Homework 3

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Homework due Feb 26, 2021 23:14 CST Past due

Due Date

Friday, 2/26 at 11:59 PM ET *(2/27 at 04:59 UTC)*

Directions:

1. Answer each of the homework problems listed below.
2. Click the **Click here to open Gradescope button** below to access Gradescope.
3. Follow the prompts to submit your PDF to the assignment **HW 3**.

   Refer to the *Submitting Assignments With Gradescope* section of this course if you need a reminder of how to submit your assignment in Gradescope.

Homework Information and Guidelines:

1. Each student must turn in their own homework assignment and complete their own calculations, coding, etc. independently.
   - However, we encourage you to use most available resources including other textbooks, information from similar courses online, discussions with class and lab mates, office hours, etc. with the exception of using old solutions to the exact problems to complete the assignment.
   - Cite all your sources including discussions with colleagues and online websites. For example, if you and a friend compare answers or work together on a problem indicate this at the end of the problem. As an example, you might have a list like this at the end of each problem: "Resources used: [1] Fourier, J.B.J., *La Theorie Analytique de la Chaleur*, F. Didot, 1822. [2] Discussion with Joseph Fourier. [3] Wikipedia: Heat Flux, https://en.wikipedia.org/wiki/Heat_flux."
   - If we find papers with identical answers and approaches that do not indicate collaboration in the resource list, this is a violation of the academic honesty policy. Similarly, if your solutions match resources available online, this is a violation of the academic honesty policy.
2. Homework will be collected via Gradescope and is due at 11:59 PM ET. A grace period of 15 minutes is allowed in case of issues during the upload. Beyond that no late homework is accepted.
3. You may submit handwritten solutions or type up your solutions. We encourage you to use computer programs of your choice to solve problems, and some problems will require computational solutions. Recall "sketch" indicates you can draw something by hand, while "plot" indicates you should quantitatively calculate the curves and will likely use a computer program to create the graph. If "sketching" a curve, make sure trends, boundary conditions, etc. are clear.

Problem 1: Lumped Capacitance

A sensor on a satellite must be cooled in order to function. It has a mass $M = 0.05$ kg and a specific heat capacity of $c = 300\ J/kg\ K$. The surface area of the instrument is $A_s = 0.02\ m^2$ and the emissivity of its surface is $\epsilon = 0.35$. The instrument is exposed to a radiative heat transfer from surroundings at $T_{sur} = 300\ K$. It is connected to a cryocooler removes $Q_{cc} = 5\ W$. The instrument is exposed to solar radiation that oscillates according to $q_s'' = \bar{q}_s'' + \Delta q_s'' \sin\omega t$, where $\bar{q}_s'' = 100\ W/m^2$, $\Delta q_s'' = 100\ W/m^2$, and $\omega = 0.02094\ rad\ s^{-1}$. The initial temperature of the instrument is $T_{int} = 300\ K$. Assume the sensor can be modeled with lumped capacitance.

be modeled with lumped capacitance.

1. Using an energy balance on the sensor, derive an ordinary differential equation that could be solved for the temperature of the sensor as a function of time ($T(t)$). Identify the initial condition.
2. Either by hand or using a computer program of your choice, solve the ODE and plot the temperature as a function of time until it reaches a steady-periodic oscillatory response (at least 5000 s). Note, if solving by hand, you will need to separate into a homogeneous and a particular ODE.
3. At approximately the average temperature of the steady-periodic oscillations, estimate a radiation heat transfer coefficient between the object and the surroundings. Then calculate an effective Biot number.

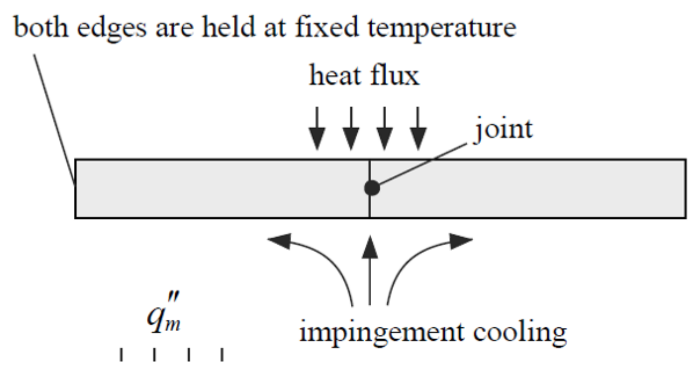
Problem 2: Semi-Infinite Solid

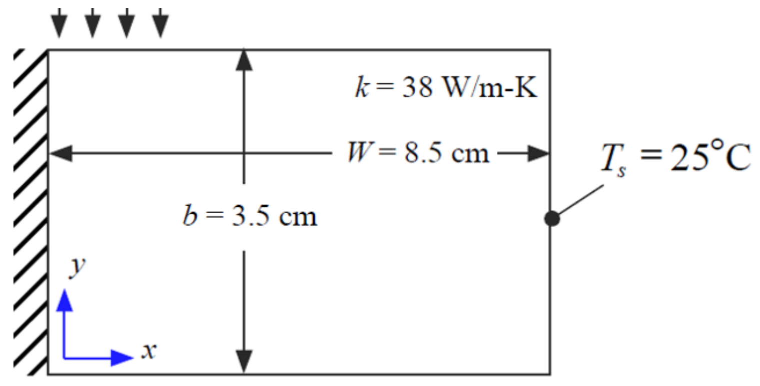
A thermocouple is embedded in a sample at a known depth while the surface is exposed to a known temperature. The transient evolution of the thermocouple temperature provides data to extract the thermal conductivity of the sample. Consider an experiment with a thermocouple embedded at $L = 10$ mm below the surface of a material with density $\rho = 2500\ kg/m^3$ and specific heat $c_p = 1000\ J/kg$ K. The surface is suddenly exposed to boiling water and reaches $T_s = 100\,^\circ C$ instantly.

1. Starting from the heat diffusion equation, simplify the differential equation as much as possible for this system. Identify boundary and initial conditions.
2. Show that your solution in (1) matches one of the cases discussed in class. Identify the correct solution for the temperature as a function of space and time.
3. The thermocouple initially reads $T_i = 20\,^\circ C$. After $\Delta t = 80$ s, the thermocouple reads $60\,^\circ C$. What is the thermal conductivity of the material?
4. How much thermal energy per unit area (J/m^2) is transferred to the material in the first 80 s?
5. Plot (a) the temperature of the thermocouple ($T(x = 10$ mm, $t)$) and (b) the heat flux at the surface ($q''(x = 0, t)$) as a function of time from 0 to 10 min.
6. If the sample is 25 mm thick, for approximately what range of times is the semi-infinite approximation a good representation of the temperatures in the sample?
7. Assuming the back side of the sample is well insulated, sketch (by hand) the temperature of the finite length sample (25 mm thick) as a function of depth at several times during the heating process: 0 s, 80 s, 200 s, and 600 s. Clearly label each line.

Problem 3: Separation of Variables - 2D Steady State

Two plates are being welded together as shown in the schematic. Both edges of the plate are clamped and effectively held at temperature $T_s = 25\,^\circ C$. The top of the plate is exposed to a spatially-varying heat flux $q''_m(x)$. The back sides of the plates are cooled with impingement cooling by a jet of fluid at $T_f = -35\,^\circ C$ with $h = 5000\ W/m^2K$. Use symmetry to break the problem in half along the centerline. The plate is $b = 3.5$ cm thick and each of the plates is $W = 8.5$ cm long. You may assume the process is steady state and two-dimensional, and you may neglect convection on the top surface. The conductivity of the plate is $k = 38\ W/mk$.





1. Simplify the heat diffusion equation inside the domain and write all needed boundary conditions.
2. Derive a symbolic expression for the steady-state temperature profile $T(x, y)$. Be sure to clearly identify the equations for the eigenvalues and the constants in the summation. Integrals need not be simplified as they can be evaluated numerically.

Assume now that $q_m''(x) = \begin{cases} q_j'' \exp -\frac{x}{L_j} & \text{for } x < L_j \\ 0 & \text{for } x > L_j \end{cases}$, where $q_j'' = 10^6\ W/m^2$ and $L_j = 2.0$ cm:

3. Calculate and plot the two-dimensional temperature distribution $T(x, y)$ as a contour map or surface plot. Make sure to include a scale bar for temperature and label your axes.
4. On one set of axes, plot the temperature as a function of x at $y = 0, 1.0, 2.0, 3.0,$ and 3.5 cm. Clearly label the axes and lines.
5. Now let's look at the impact of h. On one graph, plot the temperature at the bottom surface ($y = 0$) as a function of x for $h = 0, 200,$ and $5000\ W/m^2K$. On a second graph, plot the temperature as a function of x for the top surface ($y = 3.5$ cm for $h = 0, 200,$ and $5000\ W/m^2K$. Clearly label each graph.

Problem 4: Separation of Variables - 1D, Transient

A $L = 25$ mm thick slab of material, initially at $T_i = 20\,°C$, is suddenly heated on one surface with boiling water. The convection to boiling water can be approximated with a heat transfer coefficient $h_o = 1000\ W/m^2K$ and free stream temperature $T_o = 100\,°C$. The other surface is well insulated. Assume the density is $\rho = 2500\ g/m^3$, specific heat is $c_p = 1000\ J/kg\ K$, and thermal conductivity is $k = 1\ W/m\ K$.

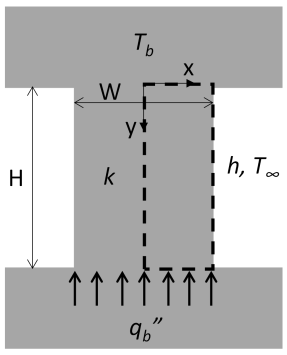
Solve the following parts symbolically:

1. What is the steady state temperature (or temperature distribution) in the slab?
2. Simplify the heat diffusion equation and convert to solving in terms of $\theta = T(x, t) - T_{ss}$. Identify boundary and initial conditions in terms of θ.
3. Using separation of variables find the expression for $\theta(x, t)$ and combine with the steady state temperature to find $T(x, t)$. Be sure to identify the eigenvalue criteria and expressions for the constants in the summation, although you do not need to simplify integrals.

Now using the numbers given in the problem statement:

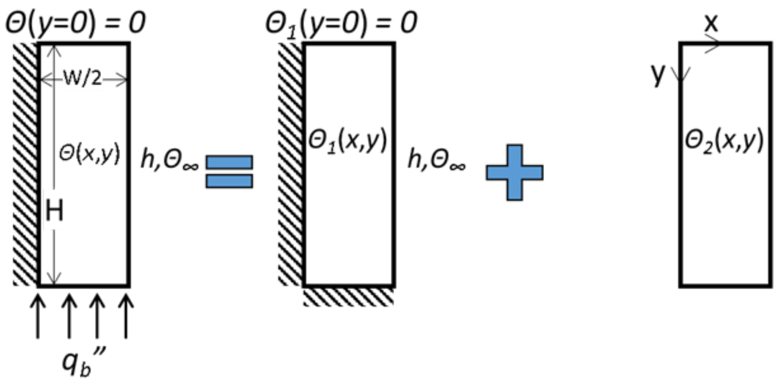
4. On one set of axes (T vs. t), plot the evolution of the surface temperatures ($T(x = 0, t)$ and $T(x = L, t)$) from $t = 0$ to approximately steady state. Clearly label the two curves.
5. On one set of axes (T vs. x), plot the temperature distribution at several times between t = 0 and the steady state final temperature distribution. Clearly label the curves with the time.
6. Qualitatively, how do your results compare with Problem 2?

Problem 5: Separation of Variables plus Superposition



A rectangular slab that is very long in the z-direction (into page) is connecting two plates. The slab has a thermal conductivity k and is placed in an ambient air at T_∞ with a convection coefficient of h. The top plate is held at a constant temperature T_b using a heat exchanger. The bottom plate provides a uniform heat flux of q_b''. Note the location of the origin and the direction of the axes shown in the figure.

1. Define $\theta = T - T_b = \theta_1(x,y) + \theta_2(x,y)$ as indicated in the schematics below and $\theta_b = T_\infty - T_b$. Find expressions for each boundary condition for the θ_2 sub-problem. [The hashmarks indicate adiabatic/symmetry boundary conditions: $q'' = 0$].



2. If q_b'' is not symmetric about $x = 0$, the domain must be extended to $-W/2 \leq x \leq W/2$. For this expanded domain, define the sub-problems that could be used with superposition to solve this new problem. In other words, show the domain and write out all boundary conditions similar to the addition sketch above. Hint, you may want to switch the definition of $\theta = T - ?$ to minimize the number of sub-problems.

For extra practice, solve these problems with separation of variables.

Problem 6: Frequency Domain

Consider the sensor of Problem 1. Here we only want to focus on the steady periodic oscillations. Recall the sensor has mass $M = 0.05$ kg, specific heat capacity of $c = 300\ J/kg\ K$, surface area of the instrument is $A_s = 0.02\ m^2$, and the emissivity of its surface is $\epsilon = 0.35$. It is exposed to a radiative heat transfer from surroundings at $T_{sur} = 300\ K$. The instrument is exposed to solar radiation that oscillates according to $q_s'' = \bar{q}_s'' + \Delta q_s'' \sin\omega t$, where $\bar{q}_s'' = 100\ W/m^2$, $\Delta q_s'' = 100\ W/m^2$, and $\omega = 0.02094\ rad\ s^{-1}$. To simplify the problem, let's consider the case where the cryocooler is off ($Q_{cc} = 0$). Assume the sensor can be modeled with lumped capacitance.

1. Estimate a radiation heat transfer coefficient h_r to the surroundings assuming an average temperature of 75 K.
2. Derive an ordinary differential equation for the temperature of the sensor as a function of time assuming the linearized form of radiation heat exchange with the surroundings.
3. Since the heat flux is varying sinusoidally, the temperature will respond sinusoidally: $T_{sensor}(t) = A \sin\omega t + \phi$, where A is the amplitude of the oscillations and ϕ is the phase delay between the heating and the temperature response. Substitute the sinusoidal temperature response into the ODE from (2). Then, solve for the amplitude and phase of the temperature response.
4. Compare the amplitude of the temperature oscillations to your results from Problem 1.

4. Compare the amplitude of the temperature oscillations to your results from Problem 4.

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Piazza

Post your questions/comments about Homework 3 to the *HW3* discussion forum in Piazza below (optional).

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