Assignment 1

STA3036S

Matthew Whall	Kiara Beilinsohn
WHLMAT002	BLNKIA001

13 September 2019

Abstract

This document outlines Plastics Inc's possibility of reaching a 20% profitability target, subject to constraints using the linear solver method in Excel.

Contents

1 Introduction **2**

2 Assumptions **2**

3 Formulation of the Linear Programme **3**
3.1 Decision Variables 3
3.2 Objective Function 3
3.3 Revenue 4
3.4 Cost 4
3.4.1 Costs per hour 5
3.4.2 Costs per sheet: 5
3.5 Constraints 6
3.5.1 Demand Constraints 6
3.5.2 Time Constraints 7
3.5.3 Integer Constraint 7

4 Final Values and Solution **8**

5 Sensitivity Report Analysis **9**
5.1 Variable Cells 12
5.2 Constraints 13
5.2.1 Hour Constraints 13
5.2.2 Overtime 13
5.2.3 Demand constraints 14

6 Conclusion **14**

7 Member Contributions **15**

8 Plagiarism Declarations **16**

1 Introduction

Plastics Inc. is a small company that contains two semi-automatic production lines, referred to as line 1 and line 2. They both produce high-quality plastics of various types, namely: plastic from recycled material and standard plastic. However, the production of standard plastic is a quicker process. It sells recycled plastic at R700,00 per sheet and standard plastic at R800,00 per sheet.
Planning at Plastics Inc is done on a quarterly and annual basis. The labour demand is based off forecasts for the future year, however changes can occur during the current year.
Plastics Inc. has been through a recent expansion at the plant and as a result there is no space for future plastic production to be stored, however inventory can be stored at a public warehouse.
Each line produces components independently, and each faces its own independent constraints such as working hours available. Each type of plastic also faces its own independent demands for the given month. Plastics Inc management wishes to determine if a 20% profitability target is possible. In this project the 20% profitability target is classified as possible or impossible.

2 Assumptions

Since the given scenario lacks various details, the linear program that has been formulated in this project is based on different assumptions that have been made. The first assumption is - given the fact that storage is allowed, I can assume that there is nothing stored at the start of month one (from May). The second assumption is that the totals for the given accounting data are for the three month period. The third assumption is that the number of plastic sheets is an integer. This assumption is ignored when interpreting the sensitivity report because one cannot generate a sensitivity report in Excel with integer constraints. Finally, in all recommendations given to the Plastics Inc. at the end of the report, they are given with the sole goal of maximising profit.

3 Formulation of the Linear Programme

3.1 Decision Variables

$X_{ijk} \equiv$ The number of sheets of type i produced on line j in month k
$I_{ik} \equiv$ The total number of sheets of type i produced in month k in inventory
$S_{ik} \equiv$ The number of sheets of type i stored in month k

Where:
$i \in 1$ (recycled) ; 2 (standard)
$j \in$ (line) 1; (line) 2
$k \in 1$ (June); 2 (July); 3 (August)

$I_{ik} = \sum_{j=1}^{2} X_{ijk} + S_{ik-1}$
$D_{ik} =$ Total demand of sheet type i in month k
$S_{ik} = I_{ik} - D_{ik}$

3.2 Objective Function

$P = R - C$

Where:
$P =$ Profit
$R =$ Revenue
$C =$ Cost

3.3 Revenue

$$\mathrm{R} = \sum_{i=1}^{2}\sum_{k=1}^{3} D_{ik} \times \mathrm{R}700{,}00 + \sum_{i=1}^{2}\sum_{k=1}^{3} D_{ik} \times \mathrm{R}800{,}00$$

3.4 Cost

$$C = \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{3}(LC_{ijk} + PO_{ijk} + OC_{ijk} + PC_{ijk} + SC_{ijk})$$

Where:

LC_{ijk} = Total Labour costs to produce sheets of type i on line j in month k.

PO_{ijk} = Total production overheads from producing sheets of type i on line j in month k.

OC_{ijk} = Total overall cost to produce sheets of type i on line j in month k.

PC_{ijk} = Total production costs to produce sheets of type i on line j in month k.

SC_{ijk} = Total cost of storing sheets of type i on line j in month k.

3.4.1 Costs per hour

The following available hours Ire given:

Process	hours per recycled plastic sheet	hours per standard plastic sheet
Line 1	3.0	1.5
Line 2	4.0 .	2.5

Total number of hours:
$H_{jk} = H =$ The total number of hours required to produce plastic on line j in month k

$$H = 3 \times \sum_{k=1}^{3} X_{11k} + 4 \times \sum_{k=1}^{3} X_{12k} + 1.5 \times \sum_{k=1}^{3} X_{21k} + 2,5 \times \sum_{k=1}^{3} X_{22k}$$

The following costs per hour Ire given:

Cost Category	Line 1	Line 2
Labour (LC)	R32,00	R32,00
Production Overheads (PO)	R24,00	R24,00

Labour costs:

$$LC_{ijk} = \text{R32,00} \times H$$

Production overheads:

$$PO_{ijk} = \text{R24,00} \times H$$

3.4.2 Costs per sheet:

The following costs per sheet Ire given:

Cost	Recycled plastic (R/sheet)	Standard Plastic (R/sheet)
Overall	R320,00	R400,00
Packaging	R80,00	R80,00
Storage	R20,00	R20,00

Overall costs:

$$OC_{ijk} = \text{R320,00} \times \sum_{j=1}^{2} \sum_{k=1}^{3} X_{1jk} + \text{R400,00} \times \sum_{j=1}^{2} \sum_{k=1}^{3} X_{2jk}$$

Packaging costs:

$$PC_{ijk} = \text{R80,00} \times \left(\sum_{j=1}^{2} \sum_{k=1}^{3} X_{1jk} + \sum_{j=1}^{2} \sum_{k=1}^{3} X_{2jk} \right)$$

Storage costs:

$$SC_{ijk} = \text{R20,00} \times \sum_{i=1}^{2} \sum_{k=1}^{3} S_{ik}$$

3.5 Constraints

3.5.1 Demand Constraints

The following demands Ire given:

Month	Recycled plastic	Standard Plastic
June	75	45
July	150	90
August	75	75

As such the formalised demand constraints are given as:

$$I_{11} \geq 75 \tag{1}$$
$$I_{12} \geq 150 \tag{2}$$
$$I_{13} \geq 75 \tag{3}$$
$$I_{21} \geq 45 \tag{4}$$
$$I_{22} \geq 90 \tag{5}$$
$$I_{23} \geq 75 \tag{6}$$

3.5.2 Time Constraints

The following available hours per month are given:

Month	Recycled plastic	Standard plastic
June	280	500
July	150	160
August	300	200

As such the time constraints are given as:

$$H_{11} \leq 280 \tag{7}$$
$$H_{12} \leq 150 \tag{8}$$
$$H_{13} \leq 300 \tag{9}$$
$$H_{21} \leq 500 \tag{10}$$
$$H_{22} \leq 160 \tag{11}$$
$$H_{23} \leq 200 \tag{12}$$

3.5.3 Integer Constraint

$$X_{ijk} \in Z \tag{13}$$

4 Final Values and Solution

The final values and the solution as generated by solver in the "No overtime" Excel sheet are:

$$
\begin{aligned}
X_{111} &= 71 \\
X_{112} &= 5 \\
X_{113} &= 62 \\
X_{121} &= 109 \\
X_{122} &= 40 \\
X_{123} &= 13 \\
X_{211} &= 44 \\
X_{212} &= 90 \\
X_{213} &= 75 \\
X_{221} &= 1 \\
X_{222} &= 0 \\
X_{223} &= 0
\end{aligned}
$$

Objective function:
Profit = Revenue - Costs
= R378 000,00 - R300 068,00
= R77 932,00

Profitability $= \frac{Profit}{Revenue} \times 100$
= R77 932,00 $\div$ R378 000,00 $\times$ 100
$= 20,62\%$

5 Sensitivity Report Analysis

In order to generate the sensitivity report, the integer constraints had to be ignored as Excel cannot generate a sensitivity report given integer constraints. Thus the values used in this analysis are not the values that generated the final answer as stated in Section 4, but rather their unrounded precursors.

The sensitivity reports for "No Overtime" and "Overtime" can be seen below.

Variable Cells

	Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
	B3	Line 1 June	70.83333333	0	-628	56	20
	C3	Line 1 July	5	0	-608	20	60
Recyclec	D3	Line 1 August	62.5	0	-588	56	56
	B4	Line 2 June	109.1666667	0	-684	20	56
	C4	Line 2 July	40	0	-664	1E+30	20
	D4	Line 2 August	12.5	0	-644	56	40
	B13	Line 1 June	45	0	-624	10	28
	C13	Line 1 July	90	0	-604	30	10
	D13	Line 1 August	75	0	-584	612	28
Standarc	B14	Line 2 June	0	-28	-680	28	1E+30
	C14	Line 2 July	0	-30.5	-660	30.5	1E+30
	D14	Line 2 August	0	-28	-640	28	1.00E+30

Constraints

	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
	B16	Inventory after production June	45	-10	45	90	10
	C16	Inventory after production July	90	-30	90	10	75
Standarc	D16	Inventory after production August	75	-612	75	75	25
	B38	June Line 1	280	18.6666667	280	327.5	47.5
	B39	July Line 1	150	25.3333333	150	315	15
	B40	August Line 1	300	18.6666667	300	37.5	112.5
	B6	Inventory after production June	180	0	75	105	1E+30
	C6	Inventory after production July	150	-40	150	12.5	37.5
Recyclec	D6	Inventory after production August	75	-644	75	37.5	12.5
	E38	June Line 2	436.6666667	0	500	1E+30	63.33333
	E39	July Line 2	160	5	160	420	63.33333
	E40	August Line 2	50	0	200	1E+30	150

Figure 1: "No overtime" Sensitivity report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Line 1 June	70.83333333	0	-628	56	20
C3	Line 1 July	5	0	-608	20	76
D3	Line 1 August	75	0	-588	588	56
B4	Line 2 June	109.1666667	0	-684	20	56
C4	Line 2 July	40	0	-664	1E+30	20
D4	Line 2 August	0	-56	-644	56	1E+30
B13	Line 1 June	45	0	-624	10	28
C13	Line 1 July	90	0	-604	58	10
D13	Line 1 August	75	0	-584	584	56
B14	Line 2 June	0	-28	-680	28	1E+30
C14	Line 2 July	0	-30.5	-660	30.5	1E+30
D14	Line 2 August	0	-56	-640	56	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B18	Inventory after production <=	45	-10	45	90	10
C18	Inventory after production <=	90	-58	90	10	28.33333333
D18	Inventory after production <=	75	-584	75	28.33333333	75
B38	June Line 1	280	18.66666667	280	327.5	47.5
B39	July Line 1	150	25.33333333	150	315	15
B40	August Line 1	337.5	0	380	1E+30	42.5
B8	Inventory after production <=	180	0	75	105	1E+30
C8	Inventory after production <=	150	-96	150	15.83333333	14.16666667
D8	Inventory after production <=	75	-588	75	14.16666667	75
E38	<= Line 2	436.6666667	0	500	1E+30	63.33333333
E39	<= Line 2	160	5	160	420	63.33333333
E40	<= Line 2	0	0	200	1E+30	200

Figure 2: "Overtime" Sensitivity report

5.1 Variable Cells

Note this is analysing the "No overtime sensitivity report".

The optimal solution consists of the following amounts of recycled from line 1: 70,833 in June, 5 in July and 75 in August. The optimal solution for line 2 consists of the following amounts 109,17 for June, 40 for July and 12,5 for August. Whereas for standard plastic the optimal solution values for line 1 are 45 for June, 90 for July and 75 for August. The standard plastic for all three months for line two is 0.

On the one hand since all of the recycled plastic from line 1 and 2 has a reduced cost of 0 which means that profit cannot be increased by changing these amounts as the definition of the the reduced cost is "how much a 1 unit change in the coefficients of the objective function will change profit." This essentially means that it is the amount by which the objective function parameters would have to improve before it would be possible for a corresponding variable to take on a positive value in the optimal solution (ie for it to become a part of the solution).

On the other hand when the reduced cost is not zero, for example with the case of standard plastic line 2 for June, the reduced cost is -28, which represents the amount by which profit will be reduced if I include a unit from line 2 in June in the solution - hence it does not make enough profit to warrant its inclusion. The profit needs to improve by at least 28. The same reasoning goes for line 2 in July and August with values of -30,5 and -28. Therefore they are not included in the solution and it would not be beneficial to add them.

On the other hand for line 2 for August by changing the right hand side of the constraint the profit will decrease by 56.

5.2 Constraints

5.2.1 Hour Constraints

In all cases, it appears that increasing the hour constraints will increase profit given that all shadow prices are ≥ 0. In particular, the available production hours on line 1 should be increased if possible. Every 1 unit increase in the available hours for June, will increase profit by R18,67; every 1 unit increase in the available hours for July, will increase profit by R25,33 and every 1 unit increase in the available hours for August, will increase profit by R18,67. The effect of changing the production capacity of line 2 is less impactful: every 1 unit increase in the available hours for July, will increase profit by R5,00, otherwise changing the available hours in June and August will have no effect on profit. As such, it would seem that the additional 80 hours that are available at the end of August, should be implemented on line 1, however this view fails to take their cost into account. The additional cost offsets the gain had by these hours and as such they should not be incorporated. Hence it is recommended that additional production hours only be included if they come at no extra cost.

5.2.2 Overtime

With reference to whether or not it is profitable to implement the extra 80 hours of Overtime it can be seen from comparing sensitivity reports of "No overtime" and "Overtime" that it is not in fact profitable to add these hours as profit would decrease as

the objective function values would actually decrease while the objective coefficients remain unchanged with the added Overtime. This can be seen as the profit with Overtime implemented is R77106,67, while the profit without Overtime is R78 006,67 (clearly this is more profitable). This is due to the cost of the Overtime as touched on in section 5.2.1.

5.2.3 Demand constraints

With reference to the "No overtime" sensitivity report it can be noted that for line 1; for standard plastic the shadow price of inventory after production for June is -10. This means that for every 1 unit increase in the demand for June the profit will decrease by R10,00. This is a similar case for inventory after production for July (with a decrease in profit of R30,00) and August (with a decrease in profit of R612,00). This essentially alludes to the fact that Plastics Inc. should not increase their demand as this will lead to a decrease in profit. Similarly for recycled plastic for line 2, no more inventory should be stored (no increase in demand is needed) as for a 1 unit for August profit will decrease by R40,00 and for August it will decrease by R644,00. However for June, since the shadow price is 0, increasing or decreasing the demand will have no effect on the optimal value of the problem.

6 Conclusion

To conclude it is possible for Plastics Inc to make a profitability target of 20% as the linear programme reveals that with No overtime added the profitability is 20.64% with a profit of R78006,67 and with Overtime added the profitability is 20.40%

with a profit of R77 106,67 - this is with the given assumptions. In summary, I recommend that production capabilities be improved particularly on line 1 as this will greatly increase profit. However, this is only recommended if it can be done at no extra cost.

7 Member Contributions

my group worked cohesively together with each member contributing substantially to the project.
Matthew Whall solved the linear programme as Ill as the additional scenario with Overtime.
Kiara Beilinsohn performed the sensitivity analysis.
Both members worked together on the abstract, introduction, assumptions and conclusion.

8 Plagiarism Declarations

Department of Statistical Sciences Plagiarism Declaration form

A copy of this form, completed and signed, to be attached to all submissions to the Statistical Sciences Department. Submissions without this form will NOT be marked.

COURSE CODE: STA3036S

COURSE NAME: Operations Research Techniques

STUDENT NAME: Matthew Whall

STUDENT NUMBER: WHLMAT002

TUTORS NAME: Zac Abel, TUT GROUP # 1

PLAGIARISM DECLARATION

1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretend that it is one's own.
2. I have used a generally accepted citation and referencing style. Each contribution to, and quotation in, this tutorial/report/project from the work(s) of other people has been attributed, and has been cited and referenced.
3. This project is my own work.
4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
5. I acknowledge that copying someone else's assignment or essay, or part of it, is wrong, and declare that this is my own work.

Note that agreement to this statement does not exonerate you from the University's plagiarism rules (http://www.uct.ac.za/uct/policies/plagiarism$_s$*tudents.pdf*).

Signature: M.R. Whall
Date: 12/09/2019

Department of Statistical Sciences Plagiarism Declaration form

A copy of this form, completed and signed, to be attached to all submissions to the Statistical Sciences Department. Submissions without this form will NOT be marked.

COURSE CODE: STA3036S

COURSE NAME: Operations Research Techniques

STUDENT NAME: Kiara Beilinsohn

STUDENT NUMBER: BLNKIA001

TUTORS NAME: Siraj Rawood, TUT GROUP # 4

PLAGIARISM DECLARATION

1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretend that it is one's own.

2. I have used a generally accepted citation and referencing style. Each contribution to, and quotation in, this tutorial/report/project from the work(s) of other people has been attributed, and has been cited and referenced.

3. This project is my own work.

4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.

5. I acknowledge that copying someone else's assignment or essay, or part of it, is wrong, and declare that this is my own work.

Note that agreement to this statement does not exonerate you from the University's plagiarism rules (http://www.uct.ac.za/uct/policies/plagiarism$_s$*tudents.pdf*).

Signature: K.M Beilinsohn

Date: 12/09/2019