Assignment 2

STA3036S

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Abstract

This document outlines how a research institute allocates its Research & Development Budget for the year in taking seven projects into consideration. It determines which projects out of the seven to pursue and how many researchers must be allocated to each, subject to various budgetary and resource constraints using the Simplex Linear Programming and GRG Non-Linear Programming methods in Excel.

Contents

1 Introduction

This project aims to determine the various decisions surrounding a research institutes budget and how they wish to allocate their funds for the following year.

Seven possible projects have been put forward to the Board for consideration. The institute must select which of the projects to pursue subject to the following information available:

- Thirty researchers are available across the seven projects
- The researchers' skills are compatible with any of the projects
- Set up costs and returns are seen in Table 1
- Set-up costs are a maximum of R1,750.00 across all of the projects.
- Each project takes 16 weeks to complete and a total of 90 project weeks are available.
- Computer Utilisation is measured as a percentage of total capacity.

Table 1

Project i	1	2	3	4	5	6	7
Max number of researchers	10	6	7	5	9	8	10
Set-up costs (in '000s)	325	200	490	175	560	620	240
Returns (in '000s)	700	1200	900	400	770	1800	800
Time to completion (wks)	16	16	16	16	16	16	16
Computer utilisation (% of total capacity)	18	25	20	28	30	23	35

2 Assumptions

Since the given scenario lacks various details, the linear program and the non linear program that has been formulated in this project is based on different assumptions that have been made:

- All prices are given in thousands of rands.
- For the linear programme, it is assumed that if a project is pursued, then the maximum amount of researchers allowed on that project are all allocated to it.
- For the time constraint to be binding, it is assumed that projects are worked on consecutively, however, for the researcher and computer utilisation constraints to be binding, it is assumed that they are worked on simultaneously. This contradiction is further explained in question 2.
- For the linear programme, we maximise profit but for the non-linear programme, we maximise revenue. This difference is not important however, because we get the same answer regardless of whether profit or revenue is maximised.

3 Question 1: Formulation and Solution of the Linear Programme

3.1 Decision Variables

$X_i \equiv$ Whether project i is selected or not
$r_i \equiv$ the total number of researches used in project i

Where:
$X_i \in [0;1]$; $0 =$ not pursued, $1 =$ pursued
$r_i \geq 0$

3.2 Objective Function

$P = R - C$
Where:
$P =$ Profit
$R =$ Revenue
$C =$ Cost

3.3 Revenue

$$R = \sum_{i=1}^{7} (R_i * Xi)$$

3.4 Cost

$$C = \sum_{i=1}^{7} (C_i * X_i)$$

Where:
$C_i =$ is the set up costs per project i in thousands of rands, as seen in row 3 of the table in the introduction.

3.5 Constraints

3.5.1 Researcher Constraints

The the total maximum number of researchers per project are constrained as follows. As such the formalised demand constraints are given as:

$$r_1 \leq 10$$
$$r_2 \leq 6$$
$$r_3 \leq 7$$
$$r_4 \leq 5$$
$$r_5 \leq 9$$
$$r_6 \leq 8$$
$$r_7 \leq 10$$

The total number of researchers allocated across all projects is as follows:

$$\sum_{i=1}^{7} r_i \leq 30$$

3.5.2 Computer usage Constraints

The following computer usages per project were given:

$$U_1 = 0.18$$
$$U_2 = 0.25$$
$$U_3 = 0.2$$
$$U_4 = 0.28$$
$$U_5 = 0.3$$
$$U_6 = 0.23$$
$$U_7 = 0.35$$

The total number of computer usage hours allocated across all projects is as follows:

$$\sum_{i=1}^{7} (U_i * X_i) = 1$$

3.5.3 Time Constraints

The following time to completion hours per project are given:

$$H_{1...7} = 16$$

The total number of hours used across all projects:

$$\sum_{i=1}^{7} H_i * X_i \leq 90$$

3.5.4 Integer Constraint

$$X_i \ , r_i \in \mathbb{Z}$$

3.6 Final Values and Solution

The final values and the solution as generated by the Simplex method in Excel solver are:

$$
\begin{aligned}
X_1 &= 0 & r_1 &= 0 \\
X_2 &= 1 & r_2 &= 6 \\
X_3 &= 1 & r_3 &= 7 \\
X_4 &= 1 & r_4 &= 5 \\
X_5 &= 0 & r_5 &= 0 \\
X_6 &= 1 & r_6 &= 8 \\
X_7 &= 0 & r_7 &= 0
\end{aligned}
$$

Objective function:

$$
\begin{aligned}
\text{Profit} &= \text{Revenue - Costs} \\
&= \text{R4,300.00 - R1,485.00} \\
&= \text{R2,815.00}
\end{aligned}
$$

3.7 Excel formulation

Below is the solution obtained in Excel using Solver:

Figure 1: Excel sheet displaying the Linear Programme

Project i	1	2	3	4	5	6	7
Max Researchers	10	6	7	5	9	8	10
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00
Time to completion (weeks)	16	16	16	16	16	16	16
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%
Project	1	2	3	4	5	6	7
Persue?	0	1	1	1	0	1	0
Researchers	0	6	7	5	0	8	0
	<=	<=	<=	<=	<=	<=	<=
	10	6	7	5	9	8	10
Revenue	R4,300.00						
Costs	R1,485.00	<=	R1,750.00				
Time	64	<=	90				
Computer utilisation	96%	<=	100%				
Researchers	26	<=	30				
Profit	R2,815.00						

4 Question 2:

The solution makes sense; however, some assumptions had to be made to arrive at this solution as the project brief lacked various details. Initially, it was not explicitly given that a pursued project must have researchers working on it. Without this information, Solver set the number of researchers equal to zero for each project regardless of whether it was pursued or not. This is illogical because a pursued project should have researchers working on it. To resolve this, we made it so that if a project was pursued, its maximum number of researchers would be allocated to it. A different way of solving it would be to have a minimum number of researchers per project.
Additionally, it is unclear as to whether all the projects can be worked on simultaneously or not. If they are worked on simultaneously, then the time constraint would be affected: all projects could be completed in 16 weeks if this was the case; however, the brief implies that it is not the case. In solving this project, we assume that projects are worked on one after the other so that the time constraint is actually binding. On the other hand, if they are worked on consecutively then the computer utilisation and researcher constraints' interpretation changes. All the available computer utilisation could be devoted to one project and the same is true for researchers: all the available researchers could be used for one project. However, given the wording of the brief, it is assumed that this is not the case, as then, these constraints would not be binding.

5 Questions 3 & 4: Formulation & Solution of the Non Linear Programme

5.1 Decision Variables

$X_i \equiv$ Whether a project is pursued or not; $X \in [0, 1]$
$r_i \equiv$ the number of researchers

Maximise:

$$\begin{aligned}\text{Returns} = \text{R}700X_1p_1 + \text{R}1200X_2p_2 \\ + \text{R}900X_3p_3 + \text{R}400X_4p_4 + \text{R}770X_5p_5 \\ + \text{R}1800X_6p_6 + \text{R}800X_7p_7\end{aligned}$$

Where:

$$p_i = 1 - 1.1e^{-a_ir_i};\ 0 \leq p \leq 1$$

Project i	1	2	3	4	5	6	7
a_i	0.2	0.5	0.3	0.4	0.25	0.3	0.2

$$r_i = \text{is the number of researchers}$$

5.2 Constraints

Subject to:

Costs:

$$R325X_1 + R200X_2 + R490X_3 + R175X_4 + R560X_5 + R620X_6 + R240X_7 \leq R1750$$

Time:

$$t_i = 15 + \frac{20}{r_i};\ \text{for } r_i > 0;\ 0 \text{ if } r_i = 0$$

$$\sum_{i=1}^{7}(t_i * X_i) \leq 90$$

Researchers:

$$\sum_{i=1}^{7}(r_i * X_i) \leq 30$$

Computer Utilisation:

$$\sum_{i=1}^{7} U_i * X_i \leq 1$$

Linking Constraint:

$$r_i - 30X_i \leq 0$$

The linking constraint ensures that if a project is not pursued, then no researchers will be allocated to it.

5.3 Final Answers

$$\begin{aligned} X_1 &= 1 & r_1 &= 9 \\ X_2 &= 1 & r_2 &= 6 \\ X_3 &= 1 & r_3 &= 7 \\ X_4 &= 0 & r_4 &= 0 \\ X_5 &= 0 & r_5 &= 0 \\ X_6 &= 1 & r_6 &= 8 \\ X_7 &= 0 & r_7 &= 0 \end{aligned}$$

Using the solution from the linear programme as a starting point, the value of our objective function came to R2471,15.

5.4 Excel formulation

Figure 2: Excel sheet displaying the Original Non Linear Programme

Project i	1	2	3	4	5	6	7	
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00	
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00	
Expected Returns ('000)	R572.72	R1,134.28	R778.77	R0.00	R0.00	R1,620.38	R0.00	
Time To completion	17.22222	18.33333	17.85714	0	0	17.5	0	
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%	
αi	0.2	0.5	0.3	0.4	0.25	0.3	0.2	
Negative probabilites	0.818171	0.945234	0.865298	-0.1	-0.1	0.90021	-0.1	
Pr(success)	0.818171	0.945234	0.865298	0	0	0.90021	0	*If statement included to eliminate negative probabilities.
Linking constraint	-21	-24	-23	0	0	-22	0	
Researchers	9	6	7	0	0	8	0	
	<=	<=	<=	<=	<=	<=	<=	
Constraint	10	6	7	5	9	8	10	
Persue?	1	1	1	0	0	1	0	
Expected returns	R4,106.15							
Costs	R1,635.00	<=	R1,750.00					
Time	70.9127	<=	90					
Computer utilisation	86%	<=	100%					
Researchers	30	<=	30					
Expected profit	R2,471.15							

6 Question 5

The table below represents the optimal solutions using randomly selected different starting points. As seen, the best optimal solution would be solution three as it generates the highest overall profit, this being R2,471.15 in the non linear programming case, with $X_1 = X_2 = X_3 = X_6 = 1$ & $X_4 = X_5 = X_6 = 0$ and $r_1 = 9; r_2 = 6; r_3 = 7; r_4 = 0; r_5 = 0; r_6 = 8; r_7 = 0$. An optimal solution can be found at multiple points as there are multiple optimal solutions for the non-convex GRG non-linear programmes. This happens to be the same value achieved in the original non-linear programme of R2,471.15. This can be attributed to the fact that this NLP is a non convex problem meaning that there can be a plethora of local maximums and depending on where the program starts it will establish its local maximum. The Generalized Reduced Gradient method was used for this NLP. It does not establish a global optima as this solver method looks at the gradient or slope of the objective function as the decision variables change and determines that it has achieved an optimum solution when the partial derivatives equal zero (Which could occur in many places).

6.1 Excel formulation

Figure 3: Excel sheet displaying the different optimal solutions at different starting points for the Non Linear Programme

Solution	Project		1	2	3	4	5	6	7	Value of objective function
1	Starting point	Pursue	0	0	0	0	0	0	0	R0.00
		No. of researchers	0	0	0	0	0	0	0	
	Solution	Pursue	0	0	0	0	0	0	0	R0.00
		No. of researchers	0	0	0	0	0	0	0	
2	Starting point	Pursue	0	1	1	0	0	1	0	R2,362.25
		No. of researchers	0	6	7	0	0	8	10	
	Solution	Pursue	0	1	0	0	0	1	1	R2,375.56
		No. of researchers	0	6	0	0	0	8	10	
3	Starting point	Pursue	1	1	1	1	1	1	1	R3,221.30
		No. of researchers	10	6	7	5	9	8	10	
	Solution	Pursue	1	1	1	0	0	1	0	R2,471.15
		No. of researchers	9	6	7	0	0	8	0	
4	Starting point	Pursue	0	1	0	1	0	1	0	R2,100.11
		No. of researchers	0	6	0	5	0	8	10	
	Solution	Pursue	0	1	0	0	0	1	1	R2,375.56
		No. of researchers	0	6	0	0	0	8	10	

Figure 4: Excel sheet displaying the second Non Linear Programme

Project i	1	2	3	4	5	6	7
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00
Expected Returns ('000)	R0.00	R0.00	R0.00	R0.00	R0.00	R0.00	R0.00
Time To completion	0	0	0	0	0	0	0
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%
αi	0.2	0.5	0.3	0.4	0.25	0.3	0.2
Non-negative probabilites	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
Pr(success)	0	0	0	0	0	0	0
Linking constraint	0	0	0	0	0	0	0
Researchers	0	0	0	0	0	0	0
	<=	<=	<=	<=	<=	<=	<=
Constraint	10	6	7	5	9	8	10
Persue?	0	0	0	0	0	0	0
Expected returns	R0.00						
Costs	R0.00	<=	R1,750.00				
Time	0	<=	90				
Computer utilisation	0%	<=	100%				
Researchers	0	<=	30				
Expected profit	R0.00						

Figure 5: Excel sheet displaying the second Non Linear Programme

Project i	1	2	3	4	5	6	7
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00
Expected Returns ('000)	R0.00	R1,134.28	R0.00	R0.00	R0.00	R1,620.38	R680.90
Time To completion	0	18.33333	0	0	0	17.5	17
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%
αi	0.2	0.5	0.3	0.4	0.25	0.3	0.2
Non-negative probabilites	-0.1	0.945234	-0.1	-0.1	-0.1	0.9002103	0.851131
Pr(success)	0	0.945234	0	0	0	0.9002103	0.851131
Linking constraint	0	-24	0	0	0	-22	-20
Researchers	0	6	0	0	0	8	10
	<=	<=	<=	<=	<=	<=	<=
Constraint	10	6	7	5	9	8	10
Persue?	0	1	0	0	0	1	1
Expected returns	R3,435.56						
Costs	R1,060.00	<=	R1,750.00				
Time	52.83333	<=	90				
Computer utilisation	83%	<=	100%				
Researchers	24	<=	30				
Expected profit	R2,375.56						

Figure 6: Excel sheet displaying the third Non Linear Programme

Project i	1	2	3	4	5	6	7
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00
Expected Returns ('000)	R572.72	R1,134.28	R778.77	R0.00	R0.00	R1,620.38	R0.00
Time To completion	17.22222	18.33333	17.85714	0	0	17.5	0
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%
αi	0.2	0.5	0.3	0.4	0.25	0.3	0.2
Non-negative probabilites	0.818171	0.945234	0.865298	-0.1	-0.1	0.90021	-0.1
Pr(success)	0.818171	0.945234	0.865298	0	0	0.90021	0
Linking constraint	-21	-24	-23	0	0	-22	0
Researchers	9	6	7	0	0	8	0
	<=	<=	<=	<=	<=	<=	<=
Constraint	10	6	7	5	9	8	10
Persue?	1	1	1	0	0	1	0
Expected returns	R4,106.15						
Costs	R1,635.00	<=	R1,750.00				
Time	70.9127	<=	90				
Computer utilisation	86%	<=	100%				
Researchers	30	<=	30				
Expected profit	R2,471.15						

Figure 7: Excel sheet displaying the fourth Non Linear Programme

Project i	1	2	3	4	5	6	7
Set up costs ('000)	R325.00	R200.00	R490.00	R175.00	R560.00	R620.00	R240.00
Returns ('000)	R700.00	R1,200.00	R900.00	R400.00	R770.00	R1,800.00	R800.00
Expected Returns ('000)	R0.00	R1,134.28	R0.00	R0.00	R0.00	R1,620.38	R680.90
Time To completion	0	18.33333	0	0	0	17.5	17
Computer utilisation (%of total capacity)	18%	25%	20%	28%	30%	23%	35%
αi	0.2	0.5	0.3	0.4	0.25	0.3	0.2
Non-negative probabilites	-0.1	0.945234	-0.1	-0.1	-0.1	0.90021	0.851131
Pr(success)	0	0.945234	0	0	0	0.90021	0.851131
Linking constraint	0	-24	0	0	0	-22	-20
Researchers	0	6	0	0	0	8	10
	<=	<=	<=	<=	<=	<=	<=
Constraint	10	6	7	5	9	8	10
Persue?	0	1	0	0	0	1	1
Expected returns	R3,435.56						
Expected costs	R1,060.00	<=	R1,750.00				
Time	52.83333	<=	90				
Computer utilisation	83%	<=	100%				
Researchers	24	<=	30				
Expected profit	R2,375.56						

7 Member Contributions

The group worked cohesively together with each member substantially to the project.
Kiara Beilinsohn solved the Linear Programme.
Matthew Whall solved the Non Linear Programme.
Both members worked together on the abstract and introduction.

8 Plagiarism Declarations

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