Homework 4

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Homework due Mar 5, 2021 23:14 CST Past due

Due Date

Friday, 3/5 at 11:59 PM ET *(3/6 at 04:59 UTC)*

Directions:

1. Answer each of the homework problems listed below.
2. Click the **Click here to open Gradescope button** below to access Gradescope.
3. Follow the prompts to submit your PDF to the assignment **HW 4**.

   Refer to the *Submitting Assignments With Gradescope* section of this course if you need a reminder of how to submit your assignment in Gradescope.

Homework Information and Guidelines:

1. Each student must turn in their own homework assignment and complete their own calculations, coding, etc. independently.
   - However, we encourage you to use most available resources including other textbooks, information from similar courses online, discussions with class and lab mates, office hours, etc. with the exception of using old solutions to the exact problems to complete the assignment.
   - Cite all your sources including discussions with colleagues and online websites. For example, if you and a friend compare answers or work together on a problem indicate this at the end of the problem. As an example, you might have a list like this at the end of each problem: "Resources used: [1] Fourier, J.B.J., *La Theorie Analytique de la Chaleur*, F. Didot, 1822. [2] Discussion with Joseph Fourier. [3] Wikipedia: Heat Flux, https://en.wikipedia.org/wiki/Heat_flux."
   - If we find papers with identical answers and approaches that do not indicate collaboration in the resource list, this is a violation of the academic honesty policy. Similarly, if your solutions match resources available online, this is a violation of the academic honesty policy.
2. Homework will be collected via Gradescope and is due at 11:59 PM ET. A grace period of 15 minutes is allowed in case of issues during the upload. Beyond that no late homework is accepted.
3. You may submit handwritten solutions or type up your solutions. We encourage you to use computer programs of your choice to solve problems, and some problems will require computational solutions. Recall "sketch" indicates you can draw something by hand, while "plot" indicates you should quantitatively calculate the curves and will likely use a computer program to create the graph. If "sketching" a curve, make sure trends, boundary conditions, etc. are clear.

Problem 1: Numerical - 1D Steady State

Chemical reactions within a hay bale release heat. This is an example of conversion of chemical to thermal energy. The amount of thermal energy generation depends on the moisture content of the hay when it is baled. Baled hay can become a fire hazard if the rate of volumetric energy generation is sufficiently high and the hay bale is sufficiently large so that the interior temperature of the bale reaches $76\,^{\circ}C$, the temperature at which self-ignition can occur.

Let's model the hay bale as an infinitely long cylinder with no variations in z or θ, such that it can be modeled as 1-D radial conduction. Consider steady state. The radius of the hay bale is $R_{bale} = 5$ ft and has a thermal conductivity of $k = 0.04\ W/mK$. The bale is wrapped in plastic (thickness $t_p = 0.045$ inch and conductivity $k_p = 0.15\ W/mK$). The bale is surrounded by air at $T_\infty = 20\,°C$ with $h = 10\ W/m^2K$. Divide the system into N nodes from $r = 0$ to R_{bale} with nodes placed on the boundaries at 0 and 5 ft. You may neglect radiation. The heat generation within the bale may be temperature-dependent: $\dot{q}(T)$.

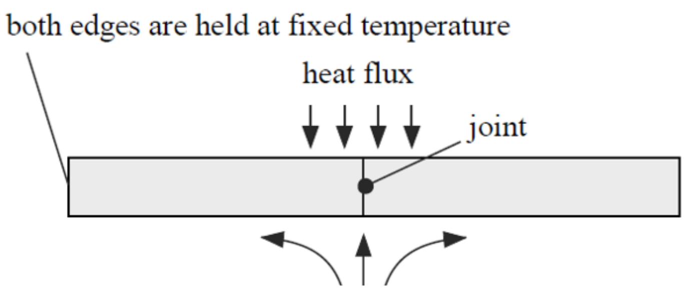
1. Derive the discretized form of an energy balance for an interior node. Note the cross-sectional area for conduction and the volume of each annular control volume varies with position. You may approximate the discretized volume as $dV = 2\pi r_i dr$, where r_i is the radius of the i^{th} node and dr is the width of the annulus.
2. Derive the discretized form of the energy balance on the first node ($r = 0$).
3. Calculate an effective thermal resistance between the outer surface of the hay and the free stream considering the plastic wrap and convection. Then derive the discretized form of the energy balance at the outermost node.
4. First, assuming $\dot{q}$ is a constant $1.5\ W/m^3$, develop a numerical model to predict the temperature distribution within the hay.
   - Plot the temperature as a function of radius for $N = 5$, 10, 25, 100, 250, and 500 nodes.
   - Plot the calculated maximum temperature as a function of the number of nodes.
   - Comment on how many nodes are required to balance computational cost and accuracy.
5. Now assume $\dot{q}$ is temperature dependent $q(T) = 1.5\left[W/m^3\right]\left[\exp\left(\frac{T}{320K}\right)\right]^{0.5}$, where T is the local temperature in K. Modify your numerical model to predict the temperature distribution within the hay. Use an appropriate number of nodes to balance computational time and accuracy.

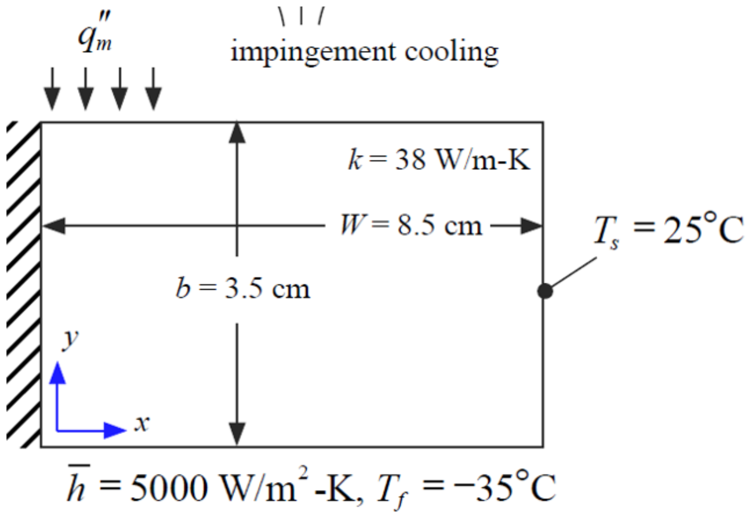
   Note, there are multiple methods to accomplish this. One approach is to use your answer from (4) to estimate $\dot{q}(T)$, then calculate the temperature distribution again. Compare the input and output temperature distributions. If they don't match sufficiently well, repeat with the new temperature distribution as the input until the solution converges.

   For your final answer, plot the temperature as a function of radius.

Problem 2: Numerical - 2D Steady State

Recall Problem 4 from Homework 3: Two plates are being welded together as shown in the schematic. Both edges of the plate are clamped and effectively held at temperature $T_s = 25\,°C$. The top of the plate is exposed to a spatially-varying heat flux $q''_m(x) = \begin{cases} q''_j \exp-\frac{x}{L_j} & \text{for } x < L_j \\ 0 & \text{for } x > L_j \end{cases}$, where $q''_j = 10^6\ W/m^2$ and $L_j = 2.0$ cm}. The back sides of the plates are cooled with impingement cooling by a jet of fluid at $T_f = -35\,°C$ with $h = 5000\ W/m^2K$. Use symmetry to break the problem in half along the centerline. The plate is $b = 3.5$ cm thick and each of the plates is $W = 8.5$ cm long. You may assume the process is steady state and two-dimensional, and you may neglect convection on the top surface. The conductivity of the plate is $k = 38\ W/mK$.





Assign a grid system with N nodes in the x-direction and M nodes in the y-direction. Place the first and last nodes in each direction directly on the boundary such that edge nodes have control volumes of half the width or height of a full control volume and corner nodes are one-quarter of a full control volume.

1. Use energy balances on the discretized control volume to find discretized forms of the relevant equations for the following nodes:

   - An interior node
   - A node on the bottom face with convection
   - A node on the top face with the specified heat flux $q''_m(x)$
   - A node on the left face with an insulated boundary condition
   - The bottom left corner with mixed boundary conditions
   - The top left corner with mixed boundary conditions

   Note that Δx may not equal Δy depending on your choice of M and N.

2. Implement your model in the computer program of your choice.

   - On one set of axes, plot the temperature as a function of x at $y = 3.5$ cm for $N = M = 5$, 10, 50, and 100 nodes in each direction, noting how long it takes to compute each solution. Clearly label the axes and lines.
   - Compare the computational time and accuracy of the different discretization levels and compare to your solution (or the solution key) from Problem 4 of Homework 3. Then select an appropriate level of discretization for finishing this problem.
   - Plot the two-dimensional temperature distribution $T(x, y)$ as a contour map or surface plot. Make sure to include a scale bar for temperature and label your axes.
   - Using the discretized temperature results, calculate the following terms to verify the energy balance:
     - The net energy into the top surface.
     - The net energy conducted out of the right face.
     - The net energy convected out of the bottom face.
     - The net energy conducted out of the left face.
     - Using the terms above, confirm energy is conserved in your system using an overall energy balance.

   Note: For each conduction term, calculate the conductive flux using a discretized form of Fourier's law and sum up the contributions from each node on the face. For the convection boundary condition, calculate Newton's law of cooling for each element and sum up the contributions from each node on the bottom face.

Problem 3: Numerical - 1D Transient

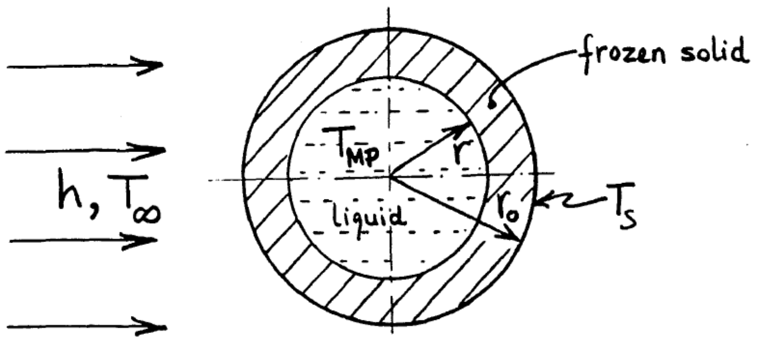
Recall Problem 4 from Homework 3: A $L = 25$ mm thick slab of material, initially at $T_i = 20\,°C$, is suddenly heated on one surface with boiling water. The convection to boiling water can be approximated with a heat transfer coefficient $h_o = 1000\ W/m^2K$ and free stream temperature $T_o = 100\,^\circ C$. The other surface is well insulated. Assume the density is $\rho = 2500\ kg/m^3$, specific heat is $c_p = 1000\ J/kg\ K$, and thermal conductivity is $k = 1\ W/m\ K$.

Now discretize the system into N nodes with the first node on the surface with boiling water and the N^{th} node on the surface with insulation.

1. Use energy balances on the discretized control volume to find discretized forms of the relevant equations for the following nodes using the explicit discretization scheme:
   - An interior node
   - The first node (on the face with boiling water)
   - The last node (on the insulated face)
2. Determine the maximum time step allowed as a function of the number of nodes.
3. Implement your model in the computer program of your choice.
   - Choose an appropriate discretization of space and time. Clearly indicate your choice and briefly justify it.
   - On one set of axes (T vs. t), plot the evolution of the surface temperatures ($T(x = 0, t)$ and $T(x = L, t)$) from $t = 0$ to approximately steady state. Clearly label the two curves.
   - On one set of axes (T vs. x), plot the temperature distribution at several times between $t = 0$ and the steady state final temperature distribution. Clearly label the curves with the time.
4. Reevaluate your energy balances in part (1) to find the discretized equations using the implicit discretization scheme. Write out the matrix equation representing the system for $N = 5$. (Optional: implement the implicit method in the computer program of your choice.)
5. Compare your results in part (2) to the results of Problem 4 of Homework 3. Comment on any advantages/disadvantages of numerical methods compared to separation of variables.

Problem 4: Solid - Liquid Phase Change

A thin walled tube of radius r_o contains a liquid phase change material that is initially at its melting point T_{MP}. The tube is exposed to convection conditions (h, T_∞) as shown in the sketch. A layer of frozen solid material develops, the radius r decreasing as time progresses. Assume the Stefan number is small, so that the temperature distribution in the solid corresponds approximately to steady state conditions (*e.g.*, conduction through the solid shell can be modeled with an appropriate thermal resistor and the temperature of the molten liquid remains uniformly at T_{MP}).



1. Sketch the resistor network between the free stream temperature and T_{MP} (the temperature at the liquid-solid interface. Then write an expression for the heat loss per unit length of pipe (q'_{loss} in W/m).
2. Using an energy balance at the (moving) liquid-solid interface, derive an implicit equation that could be solved to find the radius r of the solidification front as a function of time.
3. Find an expression for how long it will take for the outer half of the cylinder to freezer ($r = r_o/2$).
4. Now assume the liquid is water (look up appropriate thermophysical properties) in a $r_o = 10$ cm diameter pipe with $h = 25\ W/m^2K$ and $T_\infty = -5\,^\circ C$. Using a computer program of your choice, plot

the evolution of the melt front by calculating the time it takes for the melt front to reach at least 5 different radii within the pipe.

Gradescope (External resource) (100.0 points possible)

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Post your questions/comments about Homework 4 to the *HW4* discussion forum in Piazza below (optional).

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Class at a Glance

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765 total contributions
178 instructors' responses
28 students' responses

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