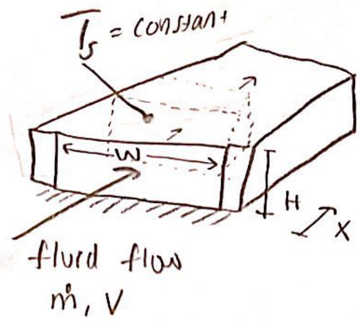
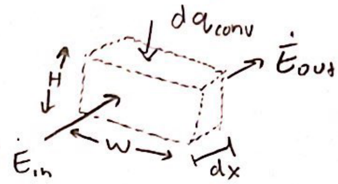
Problem 1



fully developed hydrodynamically & thermally $v = 0$, $\frac{\partial u}{\partial x} = 0$



$$V = \frac{\dot{m}}{\rho A_c}$$

$$P = 2W + 2H$$

$$A_c = WH$$

$$\dot{E}_{in} = \dot{m} c_p T_m + h(T_s - T_m) W\, dx$$

$$\dot{E}_{out} = \dot{m} c_p \left(T_m + \frac{dT_m}{dx} dx\right)$$

$$\frac{dT_m}{dx} = \frac{hW}{\dot{m} c_p}(T_s - T_m)$$

Let $\Delta T = T_s - T_m$

$$\frac{d(\Delta T)}{dx} = 0 - \frac{dT_m}{dx}$$

$$-\frac{d(\Delta T)}{dx} = \frac{hW}{\dot{m} c_p} \Delta T$$

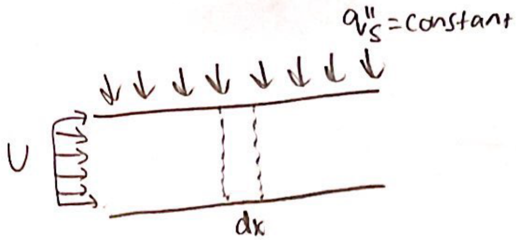
$$\frac{d(\Delta T)}{\Delta T} = -\frac{hW}{\dot{m} c_p} dx$$

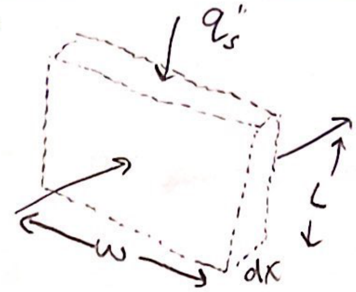
$$\ln\left(\frac{\Delta T}{\Delta T_i}\right) = -\frac{hW}{\dot{m} c_p} x$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{hW}{\dot{m} c_p} x\right]$$

$$T_m = T_s - (T_s - T_{m,i}) \exp\left[-\frac{hW}{\dot{m} c_p} x\right]$$

Problem 2





$$\dot{E}_{in} = \dot{m} c_p T_m + q''_s w\, dx$$
$$\dot{E}_{gen} = q''' w L\, dx$$
$$\dot{E}_{out} = \dot{m} c_p \left(T_m + \frac{dT_m}{dx} dx\right)$$

$$\dot{m} c_p T_m + q''_s w\, dx + q''' w L\, dx = \dot{m} c_p \left(T_m + \frac{dT_m}{dx} dx\right)$$
$$\dot{m} = \rho U A_c$$

$$\frac{dT_m}{dx} dx = \frac{q''_s W dx + q''' W L dx}{\rho U W L C_p}$$

$$\frac{dT_m}{dx} = \frac{q''_s + q''' L}{\rho U L C_p}$$

$$T_m = \frac{q'' + q''' L}{\rho U L C_p} x + C_1 \quad @\ x = 0,\ T_m = T_{m,i}$$

$$\boxed{T_m = \frac{q'' + q''' L}{\rho U L C_p} x + T_{m,i}}$$

$$u \frac{\partial T}{\partial x} + \underbrace{v \frac{\partial T}{\partial y}}_{0 \text{ (assume low speed)}} = \alpha \frac{\partial^2 T}{\partial y^2} + \underbrace{\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2}_{0 \text{ (neglect viscous dissipation)}}$$

Constant heat flux $\frac{dT_m}{dx} = \frac{dT}{dx}$

$$\frac{dT_m}{dx} = \frac{q''_s + q''' L}{\rho U L C_p}$$

$$u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$\frac{U}{\alpha}\left(\frac{q''_s + q''' L}{\rho U L C_p}\right) = \frac{\partial^2 T}{\partial y^2} \rightarrow \frac{\partial T}{\partial y} = \frac{q''_s + q''' L}{\rho \alpha L C_p} y + C_1 \rightarrow \text{BC } @\ y = 0\ \frac{dT}{dy} = 0 \text{ (insulated)}$$
$$\rightarrow C_1 = 0$$

$$T(x,y) = \left(\frac{q''_s + q''' L}{2 \alpha L C_p}\right) y^2 + C_1 y + T(x)$$

$$\frac{dT_m}{dx} = \frac{dT}{dx}$$

$$\frac{q''_s + q''' L}{\rho U L C_p} = \frac{dT}{dx} \rightarrow T(x) = \frac{q''_s + q''' L}{\rho U L C_p} x + C_1 \rightarrow \text{BC } @\ x = 0,\ T(x) = T_i$$
$$C_1 = T_i$$

$$\boxed{T(x,y) = \left(\frac{q''_s + q''' L}{2 \alpha L C_p}\right) y^2 + \left(\frac{q'' + q''' L}{\rho U L C_p} x + T_i\right)}$$

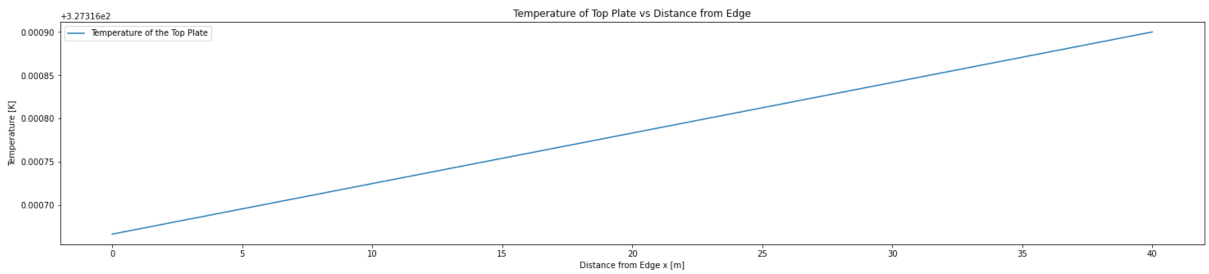
In [1]:

```
#Properties of water from engineering toolbox
Ti=25+273.15
U=5
L=0.5
qs=20
qdot=100

p=1000
cp=4800
kf=600*10**(-3)
import numpy as np
n=500
x=np.linspace(0,40,n)
T_top=np.linspace(0,n,n)
T_bottom=np.linspace(0,n,n)
T_m=np.linspace(0,n,n)
for i in range(0,n):
    T_top[i]=(qdot/kf+qs/(L*kf))*(L**2)/2+(1/(U*L*p*cp))*(qdot*L+qs)*x[i]+Ti
    T_bottom[i]=(1/(U*L*p*cp))*(qdot*L+qs)*x[i]+Ti
    T_m[i]=(1/(U*L*p*cp))*(qdot*L+qs)*x[i]+Ti
```

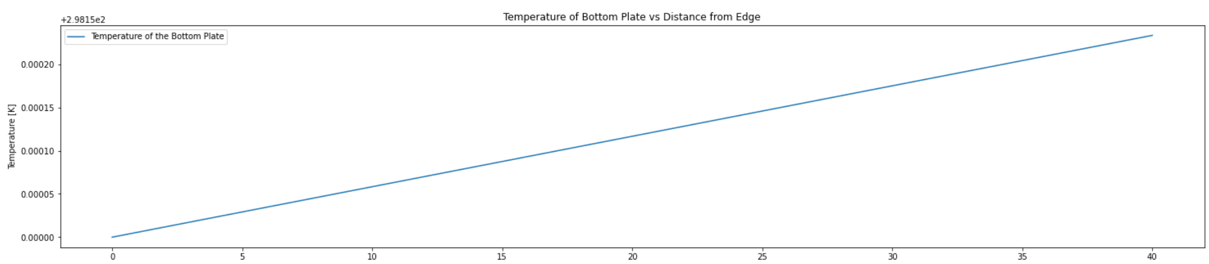
In [10]:

```
import matplotlib.pyplot as plt
plt.plot(x, T_top, label='Temperature of the Top Plate')
plt.xlabel('Distance from Edge x [m]')
plt.ylabel('Temperature [K]')
plt.title('Temperature of Top Plate vs Distance from Edge')
plt.legend()
plt.rcParams['figure.figsize']=(25,5)
plt.show()
```

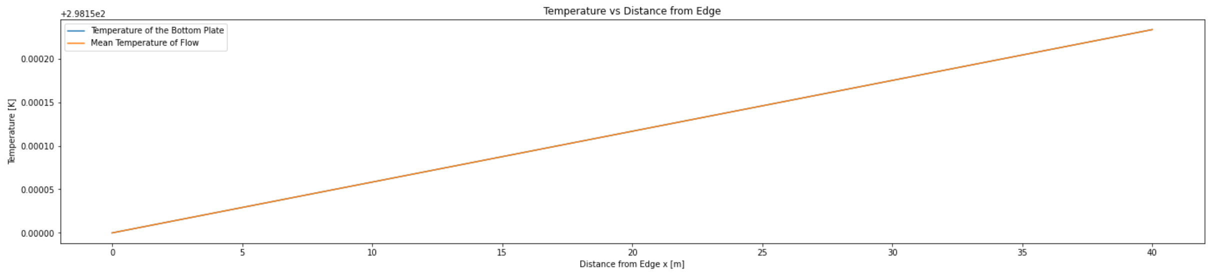


In [9]:

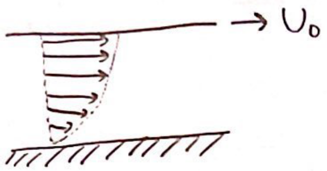
```
plt.plot(x, T_bottom, label='Temperature of the Bottom Plate')
plt.xlabel('Distance from Edge x [m]')
plt.ylabel('Temperature [K]')
plt.title('Temperature of Bottom Plate vs Distance from Edge')
plt.legend()
plt.rcParams['figure.figsize']=(25,5)
plt.show()
```



In [8]:
```
plt.plot(x, T_bottom, label='Temperature of the Bottom Plate')
plt.plot(x, T_m, label='Mean Temperature of Flow')
plt.xlabel('Distance from Edge x [m]')
plt.ylabel('Temperature [K]')
plt.title('Temperature vs Distance from Edge')
plt.legend()
plt.rcParams['figure.figsize']=(25,5)
plt.show()
```



Problem 3



Assume

1) laminar
2) No viscous dissipation
3) $\frac{dP}{dx} = 0$
4.) Steady State
5) No v
6) Fully developed - infinitely long plates

Mass Conservation

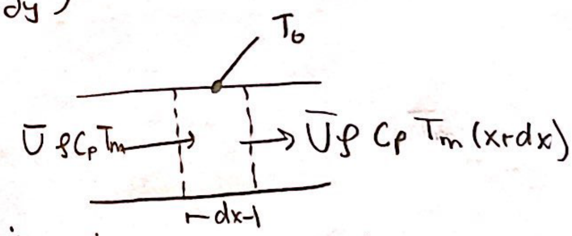
$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} = 0 \qquad \frac{\partial u}{\partial x} = 0 \rightarrow \boxed{u = u(y)}$$

Momentum Conservation

$$u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}} = \alpha\frac{\partial^2 T}{\partial y^2} + \cancel{\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2}$$

$$u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x}$$



$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}} = \cancel{\dot{E}_{st}}$$

$$\rho\bar{U}\cancel{C_p T_m(x)HW} + h(T_0 - T_m(x))W\,dx$$

$$h(T_0 - T_m(x))\cancel{dx}\,\cancel{W} = \bar{U}\rho C_p H\cancel{W}\cancel{dx}\frac{dT_m}{dx} \qquad -\bar{U}\rho C_p HW\left(\cancel{T_m(x)} + \frac{dT_m(x)}{dx}dx\right) = 0$$

$$\int_0^x \frac{h}{\bar{U}\rho C_p H}dx = \int_0^x \frac{1}{T_0 - T_m(x)}dT_m(x)$$

$$\frac{h}{\bar{U}\rho C_p H}x = \ln\left(\frac{1}{T_0 - T_m(x)}\right)\Big|_0^x = \ln\left(\frac{T_0 - T_{m,i}}{T_0 - T_m(x)}\right)$$

$$\frac{T_0 - T_m(x)}{T_0 - T_{m,i}} = \exp\left(\frac{-hx}{\bar{U}\rho C_p H}\right) \rightarrow T_0 - T_m(x) = (T_0 - T_{mi})\exp\left(\frac{-hx}{\bar{U}\rho C_p H}\right)$$

$$\frac{dT_m(x)}{dx} = (T_0 - T_{mi})\exp\left(\frac{-hx}{\bar{U}\rho C_p H}\right)\left(\frac{h}{\bar{U}\rho C_p H}\right)$$

$\rightarrow$

$$u\frac{dT}{dx} = U(T_o - T_{m,i})\left(\frac{h}{\bar{U}\rho C_p H}\right)\exp\left(\frac{-hx}{\bar{U}\rho C_p H}\right) = \alpha\frac{d^2T}{dy^2}$$

$$\frac{\rho C_p}{k}\cdot\frac{U_o}{H}y(T_o - T_{m,i})\left(\frac{h}{\bar{U}\rho C_p H}\right)\cdot\exp\left(\frac{-hx}{\bar{U}\rho C_p H}\right) = \frac{\partial^2 T}{\partial y^2}$$

$$\bar{U} = \frac{1}{H}\int_0^H \frac{U_o}{H}y\,dy \rightarrow \bar{U} = \frac{1}{H}\left(\frac{U_o}{2H}y^2\right)\Big|_0^H = \frac{U_o}{2}$$

$$\frac{1}{k}\left(\frac{U_o}{H}\right)(T_o - T_{m,i})\left(\frac{h}{(U_o/2)H}\right)\exp\left(\frac{-2hx}{U_o\rho C_p H}\right) = \frac{\partial^2 T}{\partial y^2}$$

$$\frac{2(T_o - T_{m,i})}{kH^2}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)y = \frac{\partial^2 T}{\partial y^2}$$

$$\frac{2(T_o - T_{m,i})h}{kH^2}\exp\left(-\frac{2hx}{U_o\rho C_p H}\right)\frac{y^2}{2} + C_1 = \frac{\partial T}{\partial x} \rightarrow @y=0\ \frac{\partial T}{\partial x} = 0$$

(insouliates)

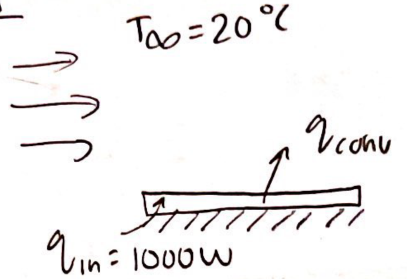
$$C_1 = 0$$

$$T(x,y) = \frac{(T_o - T_{m,i})h}{3kH^2}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)y^3 + C_2 \quad @\,y = H,\ T = T_o$$

$$T_o = \frac{(T_o - T_{m,i})h}{3k\cancel{H^2}}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)\cancel{H^3} + C_2 \rightarrow C_2 = T_o - \frac{(T_o - T_{m,i})hH}{3k}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)$$

$$\boxed{T(x,y) = \frac{(T_o - T_{m,i})h}{3kH^2}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)y^3 + T_o - \frac{(T_o - T_{m,i})hH}{3k}\exp\left(\frac{-2hx}{U_o\rho C_p H}\right)}$$

Problem 4



$$q_{in} - q_{conv} = 0$$

$$q_{in} = \bar{h}(T_s - T_\infty)$$

$$\bar{h} = \int_0^L h\,dx = \int_0^L \left(\frac{k}{x}\right) Nu_x\,dx$$

for turbulent flow over isothermal flat plate

$$Nu_x = \frac{hx}{k} = 0.0296 Re_x^{4/5} Pr^{1/3} \qquad Pr = \frac{\nu}{\alpha} = 0.7$$

$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\nu}$$

$$\bar{h} = \int_0^L \left(\frac{k}{x}\right) 0.0296 \left(\frac{U_\infty x}{\nu}\right)^{4/5} Pr^{1/3}\,dx = \left[\frac{0.0296 k U_\infty Pr^{1/3}}{\nu}\right]\int_0^L x^{-1/5}\,dx$$

$$\bar{h} = \left[\frac{0.0296 k Pr^{1/3}}{\nu}\right] U_\infty \left[\frac{5}{4} x^{4/5}\right]\Big|_0^L = \left[\frac{5(0.0296)k Pr^{1/3}}{4\nu} L^{4/5}\right] U_\infty$$

$$\bar{h} = \frac{(5)(0.0296)(0.03)(0.7)^{1/3}(0.01)^{4/5}}{4(20.92E-6)} U_\infty \rightarrow \bar{h} = 1.18 U_\infty$$

$$\frac{q_{in}}{T_s - T_\infty} = 1.18 U_\infty \rightarrow U_\infty = \frac{1}{1.18}\left(\frac{1000}{100-20}\right) = \boxed{10.6 \text{ m/s min}}$$

Assume 75 CFM PC Fan w/ D = 0.120 (Rosewill RFA-120)

$$75 \frac{ft^3}{min}\left(\frac{1}{60}\right)\left(\frac{0.0283}{1}\right) = 0.035 \text{ m}^3/\text{s}, \quad A = \pi/4\, D^2 = 0.011 \text{ m}^2$$

$$\text{Fan air velocity} = \frac{\dot{V}}{A} = 0.035/0.011 = 3.2 \text{ m/s}$$

Typical PC fan speed gives 3x less than minimum air velocity required for this application.

$\boxed{\text{Unreasonable Cooling Scheme}}$

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In [ ]:
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