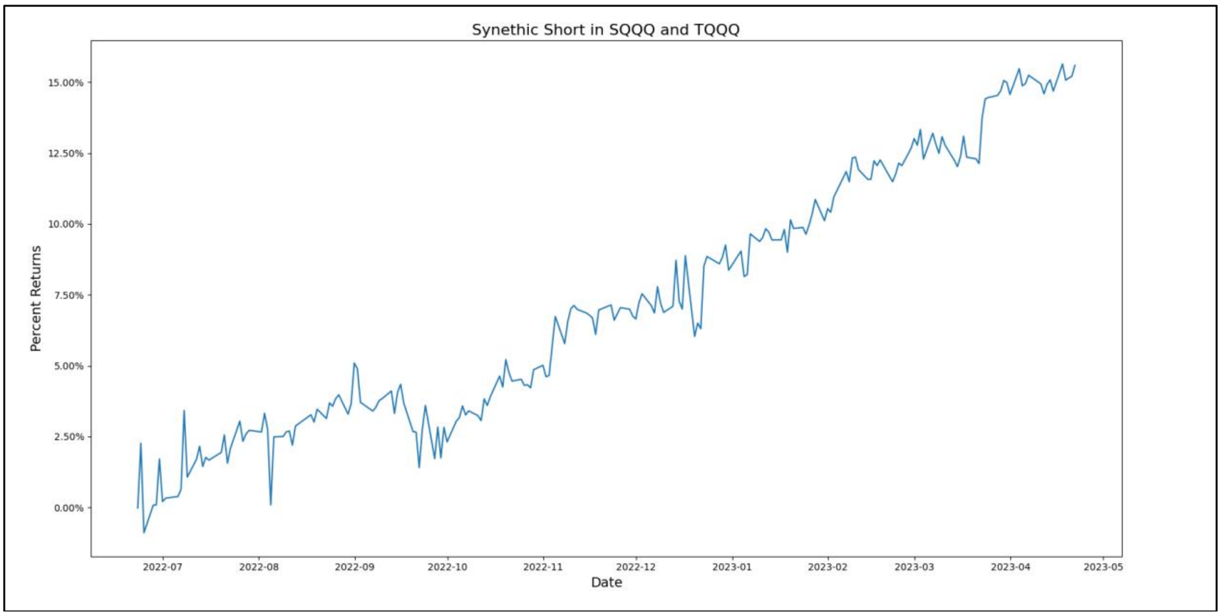

Figure XII: Overnight Returns, SPY + SPXU Portfolio

Figure XII illustrates the overnight returns of a portfolio hedging SPY and SPXU. The positive trend in overnight returns may reflect demand for an inverse instrument during the riskier night session.

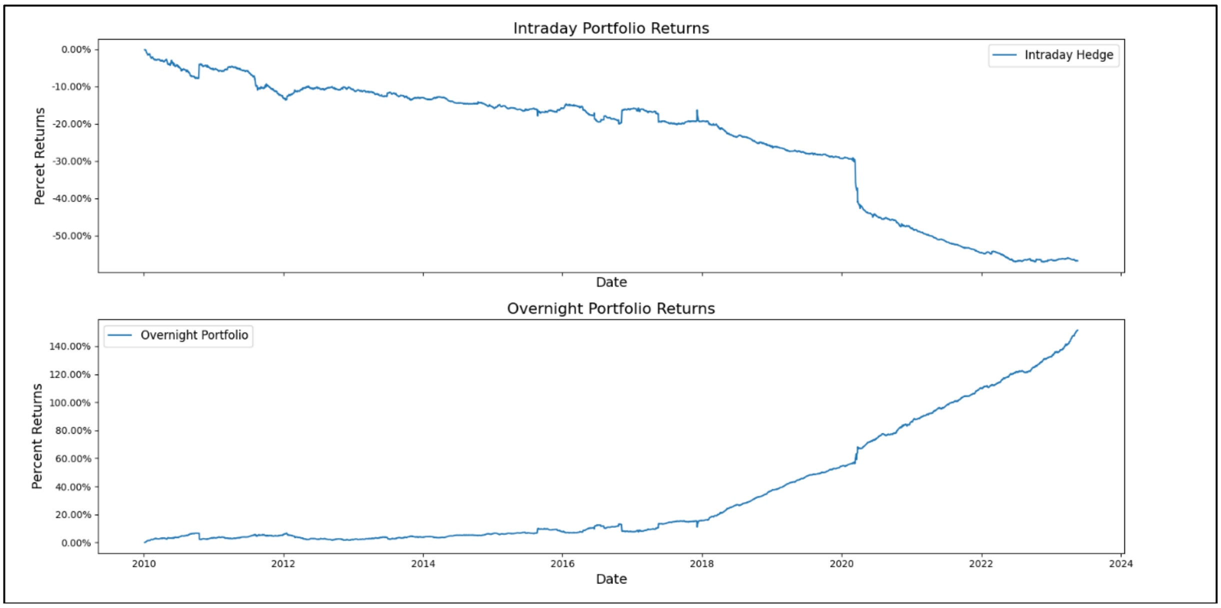

Figure XIII: Overnight Returns, SOXX + SOXS Portfolio

Figure XIII illustrates the overnight returns of a portfolio hedging SOXX and SOXS. The same positive trend observed in Figure XVI is significantly stronger here, delivering a 3000% return.

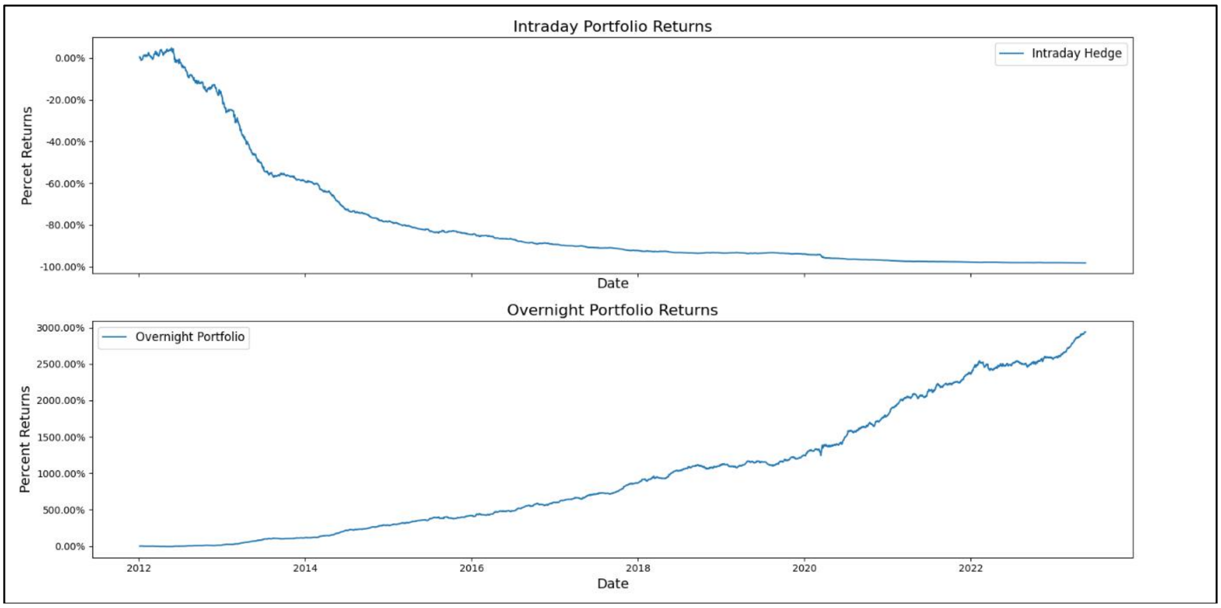

Figure XIV: SPY + SH Hedge, Option Returns, Night vs Day

Figure XIV illustrates the overnight and intraday returns of a portfolio holding synthetic long options in SPY and SH in equal dollar weight. To calculate the change in value of the options portfolio, we use a single SPY contract as a reference point and determine the appropriate number of SH contracts needed to provide a hedge.

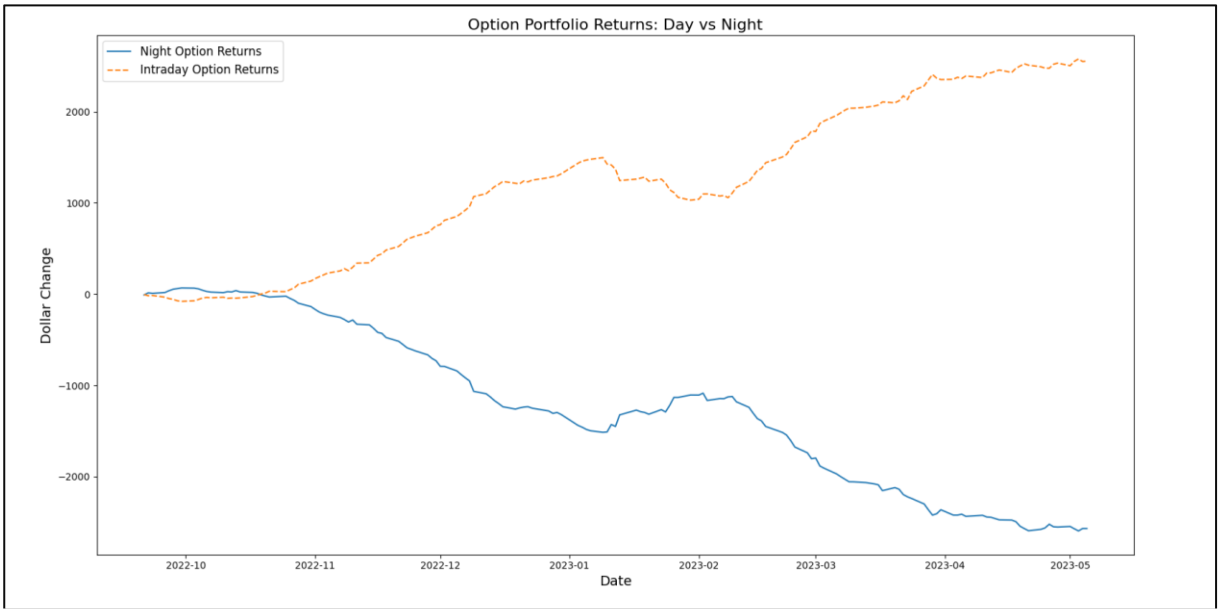

Figure XV: SPY + SH Hedge, Stock Returns, Night vs Day

Figure XV illustrates the overnight and intraday returns of a portfolio holding shares in SPY and SH in equal dollar weight. To calculate the change in value of the stock portfolio, we use a single SPY share as a reference point and determine the appropriate number of SH shares needed to provide a hedge.