Homework 2

Homework due Feb 12, 2021 23:14 CST Past due

Due Date

Friday, 2/12 at 11:59 PM ET *(2/13 at 04:59 UTC)*

Directions:

1. Answer each of the homework problems listed below.
2. Click the **Click here to open Gradescope button** below to access Gradescope.
3. Follow the prompts to submit your PDF to the assignment **HW 2**.

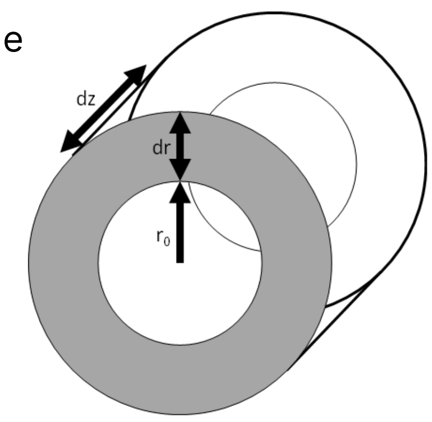
   Refer to the *Submitting Assignments With Gradescope* section of this course if you need a reminder of how to submit your assignment in Gradescope.

Homework Information and Guidelines:

1. Each student must turn in their own homework assignment and complete their own calculations, coding, etc. independently.
   - However, we encourage you to use most available resources including other textbooks, information from similar courses online, discussions with class and lab mates, office hours, etc. with the exception of using old solutions to the exact problems to complete the assignment.
   - Cite all your sources including discussions with colleagues and online websites. For example, if you and a friend compare answers or work together on a problem indicate this at the end of the problem. As an example, you might have a list like this at the end of each problem: "Resources used: [1] Fourier, J.B.J., *La Theorie Analytique de la Chaleur*, F. Didot, 1822. [2] Discussion with Joseph Fourier. [3] Wikipedia: Heat Flux, https://en.wikipedia.org/wiki/Heat_flux."
   - If we find papers with identical answers and approaches that do not indicate collaboration in the resource list, this is a violation of the academic honesty policy. Similarly, if your solutions match resources available online, this is a violation of the academic honesty policy.
2. Homework will be collected via Gradescope and is due at 11:59 PM ET. A grace period of 15 minutes is allowed in case of issues during the upload. Beyond that no late homework is accepted.
3. You may submit handwritten solutions or type up your solutions. We encourage you to use computer programs of your choice to solve problems, and some problems will require computational solutions. Recall "sketch" indicates you can draw something by hand, while "plot" indicates you should quantitatively calculate the curves and will likely use a computer program to create the graph. If "sketching" a curve, make sure trends, boundary conditions, etc. are clear.

Problem 1: Heat Diffusion Equation

Consider the toroidal-shaped control volume shown within a solid object. The thermal conductivity is constant, but anisotropic: specifically, the conductivity has one value along the axial direction (k_z) and another value in plane (k_r). Assume a spatially-varying heat generation rate $\dot{q}(r,z)$ within the medium and assume the system has reached steady state.



1. Starting with the heat diffusion equation simplify it as much as possible for the geometry and conditions described above:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_\theta \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + \dot{q}$$

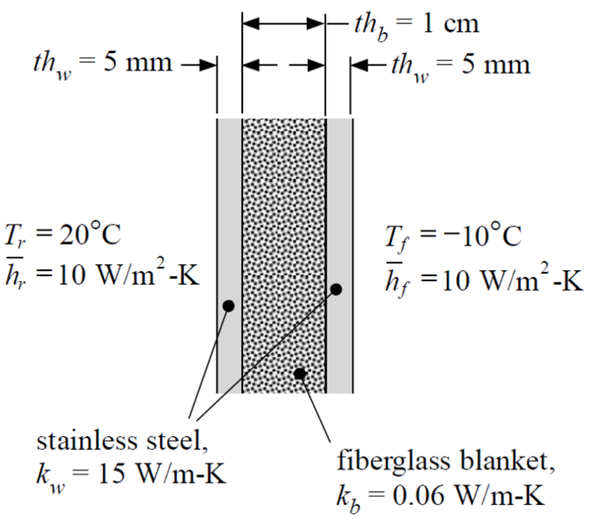
List all assumptions as you simplify the expression.

Now derive this simplified form of the heat diffusion equation from an energy balance on this control volume:

2. Write an expression for the conductive heat transfer rate, q_r, into the inner cylindrical surface (at $r = r_o$) in terms of thermal conductivity, temperatures, and any other appropriate variables.
3. Use a Taylor series expansion to evaluate the conductive heat transfer rate, q_{r+dr}, out of the outer cylindrical surface (at $r = r_o + dr$) in terms of q_r.
4. Combine your answers from (2) and (3) to find an expression for $q_r - q_{r+dr}$ in terms of thermal conductivity, temperatures, and any other appropriate variables. Consider carefully your expression for heat transfer rate in (2): what parameters change with radius?
5. Now, starting with an energy balance on the toroidal-shaped control volume shown, derive the 2D (r-z), transient heat equation for cylindrical coordinates. Show all your work (but you may reference your work in (2)-(4) as needed instead of rewriting the work).

Problem 2: 1D Steady State Heat Transfer & Thermal Resistance

You have designed a wall for a freezer as shown in the schematic. The wall separates the freezer air at $T_f = -10\,°C$ from air within the room at $T_f = 20\,°C$. The heat transfer coefficient between the freezer air and the inner wall of the freezer is $h_f = 10\ W/m^2K$. The wall is composed of a $th_b = 1.0$ cm thick layer of fiberglass blanket sandwiched between two $th_w = 5.0$ mm sheets of stainless steel. The thermal conductivity of the fiberglass and the stainless steel are $k_b = 0.06\ W/mK$ and $k_w = 15\ W/mK$, respectively. Neglect radiation from both the inner and outer walls.



1. Draw a resistance network from the room temperature T_r to the freezer temperature T_f. Calculate and label each resistor with an equation for the area normalized resistance: that is, R'' in m^2K/W.
2. Enter all inputs into the computer program of your choice (*i.e.* Excel, Jupyter, Matlab, EES, etc.) in standard base SI units. Use the computer program to calculate the total area normalized resistance (R''_{tot}) from the room air to the freezer air and the net heat flux to the freezer (q''_{net} in W/m^2).
3. Your boss wants to make a more energy efficient freezer by reducing the rate of heat transfer to the freezer. He suggests that you increase the thickness of the stainless steel wall panels in order to accomplish this.
   - a) Calculate and plot the net heat flux to the freezer as a function of the thickness of the stainless steel walls. Make sure your plot is clear with the axes labeled (with the variable and units).
   - b) Calculate and plot the net heat flux to the freezer as a function of the thickness of the fiberglass. Make sure your plot is clear with the axes labeled (with the variable and units).
   - c) Was increasing the thickness of the stainless steel a good idea? Justify your answer briefly.
4. One of the design requirements is that no condensation must form on the external surface of the freezer wall, even if the relative humidity in the room reaches 75%. This implies that the

temperature of the external surface of the freezer wall must be greater than 15°C. Does the original freezer wall satisfy this requirement? Calculate the external surface temperature (°C).

5. In order to prevent condensation, you suggest placing a heater between the outer stainless steel wall and the fiberglass.
   - a) How much heat would be required to keep condensation from forming? Assume that the heater is very thin and conductive.
   - b) Calculate and plot the required heat flux by the heater as a function of the freezer air temperature.

Problem 3: 1D Steady State Heat Transfer

A very long, hollow, plastic cylinder of inner radius r_{in} and outer radius r_{out} has a uniform thermal conductivity k and experiences a uniform rate of heat generation $\dot{q}$. The inner wall is exposed to a uniform heat flux of q''_{in} and the outer wall is maintained at T_{out}.

1. Simplify the heat diffusion equation to find an ordinary differential equation (ODE) for $T(r)$.
2. Solve the ODE to find the general form of the temperature profile. (Note, you may solve by hand or with a computer program of your choice).
3. The general form of the temperature profile has 2 unknown constant of integration that can be found from the boundary conditions. Write appropriate equations at the inner and outer wall that could be solved to find these two constants.
4. Solve for the two constants (either by hand or with a program of your choice) and find the steady-state temperature profile $T(r)$.
5. Now assume r_{in} = 5 mm, r_{out} = 25 mm, $\dot{q} = 10^6\ W/m^3$, $q''_{in} = 100\ W/m^2$, $T_{out} = 20\ ^\circ C$, and $k = 0.5\ W/mK$ in the problem and plot the temperature profile as a function of radius.

Hint: You may use Jupyter notebook "1D SS Heat Transfer (1-1-3B)" as a template for solving this problem.

Problem 4: Fins

Your company has developed a technique for forming very small fins on a plastic substrate. The width of the fins at their base is $D = 1$ mm. The ratio of the length of the fin to the base diameter is the aspect ratio, $AR = 10$. The fins are arranged in a hexagonal close packed pattern. The ratio of the distance between fin centers and to the base width is the pitch ratio, $PR = 2$. The conductivity of the plastic material is $k = 2.8\ W/mK$. The heat transfer coefficient between the surface of the plastic and the surrounding gas is $h = 35\ W/m^2K$. The base temperature is $T_b = 60\ ^\circ C$ and the gas temperature is $T_b = 35\ ^\circ C$.

You need to evaluate whether square (side width = D), triangular (equilateral triangles with side length of D), or circular (diameter = D) cross-sections provide the best performance. For each case, for a $1\ m^2$ surface:

1. Determine the number of fins. (Round down to the nearest whole number)
2. Determine the resistance of the unfinned base.
3. Calculate the resistance of 1 fin.
4. Calculate the efficiency and total resistance of the finned surface.
5. Calculate the heat transfer rate from the finned surface

Problem 5: Extended Surfaces - Advanced

A rod is extruded in a materials processing system with a diameter of $D = 5$ cm at a velocity of $u = 0.75\ ms^{-1}$. The material enters at $T_{in} = 300\ K$ and has a thermal conductivity of $k = 40\ W/mK$ and thermal diffusivity of $\alpha = 0.001\ m^2s^{-1}$. In order to precisely control the temperature of the material, the oven wall is very close to the outer diameter of the extruded material and the oven wall temperature distribution is carefully controlled. The gap between the oven wall and the material is $th = 0.6$ mm and the oven-to-material gap is filled with gas that has a thermal conductivity of $k_g = 0.03\ W/mK$. Radiation can be neglected in favor of convection through the gas from the oven wall to the material. For this situation, the heat flux experienced by the material surface can be approximately modeled according to:

$$q''_{conv} = \frac{k_g}{th}(T_w - T),$$

where T_w and T are the oven wall and material temperatures at that position. Note, this is approximating convection in the gap as a conduction: $R_{cond} = \frac{th}{k_g} = R_{conv} = \frac{1}{h_{conv}} \rightarrow h_{conv} = \frac{k_g}{th}$. The oven wall temperature may vary with position: $T_w(x)$. Assume that the oven can be approximated as being infinitely long.

1. Is an extended surface model appropriate for this problem? Define and calculate a Biot number to compare internal temperature gradients to external temperature gradients.
2. Assume that the extended surface model is appropriate, use an energy balance on a coin-shaped differential element (of length dx and circular cross-section) and Taylor Series Expansions to symbolically derive an ordinary differential equation that could be solved for $T(x)$. Recall that the rod is moving at velocity u through the system - at a position along the rod, the energy carried by the moving material is $(\rho u A_c c T)_x$.

Now assume the oven wall temperature varies with position x according to:

$$T_w = T_f - (T_f - T_{w,0})\exp\left(\frac{-x}{L_c}\right),$$

where $T_{w,0}$ is the temperature of the wall at the inlet (at $x = 0$), T_f is the temperature of the wall far from the inlet, and L_c is a characteristic length that dictates how quickly the oven wall temperature approaches T_f.

3. Assuming the rod is infinitely long, $T(x = 0) = T_{w,0}$, and the functional form of $T_w(x)$ shown above, solve the ODE for the axial temperature profile $T(x)$. (Feel free to use software tools to help solve the ODE).
4. Assuming that $T_{w,0} = 500\ K$, $T_f = 1000\ K$, and $L_c = 1$ m, plot the temperature of the rod and the wall as function of position for $0 < x < 20$ m. On a separate graph, plot the temperature gradient experienced by the material as a function of position for $0 < x < 20$ m.

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