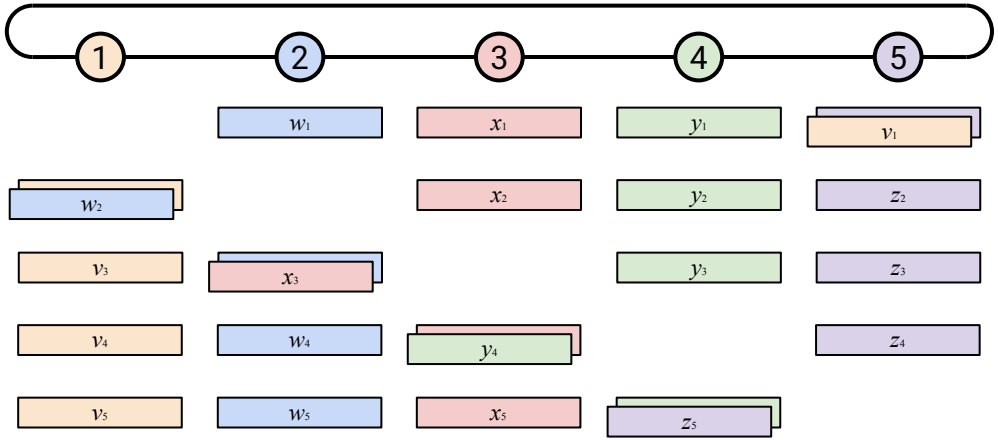


Lecture 23 (Beyond Client-Server 2)

Collective Operations

CS 168, Spring 2026 @ UC Berkeley

Slides credit: Sylvia Ratnasamy, Rob Shakir, Peyrin Kao

Motivation: Distributed AI Training

Lecture 23, CS 168, Spring 2026

Motivation: AI Training

Modern AI systems require training models on huge amounts of data.

For this class, we'll use a very simplified model of AI training.

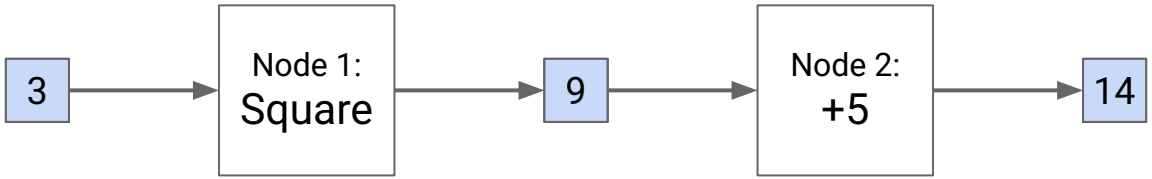
Distributed Training

Modern AI training jobs are far too large to be run on a single machine.

- The job must be **distributed** across multiple machines.

Many options for how to distribute the work. Examples:

- Each node works on a subset of the training data.
- Each node works on a subset of the model.
- Each node works on a subset of the task (pipelining).
- For this class, we don't care which option is used.

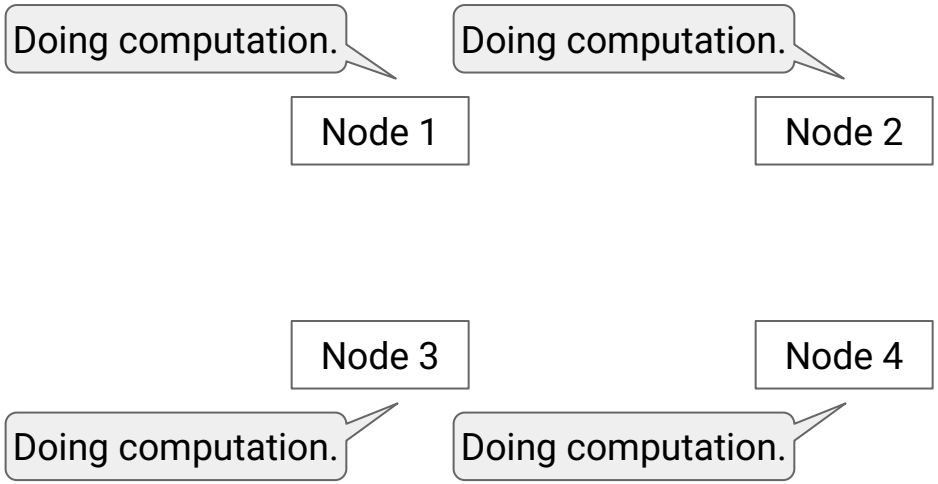


Example of pipelining. The task is to compute f(x) = x^2 + 5.
We split the task into two stages.

Nodes Exchanging State

Regardless of how you distribute the work, nodes must periodically exchange state.

- Example: Each node has a piece of the solution, and they must exchange state to reconstruct the full solution.
- Example: Node 2 needs data from Node 1 to continue its computation.



1. **Split job into sub-tasks.**
   Each node runs a sub-task.
2. After every node finishes their sub-task, the nodes exchange a large amount of state.
3. Proceed to the next task, and repeat Steps 1–2 for the next task.

Nodes Exchanging State

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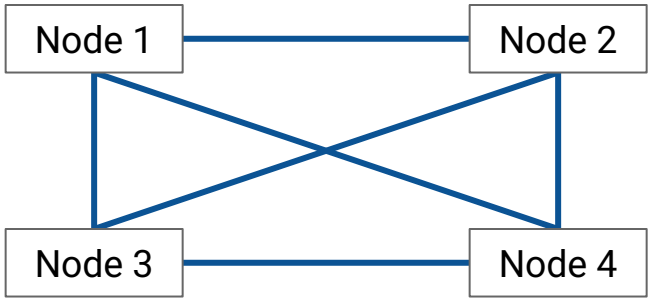

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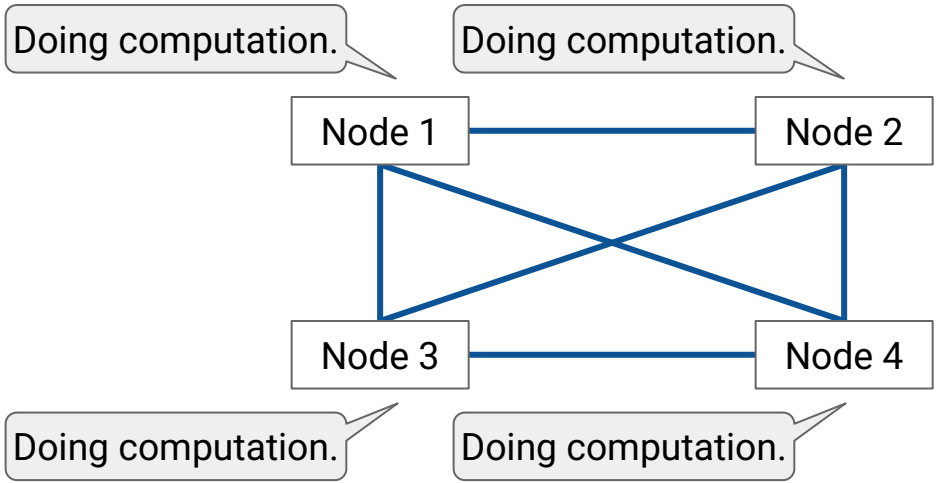

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Distributed Training Infrastructure

Lecture 23, CS 168, Spring 2026

Distributed AI Training

- Motivation
- **Infrastructure**

Defining Collectives

- Definition
- Redistribution Operations
- Consolidation Operations
- Duals and Compositions

Implementing Collectives

- Mesh Topology
- Single Root
- Tree
- Ring (Naive)
- Ring (Optimized)
- Overlay/Underlay Topologies

Distributed Training Infrastructure: GPUs, TPUs

What exactly is each "node"?

- GPUs (Graphics Processing Units): Chips that are good at AI operations.
- TPUs (Tensor Processing Units): AI-optimized chips designed by Google.

The nodes are inter-connected in a datacenter-like network.

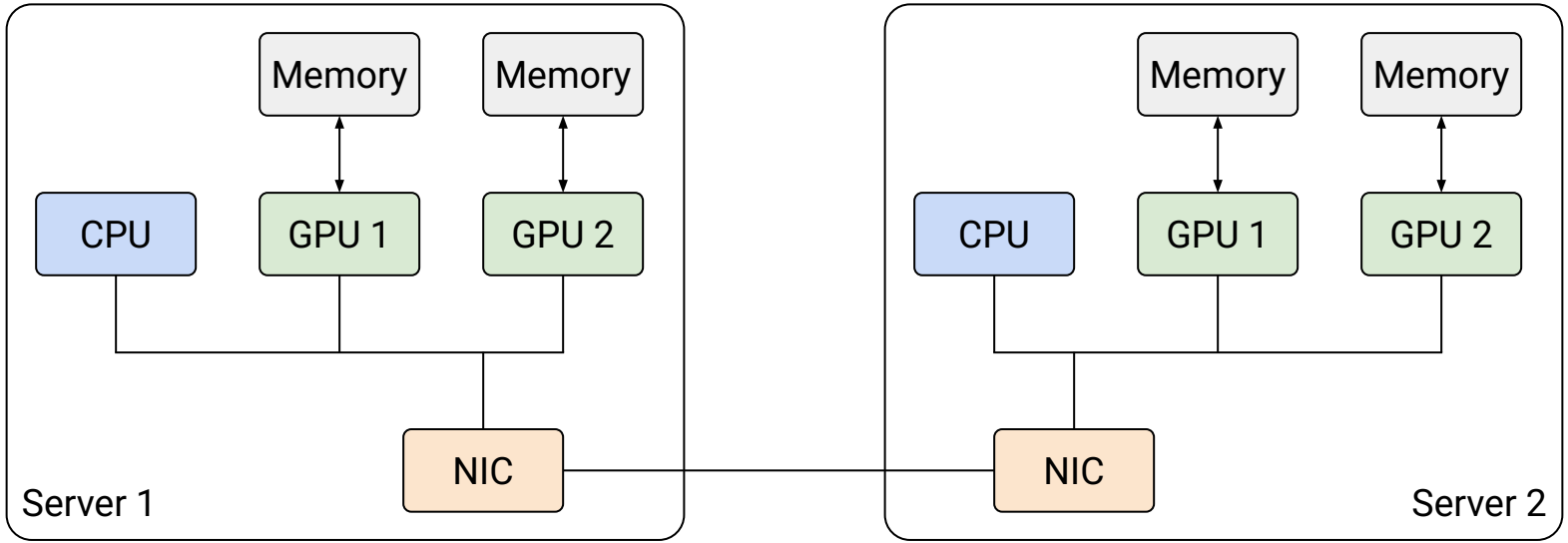
Recall the properties of a datacenter network:

- Nodes are physically close (e.g. same building).
- Nodes are organized in a structured topology (e.g. Clos network).
- Nodes are homogeneous (e.g. all the same model of GPU).
- Links have very high bandwidth.

Distributed Training Infrastructure: Servers

Each server in the datacenter has:

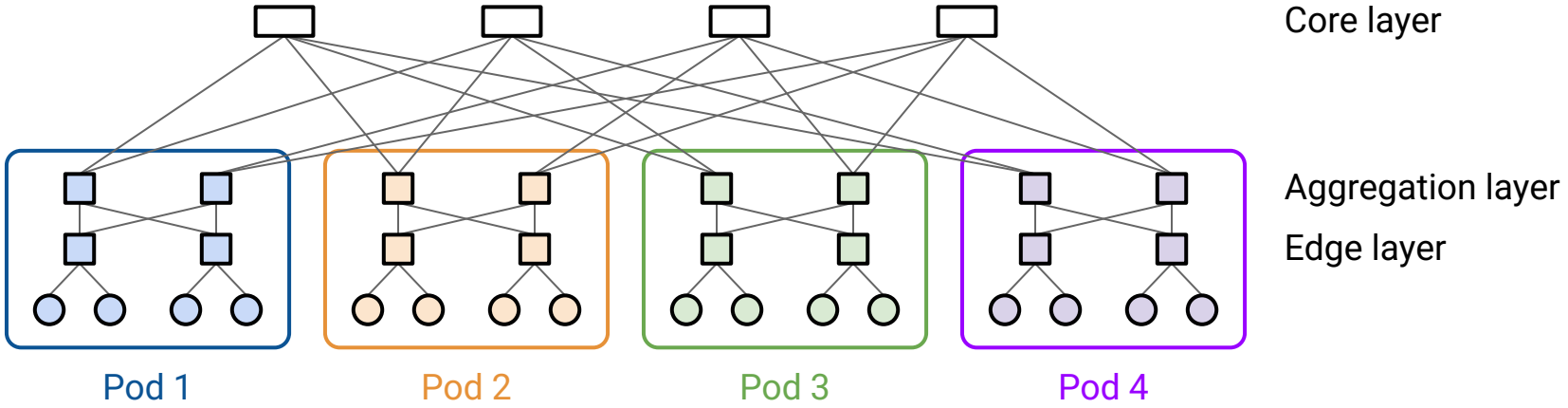
- One or more GPUs/TPUs. Used to do the AI training.
- A relatively weak CPU. Used for miscellaneous operations, not AI training.
- A NIC, shared by all the chips on this server.



Distributed Training Infrastructure: Network Topology

The servers are connected in a structured topology.

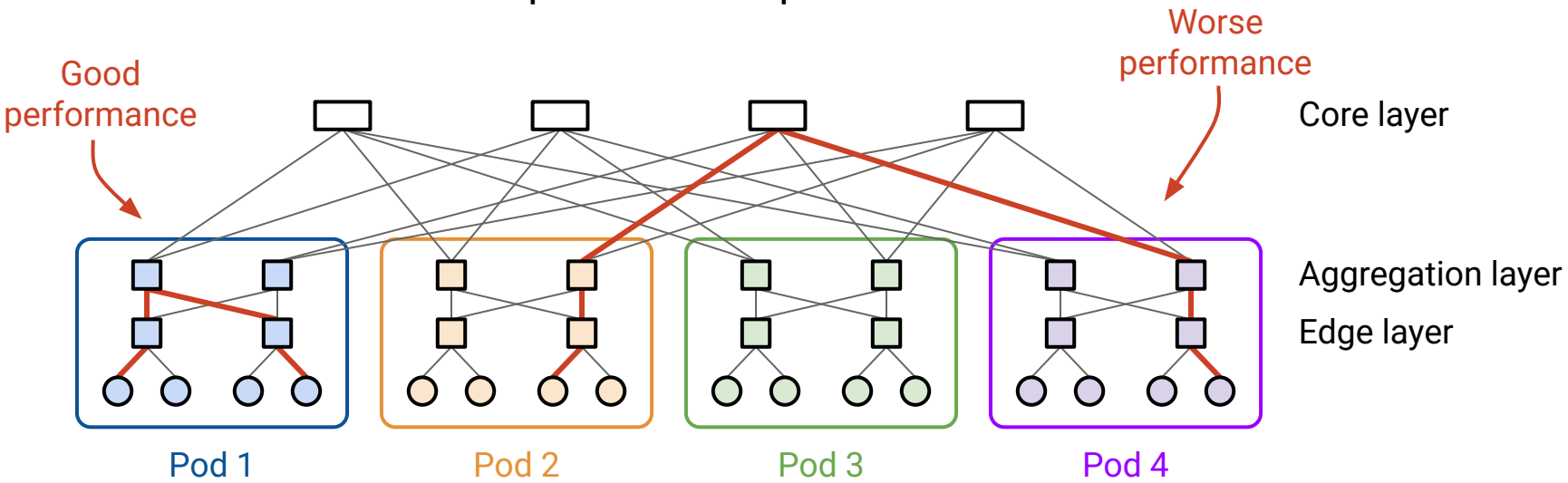
- Many different possible topologies. Example: Fat-tree Clos topology.
- Remember: Each server (○ ○ ○ ○) could have one or more GPUs/TPUs.



Distributed Training Infrastructure: Network Topology

Notice: Some pairs of nodes are more closely-connected than others.

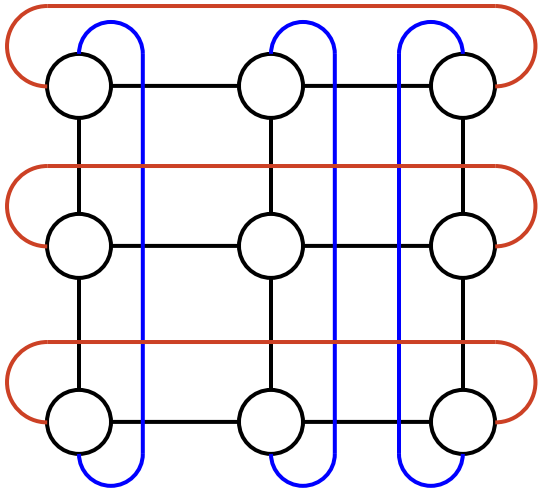
- Two nodes on the same server: Great performance.
  - Don't need to use the network at all.
  - Nodes can communicate with effectively infinite bandwidth and zero latency.
- Two nodes in the same pod: Good performance.
- Two nodes in different pods: Worse performance.



Distributed Training Infrastructure: Network Topology

Alternate topology: **2D torus**: A square, where the edges wrap around.

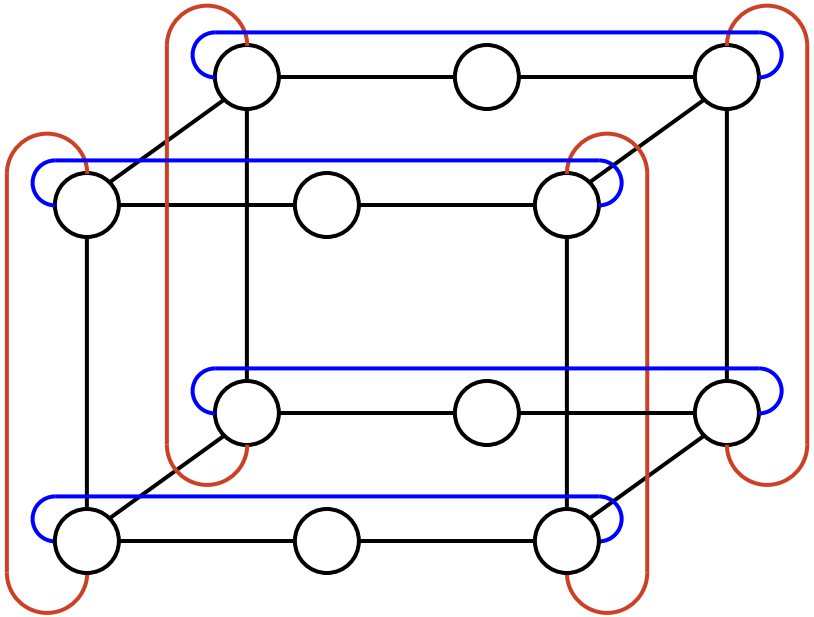
- TPUs can be directly connected to each other with ICI (Inter-Core Interconnect), with no routers needed.
- As before, some pairs of nodes are more closely-connected than others.
- "Wrap around" means: If you're at a bottom node and traverse a link down, you'll end up at the top node. (And similar for the other directions.)



Distributed Training Infrastructure: Network Topology

Alternate topology: **3D torus**: A cube, where the edges wrap around.

- TPUs can be directly connected to each other with ICI (Inter-Core Interconnect), with no routers needed.
- As before, some pairs of nodes are more closely-connected than others.



Collective Communication: Definition

Lecture 23, CS 168, Spring 2026

Collective Communication: Definition

Collective communication: A group of nodes that exchange data in a coordinated manner as part of a group computation.

- The nodes work together to achieve a common goal.
- The nodes have to exchange data during that process.

Brief history:

- Originally developed for supercomputers.
- Now an active area of research again in the context of AI training.
- Modern implementations: NCCL (Nvidia), MSCCL (Microsoft), etc.
- NCCL code is available online if you're curious.

Why is Collective Communication Different?

Highly structured communication.

- General Internet: Hosts communicate whenever they want.
- Collectives: We know about a tightly-scripted set of exchanges that must occur.

Dedicated network infrastructure.

- General Internet: Many applications/tenants/users running at the same time.
- Collectives: The AI training job is the only job running.
  This lets us plan out how much bandwidth will be used ahead of time.

Data is transformed as it's exchanged.

- General Internet: The data sent is identical to the data received.
- Collectives: Nodes perform computation as the data is forwarded across nodes.

Collective Communication: Operations

Collective operations: A set of basic communication patterns.

- Users invoke these basic operations to implement their desired communication.
- Think of these as the "API" for distributed communication.

Our focus:

- **What**: The definitions of these collectives, e.g. inputs and outputs.
- **How**: How these collectives are implemented in the network.
- **Why**: We won't discuss why AI training can be broken down into these operations.

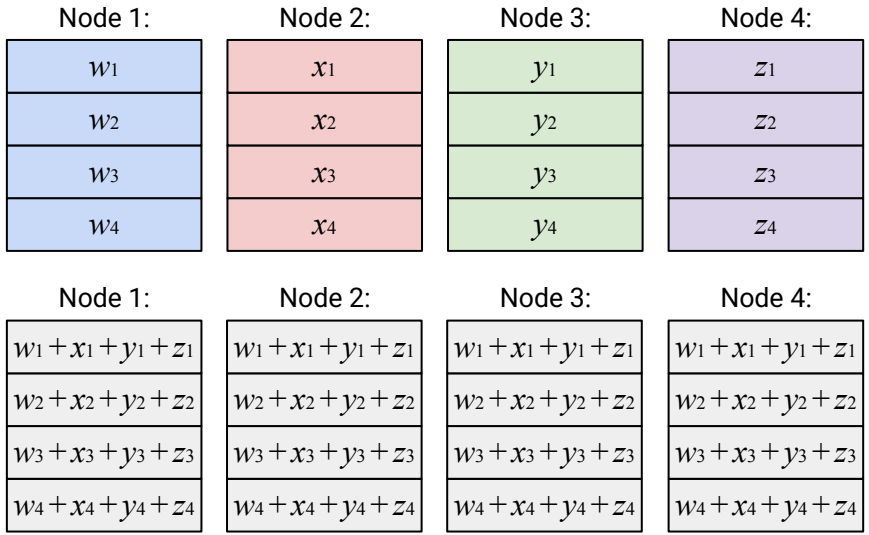
Collective Communication: Setup

There are *p* nodes. Each node specifies a *p*-element vector of input data, and receives a *p*-element vector of output data.

- Each vector element could be an integer, a chunk of training data, etc.
- Each operation specifies what output values should be sent to each node.
- The data could be transformed in the output, e.g. summing vector elements.

Input: Each node specifies a *p*-element vector of input data.

(e.g. Specify the memory address of a read buffer.)



p = 4 in this example.

Output: Each node receives a *p*-element vector of output data.

(e.g. Specify a memory address, and receive output at that write buffer.)

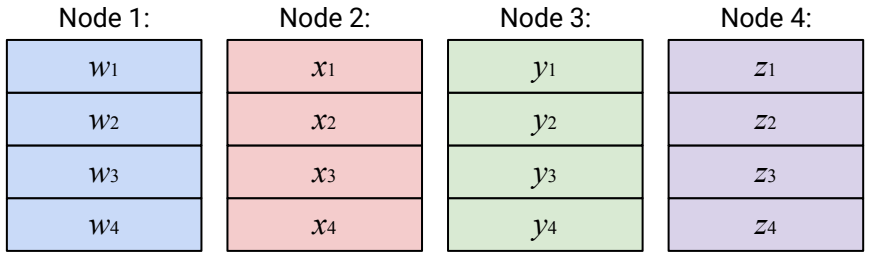
Collective Communication: Setup

Before the operation, a centralized controller does some setup.

- Tell each node its number ("you are Node 1").
- Tell each node the value of *p* ("there are 4 nodes in total").
- Specify the memory addresses of the read buffer and write buffer.

Input: Each node specifies a *p*-element vector of input data.

(e.g. Specify the memory address of a read buffer.)

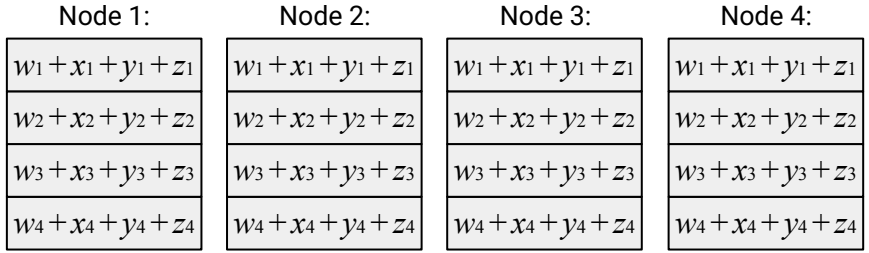


p = 4 in this example.

AllReduce

Output: Each node receives a *p*-element vector of output data.

(e.g. Specify a memory address, and receive output at that write buffer.)



Collective Communication: Setup

During the operation, nodes exchange data to produce the desired output.

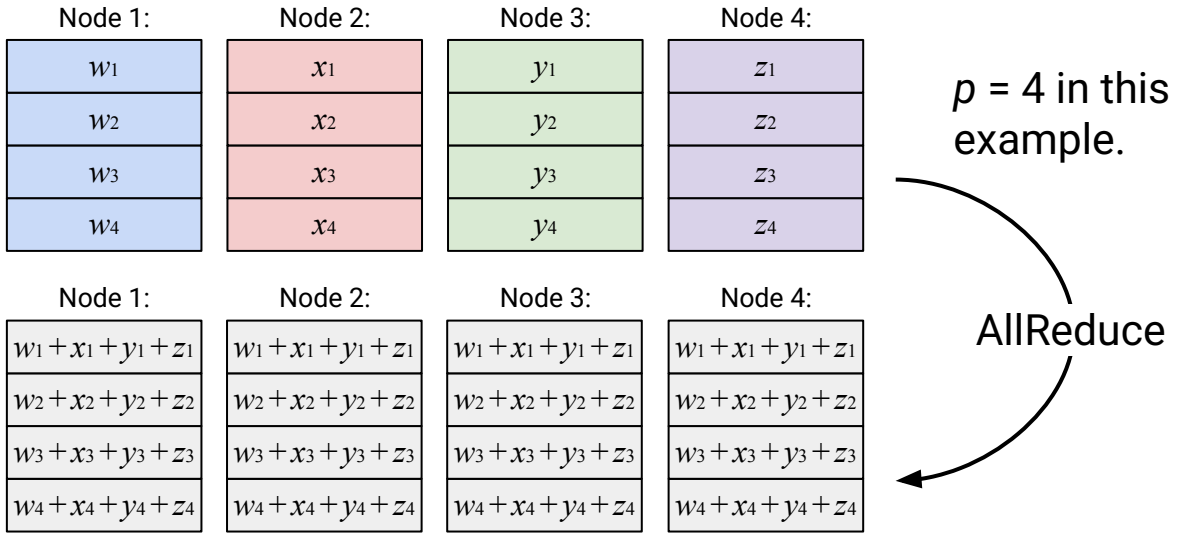
- Each node calls the operation, in parallel, at the same time.
- The operation is blocking: Must wait for all nodes to finish before we can proceed to the next task.

Input: Each node specifies a *p*-element vector of input data.

(e.g. Specify the memory address of a read buffer.)

Output: Each node receives a *p*-element vector of output data.

(e.g. Specify a memory address, and receive output at that write buffer.)



Collective Communication Definition: Recap

To recap, collective operations are:

- **Orchestrated**: A centralized controller plans out the job ahead of time.
- **Synchronized**: Every node starts the operation at the same time.
- **Blocking**: Must wait for all nodes to finish before proceeding.

We'll now define the 7 collective operations.

- First, we'll define the inputs and outputs (the *what*).
- Then, we'll look at how the operation is implemented in the network (the *how*).

The operations can roughly split into two categories:

- **Redistribution** operations move data around without transforming it.
- **Consolidation** operations aggregate many pieces of data into a single output.

Redistribution Operations

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Distributed AI Training

- Motivation
- Infrastructure

Defining Collectives

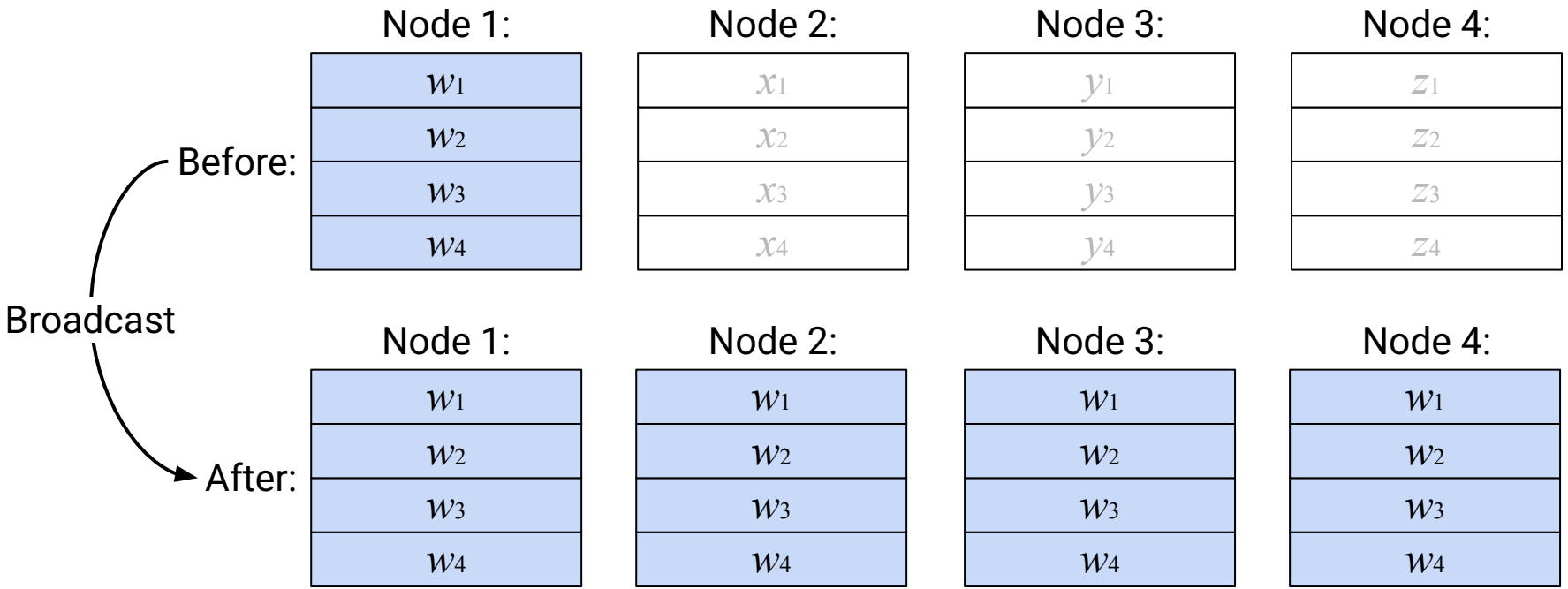
- Definition
- **Redistribution Operations**
- Consolidation Operations
- Duals and Compositions

Implementing Collectives

- Mesh Topology
- Single Root
- Tree
- Ring (Naive)
- Ring (Optimized)
- Overlay/Underlay Topologies

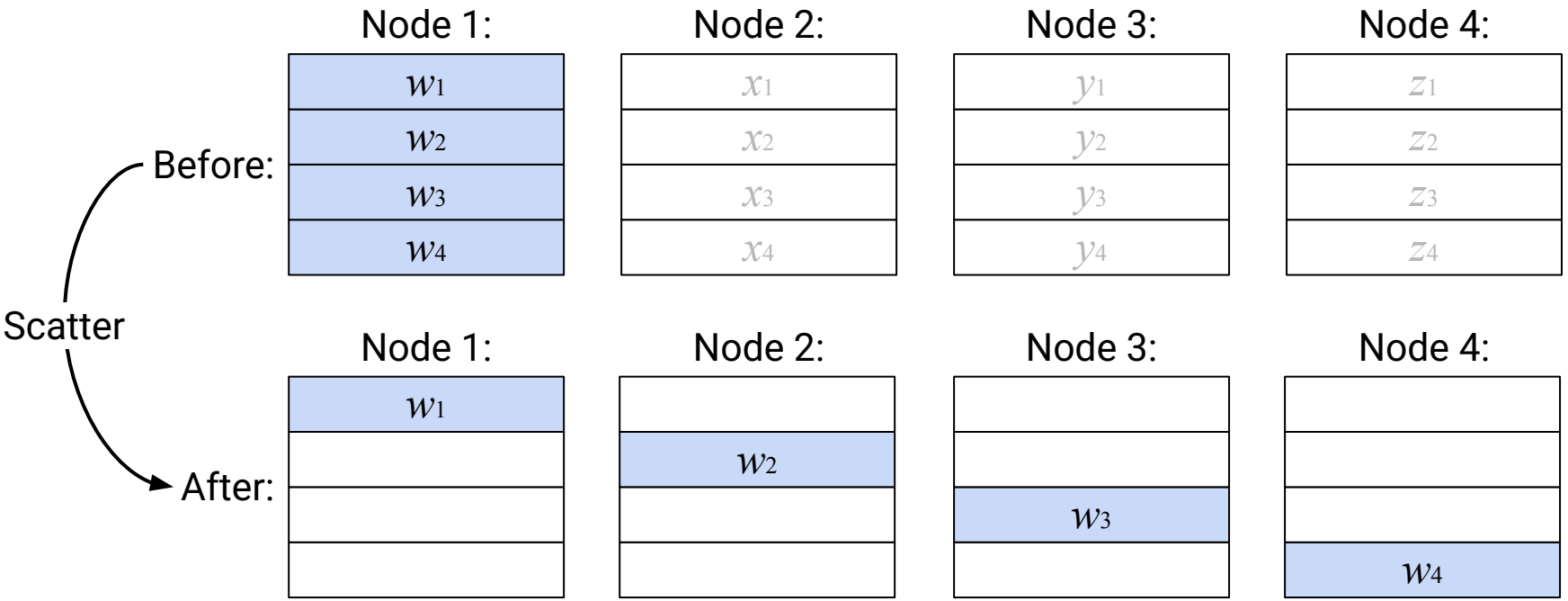
Broadcast

The target node sends its vector to everybody.



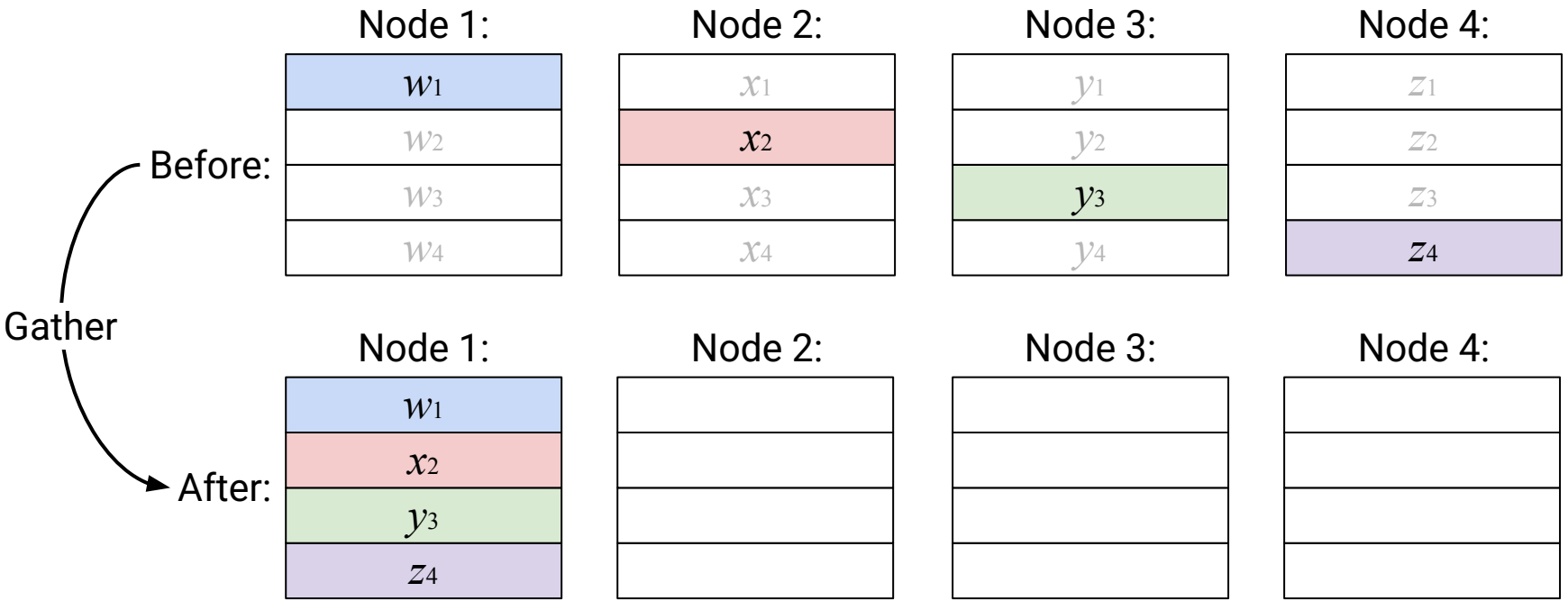
Scatter

The target node sends its *i*th element to the *i*th node.



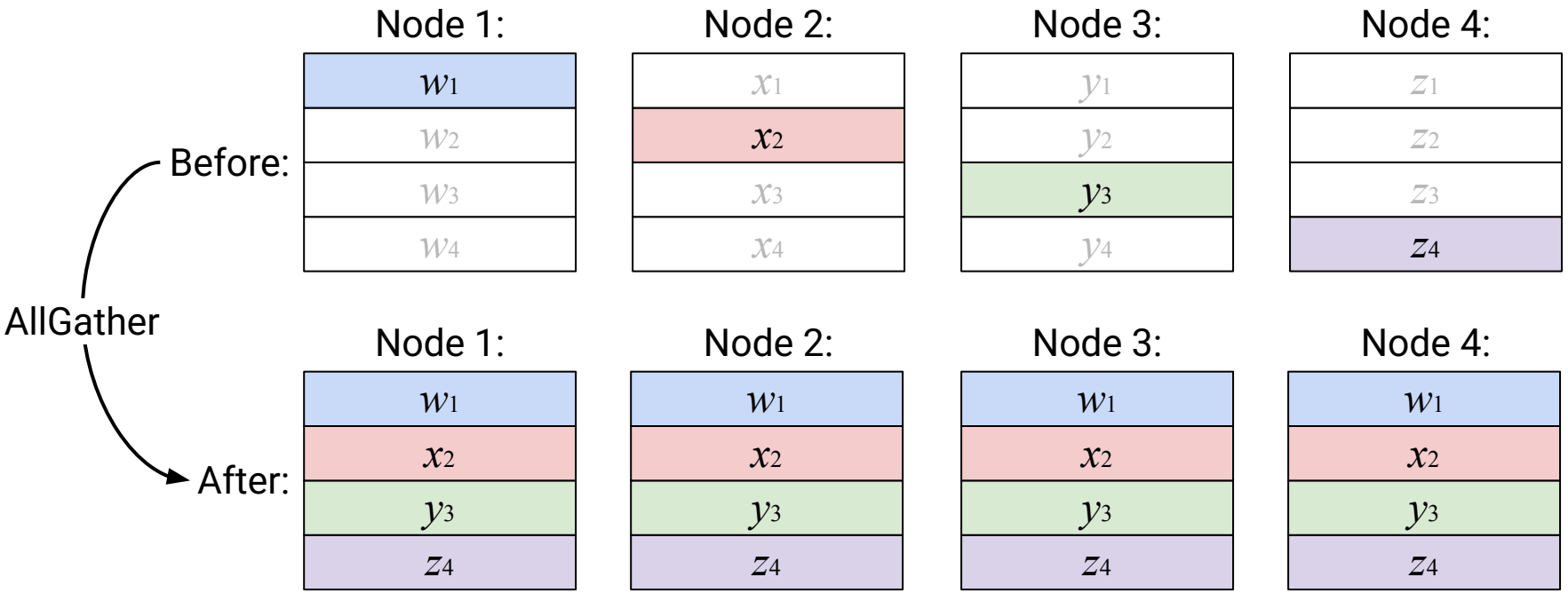
Gather

Node *i* sends its *i*th element to the target node.



AllGather

Node *i* sends its *i*th element to everybody.



Consolidation Operations

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Distributed AI Training

- Motivation
- Infrastructure

Defining Collectives

- Definition
- Redistribution Operations
- **Consolidation Operations**
- Duals and Compositions

Implementing Collectives

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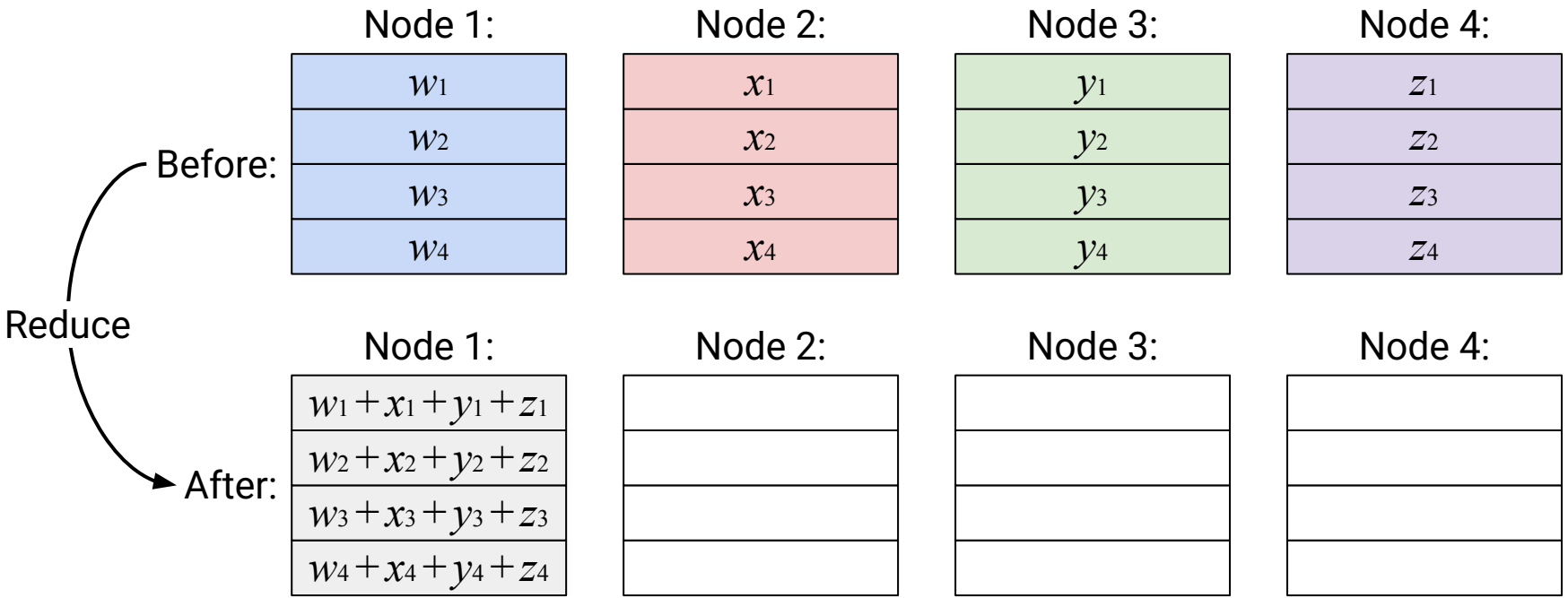
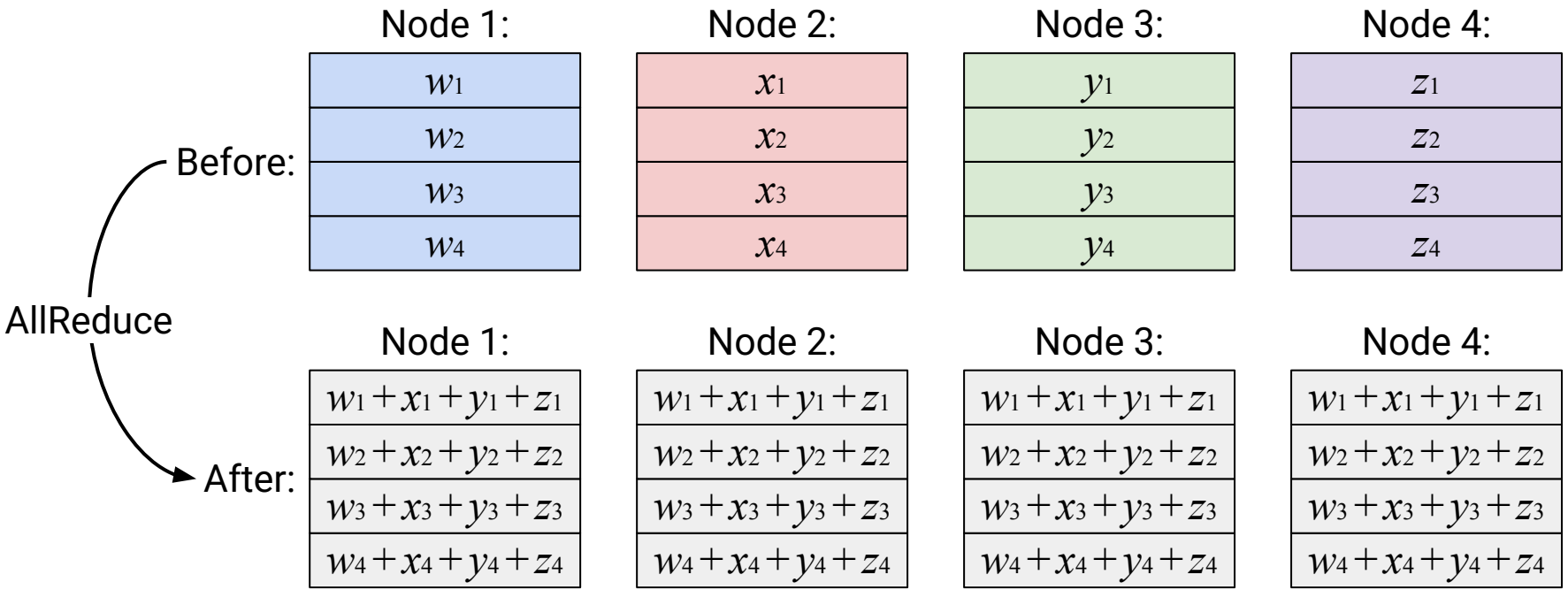
Reduce

Sum up the vectors, then send the resulting sum vector to a target node.



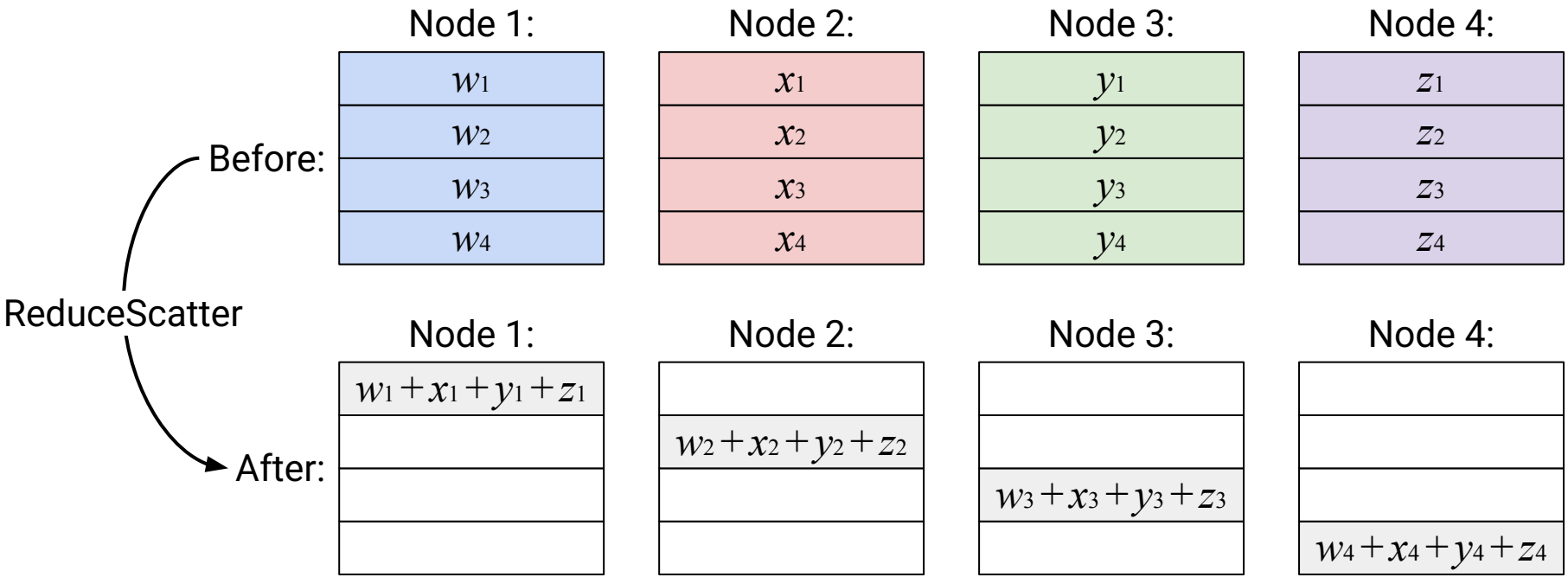
AllReduce

Sum up the vectors, then send the resulting sum vector to all nodes.



ReduceScatter

The *i*th node has the sum of the *i*th elements of each vector.



Duals and Compositions

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Distributed AI Training

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- Infrastructure

Defining Collectives

- Definition
- Redistribution Operations
- Consolidation Operations
- **Duals and Compositions**

Implementing Collectives

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Duals

Some operations are **duals** of each other: One operation is the reverse of the other.

- Analogy: In math, x^2 and $\sqrt{x}$ are duals.

When checking if operations are duals, ignore any reduction operations.

- Just look at which boxes we read from, and which boxes we write to.

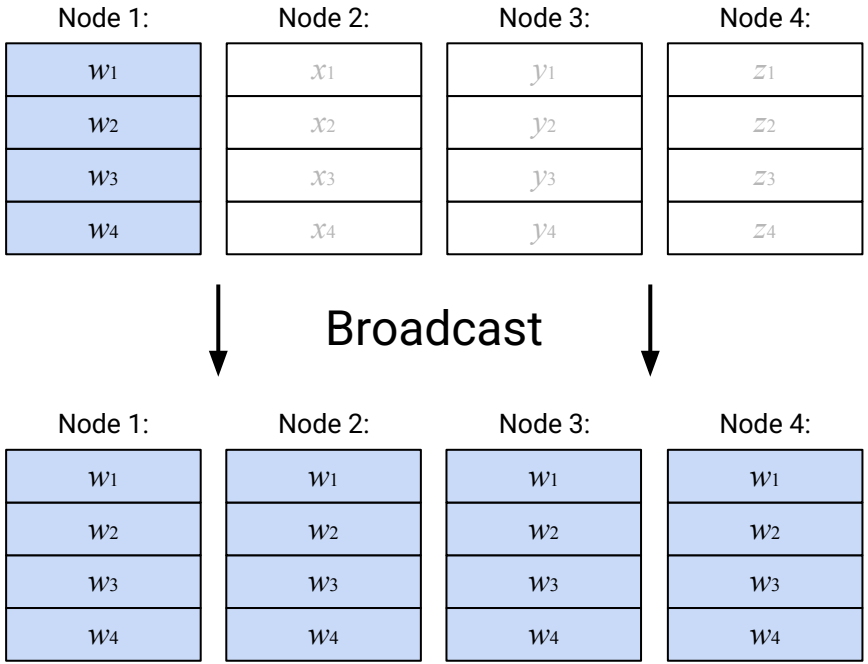
Duals are useful when we think about implementing these collectives.

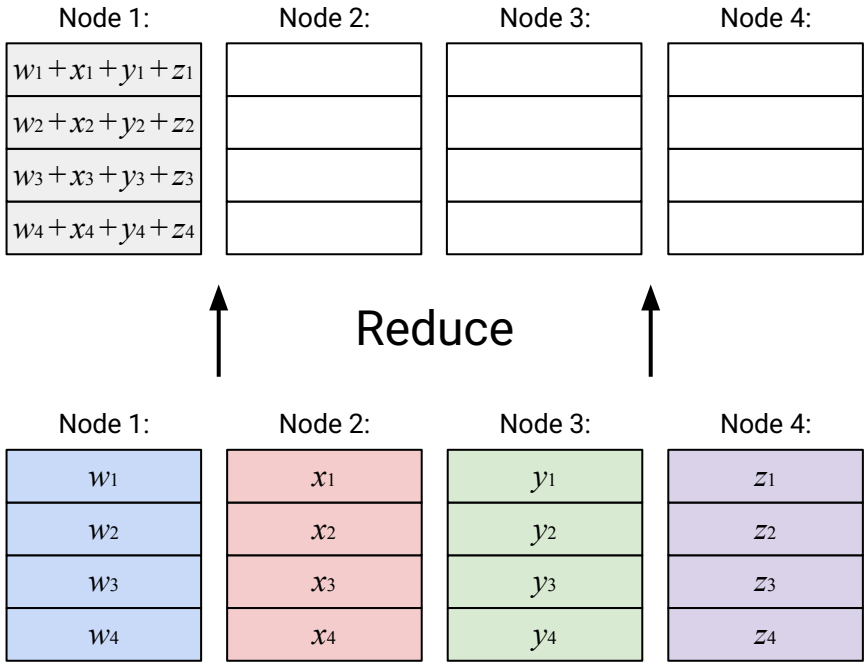
- All else equal, two dual operations use the same bandwidth.
- Total amount of data sent/received is the same in both operations, because we write/read from the same boxes.

Duals

Broadcast and Reduce are duals.

- Broadcast: Read from 4 boxes in root node. Write to all 16 boxes.
- Reduce: Read from 16 boxes. Write to 4 boxes in root node.

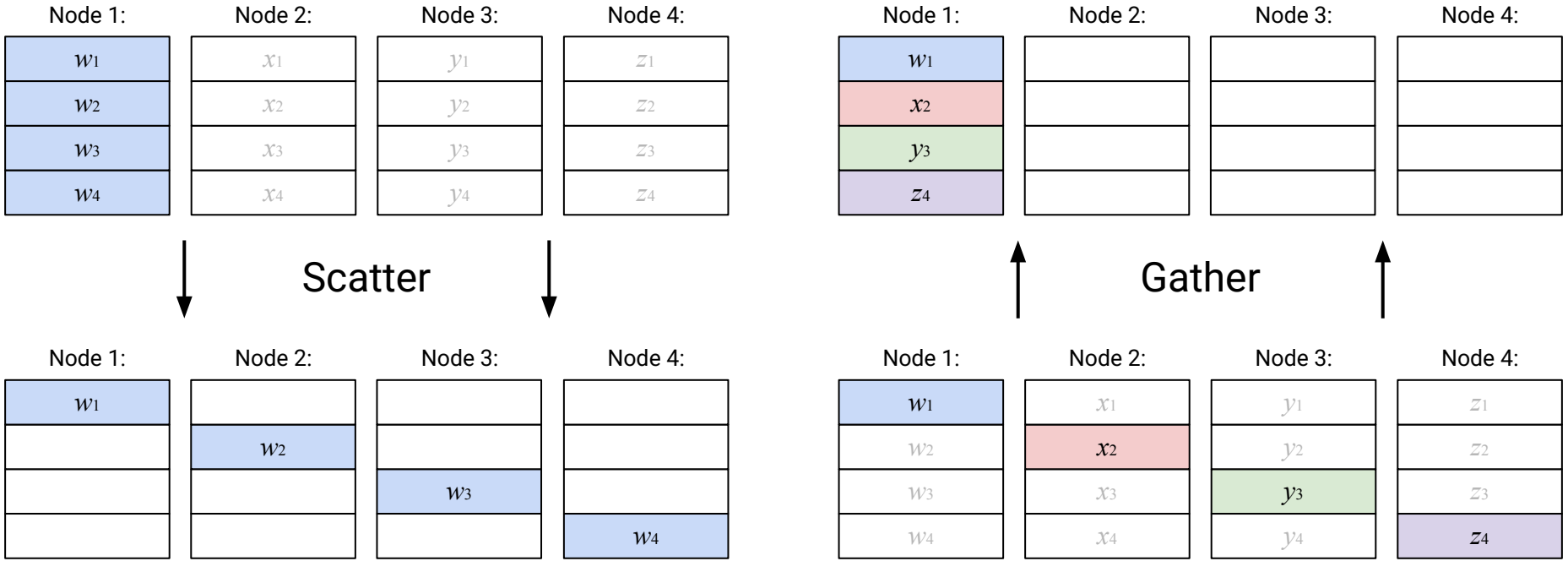




Duals

Scatter and Gather are duals.

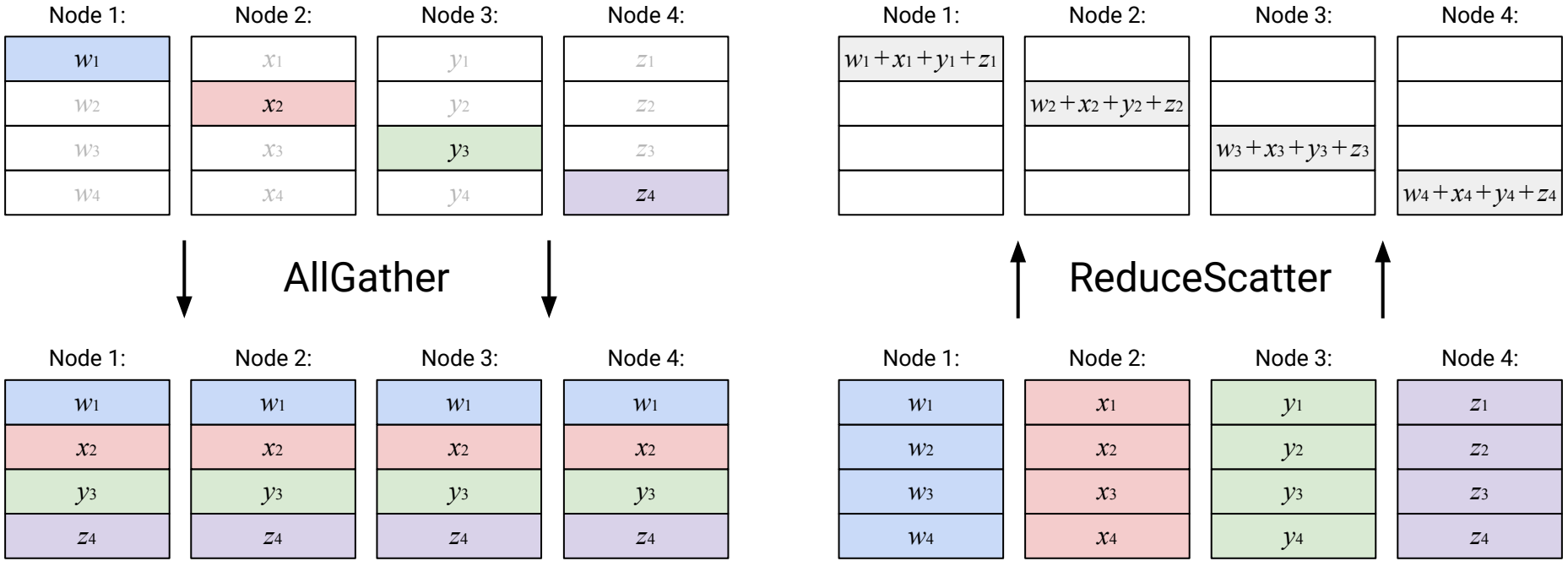
- Scatter: Read from 4 boxes in root node. Write to the *i*th box in *i*th node.
- Gather: Read from the *i*th box in *i*th node. Write to 4 boxes in root node.



Duals

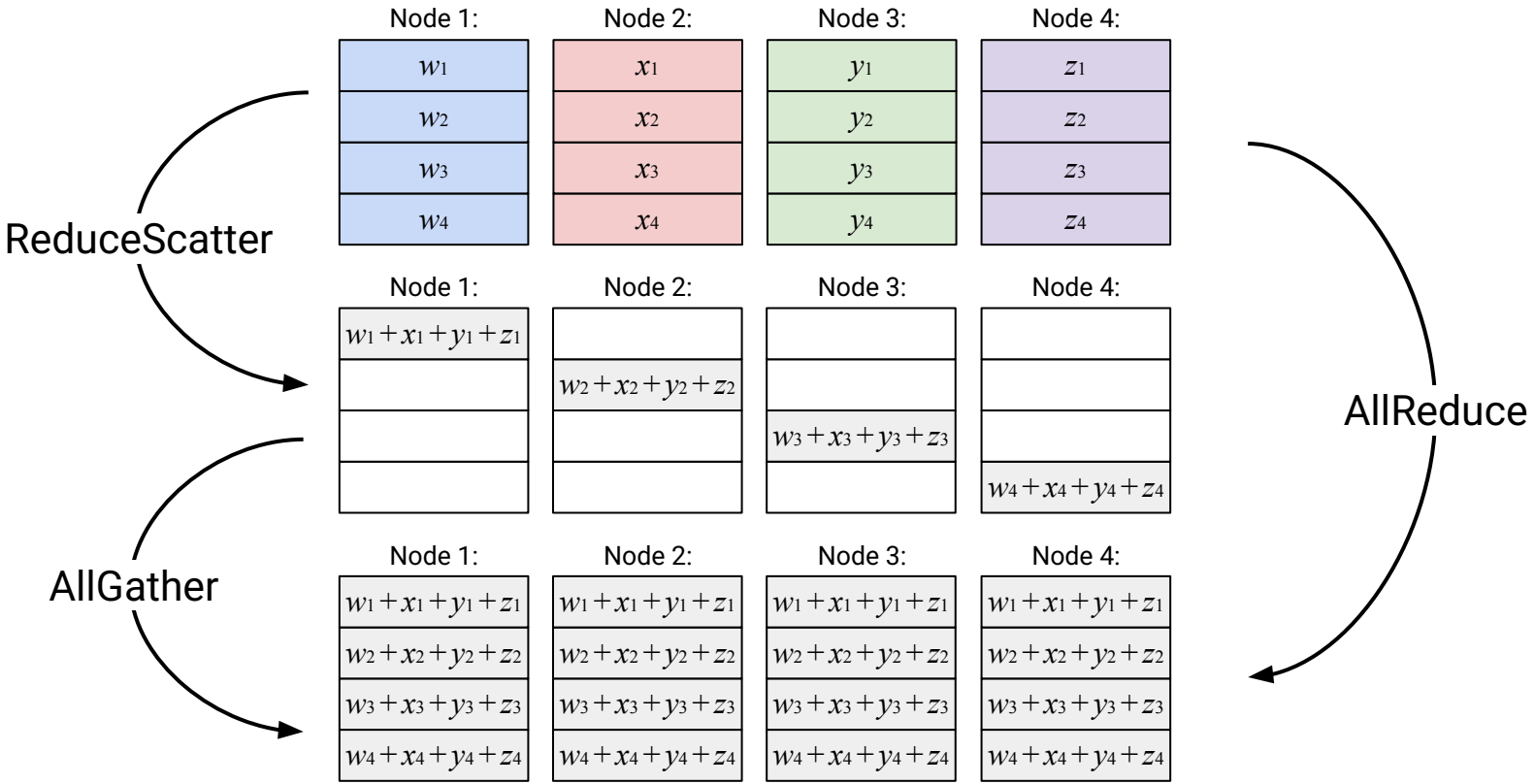
AllGather and ReduceScatter are duals.

- AllGather: Read from the *i*th box in *i*th node. Write to all 16 boxes.
- Reduce: Read from all 16 boxes. Write to the *i*th box in *i*th node.



Compositing Operations

AllReduce = ReduceScatter + AllGather.



AllReduce with Mesh Topology

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Defining Collectives

- Definition
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Implementing Collectives

- **Mesh Topology**
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Implementing Collective Operations

So far: We've defined the input and desired output of each operation.

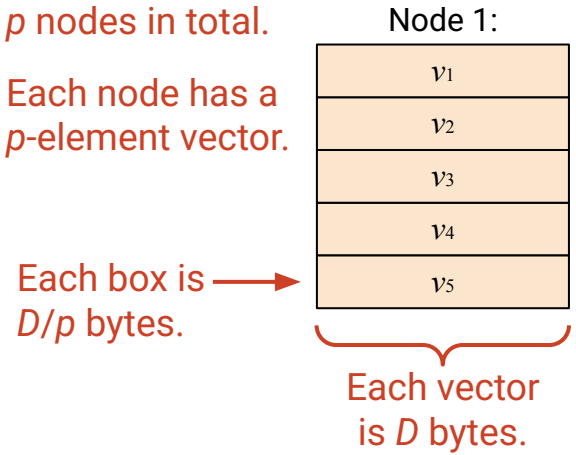
Up next: How do we implement these operations?

- What topology do we use to connect the nodes?
- What data does each node have to send to the other nodes?

Our choices will affect the performance of the operation.

- Total amount of bandwidth used?
- Number of steps for operation to complete?
- Bandwidth each node sends per step?

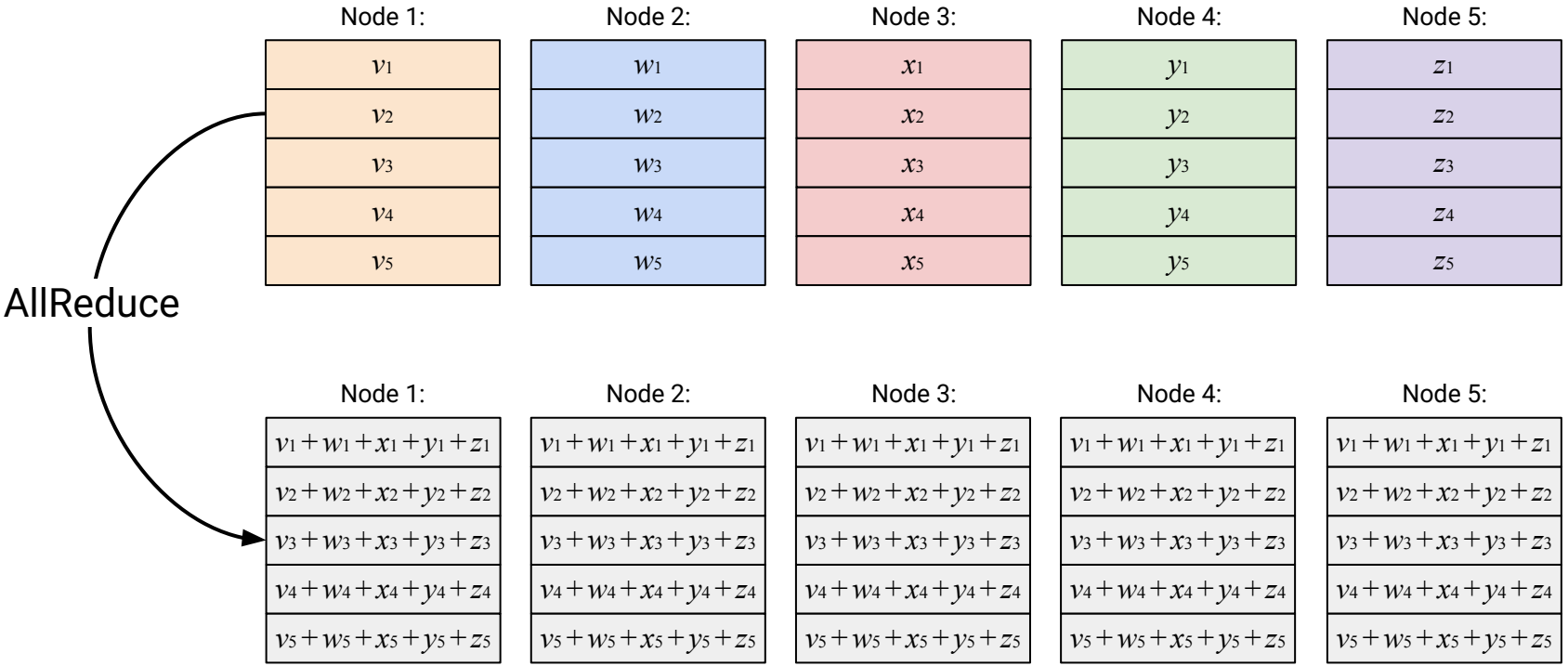
We'll use AllReduce as our running example.



Reminder: AllReduce Operation

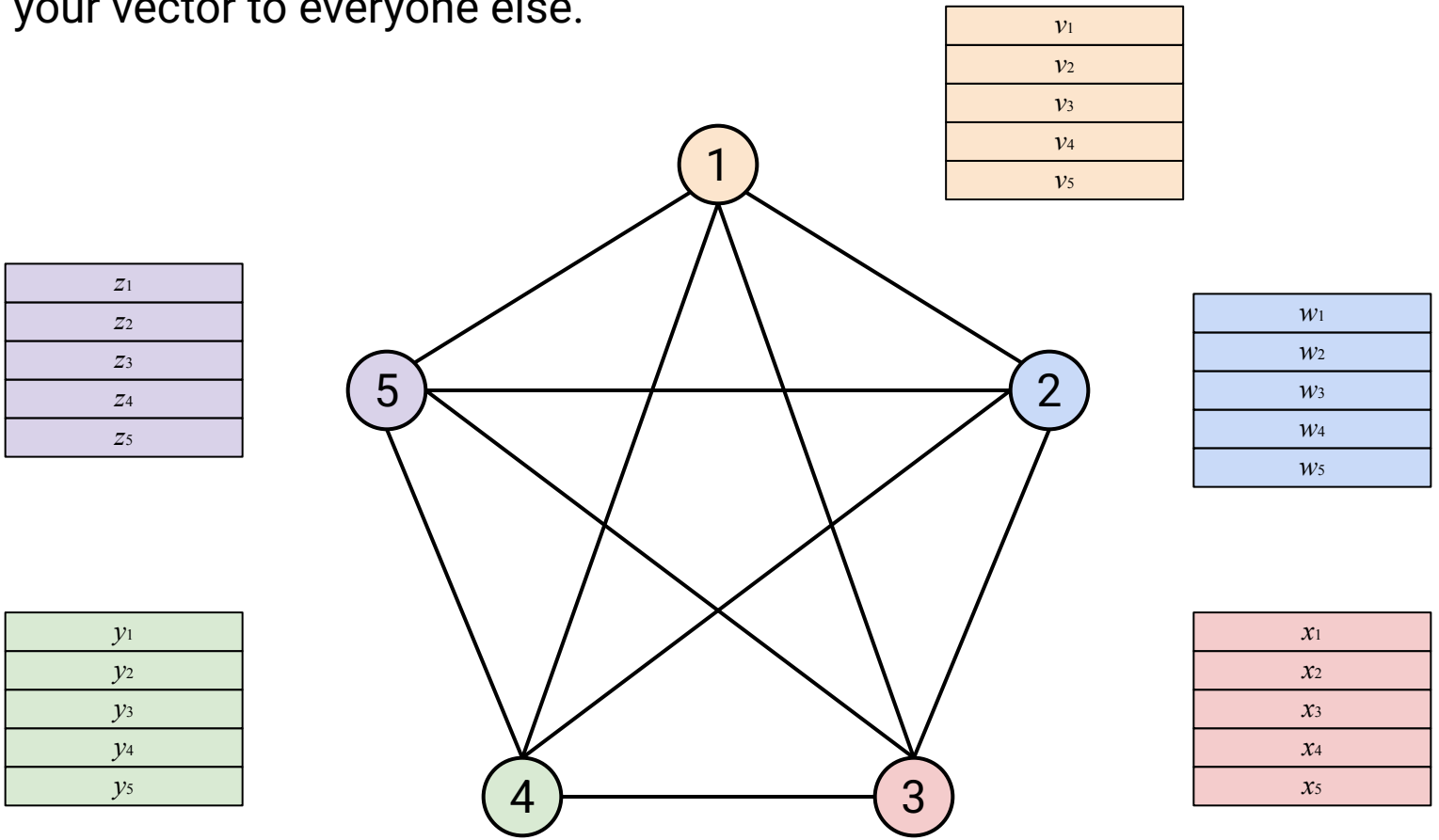
Sum up the vectors, then send the resulting sum vector to all nodes.

For this demo, we'll use $p = 5$ nodes instead of $p = 4$ nodes.



Mesh Topology

1. Send your vector to everyone else.



Mesh Topology

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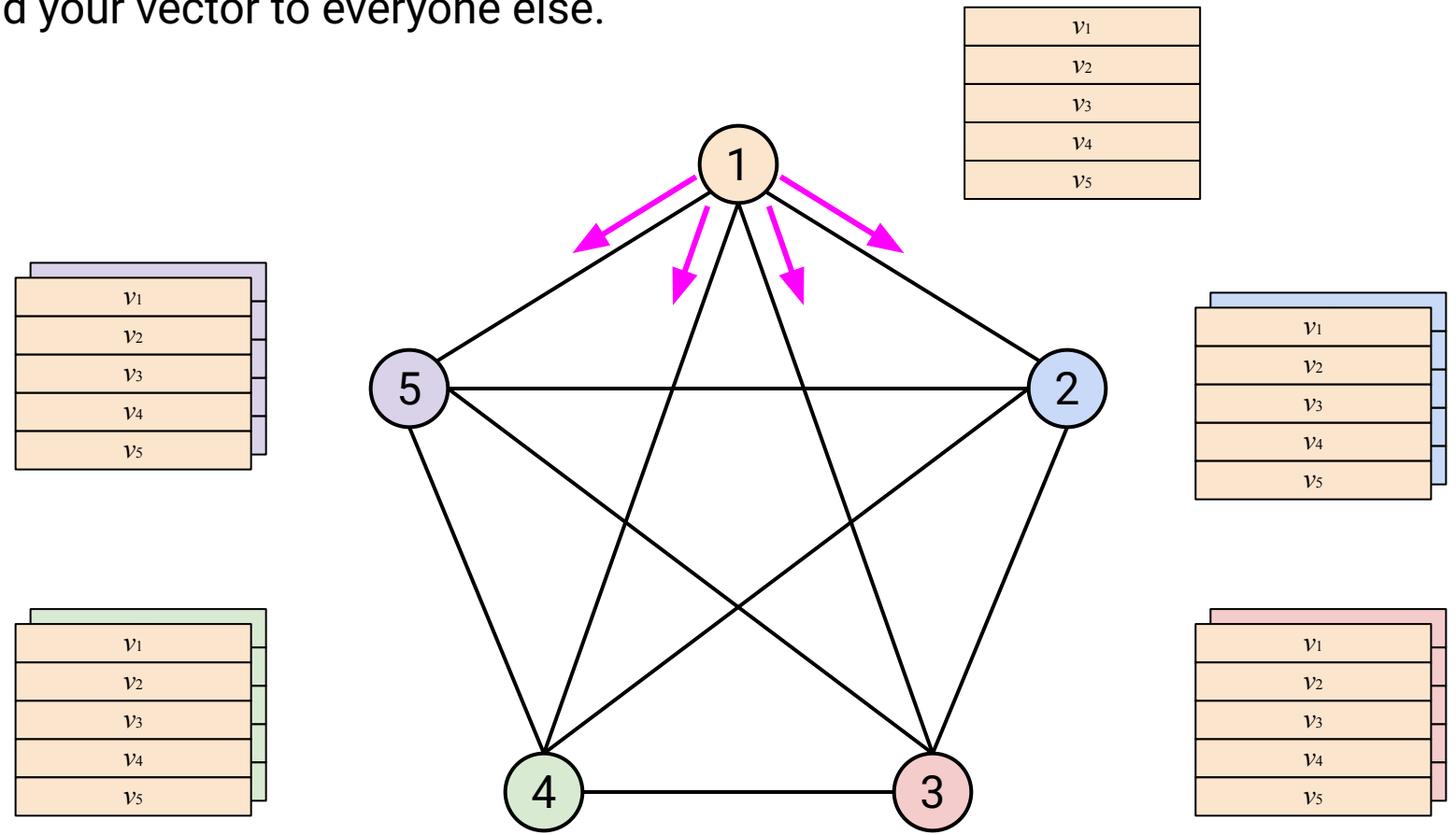

Mesh Topology

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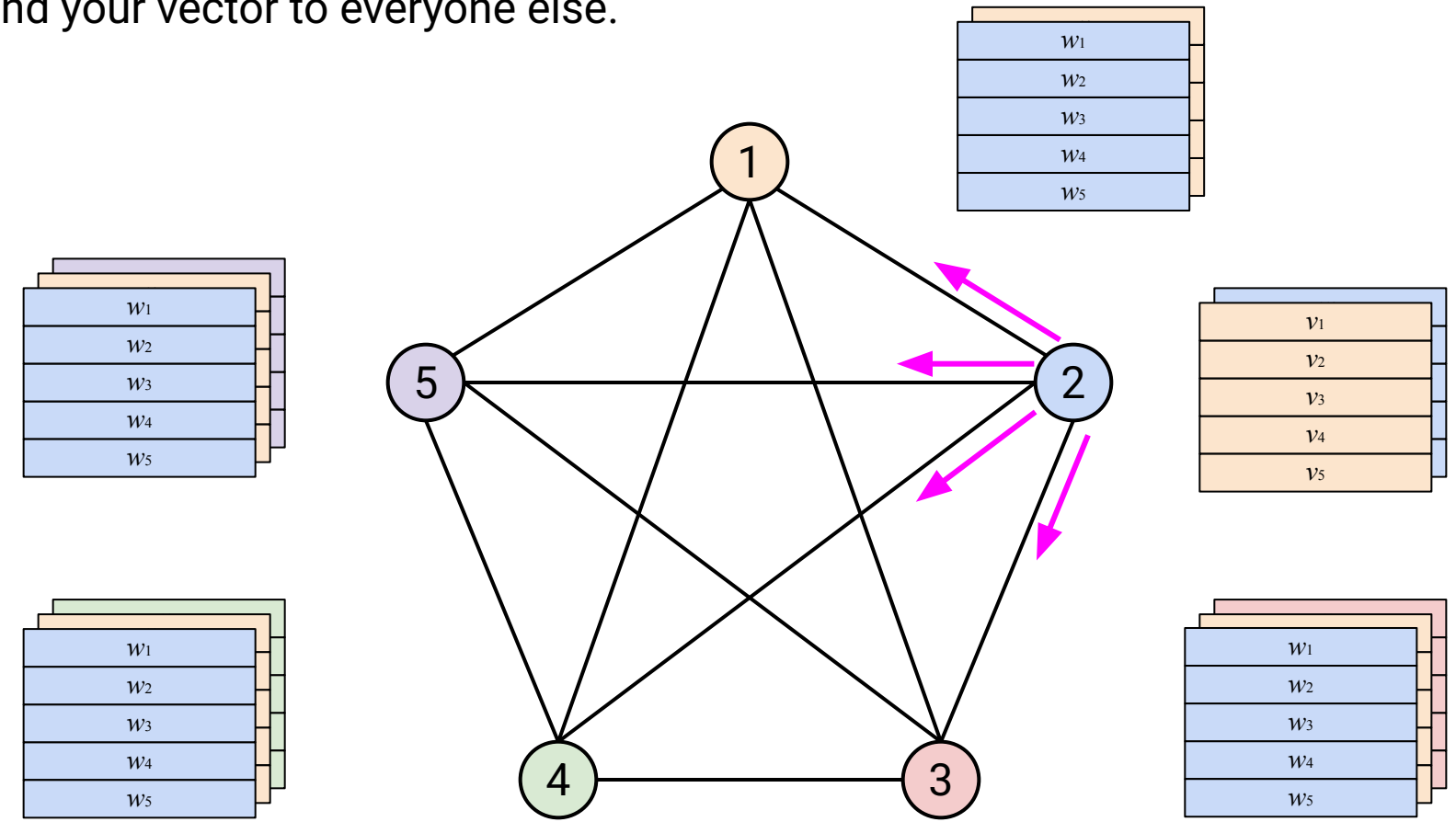

Mesh Topology

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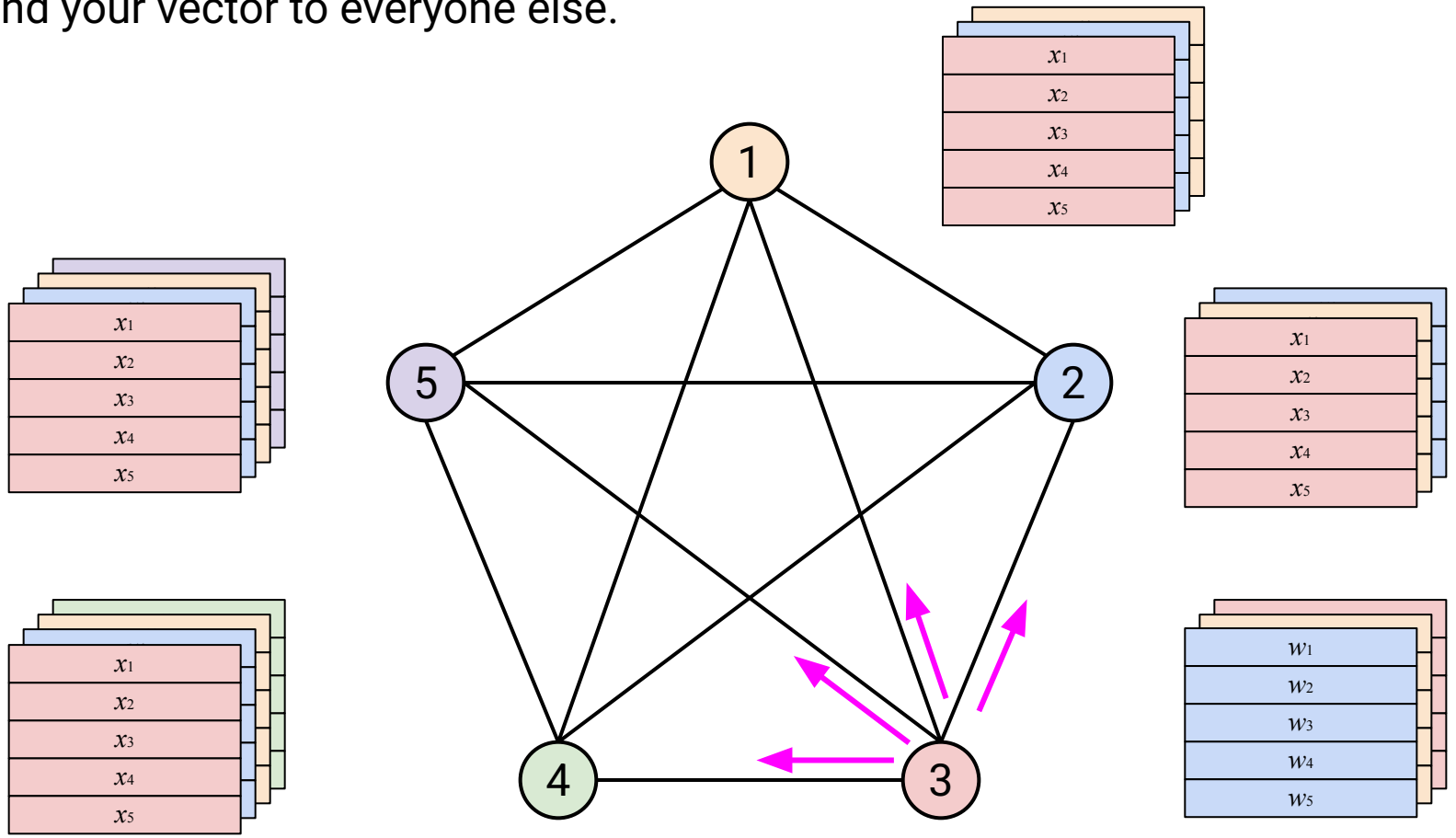

Mesh Topology

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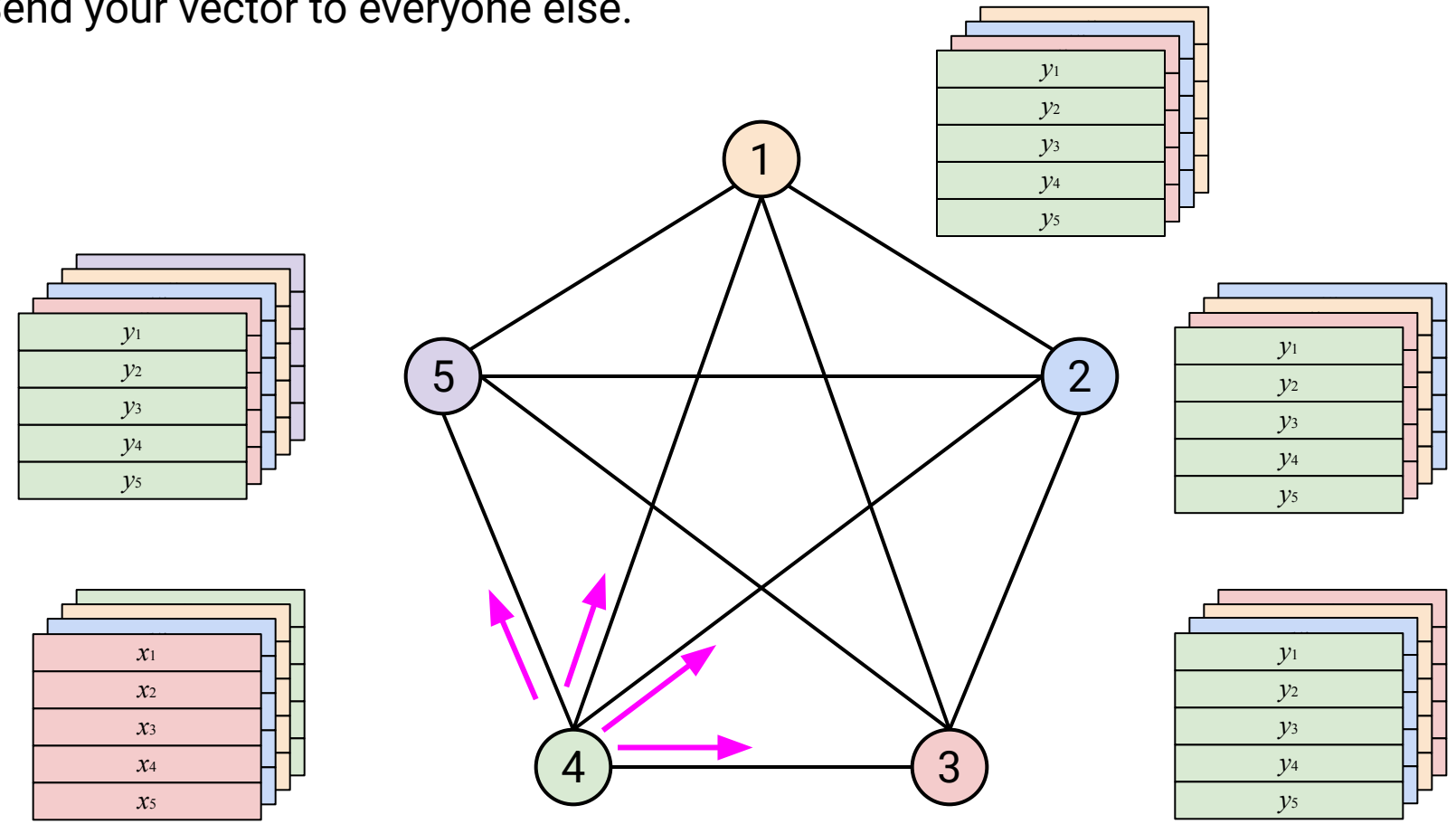

Mesh Topology

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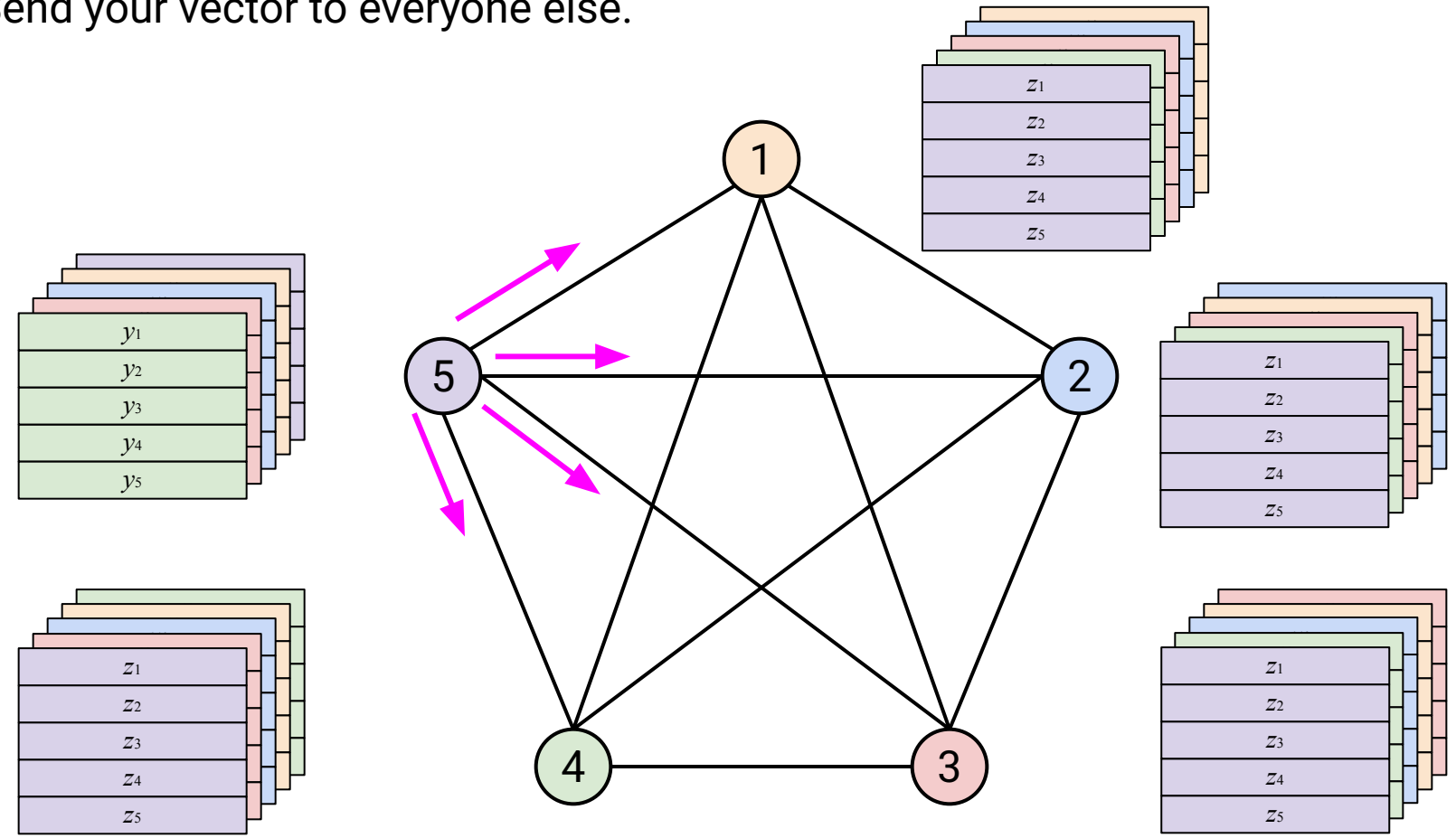
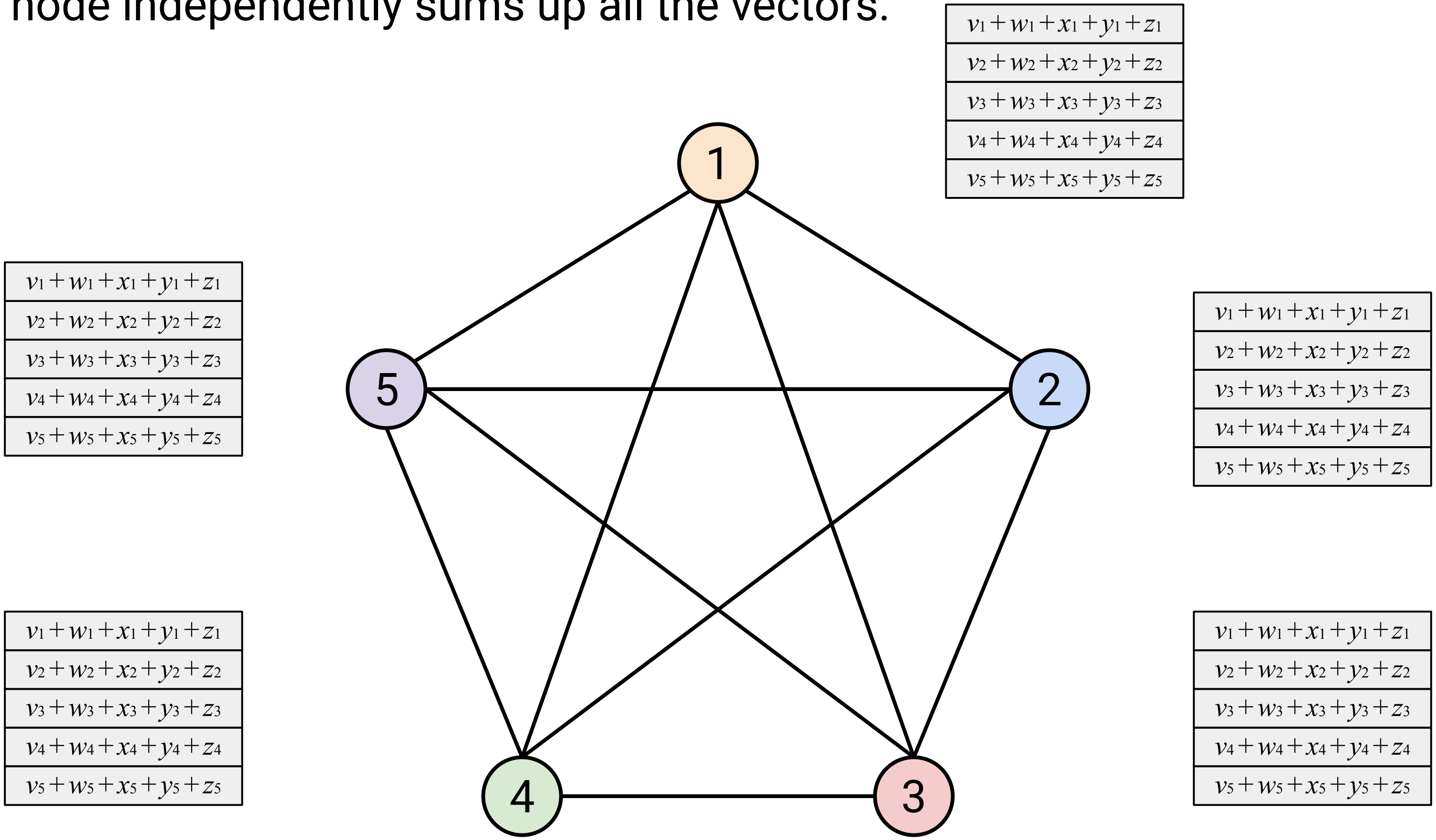


Mesh Topology

2. Each node independently sums up all the vectors.



Mesh Topology: Efficiency

Total bandwidth: Each node sends its whole vector (D bytes) to all $p-1$ other nodes. There are p nodes in total.

Number of steps: Everyone can simultaneously send their vector to everyone else.

Bandwidth per step: Send D bytes to $p-1$ other nodes.

	Total Bandwidth	Number of Steps	Bandwidth Per Step
Mesh	Dp^2	1	Dp

AllReduce with Single Root

Lecture 23, CS 168, Spring 2026

Distributed AI Training

- Motivation
- Infrastructure

Defining Collectives

- Definition
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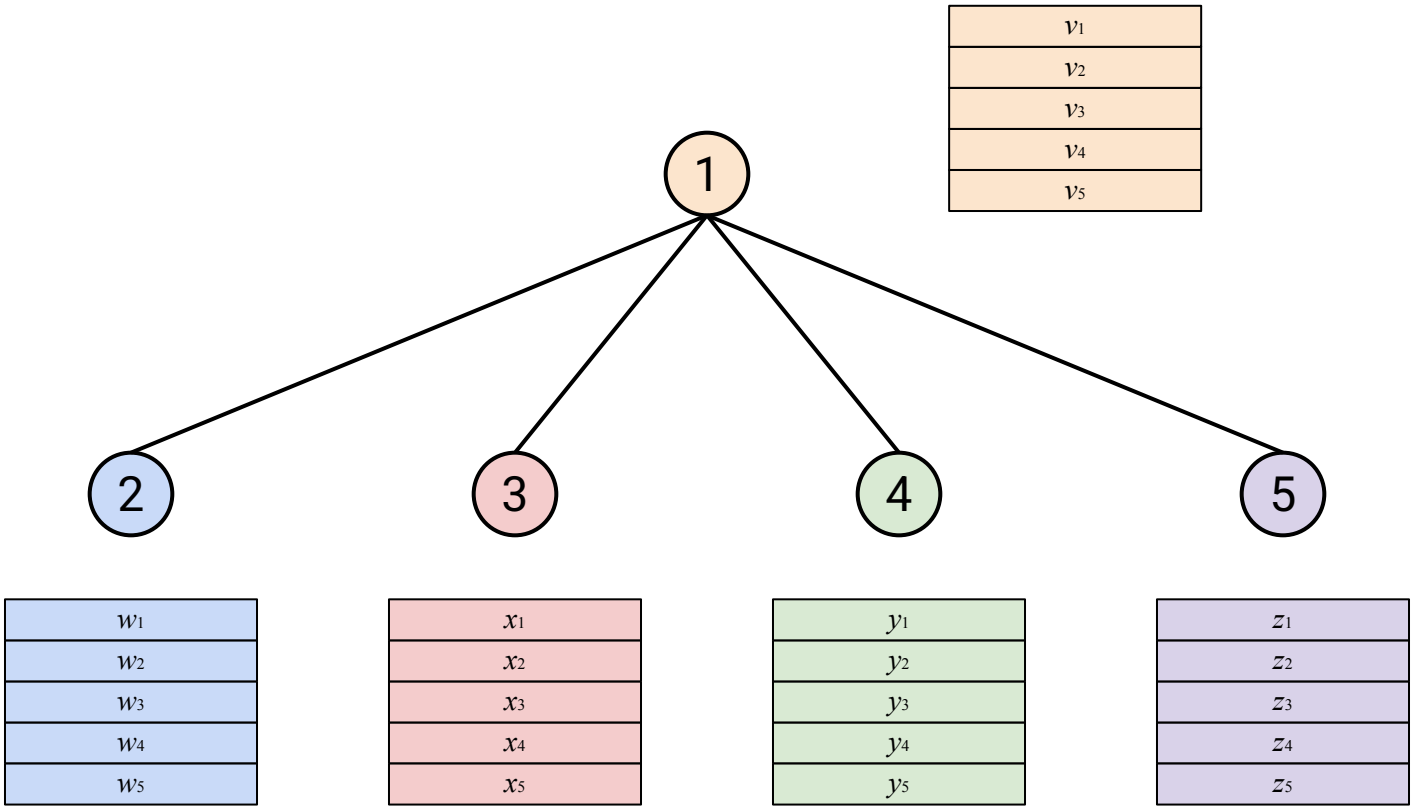
- Consolidation Operations
- Duals and Compositions

Implementing Collectives

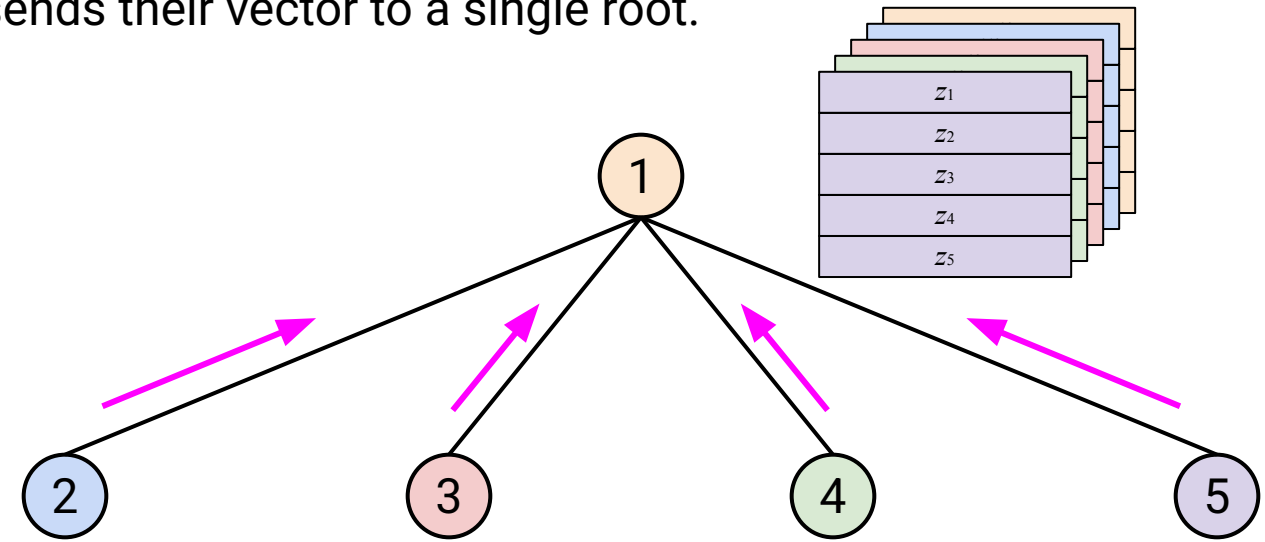
- Mesh Topology
- **Single Root**
- Tree
- Ring (Naive)
- Ring (Optimized)
- Overlay/Underlay Topologies

Mesh Topology



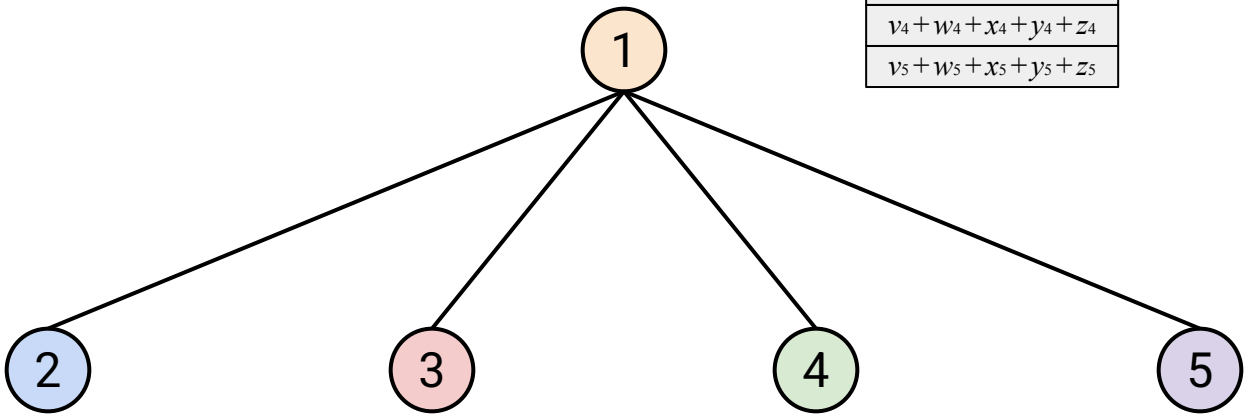
Mesh Topology

1. Everyone sends their vector to a single root.



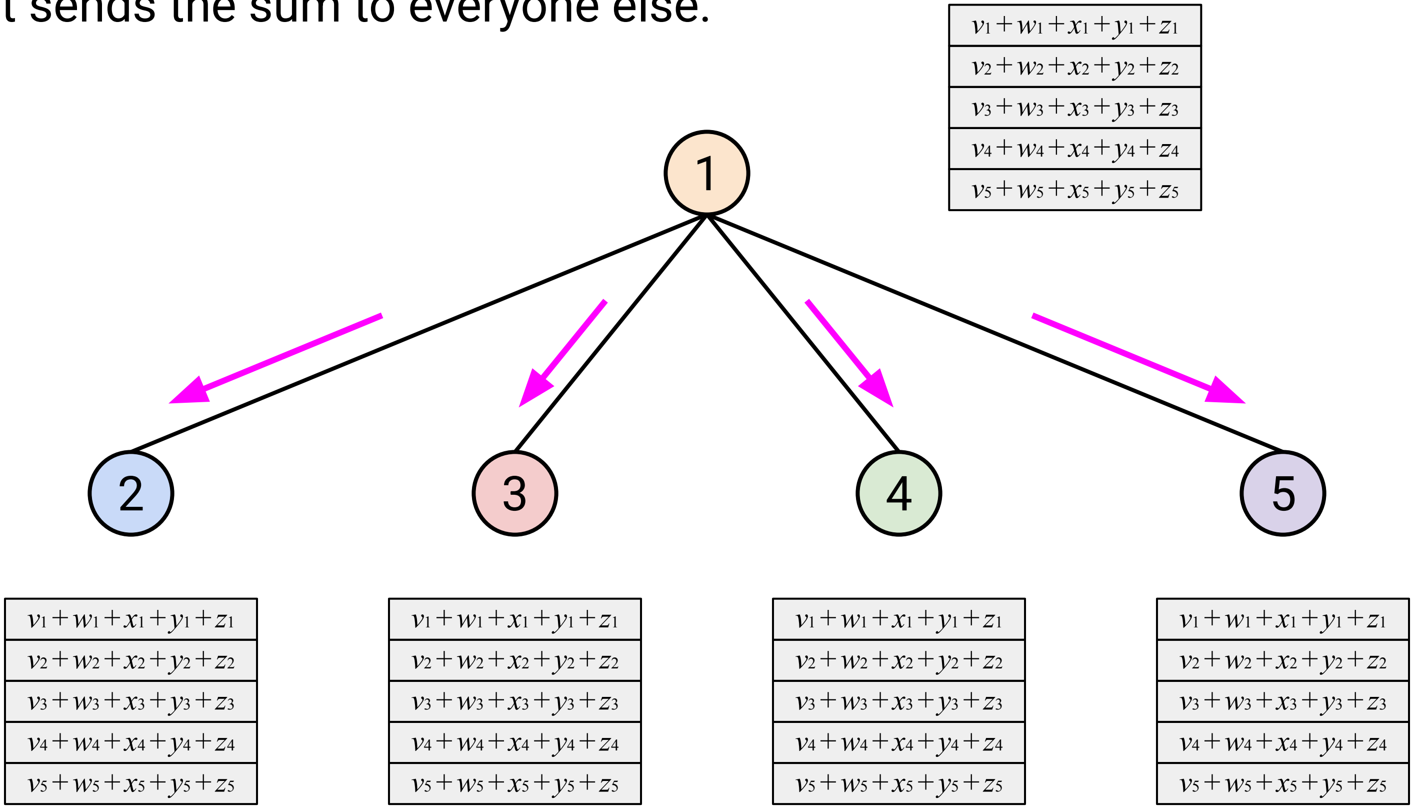
2. The root computes the sum.

$v_1 + w_1 + x_1 + y_1 + z_1$
$v_2 + w_2 + x_2 + y_2 + z_2$
$v_3 + w_3 + x_3 + y_3 + z_3$
$v_4 + w_4 + x_4 + y_4 + z_4$
$v_5 + w_5 + x_5 + y_5 + z_5$



Mesh Topology

3. The root sends the sum to everyone else.



Single Root Topology: Efficiency

Total bandwidth: Each node sends its whole vector (D bytes) to the root. There are p nodes in total.

Number of steps: One step to send to root, and one step to distribute sum from root.

Bandwidth per step: Root has to receive D bytes from the $p-1$ other nodes.

	Total Bandwidth	Number of Steps	Bandwidth Per Step
Mesh	Dp^2	1	Dp
Single Root	Dp	2	Dp

Tree-Based AllReduce

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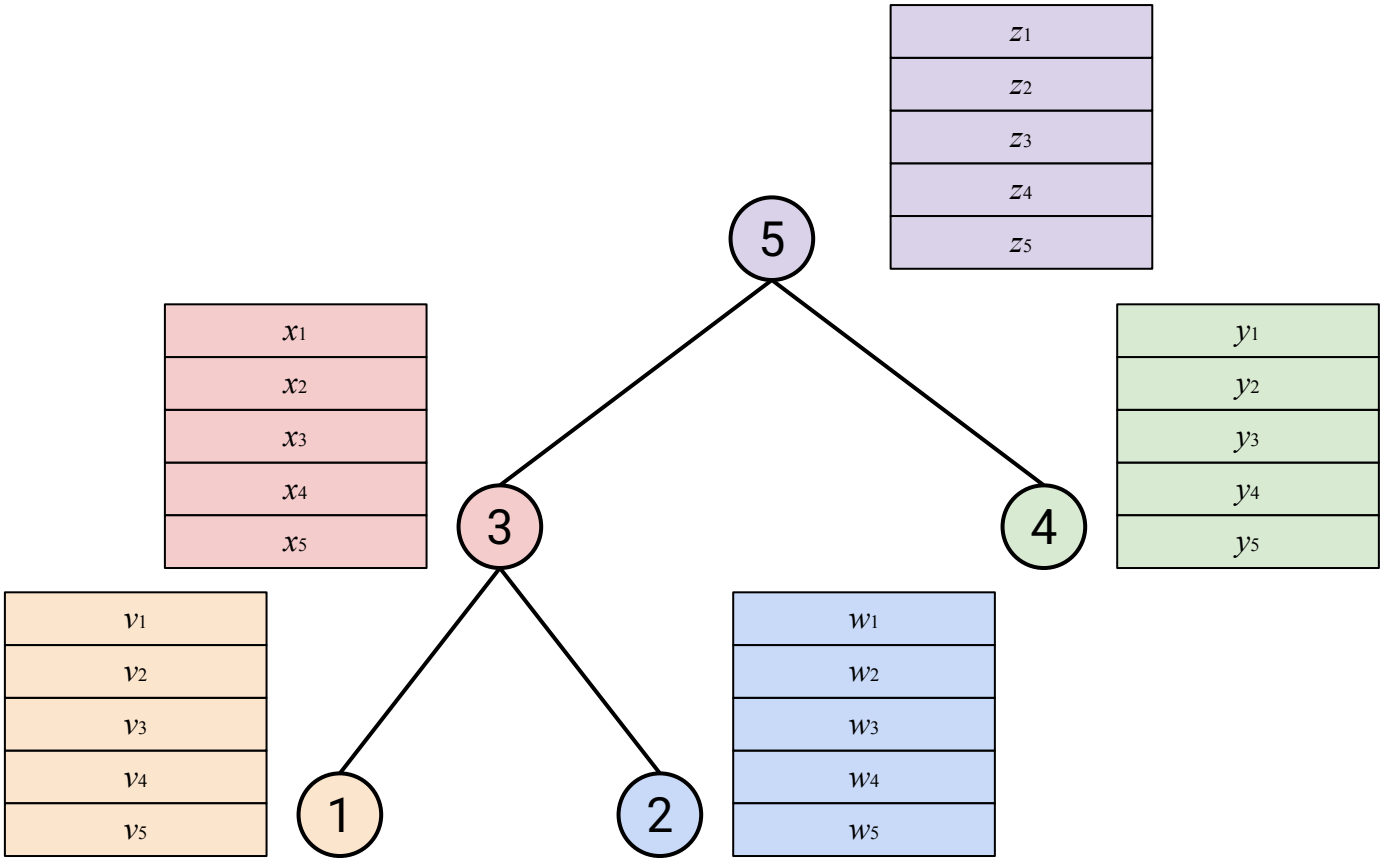
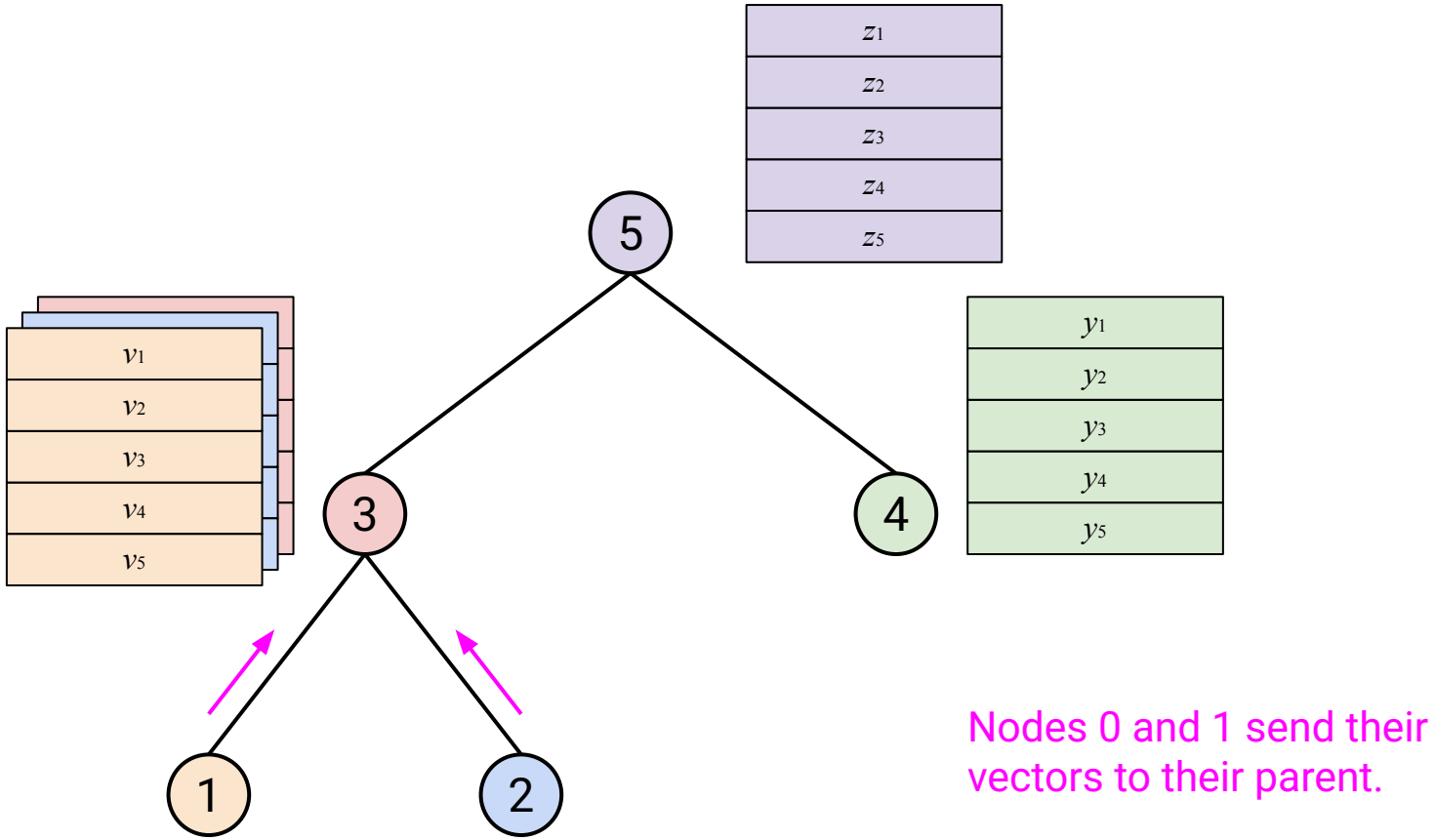
Tree-Based AllReduce

1. Send your vector to your parent. When you receive vector(s), sum them up.



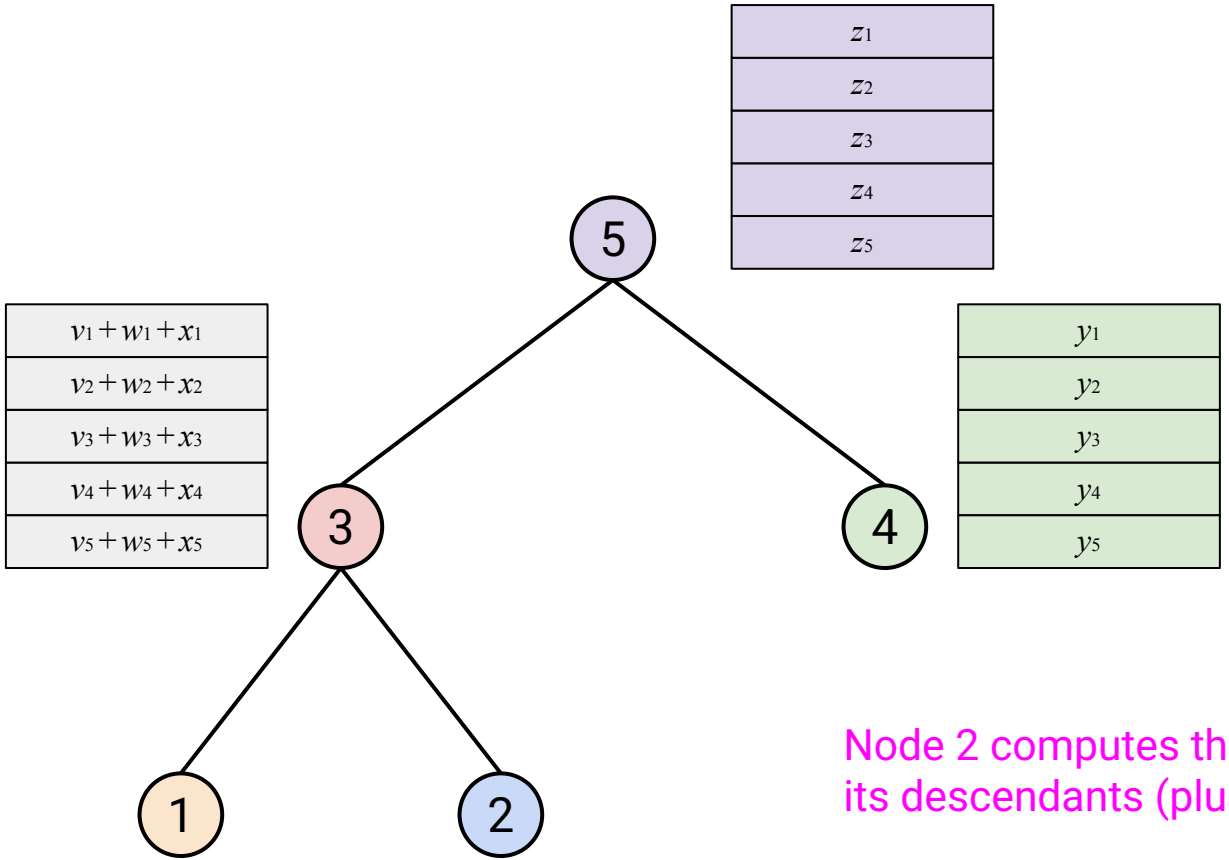
Tree-Based AllReduce

1. Send your vector to your parent. When you receive vector(s), sum them up.



Tree-Based AllReduce

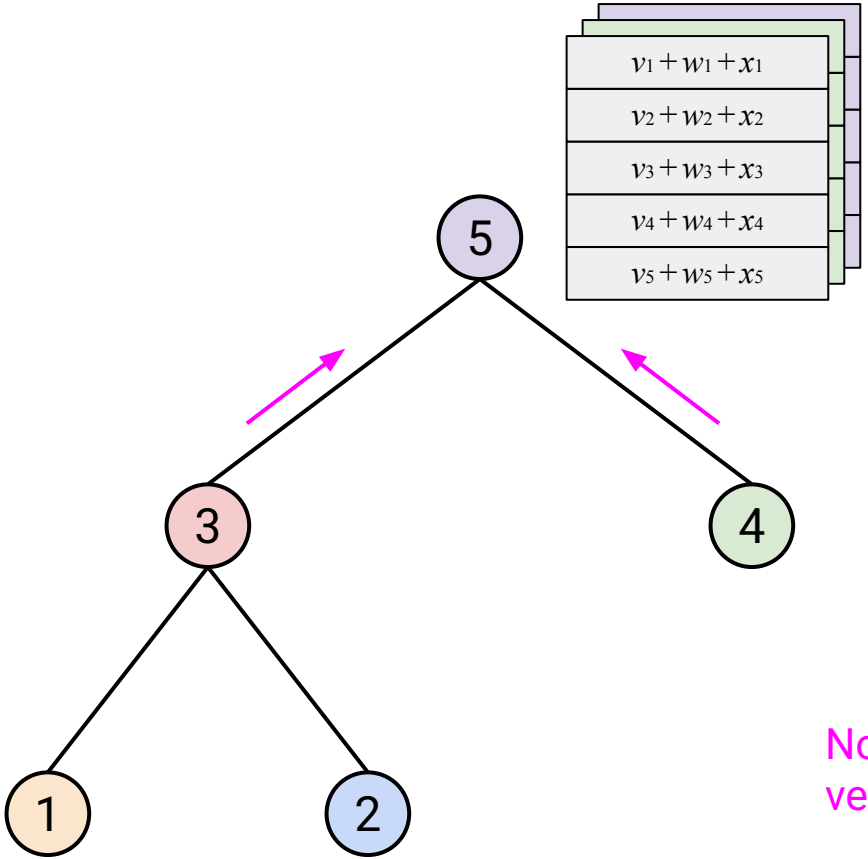
1. Send your vector to your parent. When you receive vector(s), sum them up.



Node 2 computes the sum of all its descendants (plus itself).

Tree-Based AllReduce

1. Send your vector to your parent. When you receive vector(s), sum them up.

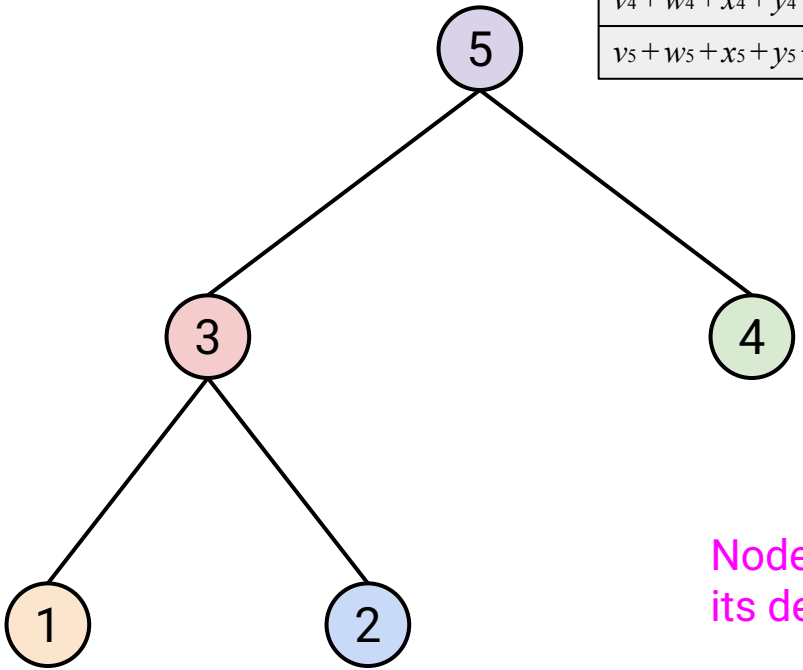


Nodes 2 and 3 send their vectors to their parent.

Tree-Based AllReduce

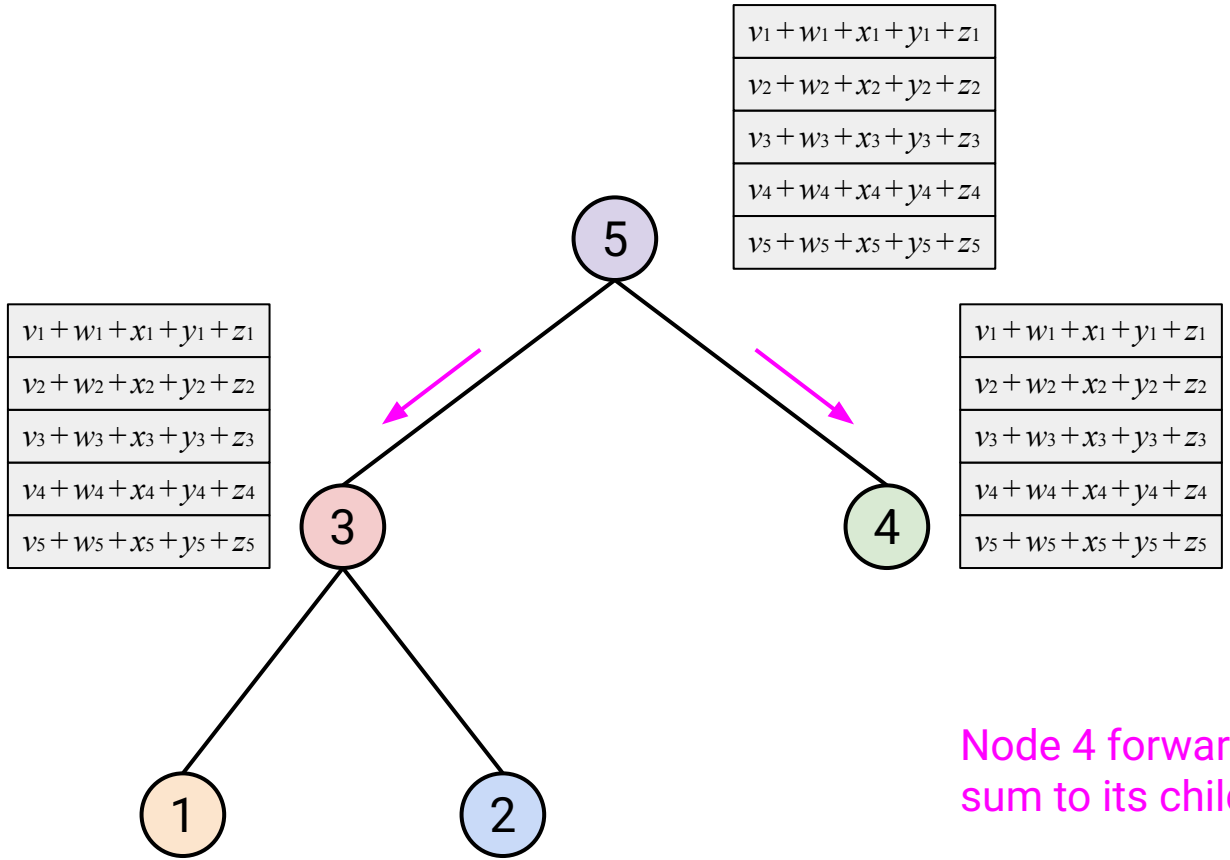
1. Send your vector to your parent. When you receive vector(s), sum them up.

$v_1 + w_1 + x_1 + y_1 + z_1$
$v_2 + w_2 + x_2 + y_2 + z_2$
$v_3 + w_3 + x_3 + y_3 + z_3$
$v_4 + w_4 + x_4 + y_4 + z_4$
$v_5 + w_5 + x_5 + y_5 + z_5$



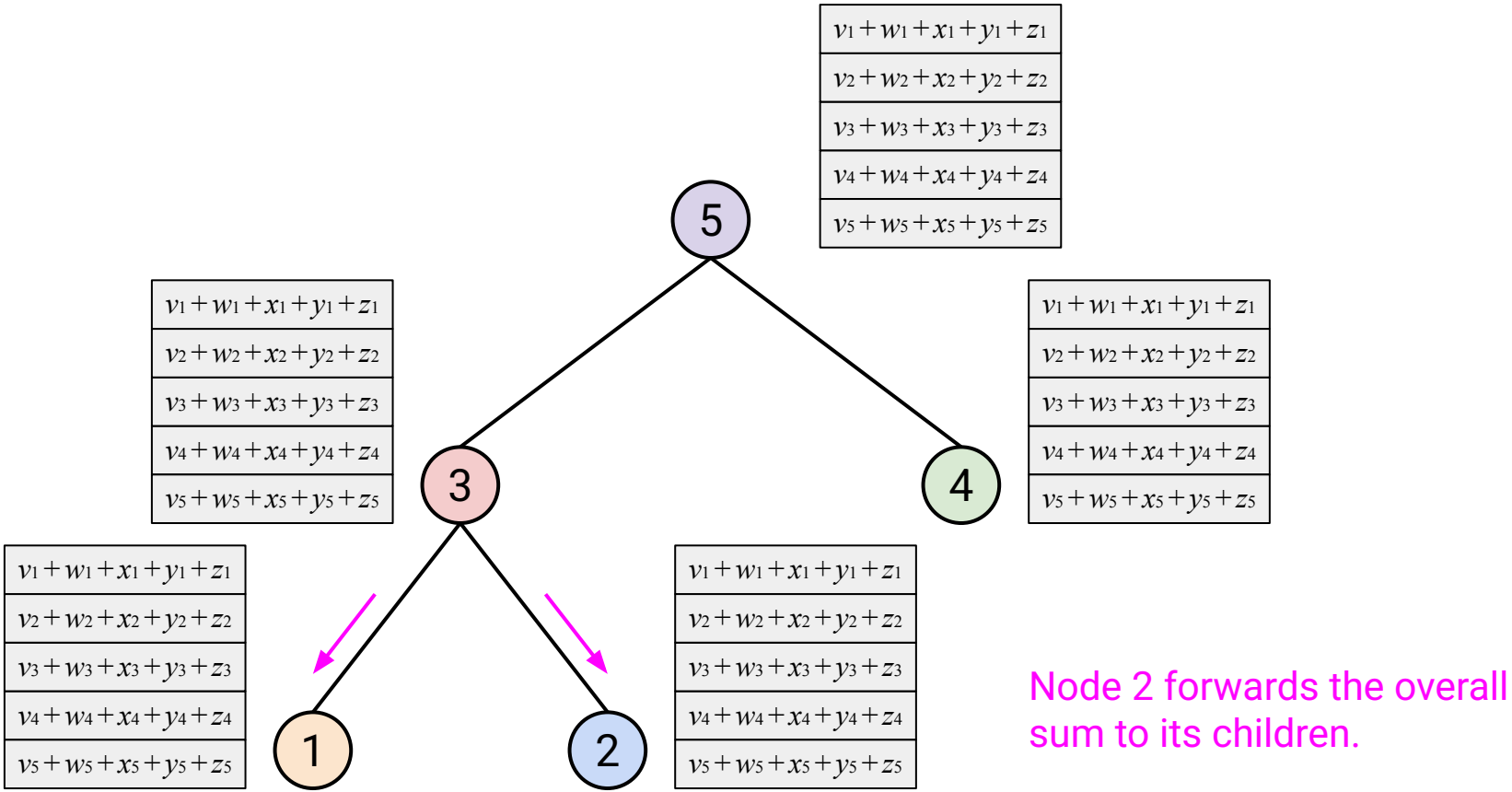
Node 4 computes the sum of all its descendants (plus itself).

2. The overall sum is sent down the tree, starting from the root.



Node 4 forwards the overall sum to its children.

2. The overall sum is sent down the tree, starting from the root.



Tree Topology: Efficiency

Total bandwidth: Each node receives up to 2 vectors (D bytes each) from children. Each node sends 1 vector (D bytes) to parent. p nodes in total.

Number of steps: Tree has p nodes, and height $\log(p)$.

Bandwidth per step: Each node receives up to 2 vectors from children, and sends up to 1 vector to parent. Each vector is D bytes.

	Total Bandwidth	Number of Steps	Bandwidth Per Step
Mesh	Dp^2	1	Dp
Single Root	Dp	2	Dp
Tree	Dp	$\log(p)$	D

Ring-Based AllReduce (Naive)

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- Ring (Optimized)
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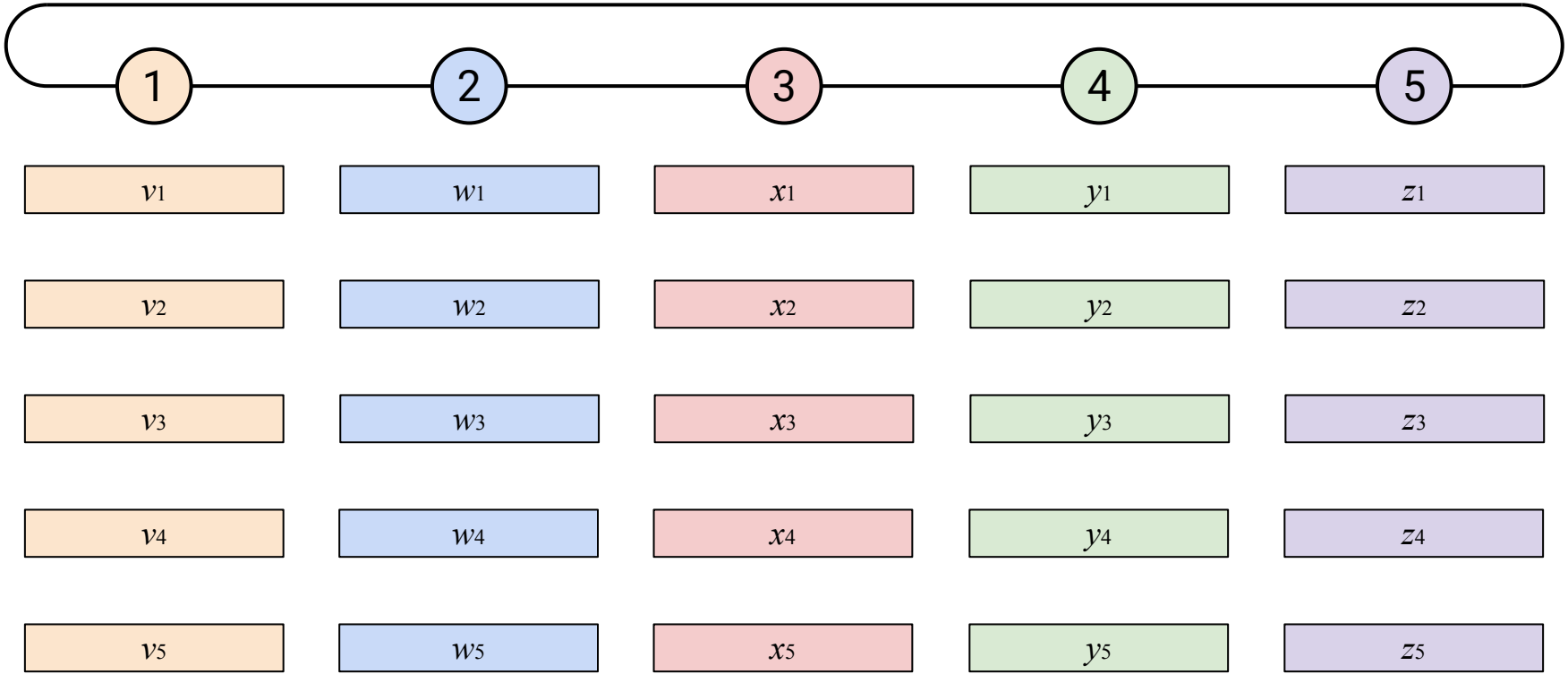
Ring-Based AllReduce (Naive)

1. Send your vector to your left. When you receive a vector, add it to yours.



Ring-Based AllReduce (Naive)

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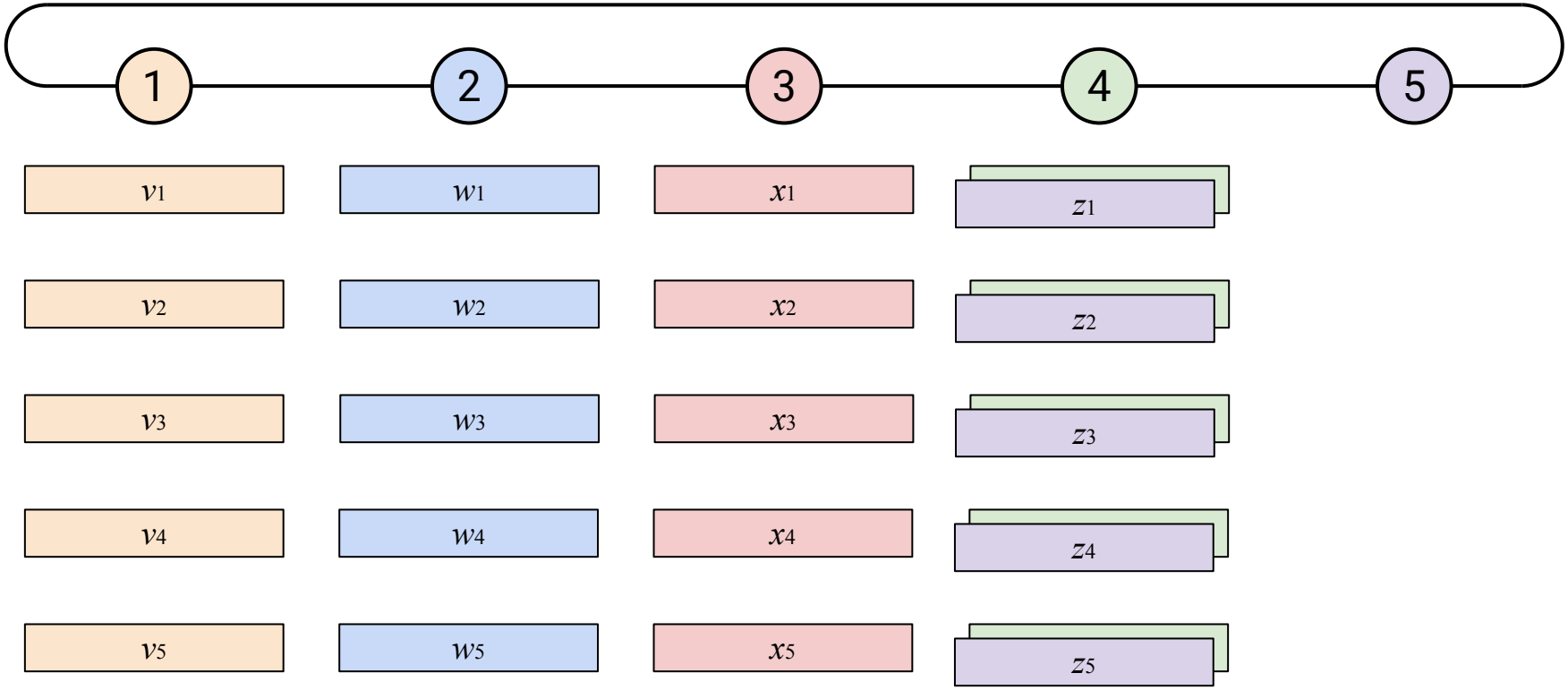

Ring-Based AllReduce (Naive)

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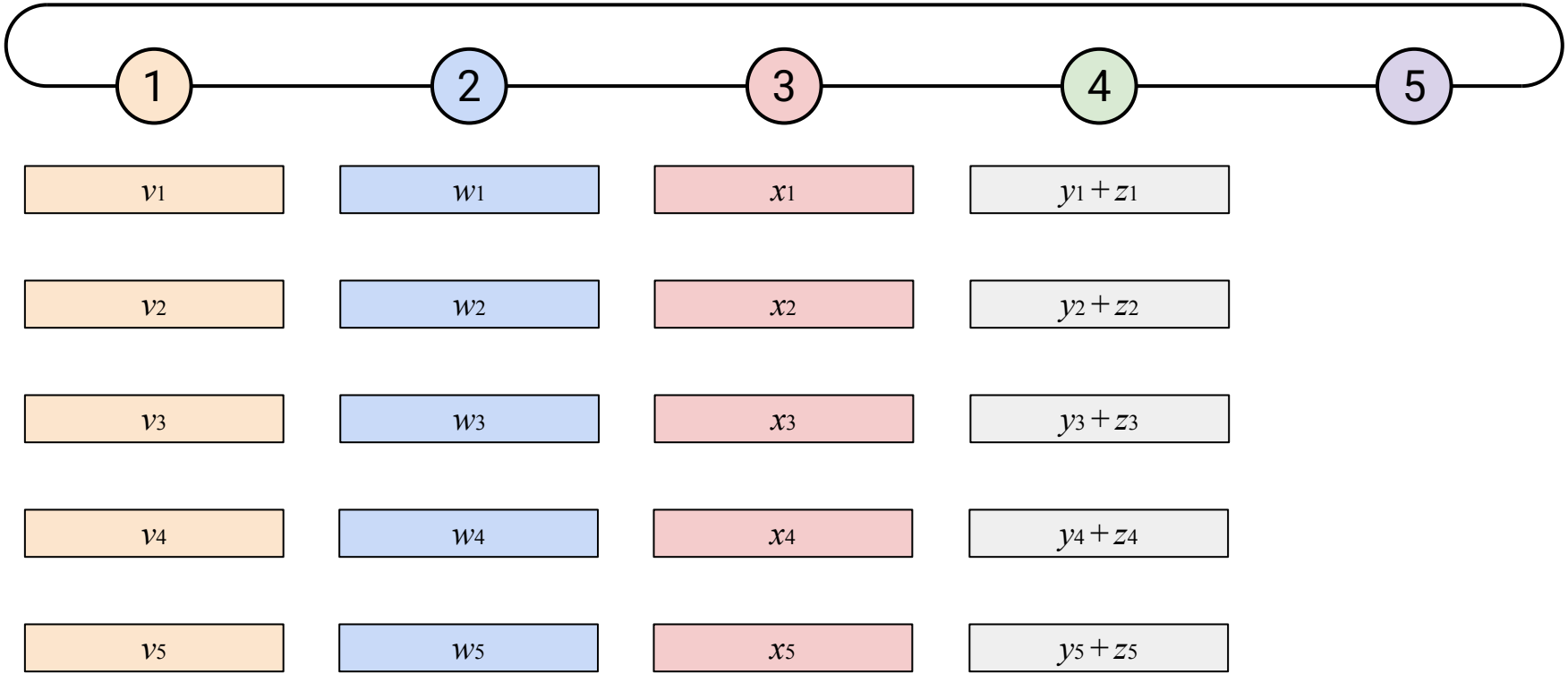

Ring-Based AllReduce (Naive)

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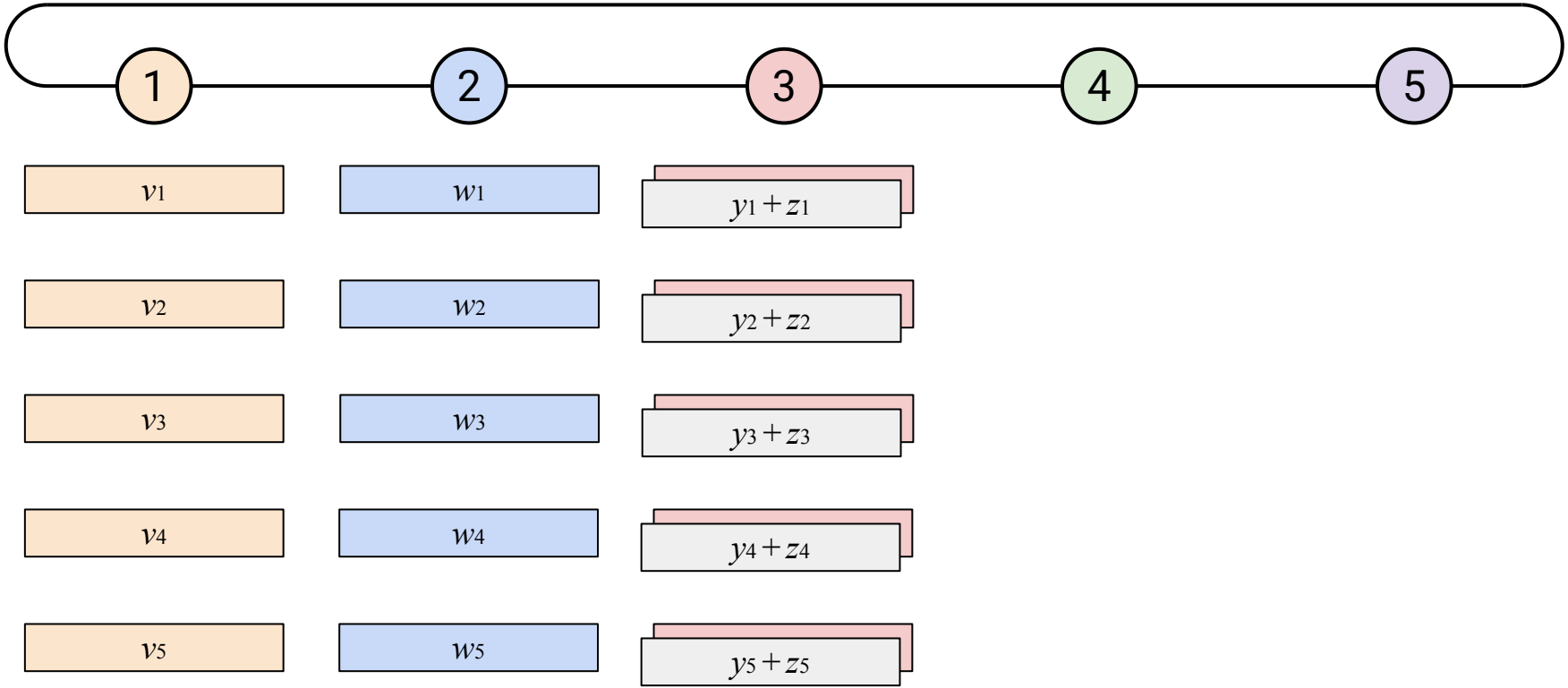

Ring-Based AllReduce (Naive)

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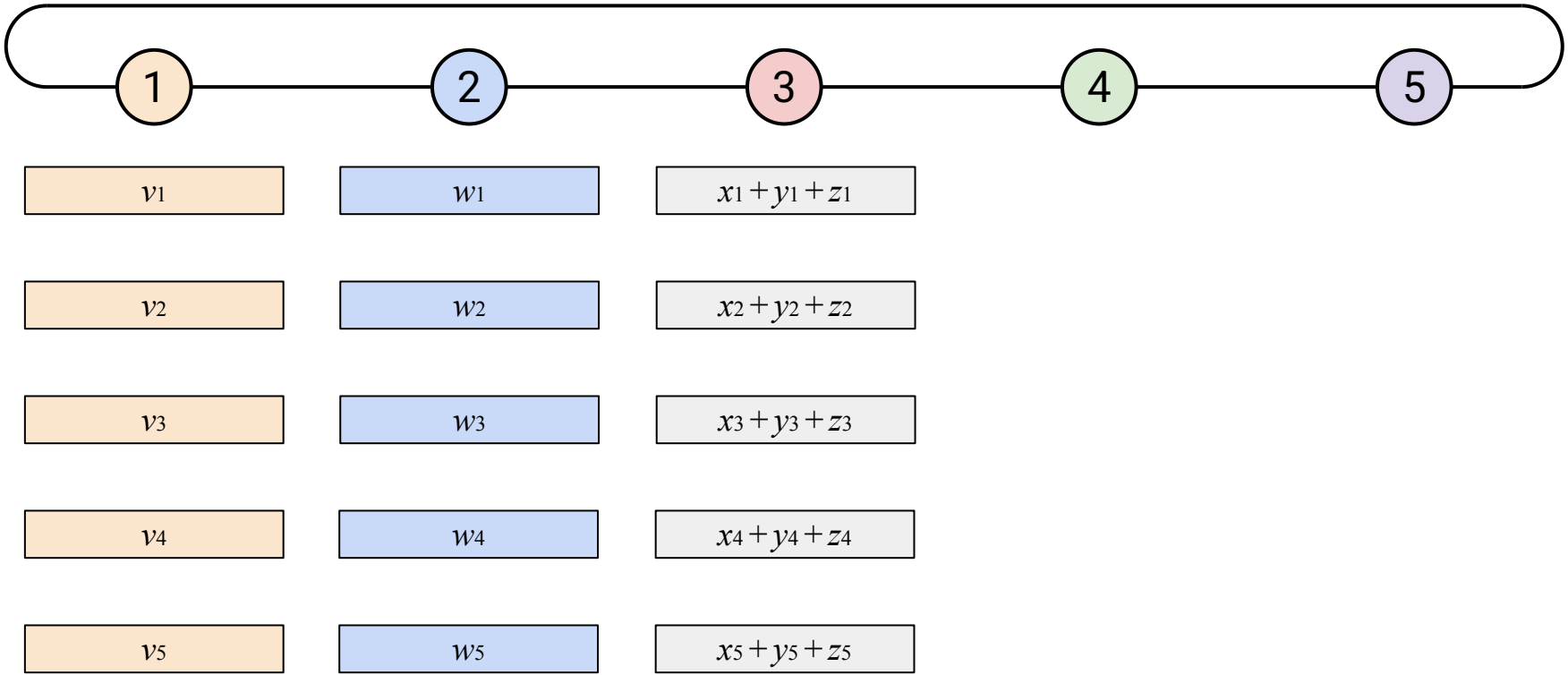

Ring-Based AllReduce (Naive)

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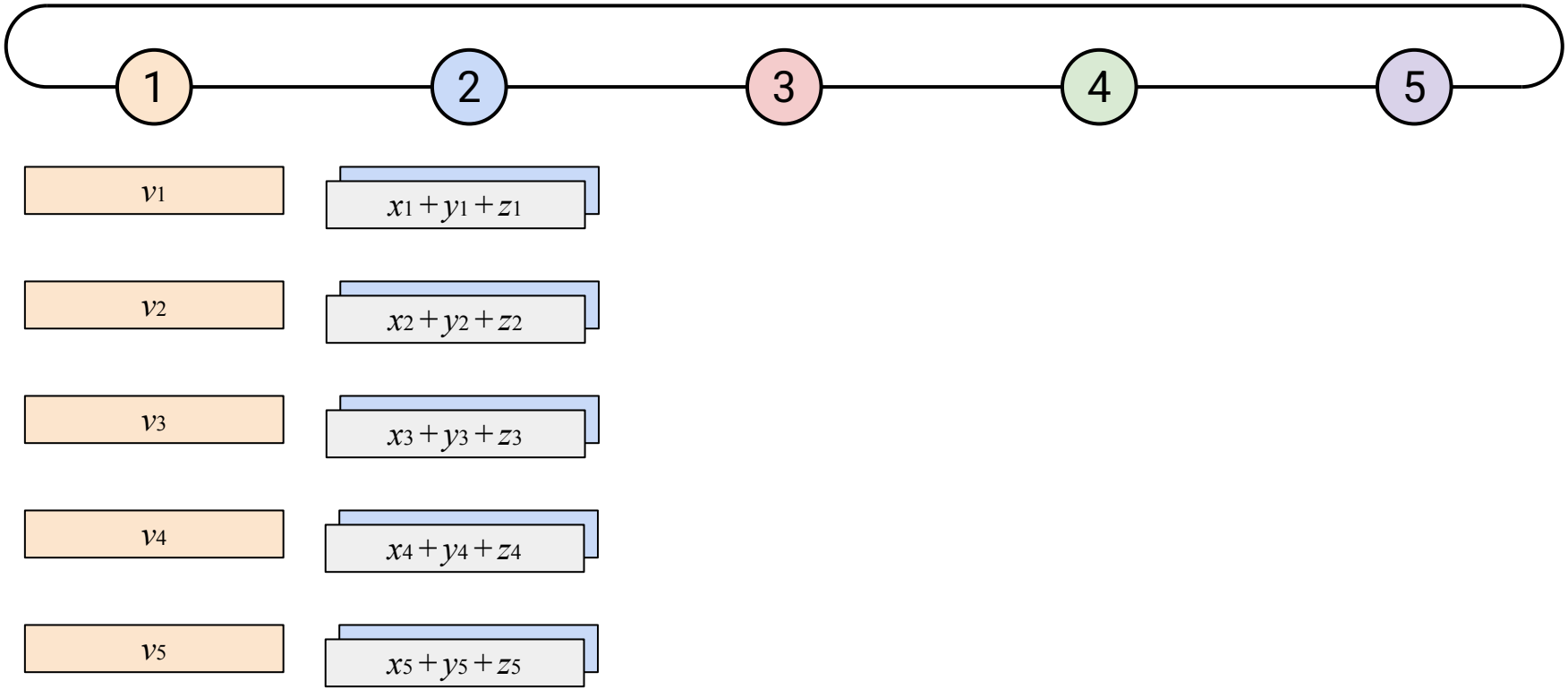
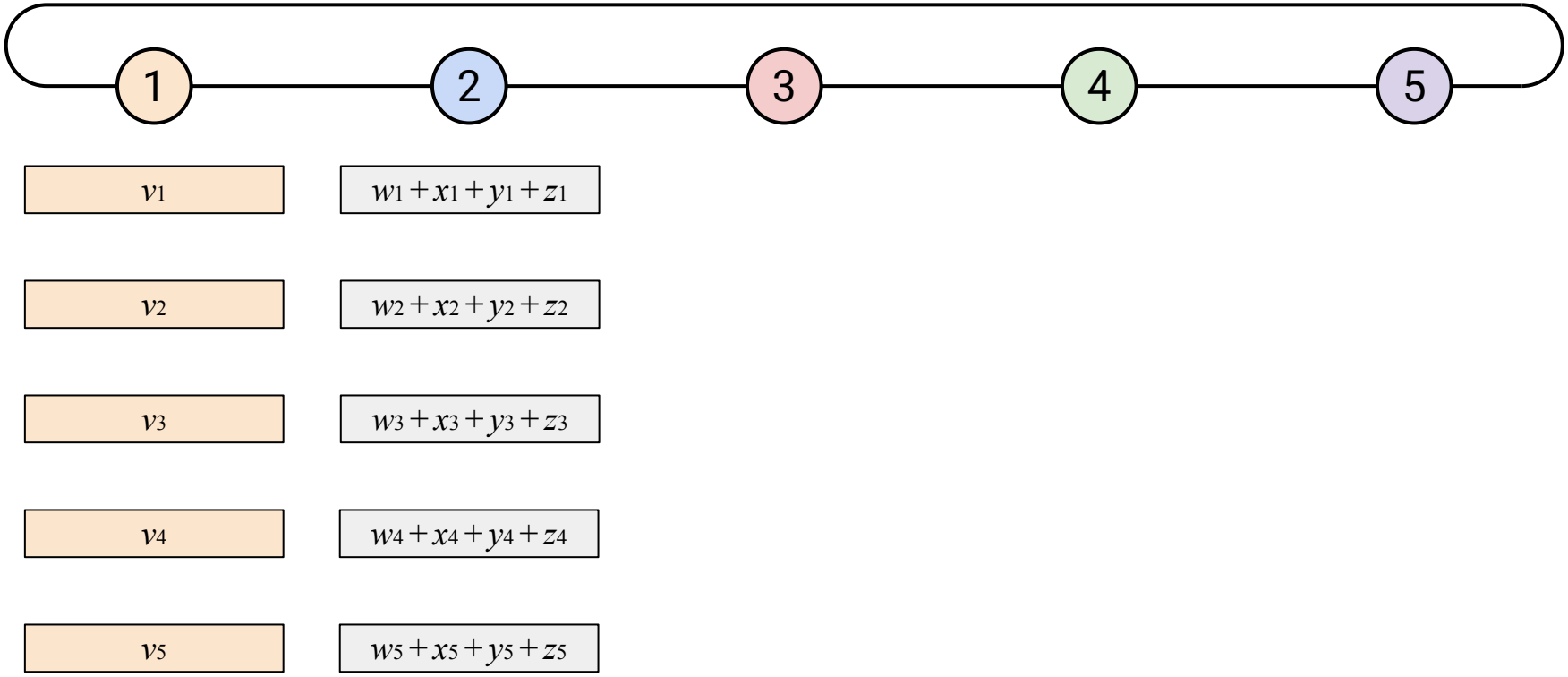


Ring-Based AllReduce (Naive)

1. Send your vector to your left. When you receive a vector, add it to yours.



Ring-Based AllReduce (Naive)

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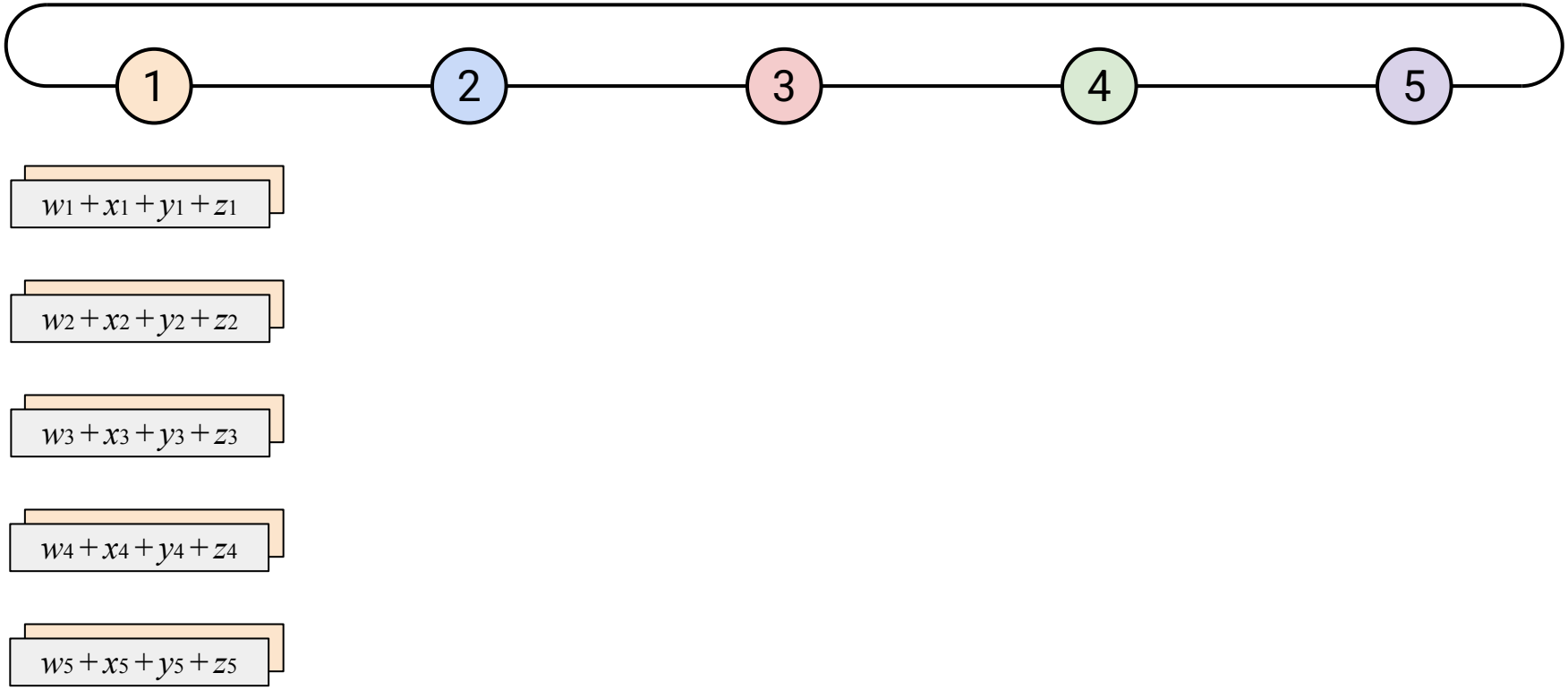

Ring-Based AllReduce (Naive)

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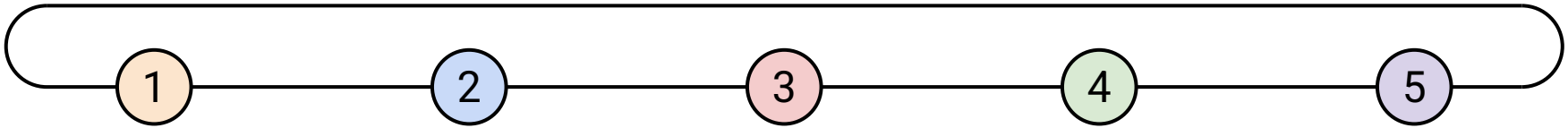


$v_1 + w_1 + x_1 + y_1 + z_1$

$v_2 + w_2 + x_2 + y_2 + z_2$

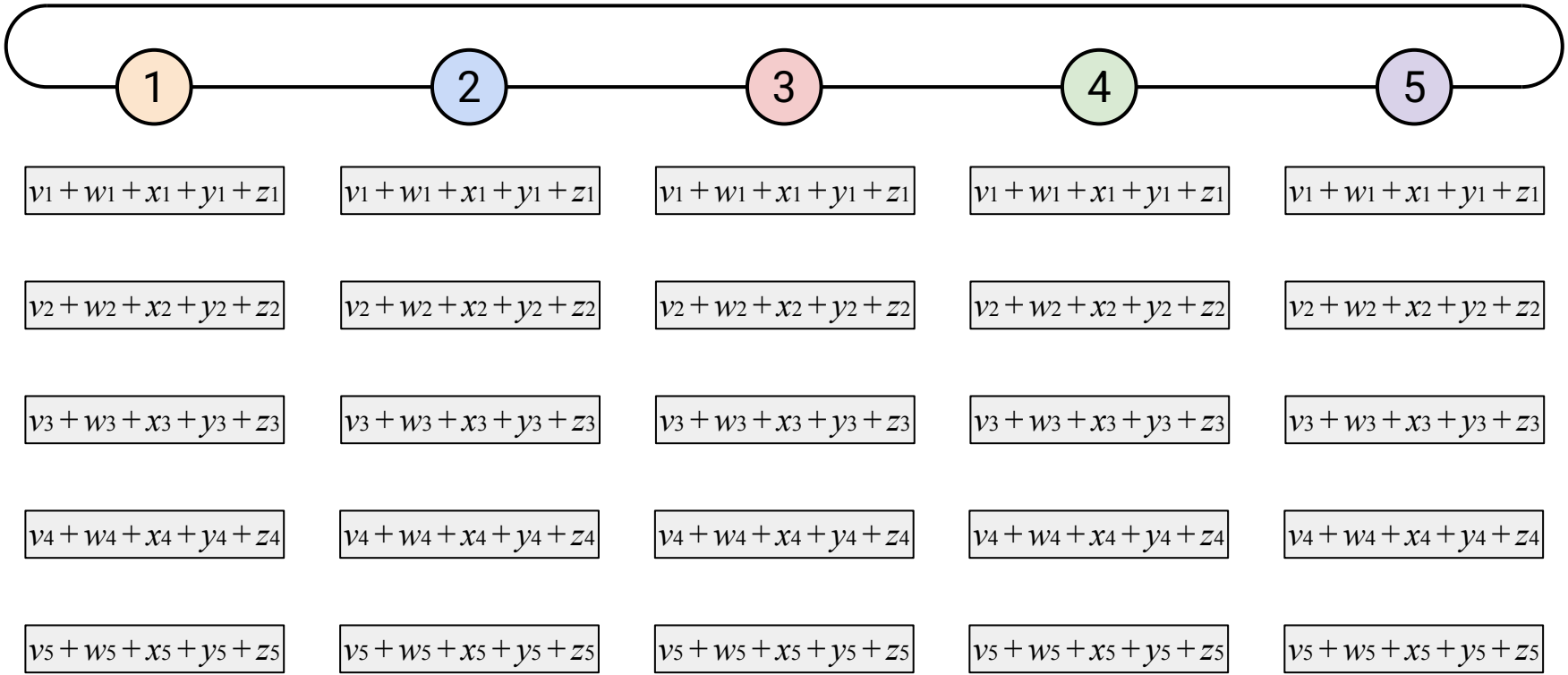
$v_3 + w_3 + x_3 + y_3 + z_3$

$v_4 + w_4 + x_4 + y_4 + z_4$

$v_5 + w_5 + x_5 + y_5 + z_5$

Ring-Based AllReduce (Naive)

2. Send the overall sum to your left, so that everyone gets a copy of it.



Naive Ring Topology: Efficiency

Total bandwidth: Each node receives 1 vector (D bytes) from the right.
Each node sends 1 vector (D bytes) to the left. p nodes in total.

Number of steps: Vector must travel around all p nodes in the ring.

Bandwidth per step: Each node receives 1 vector from the right.
Each node sends 1 vector to the left. Each vector is D bytes.

	Total Bandwidth	Number of Steps	Bandwidth Per Step
Mesh	Dp^2	1	Dp
Single Root	Dp	2	Dp
Tree	Dp	$\log(p)$	D
Ring (Naive)	Dp	p	D

Ring-Based AllReduce (Optimized)

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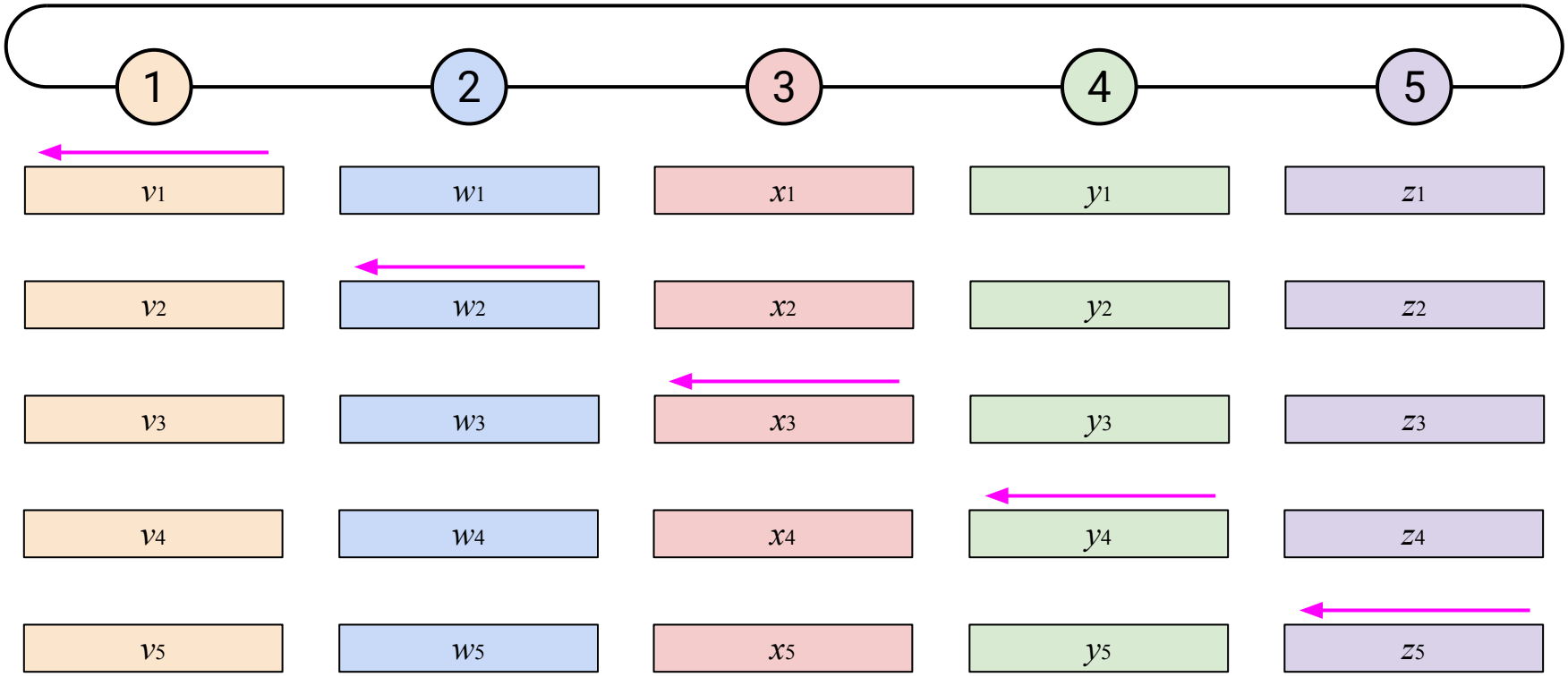
- **Ring (Optimized)**
- Overlay/Underlay Topologies

Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



Ring-Based AllReduce (Optimized)

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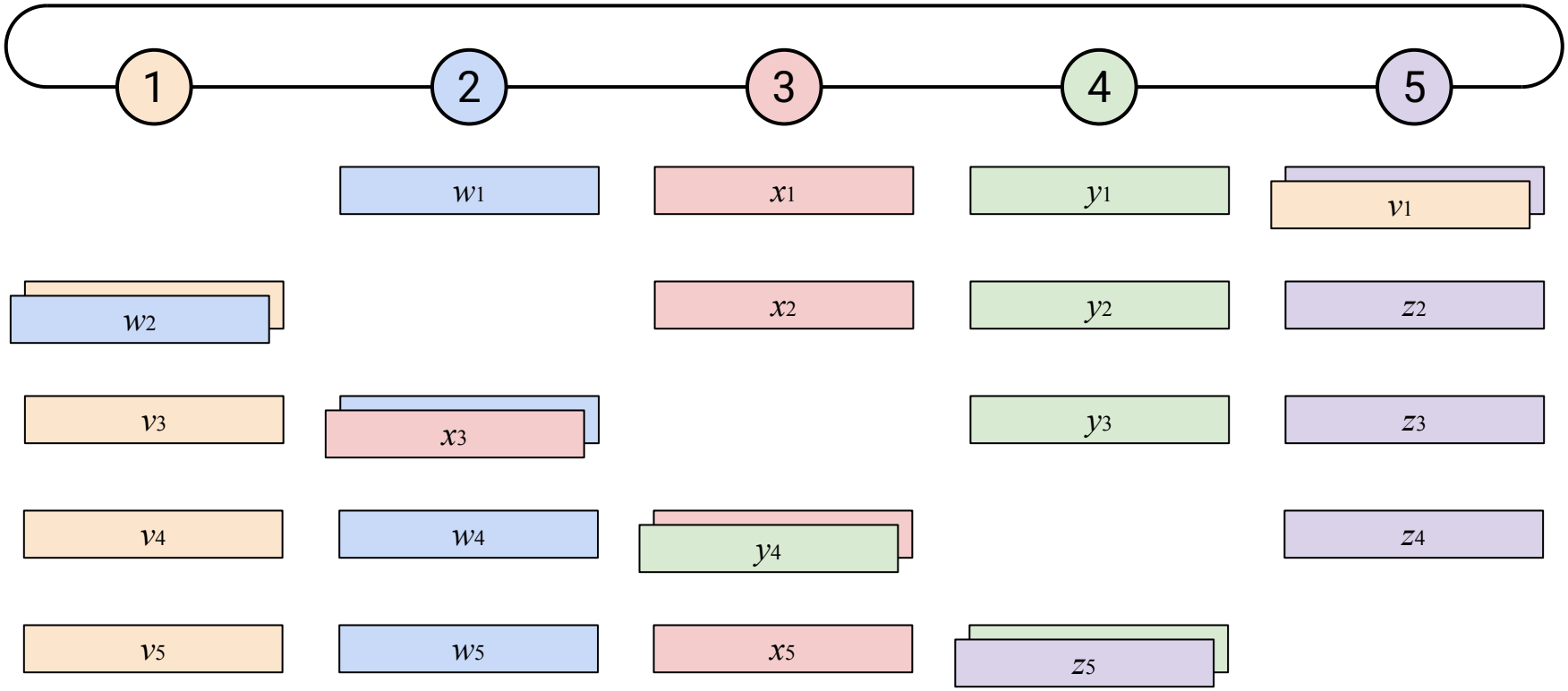

Ring-Based AllReduce (Optimized)

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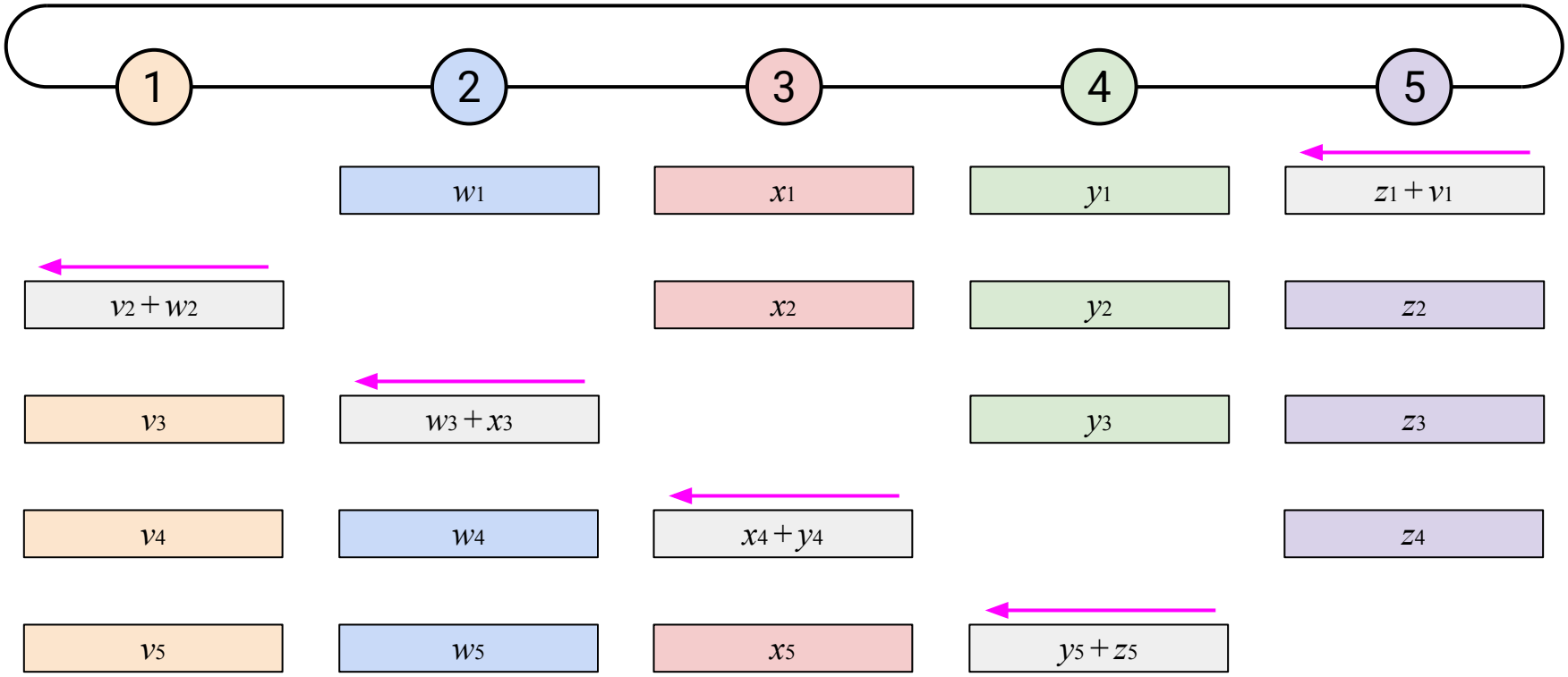

Ring-Based AllReduce (Optimized)

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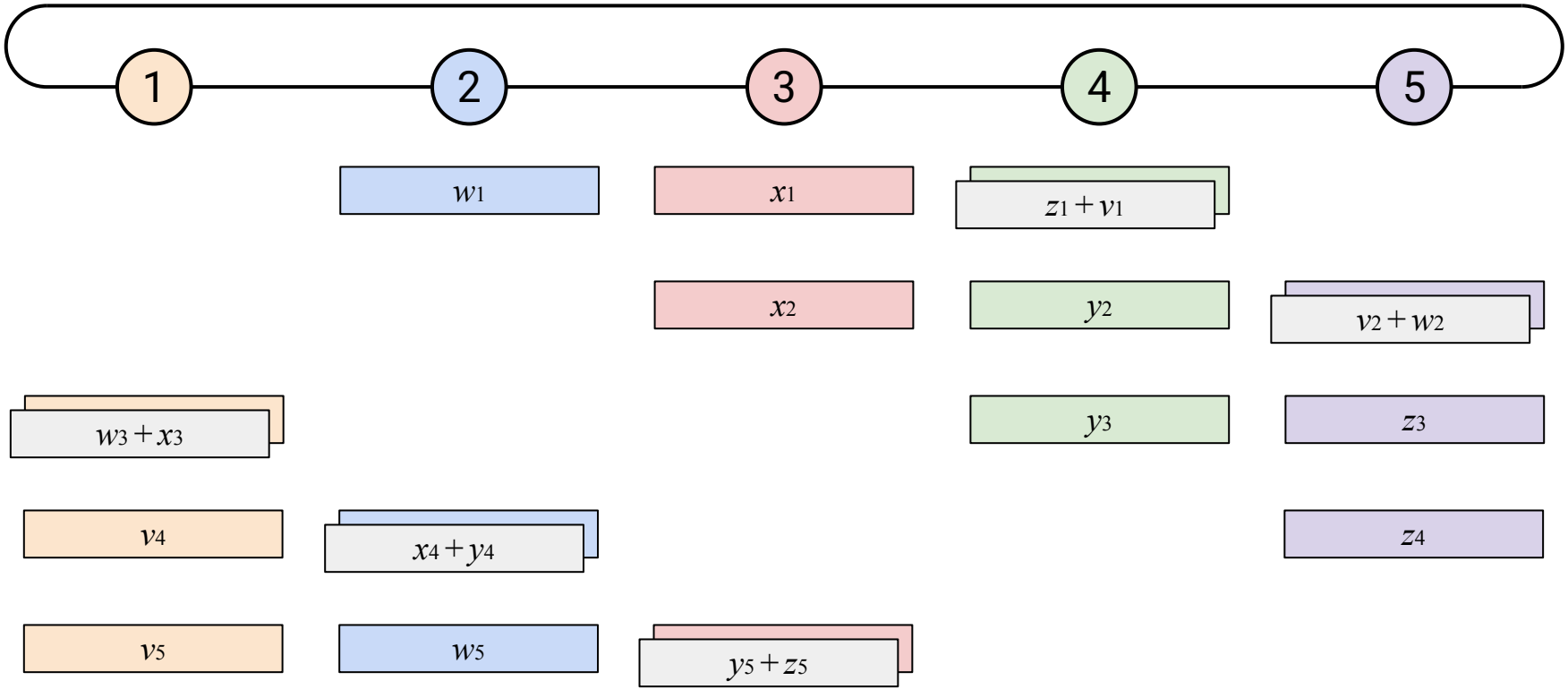
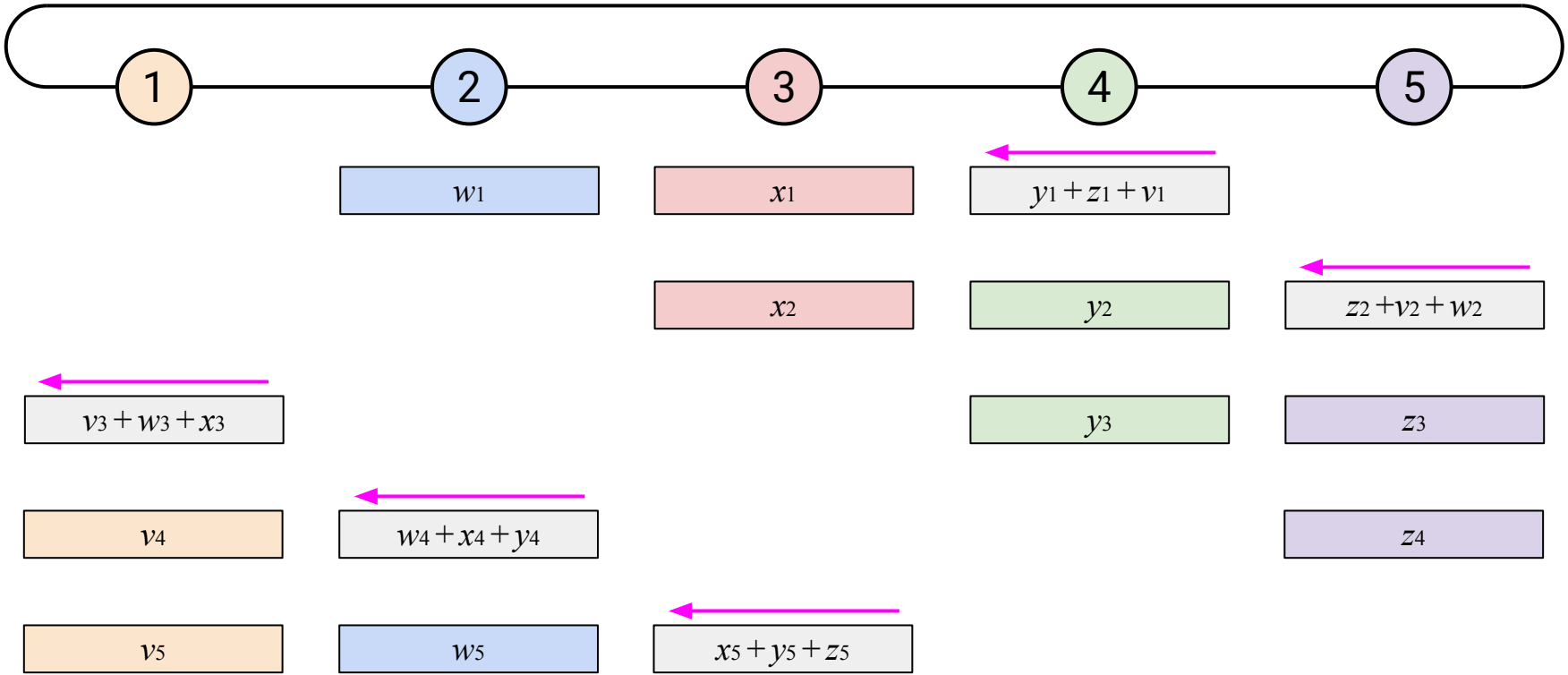


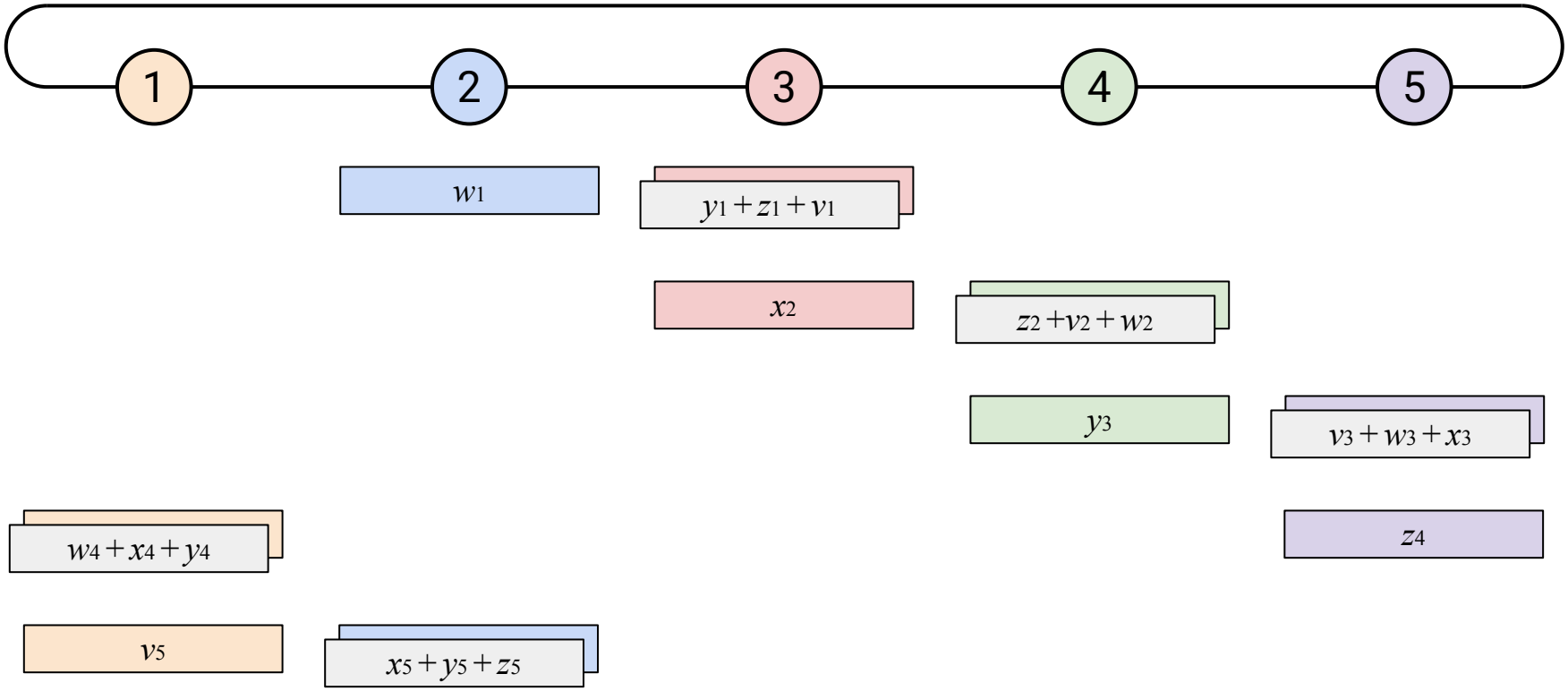
Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



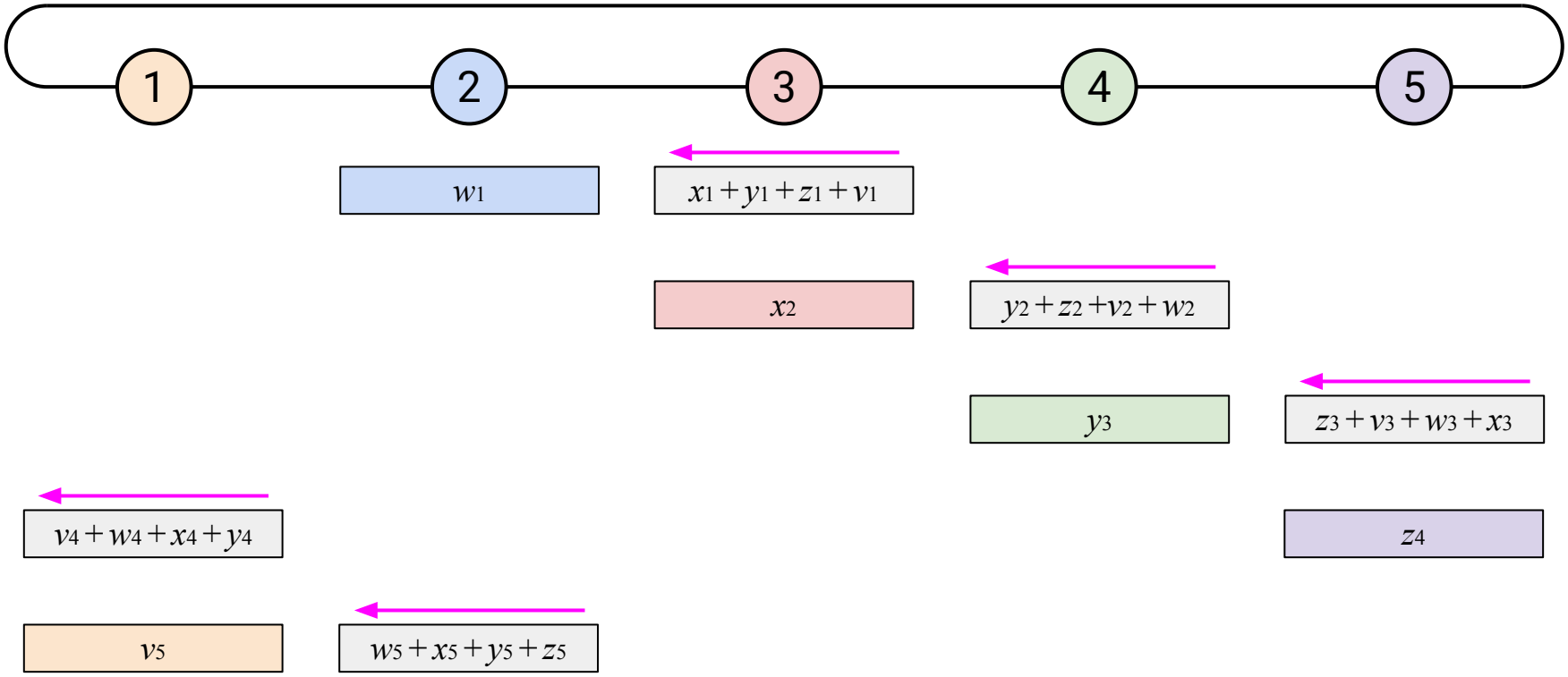
Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



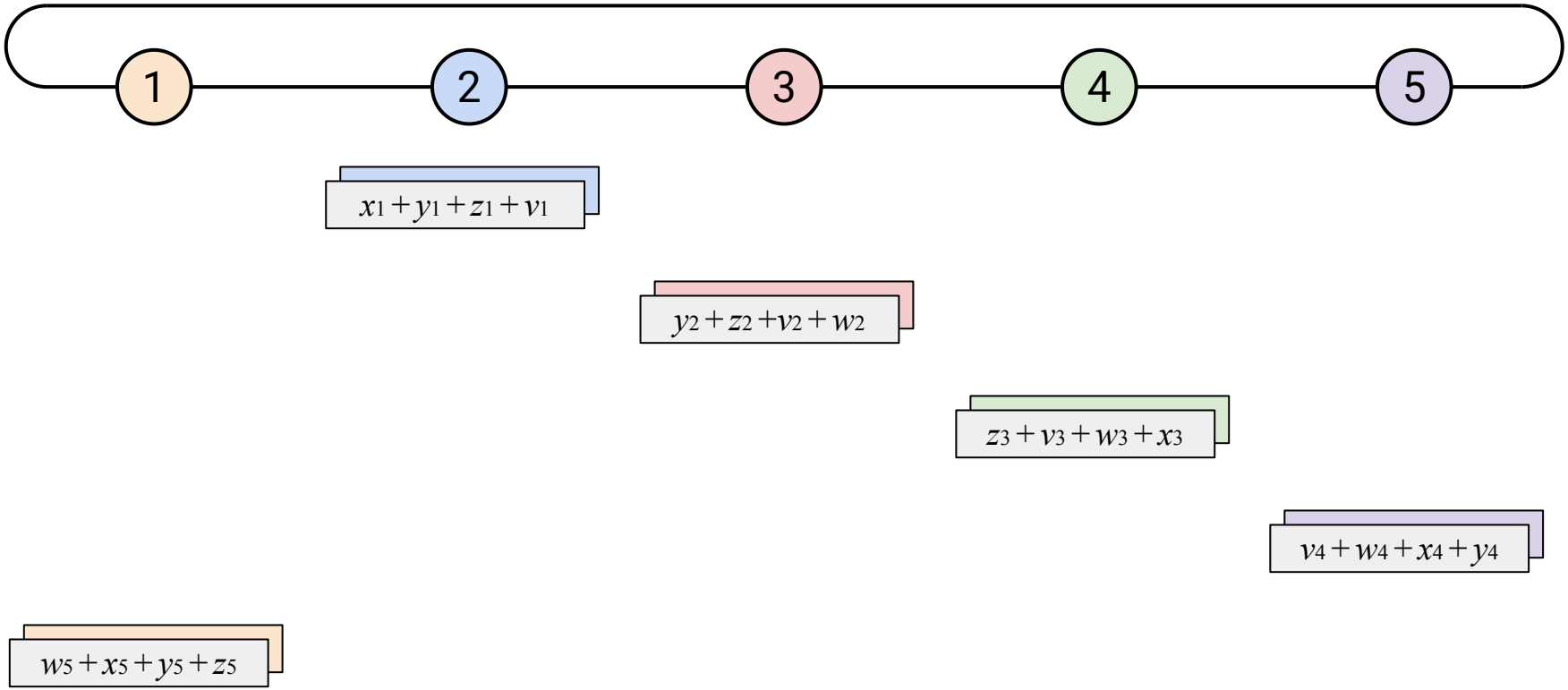
Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



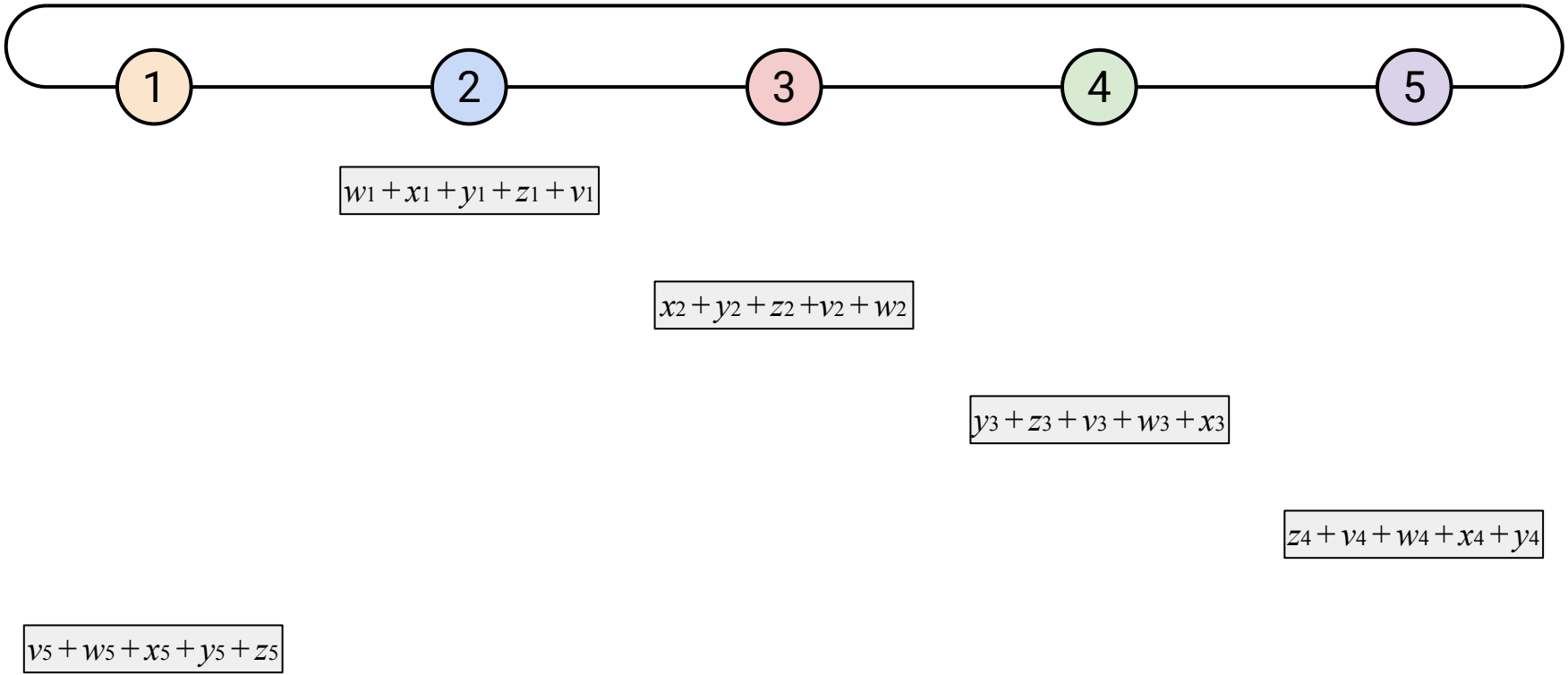
Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



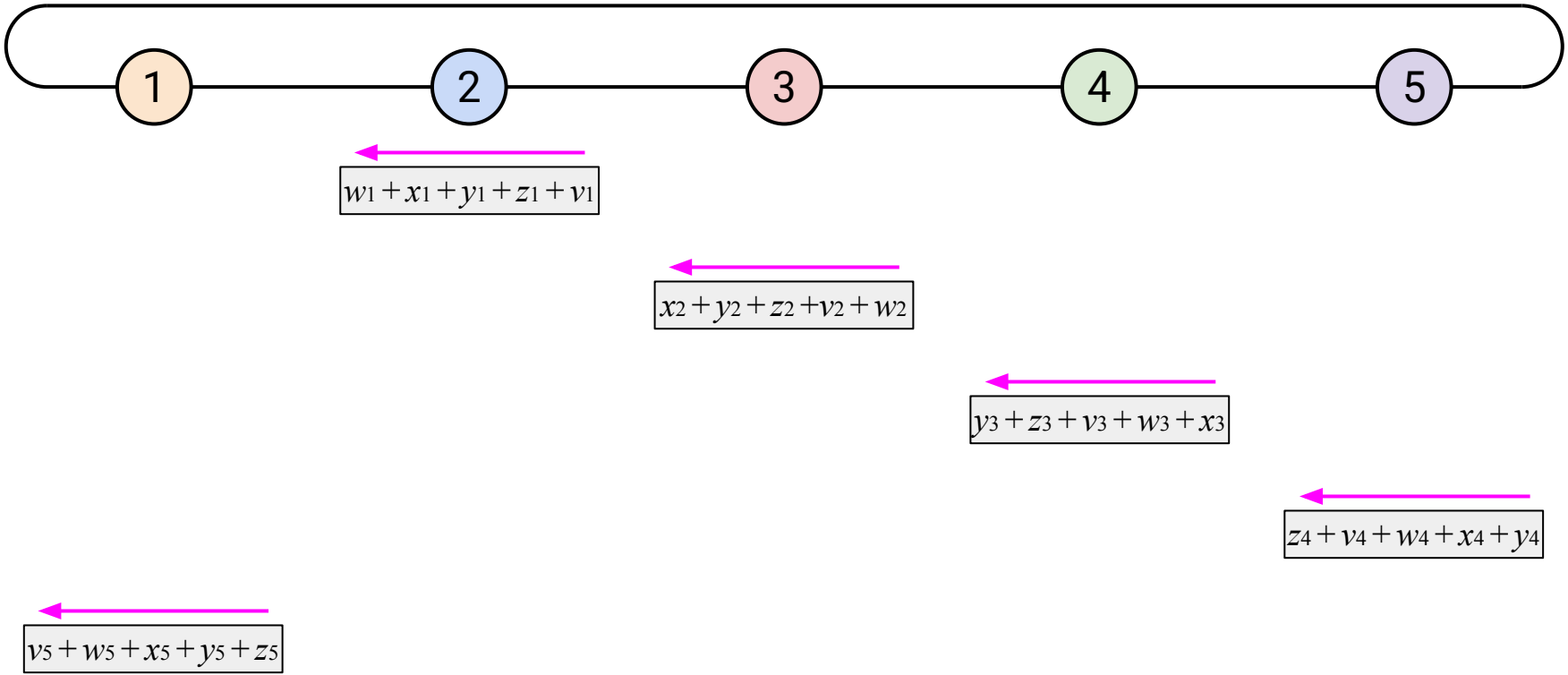
Ring-Based AllReduce (Optimized)

1. Send your vector to your left, one element at a time (i.e. delaying later elements).



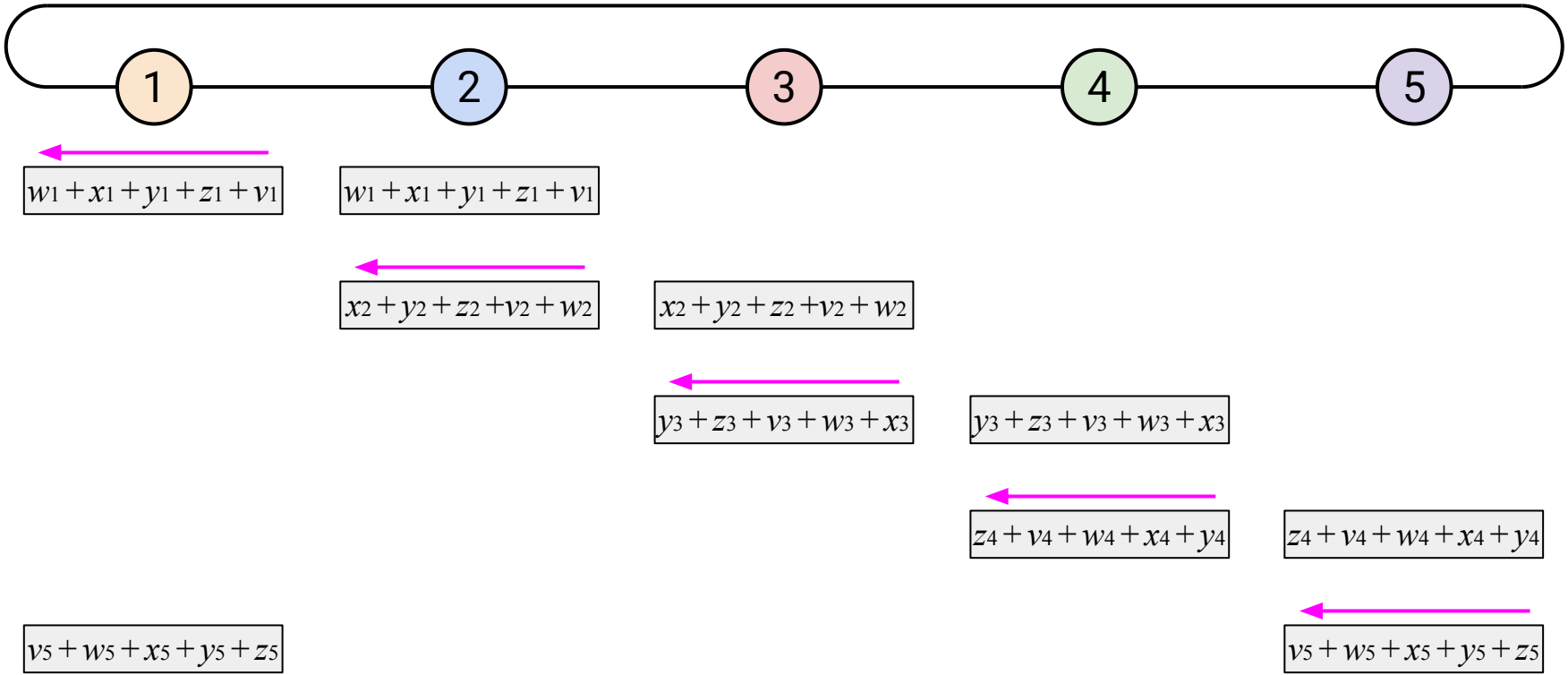
Ring-Based AllReduce (Optimized)

2. Send the overall sum to the left.



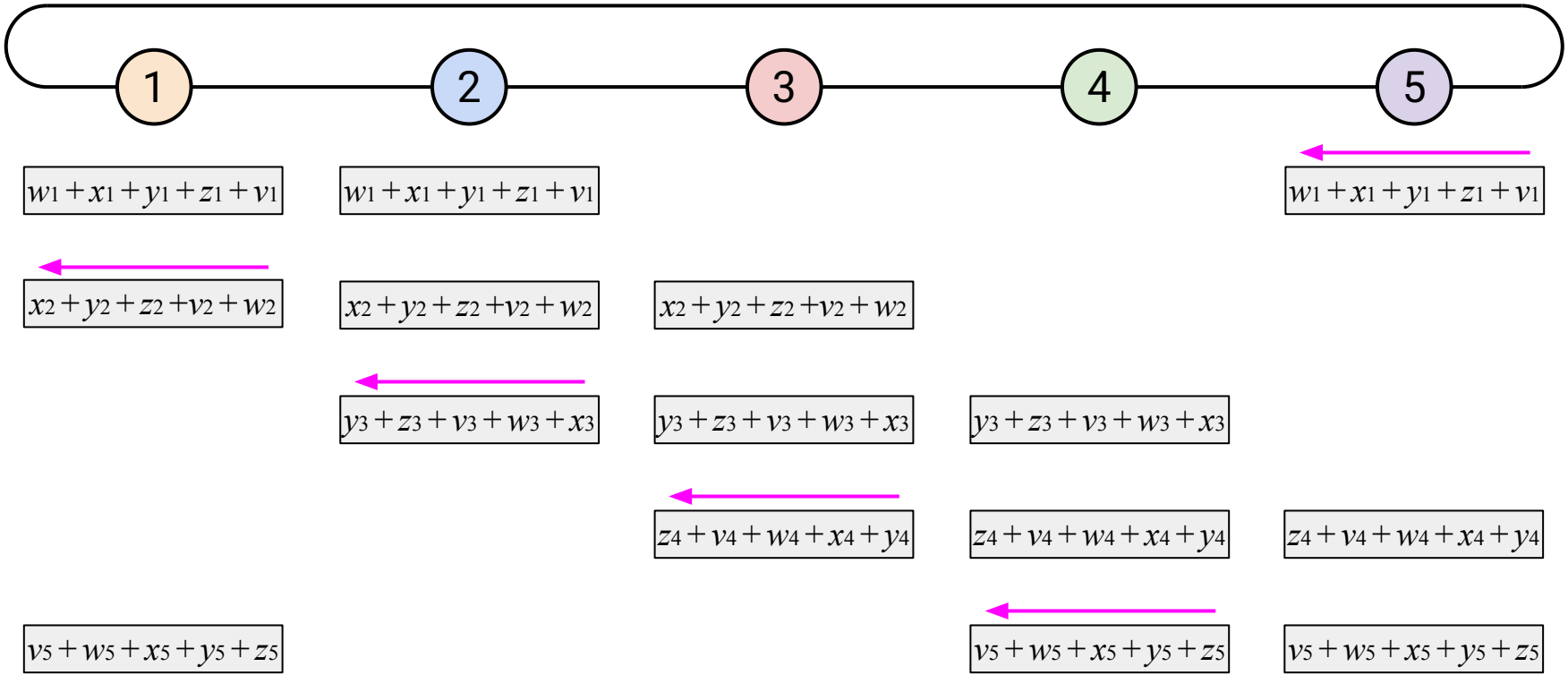
Ring-Based AllReduce (Optimized)

2. Send the overall sum to the left.



Ring-Based AllReduce (Optimized)

2. Send the overall sum to the left.



Ring-Based AllReduce (Optimized)

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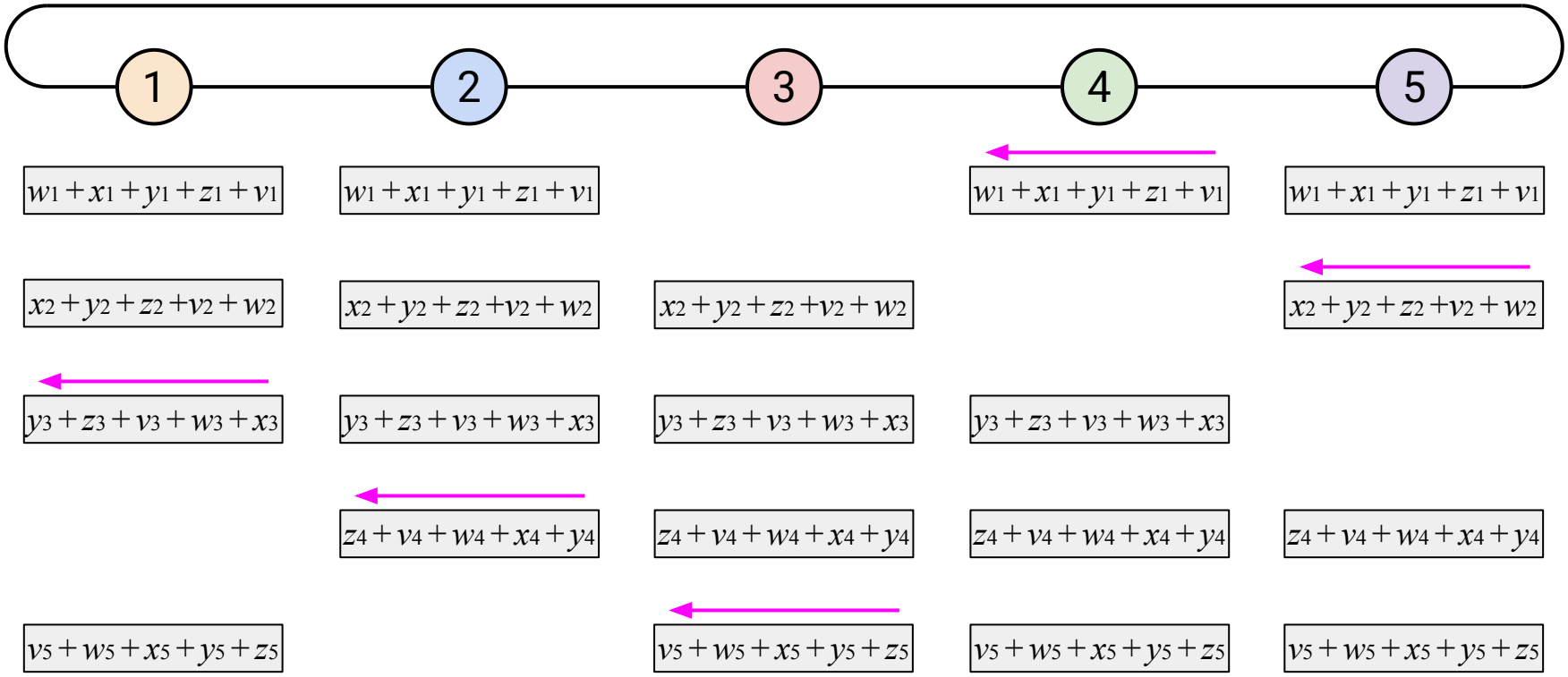
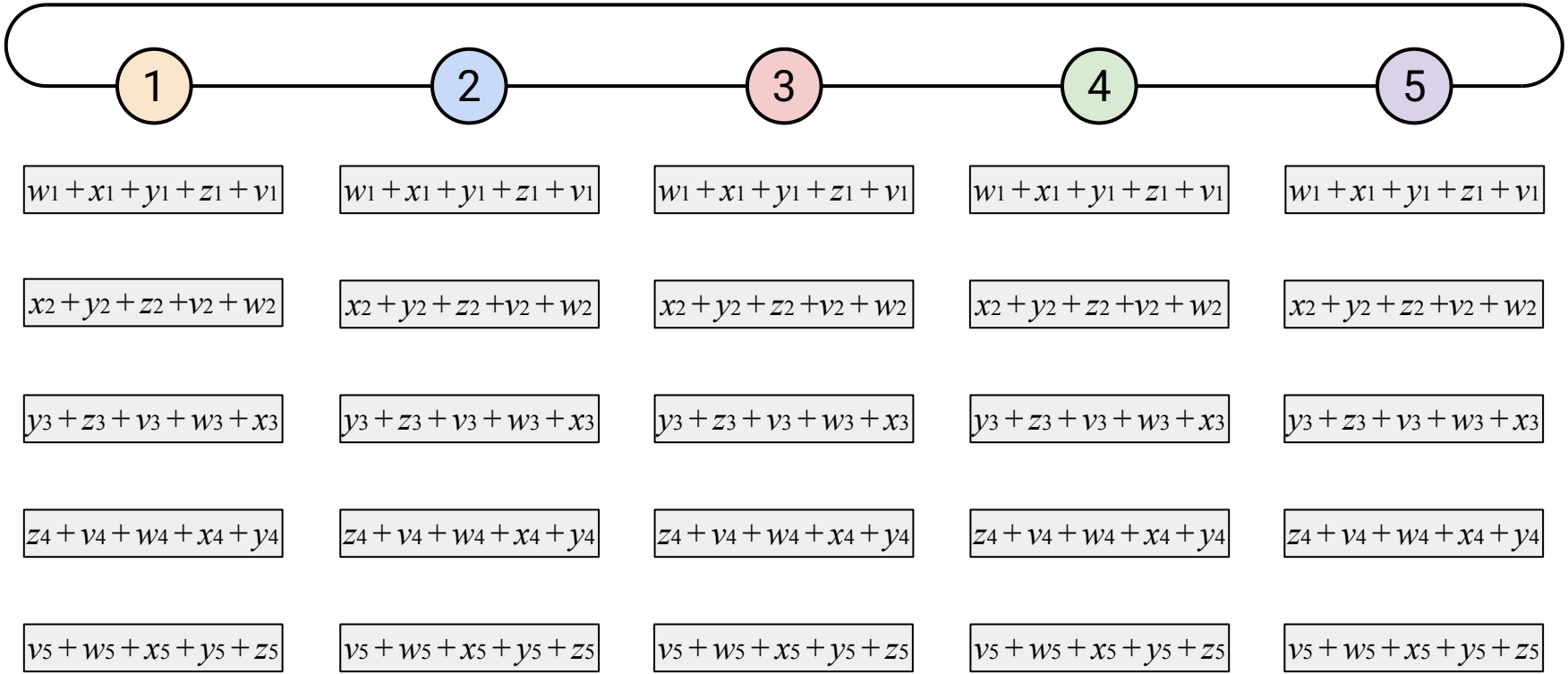


Ring-Based AllReduce (Optimized)

2. Send the overall sum to the left.



Naive Ring Topology: Efficiency

Total bandwidth: Each node receives 1 vector (D bytes) from the right.
Each node sends 1 vector (D bytes) to the left. p nodes in total.

Number of steps: Vector must travel around all p nodes in the ring.

Bandwidth per step: Each node receives 1 element (D/p bytes) from the right.
Each node sends 1 element (D/p bytes) to the left.

	Total Bandwidth	Number of Steps	Bandwidth Per Step
Mesh	Dp^2	1	Dp
Single Root	Dp	2	Dp
Tree	Dp	$\log(p)$	D
Ring (Naive)	Dp	p	D
Ring (Optimized)	Dp	p	D/p

Overlay/Underlay Topologies

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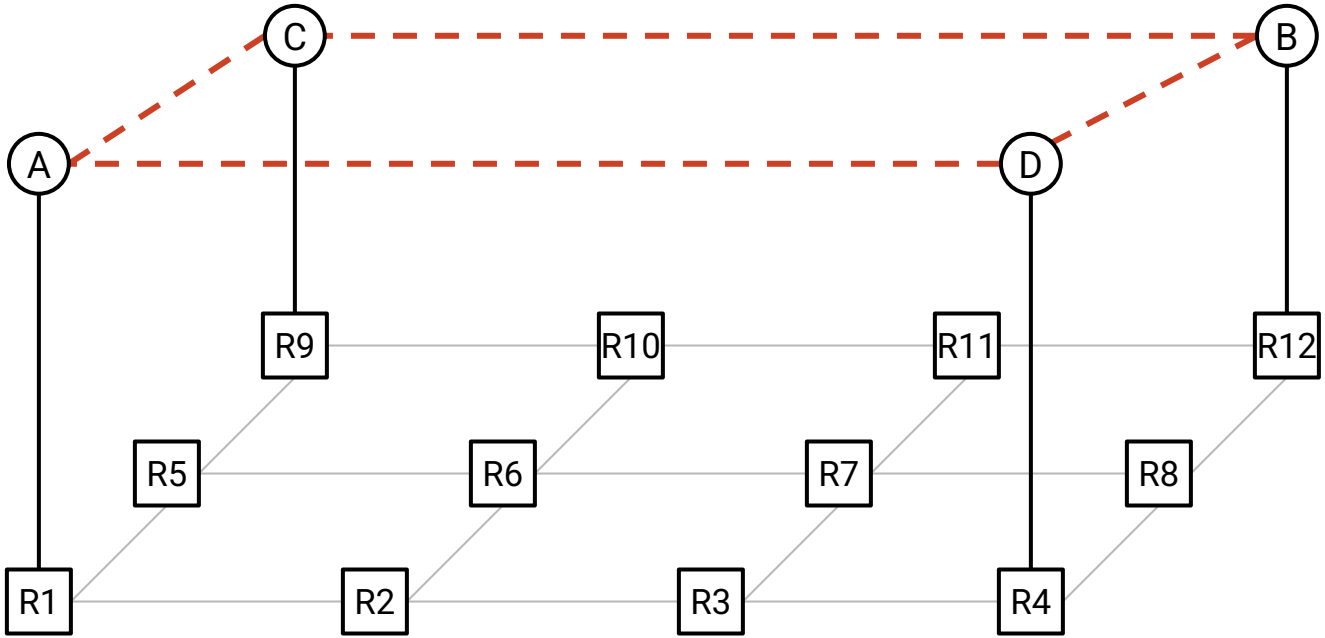
Overlay and Underlay Topologies

Suppose Hosts A, B, C, D want to run an AllReduce operation.

A, B, C, D are not in a ring topology. But, we can add virtual links to create a ring!



Different Overlay Topologies

Suppose Hosts A, B, C, D want to run an AllReduce operation.

For best performance, in what order should we connect the hosts in a ring?

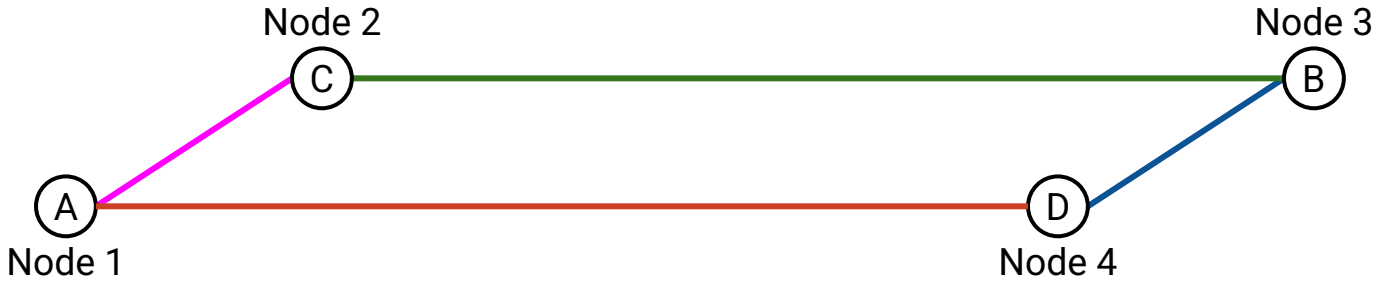
(The way we number the nodes affects the ring we draw.)

Overlay Topology 1:

- Node 1 = A
- Node 2 = C
- Node 3 = D
- Node 4 = B

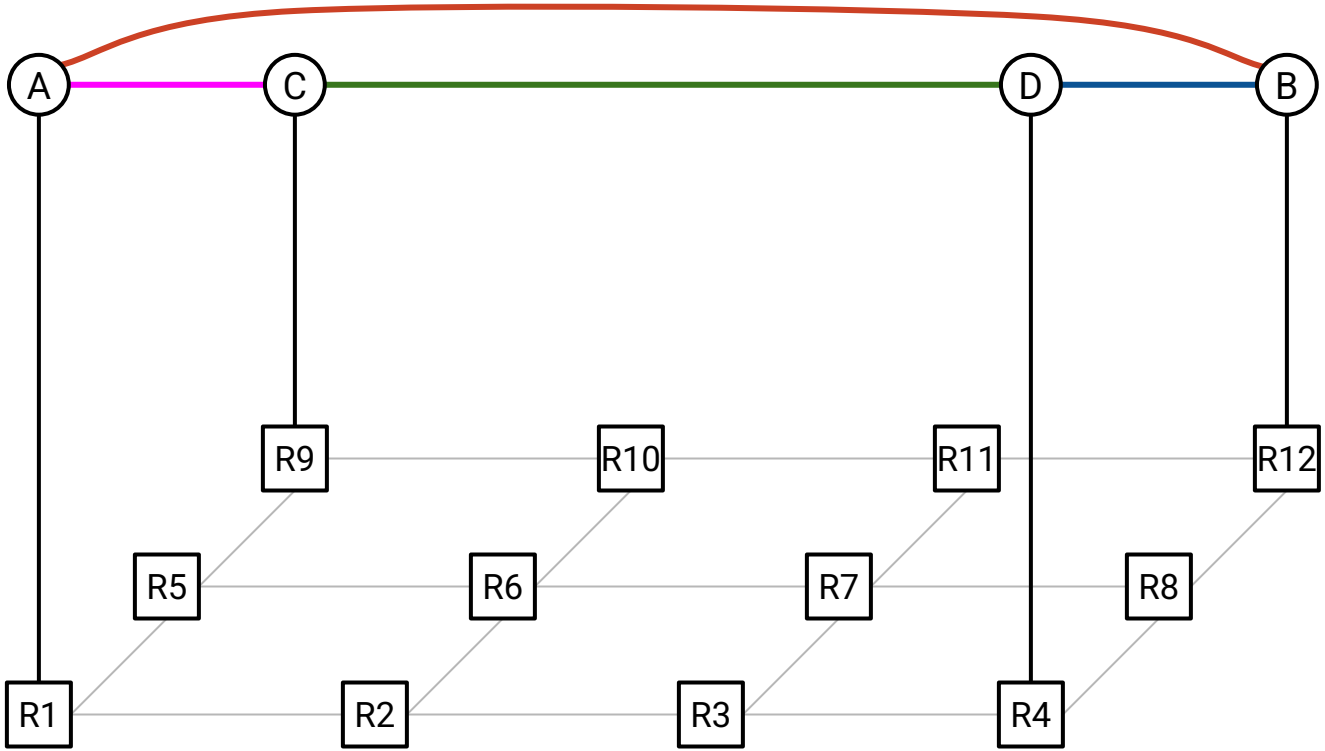
Overlay Topology 2:

- Node 1 = A
- Node 2 = C
- Node 3 = B
- Node 4 = D



Overlay Ring Topology #1

The average stretch factor depends on how the nodes are numbered.



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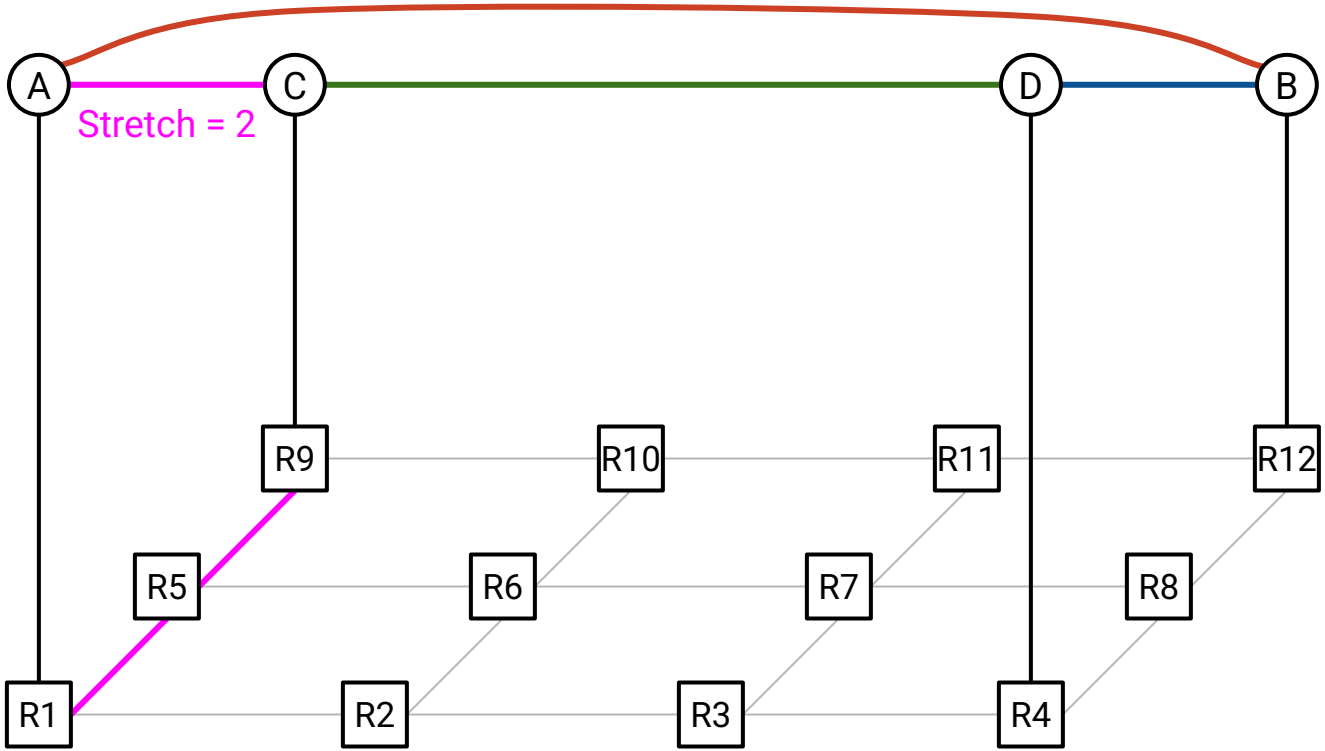

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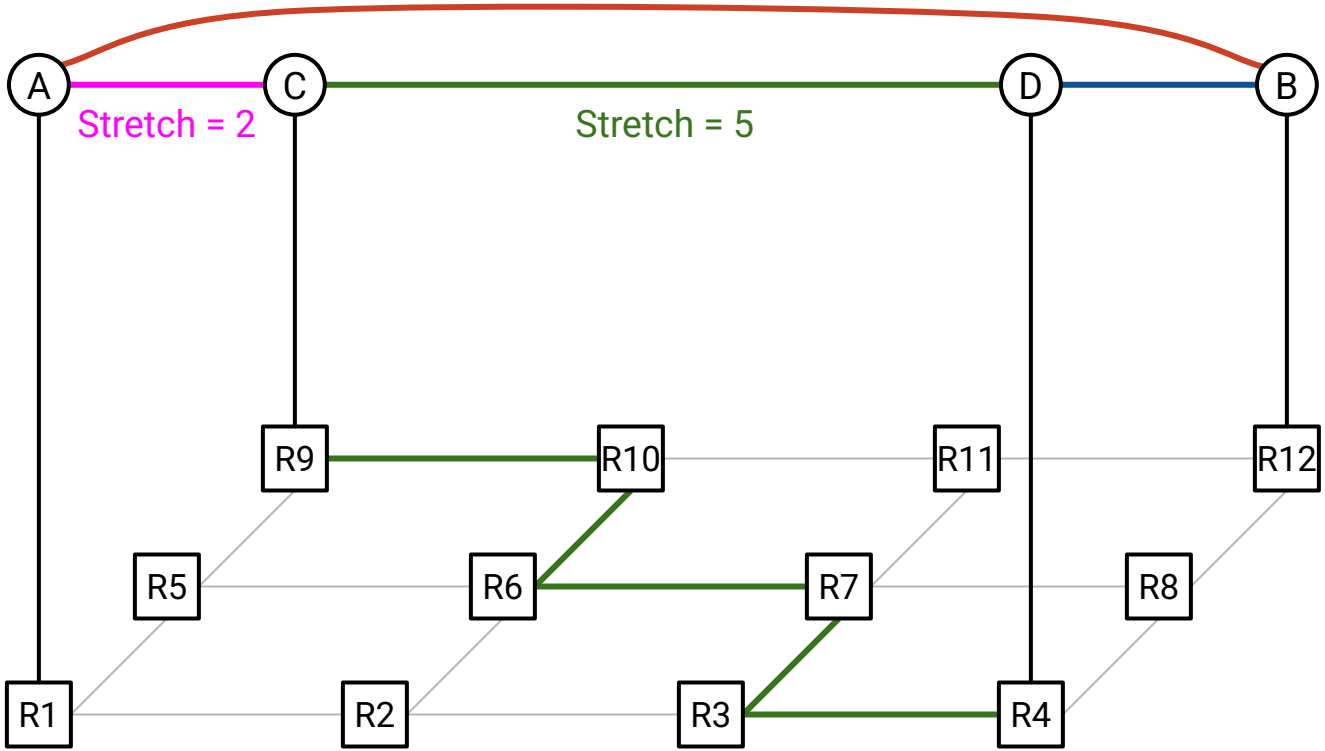

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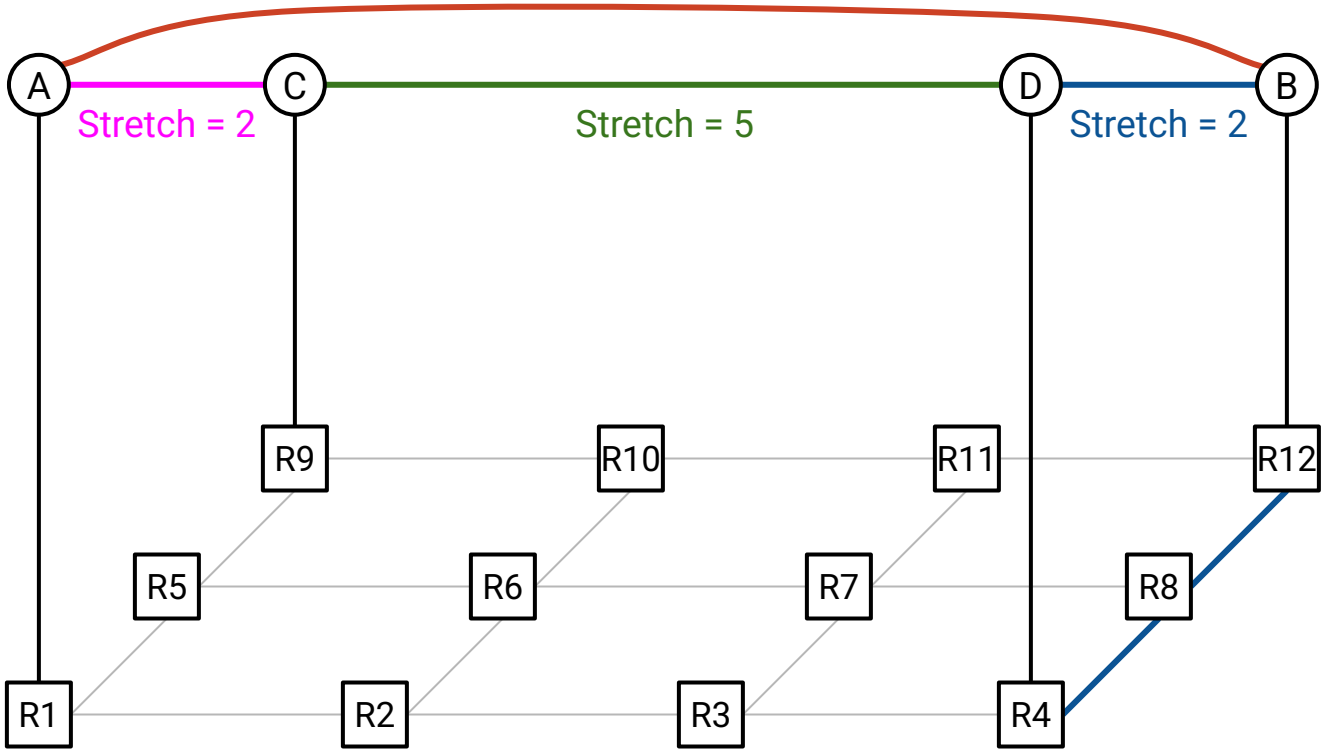

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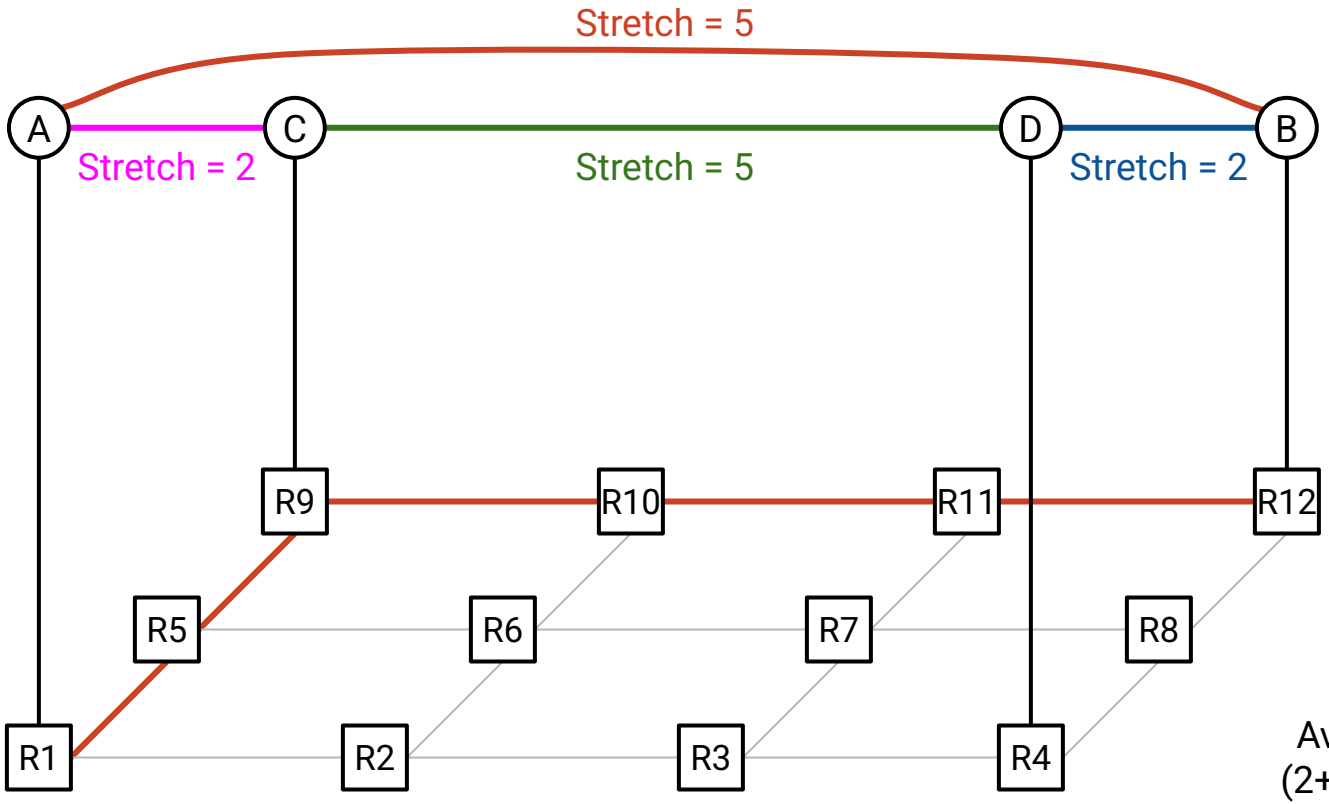
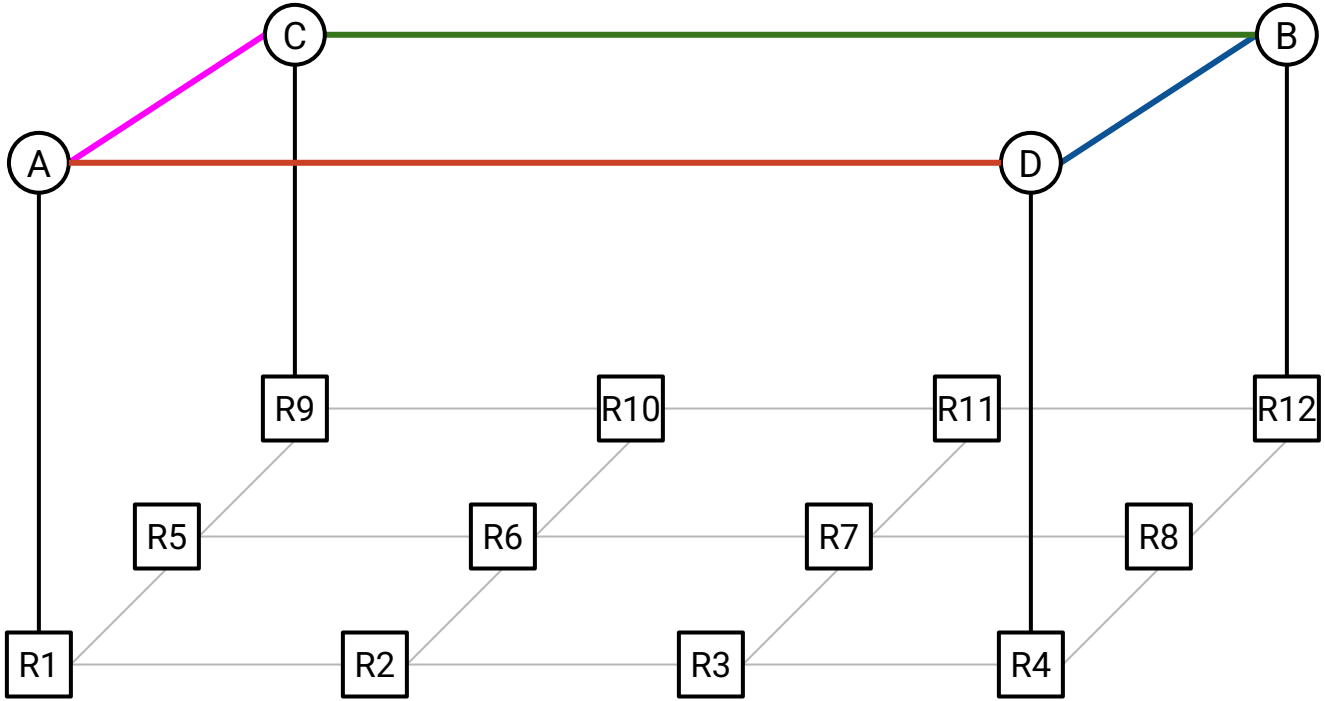


Average stretch:
(2+5+2+5)/4 = 3.5

Overlay Ring Topology #2

The average stretch factor depends on how the nodes are numbered.



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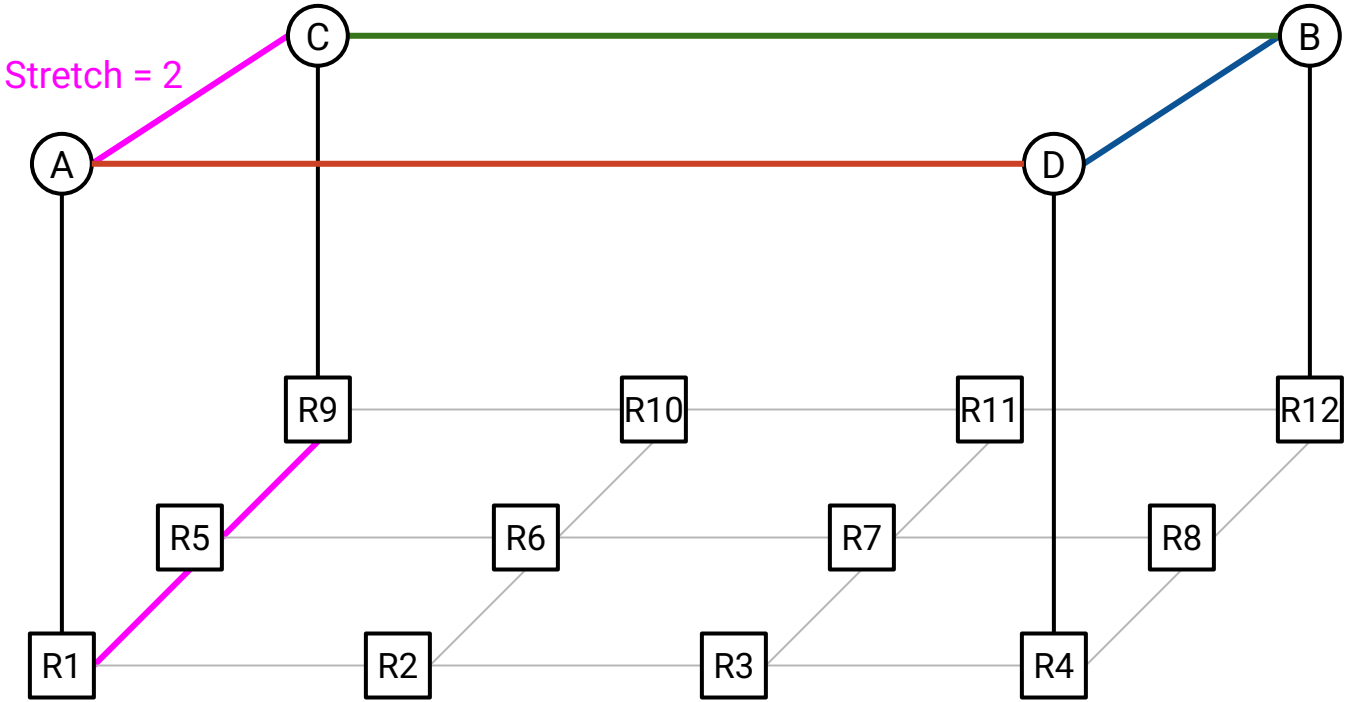

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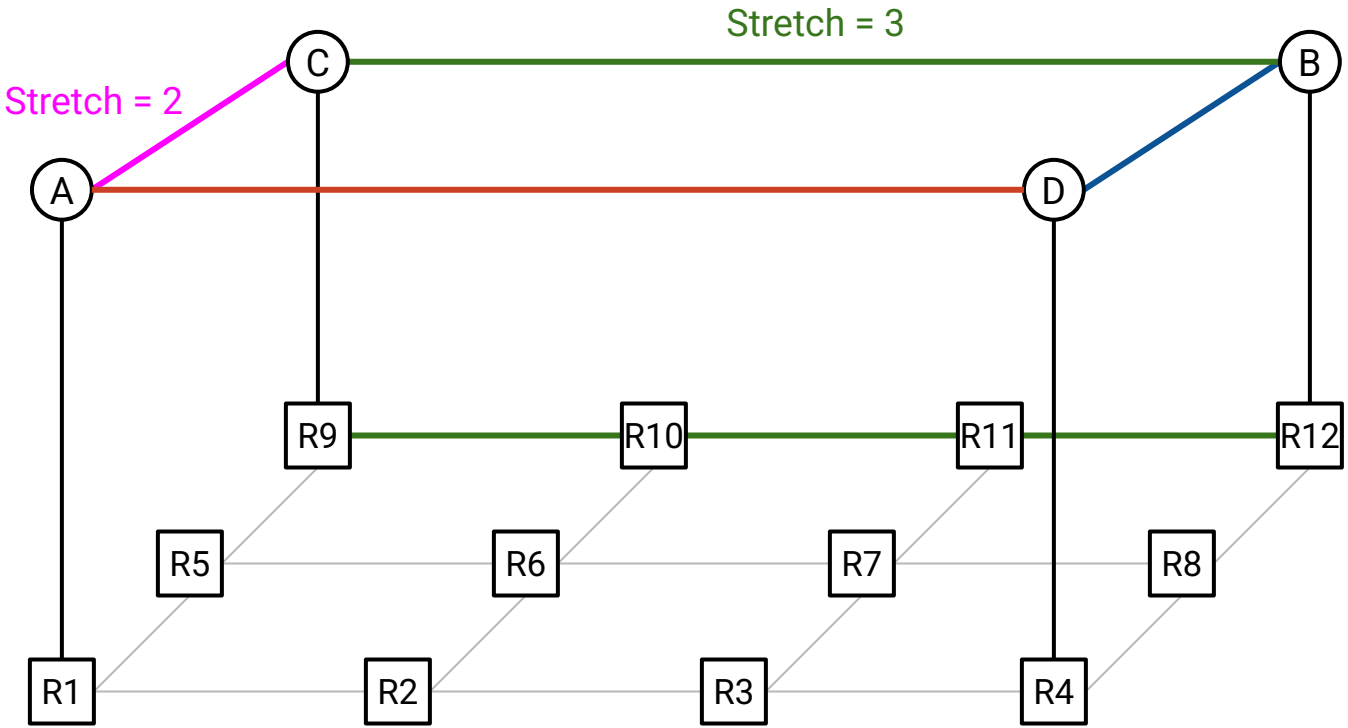

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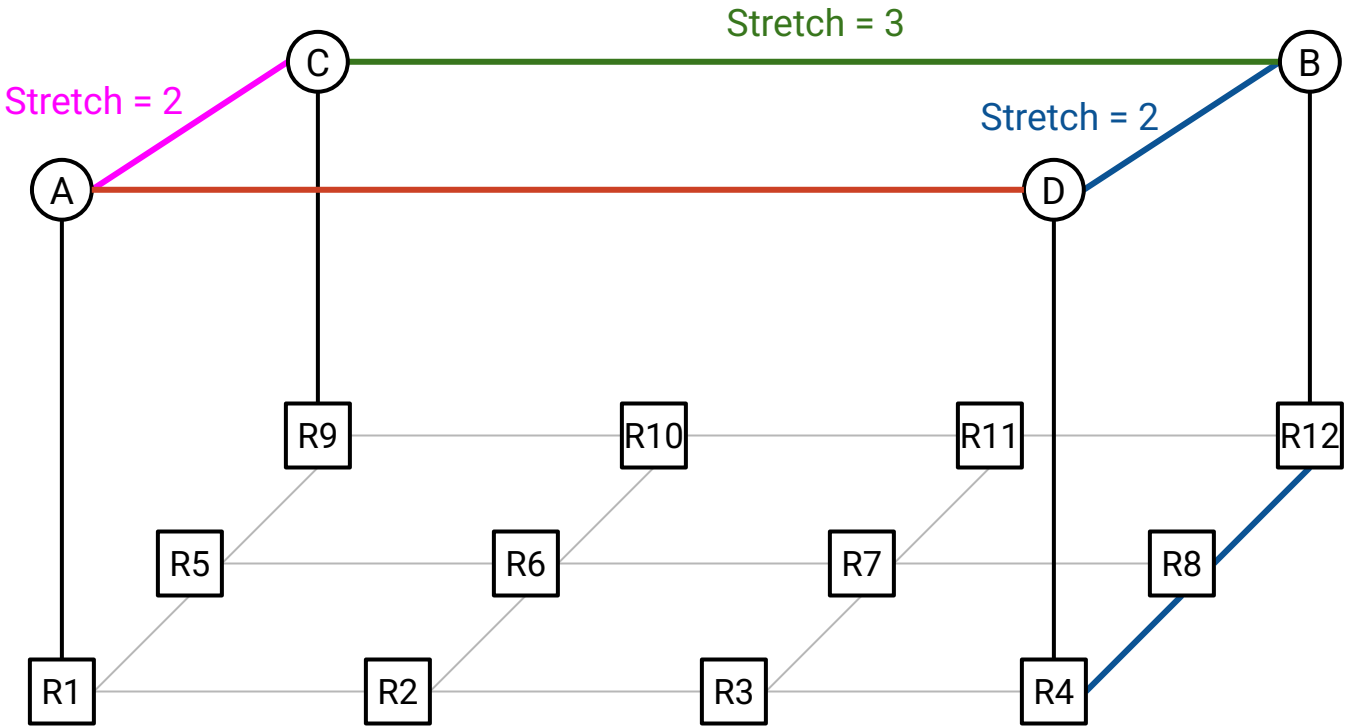

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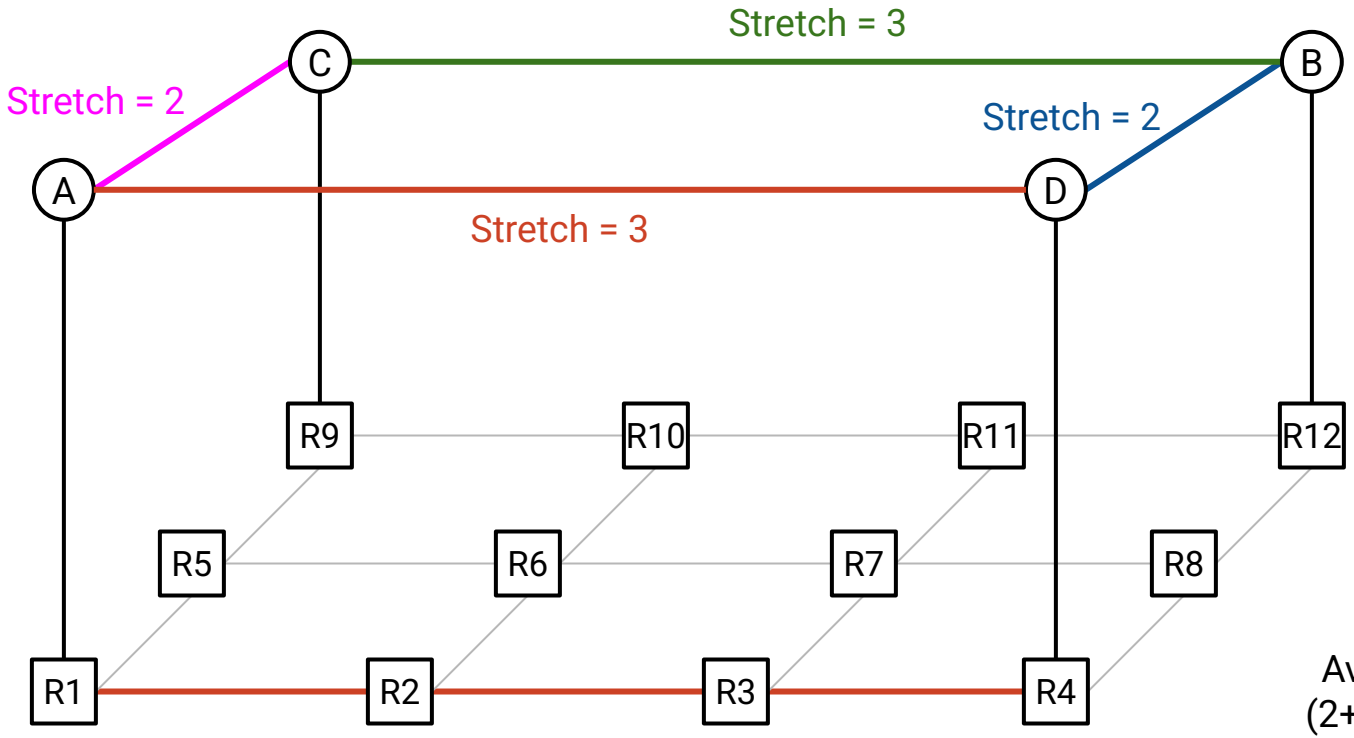


Average stretch:
(2+3+2+3)/4 = 2.5

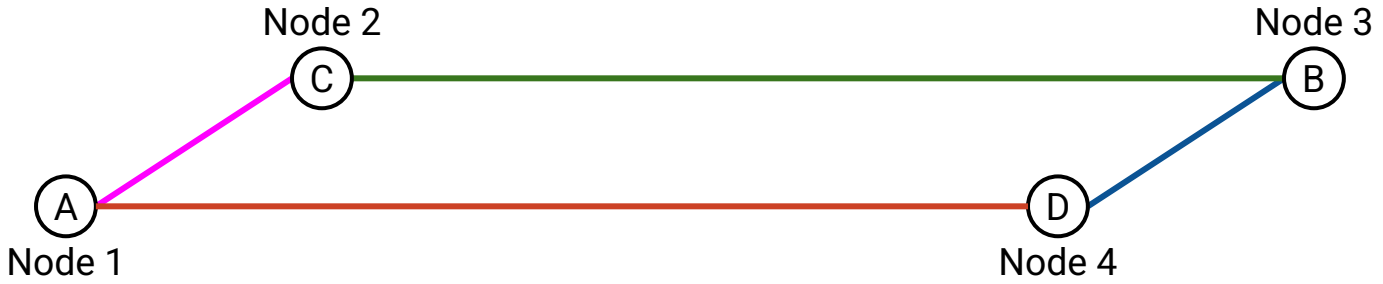
Different Overlay Topologies

Topology 2 achieves better average stretch (i.e. better matches the underlay).

Takeaway: How you draw the overlay network (e.g. the order you connect the hosts) has a big impact on performance.

Overlay Topology 1:
Node 1 = A
Node 2 = C
Node 3 = D
Node 4 = B
Average Stretch: 3.5

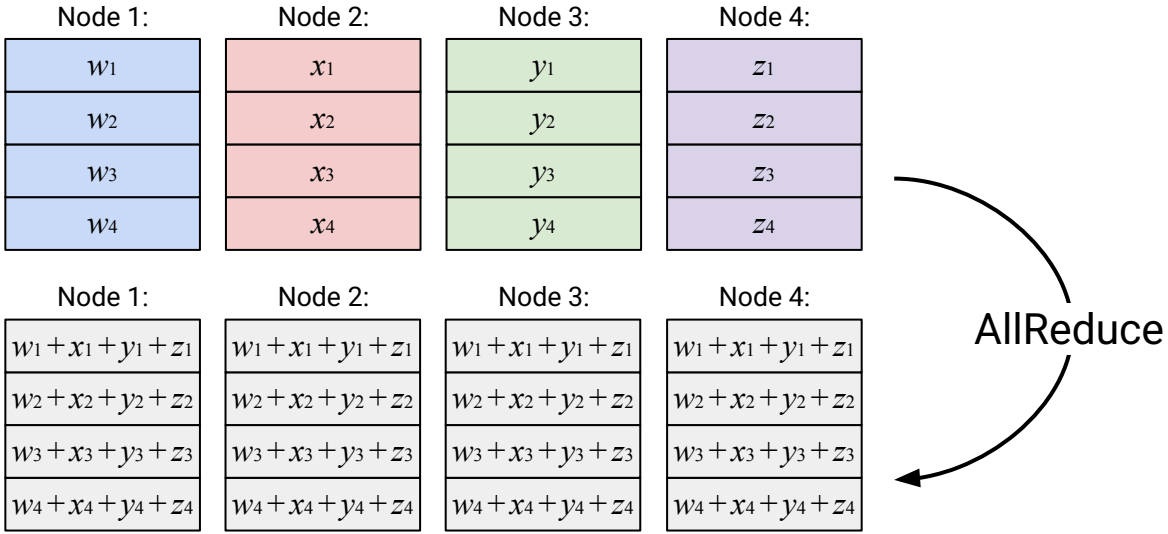
Overlay Topology 2:
Node 1 = A
Node 2 = C
Node 3 = B
Node 4 = D
Average Stretch: 2.5



Summary - Layers of Abstraction

We thought about collectives at 3 different layers of abstraction:

- **Definitions: Users just need to know the input/outputs to use the operations.**



Summary - Layers of Abstraction

We thought about collectives at 3 different layers of abstraction:

- Definitions: Users just need to know the input/outputs to use the operations.
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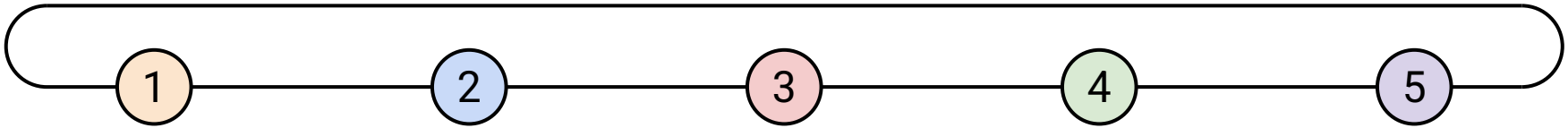

Summary - Layers of Abstraction

We thought about collectives at 3 different layers of abstraction:

- Definitions: Users just need to know the input/outputs to use the operations.
- Overlay: The data exchanged over virtual links.
- **Underlay: How the virtual links correspond to actual physical links.**