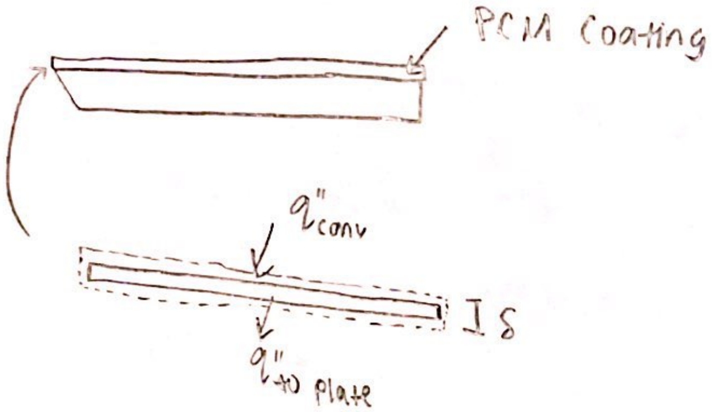
Problem 1

(a)



__Heat Transfer Rates__

$$q''_{convection} = h(T_\infty - T_s)$$

$$q''_{sub} = \rho L_s \Delta\delta\, l\, w$$

__Assumptions__

- No radiation
- Thin Plate
- no heat generation
- T_s constant for Plate
- T_s constant for PCM

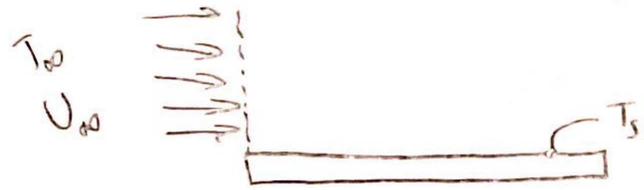
(b) $$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q''_{conv} \cdot l \cdot w - q''_{to\ plate} \cdot l \cdot w = \rho L_s \Delta\delta \cdot l \cdot w$$

$$\boxed{h(T_\infty - T_s) - \rho L_s \Delta\delta = q''_{to\ plate}}$$

heat transfer rate applied to plate

Problem 2



(a)

Assumptions

- Flat plate
- $\frac{\nu}{C_p}\left(\frac{\partial v}{\partial y}\right)^2 << \alpha \frac{\partial^2 T}{\partial y^2}$ $\Big\} \rightarrow T^* = \frac{T - T_s}{T_\infty - T_s} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$
- Constant T_s
- Steady State
- No heat generation
- Pr is very small so $\delta << \delta_t \rightarrow U = U_\infty$

$T^*(y=0) = 0$

$T^*(y=\delta) = 1$

$\left.\frac{\partial T^*}{\partial y}\right|_{y=\delta} = 0$

$0 = a$

$1 = b + c$

$0 = b/\delta + \frac{2c}{\delta^2}\delta = \frac{b}{\delta} + \frac{2c}{\delta}$

$0 = b + 2c$

$b = -2c$

$1 = -2c + c$

$c = -1$

$\rightarrow b = 2$

$$\boxed{\frac{T - T_s}{T_\infty - T_s} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2}$$

(b) velocity field is constant relative to thermal boundary layer being larger with small Pr $U = U_\infty$

$$\frac{d}{dx}\left[\int_0^\delta (T_\infty - T) U_\infty \, dy\right] = \alpha \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$T = (T_\infty - T_s)\left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right) + T_s$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left(2/\delta - \frac{2}{\delta}\left(\frac{0}{\delta}\right)\right) = (T_\infty - T_s)\left(\frac{2}{\delta}\right)$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta \left(T_\infty - (T_\infty - T_s)\left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right) + T_s\right)\right] U_\infty \, dy = \alpha (T_\infty - T_s)\left(\frac{2}{\delta}\right)$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta (T_\infty - T_s) - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)(T_\infty - T_s)\right] U_\infty \, dy = \alpha (T_\infty - T_s)\left(\frac{2}{\delta}\right)$$

Scanned with CamScanner

$$\frac{d}{dx}\left[\underbrace{(T_\infty - T_s)}\int_0^\delta \left(1-\left[2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right]\right)U_\infty dy\right] = \alpha\,\underbrace{(T_\infty - T_s)}\left(\frac{2}{\delta}\right)$$

$$\frac{d}{dx}\left(U_\infty \int_0^{\delta(x)} 1 - \frac{2y}{\delta(x)} + \left(\frac{y}{\delta(x)}\right)^2 dy\right) = \alpha\frac{2}{\delta(x)}$$

$$U_\infty \frac{d}{dx}\left(y - \frac{y^2}{\delta(x)} + \frac{y^3}{3(\delta(x))^2}\Big|_0^{\delta(x)}\right) = \alpha\frac{2}{\delta(x)}$$

$$U_\infty \frac{d}{dx}\left(\underbrace{\delta(x) - \delta(x)}_{0} + \frac{1}{3}\delta(x) - 0\right) = \alpha\frac{2}{\delta(x)}$$

$$\frac{U_\infty}{3}\frac{\partial \delta(x)}{\partial x} = \alpha\frac{2}{\delta(x)}$$

$$\int_0^{\delta(x)} z\,dz = \int_0^x 6\frac{\alpha}{U_\infty}dx$$

$$\frac{1}{2}z^2\Big|_0^{\delta(x)} = 6\frac{\alpha}{U_\infty}x\Big|_0^x$$

$$\boxed{\delta(x) = \left(\frac{12\alpha x}{U_\infty}\right)^{1/2}}$$

$$\alpha = \frac{k}{\rho c_p}$$

(c)

$$Nu_x = \frac{hx}{k} = \frac{\partial T^*}{\partial y}\Big|_{y=0} = \frac{\partial}{\partial y}\left(\frac{2y}{\delta(x)} - \left(\frac{y}{\delta(x)}\right)^2\right)\Big|_{y=0} = \frac{2}{\delta(x)} = 2\left(\frac{U_\infty}{12\alpha x}\right)^{1/2}$$

$$Re_x = \frac{U_\infty x}{\nu} \qquad Pr = \frac{\nu}{\alpha}$$

$$Nu_x = \frac{2}{\sqrt{12}}\left(\frac{x}{1}\right)^{-1/2}(U_\infty)^{1/2}\left(\frac{1}{\alpha}\right)^{1/2} = \frac{2}{\sqrt{12}}\left(\frac{U_\infty x \nu}{U_\infty \nu}\right)^{-1/2}(U_\infty)^{1/2}\left(\frac{1}{\alpha}\right)^{1/2}$$

$$Nu_x = \frac{2}{\sqrt{12}}\left(\frac{xU_\infty}{\nu}\right)^{-1/2}\left(\frac{\nu}{U_\infty}\right)^{-1/2}(U_\infty)^{1/2}\left(\frac{1}{\alpha}\right)^{1/2} = \frac{2}{\sqrt{12}}Re^{-1/2}\frac{U_\infty}{\nu^{1/2}}\cdot\frac{\nu^{1/2}}{\nu^{1/2}}\left(\frac{1}{\alpha}\right)^{1/2}$$

$$= \frac{2}{\sqrt{12}}Re^{-1/2}\left(\frac{U_\infty}{\nu}\right)\left(\frac{\nu}{\alpha}\right)^{1/2}$$

$$\boxed{Nu_x = \frac{2}{\sqrt{12}}\frac{U_\infty}{\nu}Re^{-1/2}Pr^{1/2}}$$

Scanned with CamScanner

Problem 3

$$\frac{d}{dx}\left[\int_0^\delta (U_\infty - U) U \, dy\right] = \nu \frac{\partial U}{\partial y}\Big|_{y=0}$$

$$\tau_s = \mu \frac{\partial U}{\partial y}\Big|_{y=0} \rightarrow \frac{\tau_s}{\mu} = \frac{\partial U}{\partial y}\Big|_{y=0}$$

$$\frac{\nu \tau_s}{\mu} = \nu \frac{\partial U}{\partial y}\Big|_{y=0} \qquad \nu = \frac{\mu}{\rho} \Rightarrow \mu = \nu\rho \rightarrow \frac{\tau_s}{\rho} = \nu \frac{\partial U}{\partial y}\Big|_{y=0}$$

$$\frac{\partial}{\partial x}\left[\int_0^\delta (U_\infty - U) U \, dy\right] = 0.0228 \rho U_\infty^2 \left(\frac{U_\infty \delta}{\nu}\right)^{-1/4} \frac{1}{\rho} = 0.0228 U_\infty^2 \left(\frac{U_\infty \delta}{\nu}\right)^{-1/4}$$

$$U = U_\infty \left(\frac{y}{\delta}\right)^{1/7}$$

$$\frac{d}{dx}\left[\int_0^\delta U_\infty U - U^2 \, dy\right] = \frac{d}{dx}\left[\int_0^\delta U_\infty^2 \left(\frac{y}{\delta}\right)^{1/7} - U_\infty^2 \left(\frac{y}{\delta}\right)^{2/7} dy\right]$$

$$= 0.0228 U_\infty^2 \left(\frac{U_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{d}{dx}\left(\frac{7}{8}\left(\frac{1}{\delta}\right)^{1/7} (y)^{8/7} - \frac{7}{9}\left(\frac{1}{\delta}\right)^{2/7} (y)^{9/7}\right)\Big|_0^\delta = 0.0228 \left(\frac{U_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{d}{dx}\left(\frac{7}{8}\left(\frac{1}{\delta}\right)^{1/7} (\delta)^{8/7} - \frac{7}{9}\left(\frac{1}{\delta}\right)^{2/7} (\delta)^{9/7}\right) = \frac{\partial}{\partial x}\left(\frac{7}{8}\delta - \frac{7}{9}\delta\right) = \frac{d}{dx}\left(\frac{7}{72}\delta\right)$$

$$= 0.0228 \left(\frac{U_\infty \delta}{\nu}\right)^{-1/4}$$

$$\frac{\partial \delta}{\partial x} = (0.0228)\left(\frac{72}{7}\right)\left(\frac{U_\infty}{\nu}\right)^{-1/4} \delta^{-1/4}$$

$$\int_0^\delta \delta^{1/4} \, d\delta = \int_0^x 0.0228 \left(\frac{72}{7}\right)\left(\frac{U_\infty}{\nu}\right)^{-1/4} dx$$

$$\frac{4}{5}\delta^{5/4} = 0.0228\left(\frac{72}{7}\right)\left(\frac{U_\infty}{\nu}\right)^{1/4} x \rightarrow \delta = \left((0.0228)\left(\frac{5}{4}\right)\left(\frac{72}{7}\right)\left(\frac{U_\infty}{\nu}\right)^{-1/4} x\right)^{4/5}$$

$$\boxed{\delta = 0.3747 \left(\frac{U_\infty}{\nu}\right)^{-1/5} x^{4/5}}$$

Scanned with CamScanner

(b)

$$\frac{d}{dx}\left[\int_0^{\delta_t}(T_\infty - T)u\,dy\right] = \alpha \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$T = (T_s - T_\infty)\left(1-\left(\frac{y}{\delta_t}\right)^{1/7}\right) + T_\infty$$

$$\frac{\partial}{\partial x}\left[\int_0^{\delta_t}\left(T_\infty - (T_s - T_\infty)\left(1-\left(\frac{y}{\delta_t}\right)^{1/7}\right) - T_\infty\right)u\,dy\right] = \frac{h}{\rho C_p}(T_\infty - T_s)$$

$$\frac{d}{\partial x}\left[\int_0^{\delta_t}(T_\infty - T_s)\left(1-\left(\frac{y}{\delta_t}\right)^{1/7}\right)U_\infty\left(\frac{y}{\delta}\right)^{1/7}dy\right] = \frac{h}{\rho C_p}(T_\infty - T_s)$$

$$\frac{d}{dx}\left[\int_0^{\delta_t}\left(\frac{1}{\delta}\right)^{1/7}(y)^{1/7} - \left(\frac{1}{\delta_t}\right)^{1/7}\left(\frac{1}{\delta}\right)^{1/7}y^{2/7}\,dy\right] = \frac{h}{\rho C_p U_\infty}$$

$$\frac{\partial}{\partial x}\left[\left(\frac{1}{\delta}\right)^{1/7}\left(\frac{7}{8}\right)y^{8/7} - \left(\frac{1}{\delta_t}\right)^{1/7}\left(\frac{1}{\delta}\right)^{1/7}\left(\frac{7}{9}\right)y^{9/7}\right]\Bigg|_0^{\delta_t} = \frac{h}{\rho C_p U_\infty}$$

$$\frac{d}{dx}\left[\frac{7}{8}\frac{(\delta_t)^{8/7}}{(\delta)^{1/7}} - \frac{7}{9}\frac{\delta_t^{8/7}}{\delta^{1/7}}\right] = \frac{7}{72}\frac{d}{dx}\left(\frac{\delta_t^{8/7}}{\delta^{1/7}}\right) = \frac{h}{\rho C_p U_\infty}$$

For turbulent flow $\delta = \delta_t$ *Assume

$$\int_0^{\delta_t} 1\,d\delta_t = \int_0^x \frac{72}{7}\frac{h}{\rho C_p U_\infty}dx \rightarrow \delta_t = \frac{72}{7}\frac{h}{\rho C_p U_\infty}x$$

$$\frac{7\delta_t \rho C_p U_\infty}{72 k} = \frac{hx}{k}$$

$$Nu_x = \frac{hx}{k}$$

$$Nu_x = \frac{7}{72}\frac{\rho C_p}{k}\frac{U_\infty}{\nu}\cdot\frac{\nu}{1}\delta_t$$

Assume $\delta = \delta_t \rightarrow \delta_t = 0.3747\left(\frac{U_\infty}{\nu}\right)^{-1/5}x^{4/5}$

$$Nu_x = \frac{7}{72}Pr\frac{U_\infty}{\nu}(0.3747)\left(\frac{U_\infty}{\nu}\right)^{-1/5}x^{4/5}$$

$$Nu_x = \left(\frac{7}{72}\right)(0.3747)Re^{4/5}Pr = 0.03642\,Re_x^{4/5}Pr$$

$$\boxed{Nu_x = 0.03642\,Re_x^{4/5}Pr}$$

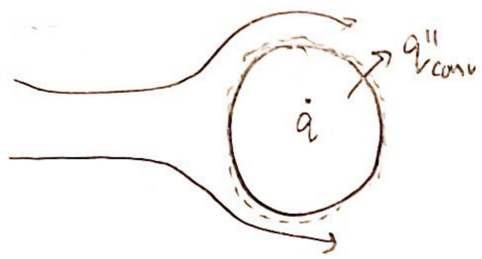
$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\overline{Nu}_x = \int_0^x Nu_x\, dx \left(\frac{1}{x}\right) = 0.03642\, Pr \left(\frac{U_\infty}{\nu}\right)^{4/5} \int_0^x \omega^{4/5}\, d\omega\, \frac{1}{x}$$

$$= 0.03642\, Pr \left(\frac{U_\infty}{\nu}\right)^{4/5} \left(\frac{5}{9}\, \frac{x^{9/5}}{x}\right)$$

$$\boxed{\overline{Nu}_x = 0.02023\, Re_x^{4/5}\, Pr}$$

Problem 4



$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$

Assume
SS, no rad

$\dot{E}_{gen} = \dot{E}_{out}$

$\dot{q} = q''_{conv} \pi D L$

$\dot{q} = \frac{E^2}{R_w}$ $\quad q''_{conv} = \bar{h}(T_w - T_\infty)$

$\frac{E^2}{R_w} = \bar{h}(T_w - T_\infty)\pi D L$ $\quad \dot{q} = \frac{E^2}{R_w}$

$\bar{h} = \frac{E^2}{R_w(T_w - T_\infty)\pi D L}$

$\overline{Nu} = \frac{\bar{h}D}{k} = 0.0296 \, Re_D^{4/5} \, Pr^{1/3}$

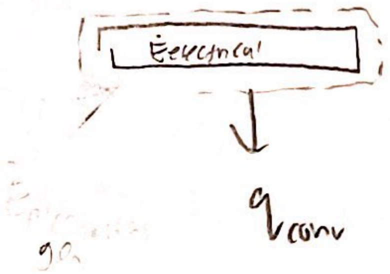
$\bar{h} = \frac{0.0296 (U_\infty)^{4/5} D^{4/5} k (\nu)^{1/3}}{\nu^{4/5} D (\alpha)^{1/3}} = \frac{E^2}{R_w(T_w - T_\infty)\pi D L}$

$U_\infty^{4/5} = \frac{D \alpha^{1/3} \nu^{4/5} E^2}{0.0296 R_w (T_w - T_\infty) k \nu^{1/3} D^{4/5} D L \pi} \rightarrow U$

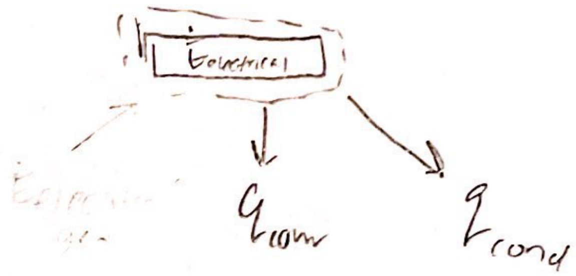
$$\boxed{U_\infty = \left(\frac{\alpha^{1/3} \nu^{7/15} E^2}{0.0296 R_w (T_w - T_\infty) k D^{4/5} L \pi} \right)^{5/4}}$$

(b)

Case 1



Case 2



Since T_w = constant, the energy leaving both systems is constant

$$q_{conv,1} = q_{conv\,2}$$

$$\cancel{\dot{E}_{in}}^{0} - \dot{E}_{out} + \dot{E}_{gen} = \cancel{\dot{E}_{st}}^{0}$$

$$\dot{E}_{electrical} = q_{conv}$$

$$\cancel{\dot{E}_{in}}^{0} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$\dot{E}_{electrical} = q_{conv} + q_{cond}$$

$$\dot{E}_{electrical,1} = \dot{E}_{electrical\,2} - q_{cond}$$

In case 2 w/ losses, the instrument will need to supply energy that isn't lost to the fluid. This causes the system to predict a higher heat transfer than expected. This would cause an over prediction in U_0