H_∞ and H_2 Robust Control Approach To a Typical Regulation Problem - Robust Control Final Exam

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Abstract

A typical regulation system with multiplicative uncertainty is given in this robust control exam and it is changed into a mixed sensitivity problem. Then, the MK configuration of the system is achieved by state space realization of the given system. For this system, H_∞ and H_2 syntheses are utilized to ensure robust stability and performance.

Contents

List of Tables

List of Figures

Chapter 1

System Definition

In this chapter, the state space representations of the original system is derived based on its block diagram. In order to analyze the robustness of the system and perform the robust synthesis of H_2 and H_∞, the MK configuration is considered.

1.1 Original System State Space Representation

The original system with multiplicative uncertainty given in the exam is shown in figure 1.1. In this block diagram, P represents the plant, the controller is shown as the block C and Δ is the plant uncertainty parameter. Additionally, W_s, W_u and W are performance weighting functions to insure robust stability and performance.

The transfer functions of $P(s)$, Ws, Wu and W are given in the following :

$$P(s) = \frac{1}{s^2 + 0.2s + 1} \tag{1.1}$$

$$W_s(s) = \frac{2(s+5)}{(s+0.5)(s+10)} \tag{1.2}$$

$$W_u(s) = 0.9\frac{1+0.1s}{1+s} \tag{1.3}$$

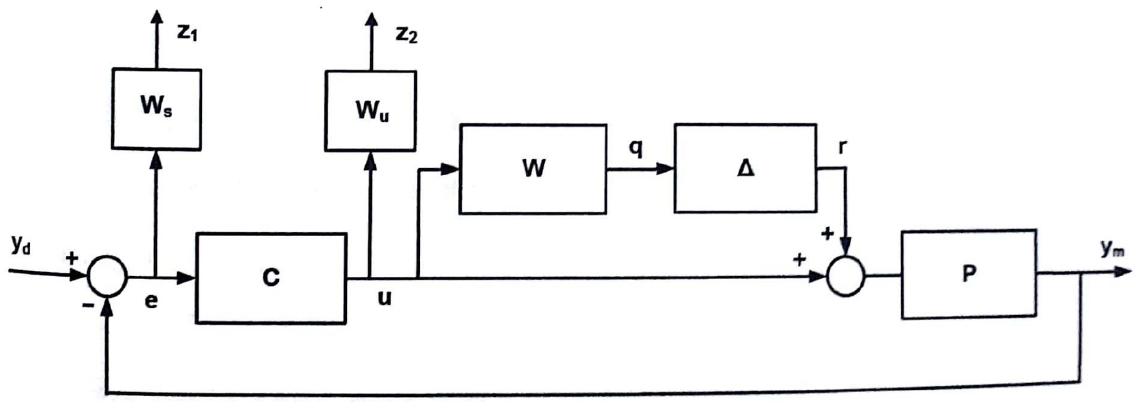

Figure 1.1: Original system block diagram

$$W(s) = 500(s+10)^2 \tag{1.4}$$

Due to the proper characteristic of the M matrix in MK robust configuration, equation 1.4 cannot be used solely because it is improper and therefore, it should be used with the plant as an augmented function PW. Consequently, the system shown in figure 1.1 is altered to an equivalent system without the uncertainty parameter depicted in figure 1.2 which yields the same results as the first one. The new system is also known as the 1 DOF mixed sensitivity problem and we derive its state space realization by taking the augmented plant as a whole subsection.

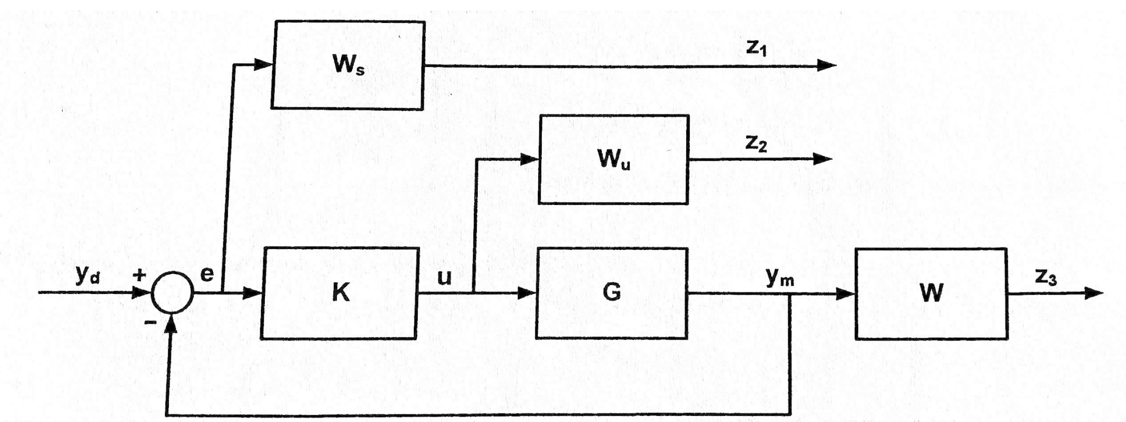

Figure 1.2: equivalent system block diagram

The first thing that needs doing is to form the PW transfer function shown in equation 1.5. To achieve the state space realization of this augmented plant, the constant is separated from equation 1.5 which leads to equation 1.6.

$$P(s)W(s) = \frac{500s^2 + 10000s + 50000}{s^2 + 0.2s + 1} \tag{1.5}$$

$$P(s)W(s) = 500 + \frac{9900s + 49500}{s^2 + 0.2s + 1} \tag{1.6}$$

Subsequently, the controllable state space realization of equation 1.6 is given by equation 1.7 and Z_3 is defined as equation 1.8.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{1.7}$$

$$Z_3 = \begin{bmatrix} 0.198 & 0.0396 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0.002]u \tag{1.8}$$

Next, the transfer function of W_u from 1.3 is also simplified by taking out the constant and forming equation 1.9. Knowing that, the controllable state space representation for this transfer function is calculated in equation 1.10 and Z_2 is defined accordingly in equation 1.11.

$$W_u(s) = 0.09 + \frac{0.81}{s+1} \tag{1.9}$$

$$\dot{x}_3 = [-1]x_3 + [1]u \tag{1.10}$$

$$Z_2 = [0.81]x_3 + [0.09]u \tag{1.11}$$

The last weighting function W_s is represented in the controllable state space realization of equation 1.2 that is calculated in equation 1.12. Consequently, Z_1 is derived from equation 1.13. In addition, the input of the controller denoted by y is found in equation 1.14.

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -10.5 \end{bmatrix} \cdot \begin{bmatrix} x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y_d - x_1) \tag{1.12}$$

$$Z_1 = 10x_4 + 2x_5 \tag{1.13}$$

$$y = y_d - x_1 \tag{1.14}$$

By Combining all the previous state space representations, the following matrices are obtained for the MK configuration.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -5 & -10.5 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{1.15}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 10 & 2 \\ 0 & 0 & 0.81 & 0 & 0 \\ 49500 & 9900 & 0 & 0 & 0 \end{bmatrix} D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} D_{12} = \begin{bmatrix} 0 \\ 0.09 \\ 500 \end{bmatrix} \tag{1.16}$$

$$C_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \end{bmatrix} D_{21} = \begin{bmatrix} 1 \end{bmatrix} D_{22} = \begin{bmatrix} 0 \end{bmatrix} \tag{1.17}$$

With this complete state space realization of figure 1.2 system, we can find the controller by performing the H_∞ and H_2 synthesis in the next chapter.

Chapter 2

H_∞ And H_2 Synthesis

This chapter outlines the H_∞ and H_2 synthsis and robust analysis of the original system by using the state space realization found in the previous chapter.

2.1 H_∞ Synthesis

By implementing the H_∞ Synthesis on this system, the controller transfer function is found in 2.1 with $\gamma_{opt} = 2$.

$$K(s) = \frac{3.751e-08s^4 + 4.059e-07s^3 + 4.78e-07s^2 + 4.705e-07s + 3.608e-07}{s^5 + 31.5s^4 + 345.5s^3 + 1465s^2 + 1650s + 500} \tag{2.1}$$

Also, the bode diagram, the step response and the performance parameters diagram for the closed loop system are achieved in figures 2.1, 2.2 and 2.3 respectively.

2.2 H_2 Synthesis

By implementing the H_2 Synthesis on this system, the controller transfer function is found in 2.2 with $\gamma_{opt} = 1.069$.

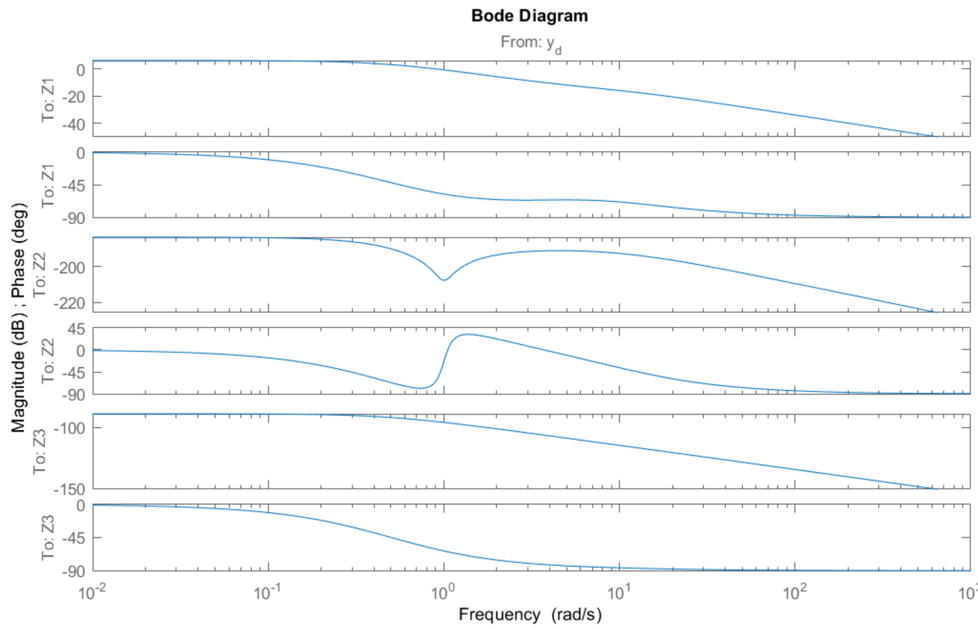

Figure 2.1: closed-loop H_∞ system bode diagram

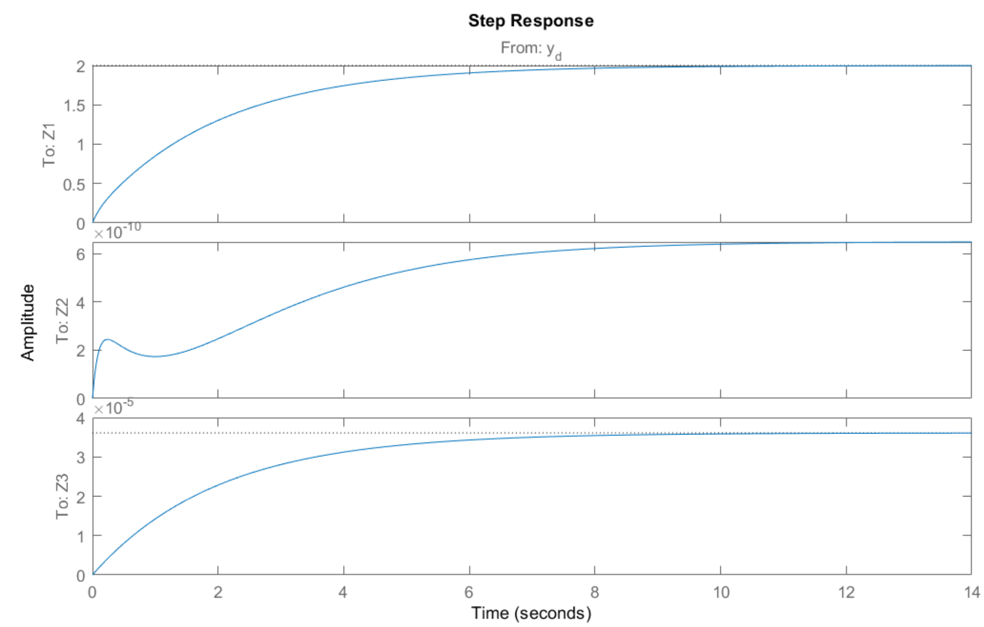

Figure 2.2: closed-loop H_∞ system step response

$$K(s) = \frac{3.756e-08s^4 + 4.064e-07s^3 + 4.787e-07s^2 + 4.707e-07s + 3.608e-07}{s^5 + 31.5s^4 + 345.5s^3 + 1465s^2 + 1650s + 500} \tag{2.2}$$

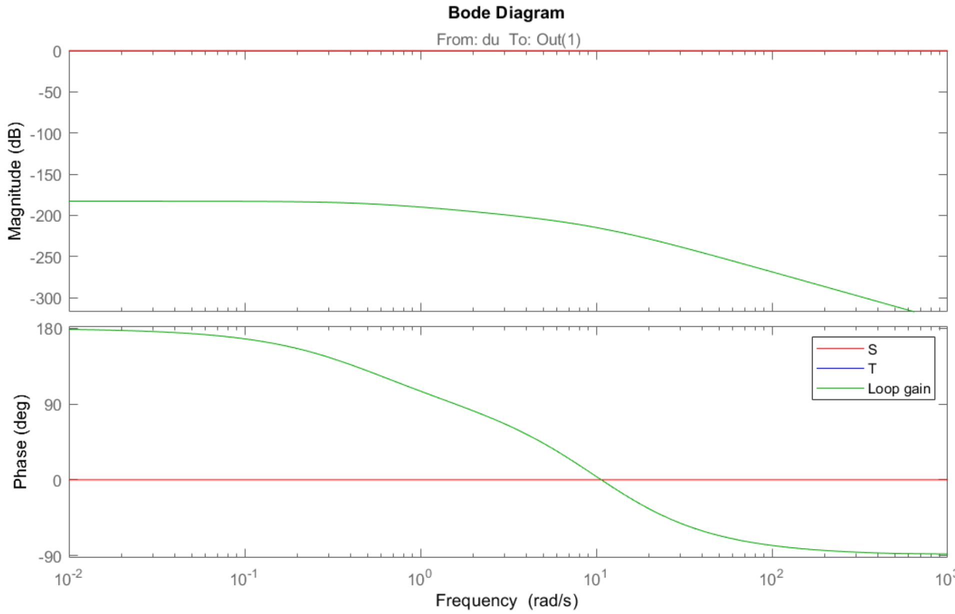

Figure 2.3: closed-loop H_∞ system performance measurements

Also, the bode diagram, the step response and the performance parameters diagram for the closed loop system are achieved in figures 2.4, 2.5 and 2.6 respectively.

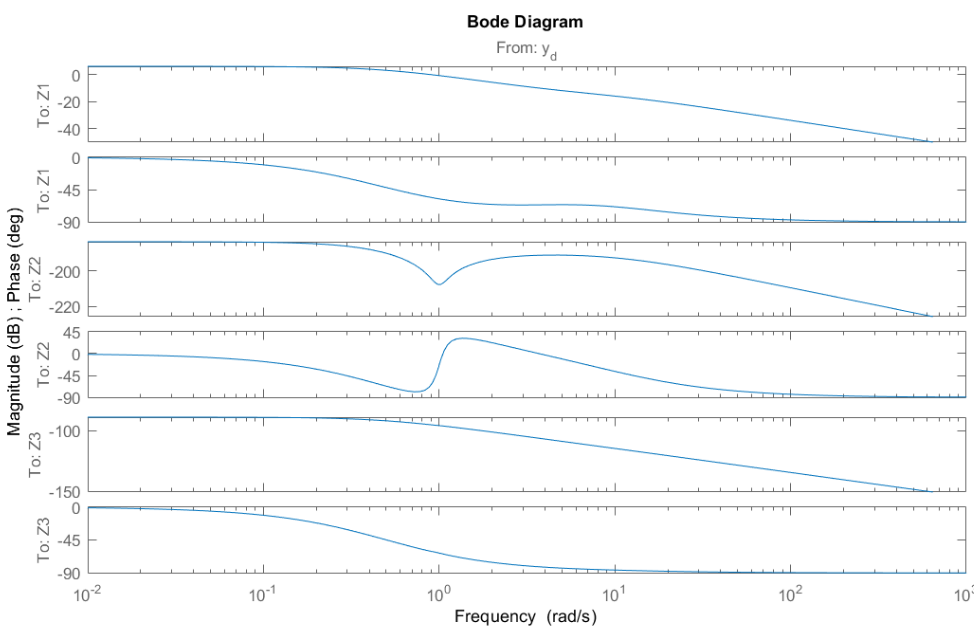

Figure 2.4: closed-loop H_2 system bode diagram

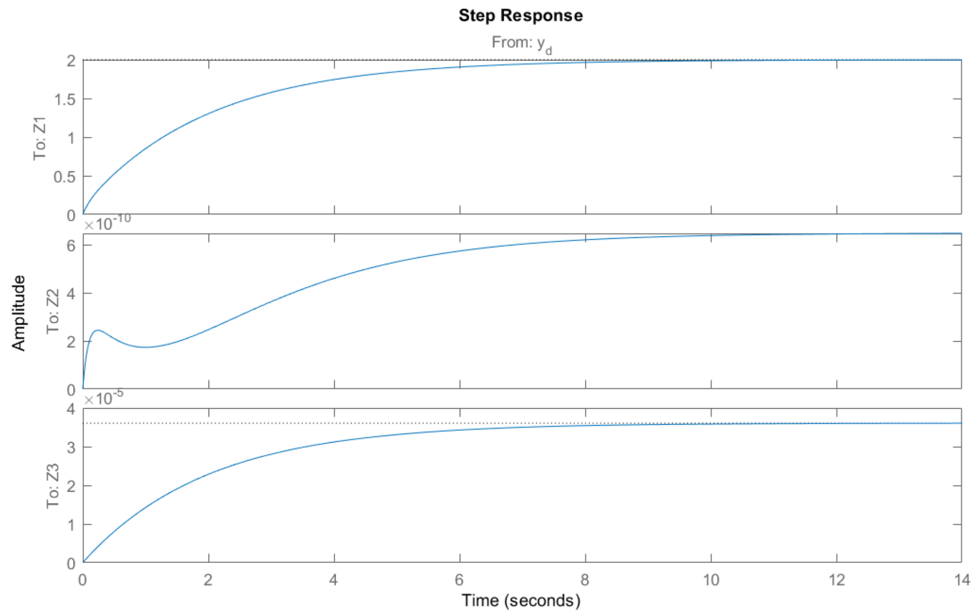

Figure 2.5: closed-loop H_2 system step response

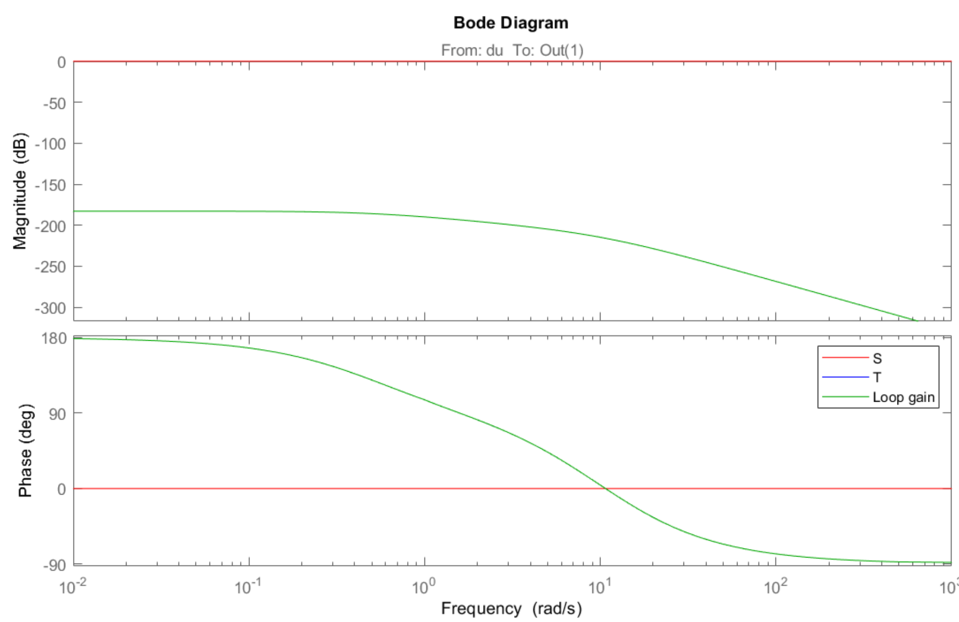

Figure 2.6: closed-loop H_2 system performance measurements

2.3 Analysis And Comparisons

In both syntheses, it is implied that the system is stable on the account of the step responses but not robustly stable because the gamma in H_∞ is larger than 1 and the gamma in H_2 does not prove robust stability even if goes lower than 1 even which is not the case here. The bode and performance diagrams show a very bad performance following the fact that the sensitivity of the system has remained at almost 1 on every frequency.

Chapter 3

Improved System H_∞ And H_2 Synthesis

In this chapter, the previous system is improved to have better robust performance and stability.

3.1 Improved System State Space Representation

In order to improve this system, the weighting function W from equation 1.4 must be modified to guarantee robust stability and therefore, robust performance. The altered version of this function is given by equation 3.1 where dc gain is much lower.

$$W(s) = 10^{-3}(s+10)^2 \tag{3.1}$$

The new state space realization for this transfer function in junction with the plant yields an improved state space representation for the MK configuration and it is given by the following matrices.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -5 & -10.5 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.2}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 10 & 2 \\ 0 & 0 & 0.81 & 0 & 0 \\ 0.099 & 0.0198 & 0 & 0 & 0 \end{bmatrix} D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} D_{12} = \begin{bmatrix} 0 \\ 0.09 \\ 0.001 \end{bmatrix} \tag{3.3}$$

$$C_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \end{bmatrix} D_{21} = \begin{bmatrix} 1 \end{bmatrix} D_{22} = \begin{bmatrix} 0 \end{bmatrix} \tag{3.4}$$

3.2 H_∞ Synthesis

By implementing the H_∞ Synthesis on this system, the controller transfer function is found in 3.5 with $\gamma_{opt} = 0.8288$.

$$K(s) = \frac{37.14s^4 + 407.3s^3 + 479.9s^2 + 472.4s + 362.8}{s^5 + 25.51s^4 + 218.1s^3 + 676.5s^2 + 475.8s + 94.49} \tag{3.5}$$

Also, the bode diagram, the step response and the performance parameters diagram for the closed loop system are achieved in figures 3.1, 3.2 and 3.3 respectively.

3.3 H_2 Synthesis

By implementing the H_2 Synthesis on this system, the controller transfer function is found in 3.6 with $\gamma_{opt} = 0.9353$.

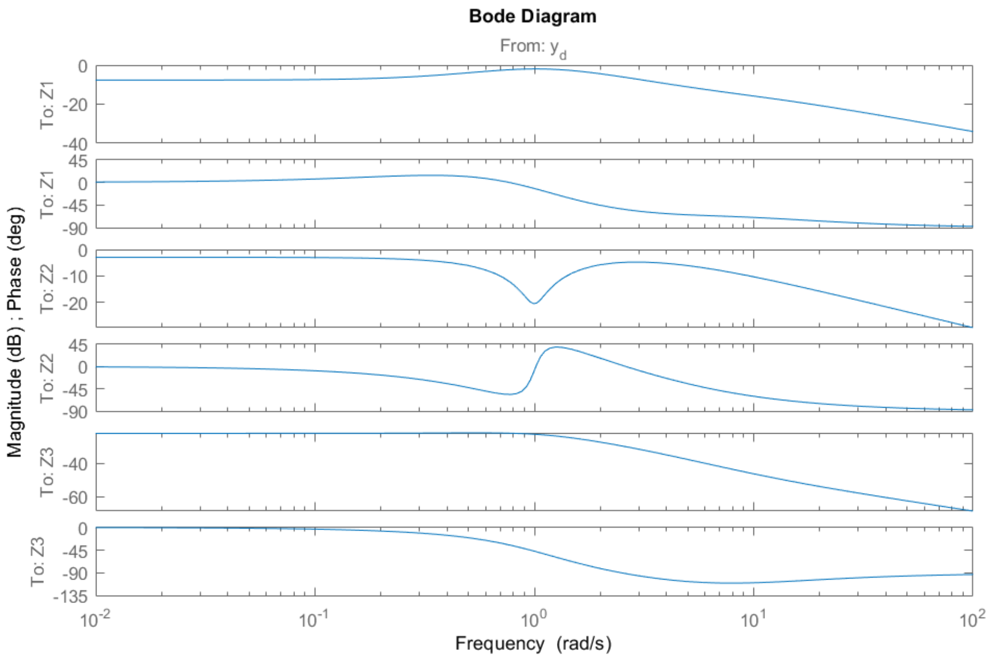

Figure 3.1: closed-loop H_∞ system bode diagram

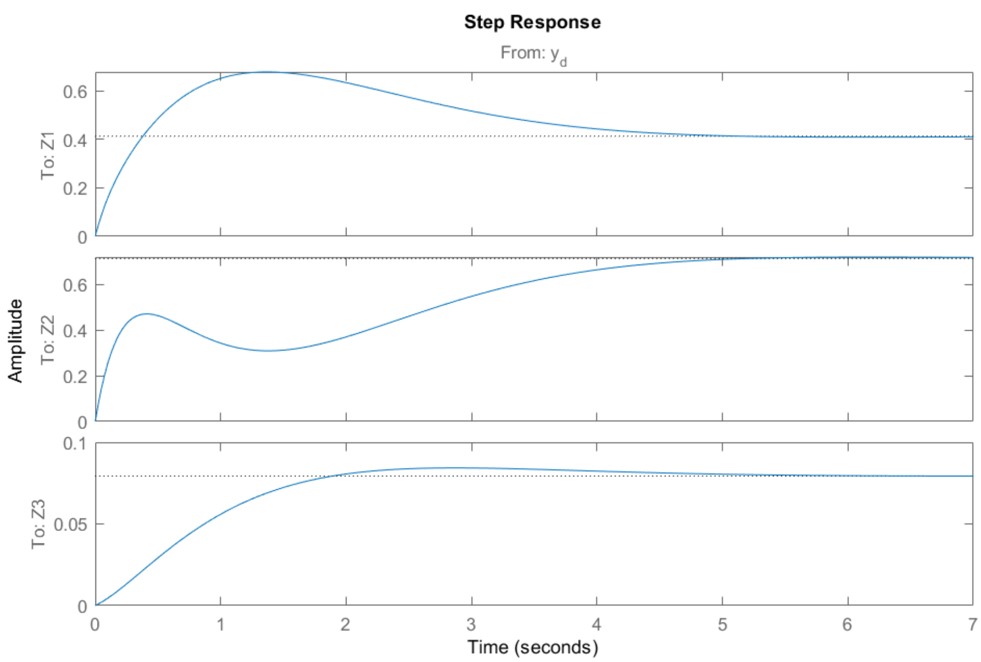

Figure 3.2: closed-loop H_∞ system step response

$$K(s) = \frac{5.832s^4 + 64.47s^3 + 75.97s^2 + 74.8s + 57.48}{s^5 + 21.82s^4 + 138.9s^3 + 227.4s^2 + 210.6s + 64.49} \tag{3.6}$$

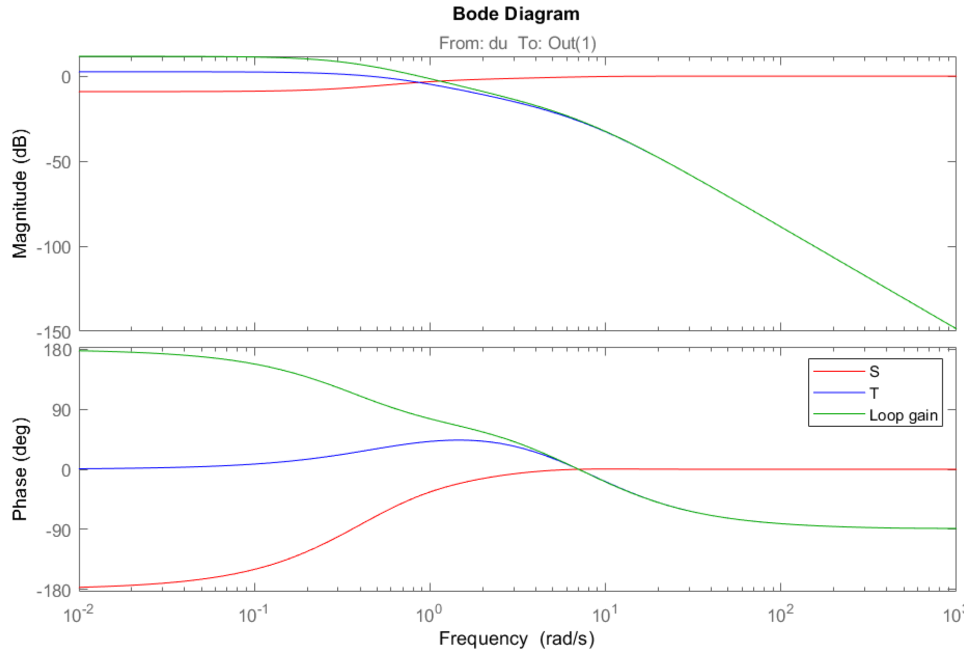

Figure 3.3: closed-loop H_∞ system performance measurements

Also, the bode diagram, the step response and the performance parameters diagram for the closed loop system are achieved in figures 3.4, 3.5 and 3.6 respectively.

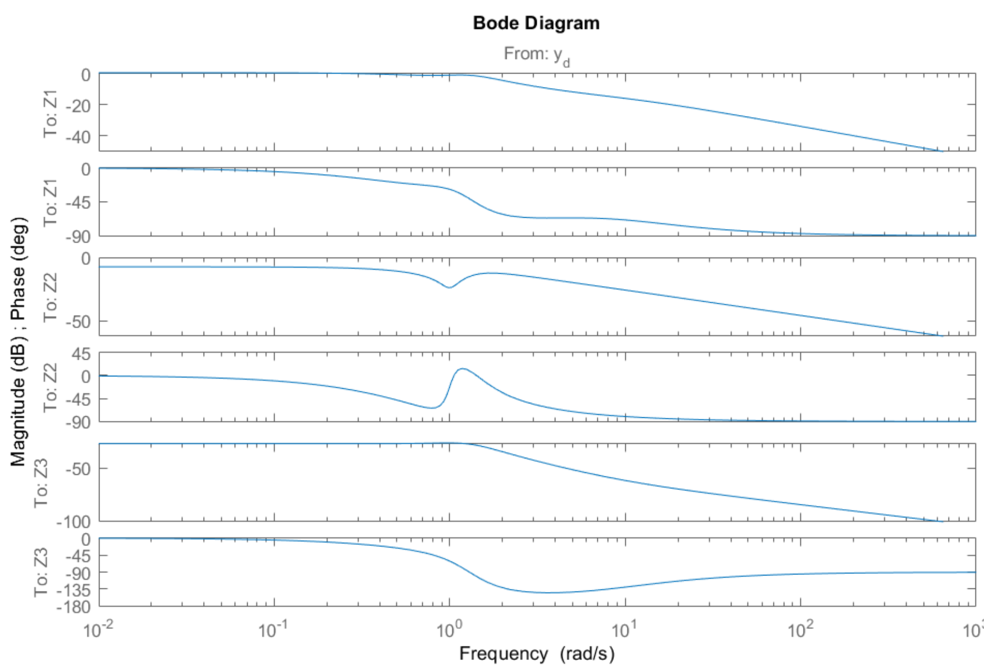

Figure 3.4: closed-loop H_2 system bode diagram

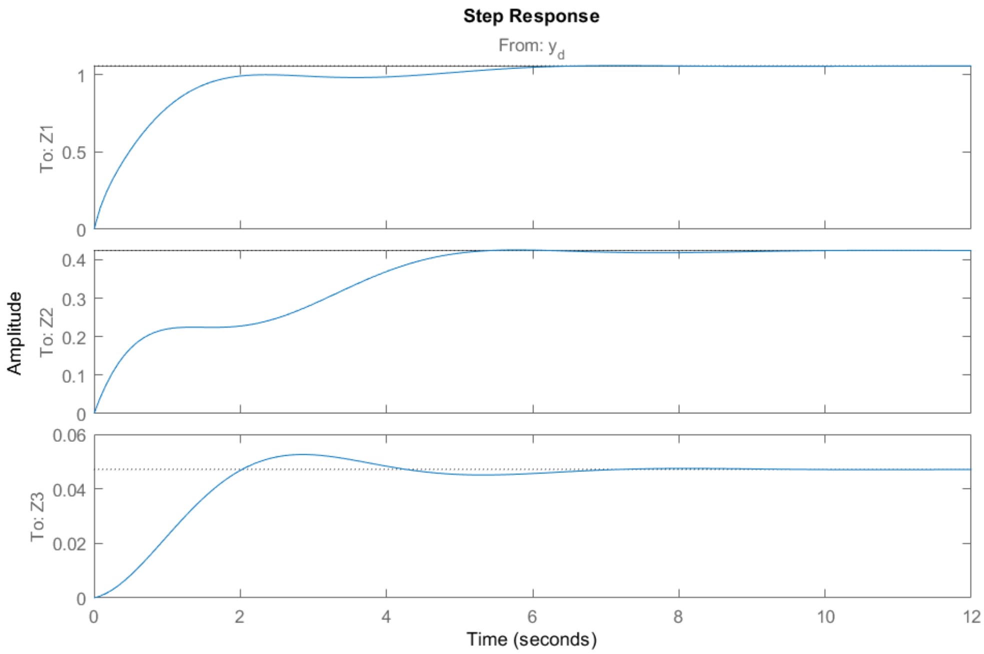

Figure 3.5: closed-loop H_2 system step response

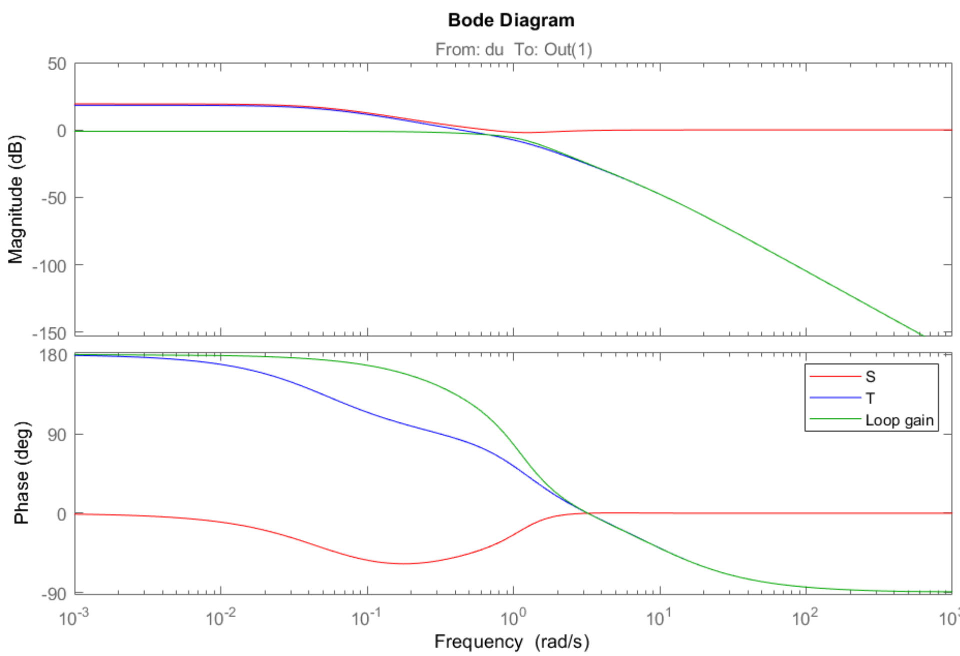

Figure 3.6: closed-loop H_2 system performance measurements

3.4 Analysis And Comparisons

In both syntheses, it is implied that the system is robustly stable on the account of the step responses and because the gamma in H_∞ is lower than 1 but the gamma in H_2 even though less than 1 does not prove robust stability. The bode and performance diagrams show an excellent performance following the fact that the sensitivity of the system has a fairly good range of being low at lower frequencies whereas the sensitivity complement goes low in higher frequencies to suppress noise. This is also indicated by the fact that a lower than 1 gamma improves performance in addition to stability.

Chapter 4

Conclusion

A typical regulation system with multiplicative uncertainty was given in this robust control exam and it was changed into a mixed sensitivity problem. Then, the MK configuration of the system was achieved by state space realization of the given system.

As expected, this system was synthesized with H_∞ and H_2 with a very poor robust performance and stability result and in order to improve that the dc gain of the closed loop system weighting function was significantly reduced. As a result, both robust stability and performance were achieved. Although, the H_2 synthesis unlike H_∞ cannot prove robust stability even with a low gamma.

4.1 Future Works

The weighting functions in this model can still be improved to yield better results however, this should be done in a way that the performance of the system is not weakened since it already is stable.