Problem 1. A

(a)

$A_1 = A_2 = 1E-4$

$\frac{A}{r^2} << 1$ ✓

$A_m = 2E-4$

$r^2 = 0.02$

$E_b = \sigma T^4 = (5.67E-8)(3000)^4 = 4.59E6 \; W/m^2$

$I_b = \frac{E_b}{\pi} = 1.46E6$

from 1 to mirror

$\omega_{m-1} = \frac{A_m \cos\theta_m}{r^2} = \frac{(2E-4)\cos 45°}{0.02}$

$= 0.00707$

$q_{1-m} = I A_1 \cos\theta_1 \, \omega_{m-1}$

$q_{1\to m} = (1.46E6)(1E-4 \cos(45))(0.00707)$

$q_{1\to m} = 0.73 W$

$\rho = (0.5)(F_{0-\lambda}) + 1(1 - F_{0-\lambda}) = (0.5)(.273) + (1 - 0.273$

$\rho = 0.863$

$E_M = \frac{.73 - 0.863}{A_m} \rightarrow E_m = 3150 \; \frac{W}{m^2}$

From Mirror to 2

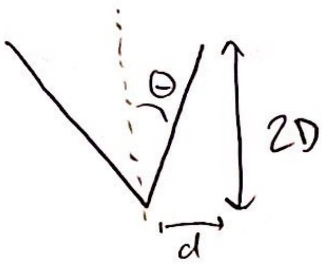
$I_m = \frac{E_m}{\pi} = \frac{3150}{\pi} = 1003 \; \frac{W}{m^2 \cdot sr}$

$\omega_{2-m} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(1E-4)\cos 45}{0.02} = 0.003535$

$q_{m\to 2} = I_m A_m \cos\theta_m \, \omega_{2-m} = 1003(2E-4)\cos(45)(0.003535)$

$\boxed{q_2 = 5E-3 W}$

(b) $q_i = 10E3$



<u>Assume</u>

① ignore reflection

② SS → $q_{emit} = q_{abs}$ (energy balance)

$$\varepsilon E = \alpha G$$

$$\alpha = \varepsilon(\lambda = 10.6\,\mu m) = 0.8\cos^2\theta$$

$$\alpha = (0.8)\left(\frac{d^2}{5d^2}\right) = 0.16$$

$$\boxed{\alpha = 0.16}$$

Problem 2

$$J = \varepsilon \sigma T^4 + \rho G$$

$T = 1000K$

$G = 1000 \cos 30$

$$\varepsilon = 0.2 F_{0\to 1} + 0.8 (1 - F_{0\to 1})$$

$$\varepsilon = (0.2)(0.000321) + (0.8)(1 - 0.000321)$$

$$\varepsilon = 0.8$$

$\alpha = \varepsilon$ (SS)

$\rho = 1 - \alpha = 0.2$

$$J = (0.8)(5.67E-8)(1000)^4 + (0.2)(1000 \cos(30))$$

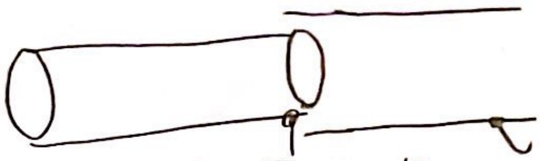
$$\boxed{J = 45,553 \ W/m^2}$$

$$q_{p-d} = I_p A_p \cos \cancel{\theta_p}^{\,1} \omega_{d-p} \qquad \omega_{d-p} = \frac{A_d \cancel{\cos \theta_d}^{\,1}}{r^2} = \frac{1E-5}{1^2}$$

$$= \frac{J}{\pi} A_p \omega_{d-p} = \frac{45553}{\pi} (1E-4)(1E-5)$$

$$\boxed{q_{p-d} = 1.45E-5 W}$$

Problem 3



$T_s = 300\text{ K}$ $\qquad T_{furn} = 1000\text{ K}$

Air @ 1000 K (table A.4)

$\rho = 0.3482 \text{ kg/m}^3 \quad \nu = 121.9E-6 \text{ m}^2/\text{s}$

$C_p = 1.141E3 \text{ J/kgK} \quad k = 66.7E-3 \text{ W/mK}$

$\eta = 424.4E-7 \text{ Ns/m}^2 \quad \alpha = 168E-6 \text{ m}^2/\text{s}$

$Pr = 0.726$

$$Ra_D = \frac{g\beta(T_\infty - T_s)D^3}{\nu\alpha} = \frac{(9.81)\left(\frac{1}{1000}\right)(1000-300)(15E-3)^3}{(121.9E-6)(168E-6)} = 1131.69$$

$$\overline{Nu}_D = \left[0.6 + \frac{0.387\, Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right]^2 = 2.686 = \frac{\bar{h}D}{k}$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \boxed{11.944}$$

$$\varepsilon = (0.4)\underbrace{F(0\to2)}_{@\,\lambda T = 600 \,=\, 0} + (0.85)(1 - F(0\to2)) = 0.85$$

$$E = (0.85)(5.67E-8)(300^4) = 390.38 \text{ W/m}^2$$

$$q''_{conv(in)} = (11.944)(1000-300) = 8360.86 \text{ W/m}^2$$

$$G = \alpha\sigma T_{furn}^4$$

$\alpha = 1 - \varepsilon$ (opaque) $= 0.15$

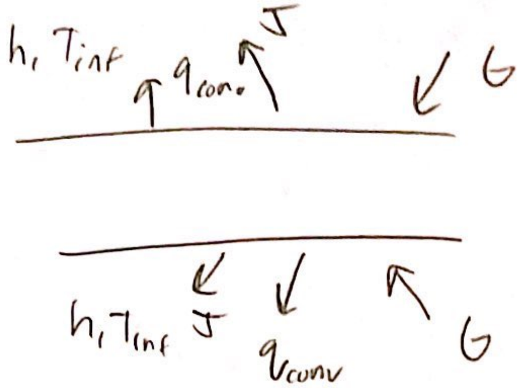
$$G = (0.15)(5.67E-8)(1000^4) = 8505$$

$$q'_{net} = P[q''_{conv} - E + G] = \pi(15E-3)[8360.86 - 390.38 + 8505]$$

$\boxed{q'_{net} = 776.4 \text{ W/m}}$ entering the rod

Problem 4

SS, Uniform surface



$\dot{E}_{in} = \dot{E}_{out}$

$\cancel{2}G = \cancel{2}q_{conv} + \cancel{2}J$

$$G = 40(350-300) + 5000 = \boxed{7000 \ \frac{W}{m^2}}$$

$$\varepsilon = \frac{E}{E_b}$$

$$J = E + \rho G + \tau G = E + (\rho + \tau) G$$

$$\rho + \tau = 1 - \alpha = 1 - 0.4$$

$$J = E + (0.6) G$$

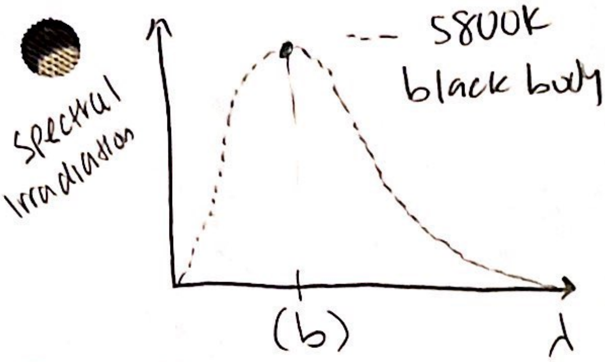
$$5000 = E + (0.6)(7000) \rightarrow E = 800$$

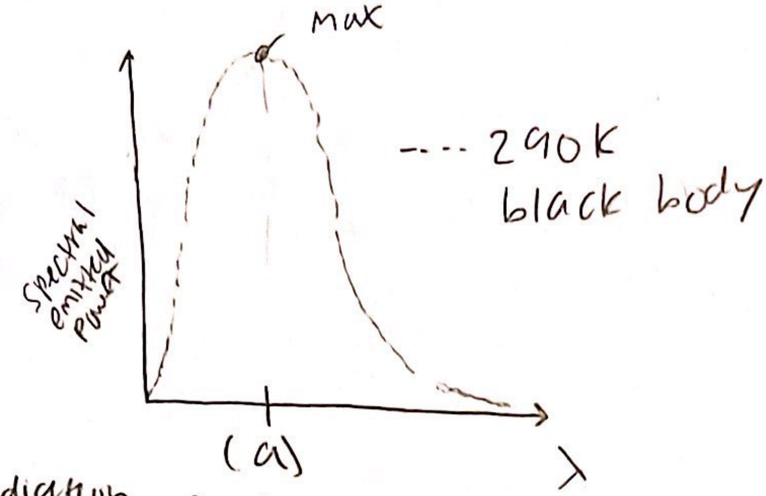
$$\varepsilon = \frac{800}{(5.67E{-8})(350^4)} = \boxed{0.94}$$

The plate is not grey because $\varepsilon \neq \alpha$

Problem 5

(a)





Point (a): max spectral irradiation for black body surface

$$2898\ \mu K = (290)\lambda_a \rightarrow \boxed{\lambda_a = 9.99\ \mu m}$$

Point (b): max spectral irradiation for surface

$$(5800K)\lambda_b = 2898 \Rightarrow \boxed{\lambda_b = 0.5\ \mu m}$$

(b)

Eq 12.32 & Table 12.1

$$E_\lambda(\lambda,T) = \varepsilon_\lambda(\lambda)\, E_{\lambda,b}(\lambda,T) = (\varepsilon_\lambda(\lambda))\left(\frac{C_1}{\lambda^5\left(\exp\left[\frac{C_2}{\lambda T}\right]-1\right)}\right)$$

$C_1 = 3.742E8$ $C_2 = 1.439E4$

$$E_\lambda(\lambda=1, T=290) = (0.9)\left(\frac{3.742E8}{(1)^5\left(\exp\left(1.439E4/(1)(290)\right)-1\right)}\right)$$

$$\boxed{E_\lambda(\lambda=1, T=290) = 1.34E-13}$$

(c)

$$\alpha_s = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, E_{\lambda,b}(\lambda, 5800)\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, 5800)\, d\lambda}$$

$$\alpha_s = \frac{(0.9)\int_0^1 E_{\lambda,b}(5800)\, d\lambda}{E_b(5800)} + \frac{(0.1)\int_1^\infty E_{\lambda,b}(5800)\, d\lambda}{E_b(5800)}$$

$$= (0.9)F(0-1\mu m) + (0.1)F(1 - F(0-1\mu m))$$

$$\alpha_s = (0.9)(0.720158) + (0.1)(1-0.720158) \rightarrow \boxed{\alpha_s = 0.676126 4}$$