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SIPLIB 2.0

Stochastic Integer Programming Library version 2.0

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Abstract We present a collection of stochastic integer programming problem instances.

Keywords Stochastic Integer Programming · Problem Instances

1 Introduction

`SIPLIB` [2] is an abbreviated term of the Stochastic Integer Programming (SIP) LIBrary firstly contructed in 2002 by Shabbir Ahmed and his colleagues. The library has been providing a collection of test instances to facilitate computational and algorithmic research in SIP. Some new test problems with instances have been added to `SIPLIB` gradually and now it contains nine different problems in total. The instances are basically given in the standard `SMPS` format accompanied with additional information including parameter data, size of the instance in terms of the number of rows, columns, and integers, benchmarking information such as best known objective value or bounds, optimality gap, and solution time.

At the time `SIPLIB` appeared, it provided enoughly large-sized instances that is reasonable to argue that the performance of algorithm is remarkable if it solves the instances. The state-of-art in SIP combined with the speedup in computing machinery, however, makes many instances of `SIPLIB` trivial so that we have no more basis to use `SIPLIB` for showing the excellence of

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new solution methods. At this point, we are motivated to develop the second version of SIPLIB, say SIPLIB 2.0 that provides larger-sized test instances with higher degree of tailorability, e.g., users can expand the size of instance as much as they want in terms of the number of scenarios included.

Stochastic programming (SP) is a framework for modeling optimization problems that involve uncertainty. Whereas optimization problems are typically formulated with known parameters, the problems in real world contain some unknown parameters in many cases. For details on SP, see, e.g., [3,4]. SIP is a branch of SP that indicates any type of SP including at least one integer decision variable. The integers can be placed anywhere in general SIP. However, we restrict our focus on two-stage SIP that contains integer variables (including binary) in its second stage throughout this paper and SIPLIB 2.0. The main reason is that the class of SIP is most widely used to model real world problems. Moreover, two-stage SIP itself has enough difficulties that have not been conquered yet even without any other details like chance-constraints and multi-stages. The main difficulty in solving two-stage SIP is that the second-stage value function is not necessarily convex, but only lower semicontinuous. Thus, the standard decomposition approaches that work nicely for stochastic *linear* programs, break down when the second stage integer variables are present [5]. Hereinafter, we use the term SIP to indicate the two-stage SIP that contains integer variables in its second stage.

We provide the test sets in two formats: SMPS files (*.cor, *.tim, *.stoch) and Julia files (*.jl). SMPS is widely used to describe stochastic linear and quadratic programs. Once having SMPS files of a problem instance, we can directly solve it using various mixed integer linear program (MILP) solvers such as CPLEX, GUROBI, and CBC. One can also use the existing open-source SIP solvers like DSP [6], PySP [7], and SMI [8] given that SMPS files. A drawback of SMPS format is its low readability by human, which we decided to provide Julia files to let users be able to easily read problems and tailor the instances.

Julia is an open source high-level, high-performance dynamic programming language for numerical computing. It is also known as its good performance, approaching that of statically-compiled languages like C [9]. The syntax of Julia is simple and should feel familiar to anyone who has experienced in another high-level languages like MATLAB or Python. A Julia package called JuMP (Julia for Mathematical Programming [10]) provides a domain-specific modeling language for mathematical optimization embedded in Julia. JuMP enables us to easily translate mathematical model to JuMP.Model-type object. Some structured mathematical models like SIP can also be translated to the JuMP.Model-type object combined with the package StructJuMP [11]. Once we have a Julia code for constructing JuMP.Model-type object, it is easy to generate instances whenever we need to modify the original mathematical model. For each problem in SIPLIB 2.0, we provide a Julia script for constructing JuMP.Model-type object. We also provide a Julia script (SmpsWriter.jl) for converting any JuMP.Model-type objects to SMPS files for users' convenience. Those who feel the given instances are not large enough can simply generate more scenario data by just modifying the parameter in `Julia` script corresponding to the number of scenarios.

By `SIPLIB 2.0`, we provide 1) richer collection of test instances for computational and algorithmic research in SIP with benchmarking computational results, 2) not only `SMPS` files but also `Julia` files that are easily readable/tailorable. Hence, the users can obtain as large-sized instances as they need by generating new scenarios and including them into instances. For those who want to utilize the instances in the legacy `SIPLIB` with strong tailorability provided by `SIPLIB 2.0`, we include the original `SIPLIB` instances as well.

2 Stochastic integer programming

In this section, we explain general description of SIP. This includes formal mathematical formulation, existing general solution methods to solve the SIPs, and currently available software libraries.

2.1 Formulation

In this subsection, we introduce the form of SIP of interest. The notations and dimensional information are summarized in Table 1. We are interested in finding solution for two-stage SIP of the form:

$$z := \min_{x \in X} \left\{ c^\top x + \mathcal{Q}(x) : \; Ax \geq b \right\}, \tag{1}$$

where $\mathcal{Q}(x) := \mathbb{E}_{\boldsymbol{\xi}} \left[\phi \left(h(\boldsymbol{\xi}) - T(\boldsymbol{\xi}) x \right) \right]$ is the recourse function associated with the random variable (r.v.) $\boldsymbol{\xi}$. We assume that $\boldsymbol{\xi}$ follows a known discrete probability distribution with the finite realizations, called *scenarios*, $\xi_1, \cdots, \xi_r$ and respective nonnegative probabilities $p_1, \cdots, p_r$, i.e., $p_i := \mathbb{P}[\boldsymbol{\xi} = \xi_i]$ for $i \in \{1, \ldots, r\}$. When the distribution is continuous, we can aproximate it by a suitably discretized distribution. The real-valued map $\phi_{\xi_j} : \mathbb{R}^{m_2} \to \mathbb{R}$ is the optimal value of the second-stage problem defined by

$$\phi_{\xi_j}(s) := \min_{y_j \in Y} \left\{ q(\xi_j)^\top y_j : \; W(\xi_j) y_j \geq s \right\}, \; s \in \mathbb{R}^{m_2}, \tag{2}$$

where ξ_j is an arbitrarily realized scenario. The sets $X \subseteq \mathbb{R}^{n_1}$ and $Y \subseteq \mathbb{R}^{n_2}$ represent integer or binary restrictions on a subset of the decision variables x and y_j, respectively. The first-stage problem data comprise A, b, and c. The second-stage data are given by $T(\xi_j)$, $W(\xi_j)$, $h(\xi_j)$, and $q(\xi_j)$ (for dimensional information refer to Table 1). Hereinafter, we use the simplified notations

(T_j, W_j, h_j, q_j). The SIP (1) can be rewritten in the extensive form

$$
\begin{aligned}
z = \min_{x, y_j}\ & c^\top x + \sum_{j=1}^{r} p_j (q_j^\top y_j), && \text{(3a)}\\
\text{s.t. } & Ax \geq b, && \text{(3b)}\\
& T_j x + W_j y_j \geq h_j, \quad \forall j \in \{1, \ldots, r\}, && \text{(3c)}\\
& x \in X, && \text{(3d)}\\
& y_j \in Y, \quad \forall j \in \{1, \ldots, r\}. && \text{(3e)}
\end{aligned}
$$

Table 1 Summary of notations in SIP formulation

Scalas	
$\boldsymbol{\xi}$	r.v. denoting scenario that can have value in the set $\{\xi_1, \cdots, \xi_r\}$
$z \in \mathbb{R}$	optimal objective value of the SIP
$r \in \mathbb{N}$	number of scenarios
$j \in \{1, \cdots, r\}$	index denoting scenarios
$p_j \in [0, 1]$	probability that the scenario j happens, i.e., $\mathbb{P}[\boldsymbol{\xi} = \xi_j]$
Sets	
$X \subseteq \mathbb{R}^{n_1}$	first-stage polyhedral set (continuous, integer, binary)
$Y \subseteq \mathbb{R}^{n_2}$	second-stage polyhedral set (continuous, integer, binary)
Vectors	
$x \in \mathbb{R}^{n_1}$	first-stage decision vector
$c \in \mathbb{R}^{n_1}$	first-stage cost vector
$b \in \mathbb{R}^{m_1}$	first-stage RHS vector
$y_j \in \mathbb{R}^{n_2}$	second-stage decision vector under scenario ξ_j
$q_j \equiv q(\xi_j) \in \mathbb{R}^{n_2}$	second-stage cost vector
$h_j \equiv h(\xi_j) \in \mathbb{R}^{m_2}$	second-stage RHS vector
Matrices	
$A \in \mathbb{R}^{m_1 \times n_1}$	first-stage constraint matrix corresponds to the decision vector x
$W_j \equiv W(\xi_j) \in \mathbb{R}^{m_2 \times n_2}$	second-stage constraint matrix corresponds to the decision vector y_j
$T_j \equiv T(\xi_j) \in \mathbb{R}^{m_2 \times n_1}$	second-stage constraint matrix corresponds to the decision vector x_j
Functions	
$\phi_{\xi_j} : \mathbb{R}^{m_2} \to \mathbb{R}$	second stage program optimal value given the realization of scenario ξ_j
$\mathcal{Q} : \mathbb{R}^{n_1} \to \mathbb{R}$	recourse function (the expectation of $\phi\,(h(\boldsymbol{\xi}) - T(\boldsymbol{\xi})x)$ over the r.v. $\boldsymbol{\xi}$)

2.2 Solution methods

2.2.1 Stage-wise decomposition algorithm

2.2.2 Scenario-wise decompostion algorithm

2.3 Software libraries

2.3.1 Modeling languages

2.3.2 Solvers

3 Summary of the test sets

3.1 Type of problems

Table 2 The type of problems in `SIPLIB 2.0`

Problem name	Description	Reference
`DCAP`	Dynamic capacity planning with stochastic demand	Ahmed and Garcia
`MPTSPs`	Multi-path traveling salesman problem with stochastic travel costs	[12,13,14]
`SMKP`	Stochastic multiple knapsack problem	Angulo et al.
`SIZES`	Optimal product substitution with stochastic demand	Jorjani et al.
`SSLP`	Stochastic server location problem	Ntaimo and Sen
`WECC`	Wind power stochastic unit commitment	Papavasiliou and Oren

Table 3 The components of problems in `SIPLIB 2.0`

	1st stage		2nd stage	
Problem	Variable	Constraint	Variable	Constraint
`DCAP`	$\mathbb{C}$, $\mathbb{B}$		$\mathbb{B}$	`PAR`, `MO11`
`MPTSPs`	$\mathbb{C}$, $\mathbb{B}$	`PAR2`, `GEN1`	$\mathbb{B}$	`GEN2`
`SMKP`	$\mathbb{B}$	`KNA1`	$\mathbb{B}$	`KNA1`
`SIZES`	$\mathbb{I}$	`VBD1`, `GEN2`	$\mathbb{B}$, $\mathbb{I}$	`IKN1`
`SSLP`	$\mathbb{B}$	`IVK1`, `GEN1`	$\mathbb{C}$, $\mathbb{I}$	`GEN2`
`WECC`				

3.2 Instance catalog

3.2.1 Instance naming rule

Table 4 Problem-specific instance naming rules

Problem	Instance name	Description
DCAP		
MPTSPs	MPTSPs_Dx_Ny_Sz	node distribution strategy Dx, number of nodes y, and number of scenarios z
SMKP		
SIZES	SIZESz	number of scenario is z
SSLP		
WECC		

4 How to run a test, generate new instance, and convert to SMPS

We explain the structure of SIPLIB 2.0. We explain procedure to generate new instances with user-generated scenario data using Julia scripts. The problem-specific descriptions are given in Section 6. We explain how to convert JuMP.Model-type object to SMPS files.

5 Implementation of SMPS Writer

We describe our Julia implementation, how to model SIP and generate SMPS files..

6 Problem descriptions

In this section, we introduce each problem in SIPLIB 2.0. For each problem, we provide various size of instances as a default. We also explain the scenario data generation procedures. Due to limited access to the original data and inconsistencies present in reference papers, we selectively choose the methods from the references and modify some of them without harming validity. Also, we guess some parameters about scenario generation to make the procedure clear.

6.1 SIZES: Selection of an optimal subset of sizes

SIZES is a simplified version of the cutting stock problem with multi-period stochastic demand. In this problem, the term *period* can be exchangeably used with *stage*. The first period (first stage) demand is deterministic whereas demands for the other periods (stages) are stochastic with scenarios. We only consider the two-periods (i.e., two stage) model to follow [16] as well as the SIP of interest discussed in Section 2.

In `SIPLIB`, only three instances are available in `SMPS` files. We refer to the mathematical formulation in [16] to construct `JuMP.Model`. Due to some unclear explanations (or typo), we slightly modify the formulation and use it for `SIPLIB 2.0`.

6.1.1 Mathematical formulation

Table 5 Notations for `SIZES`

Index sets	
N	index set of items ($i, j \in N$)
T	index set of time periods ($t \in T$)
$\mathcal{L}$	index set of scenarios ($l \in \mathcal{L}$)
Parameters	
d_{itl}	demand for item i at time t under scenario l
p_i	unit production cost for item i
s	setup cost for producing any item
r	unit cutting cost
c_{tl}	production capacity at time t under scenario l
$\mathbb{P}(l)$	the probability of occurence of scenario l
Variables	
y_{it} (1st stage)	number of units of size i produced at time t
x_{ijtl} (2nd stage)	number of units of size i cut to meet demand for smaller size j at time t under scenario l
z_{itl} (2nd stage)	1 if we produce size i at time t under scenario l, 0 otherwise

$$
\begin{aligned}
(\texttt{SIZES})\ \min\ & \sum_{t\in T}\sum_{i\in N} p_i y_{it} + \sum_{l\in\mathcal{L}} \mathbb{P}(l) \sum_{t\in T}\left[\sum_{i\in N} s z_{itl} + r \sum_{i\in N\setminus\{1\}}\sum_{j=1}^{i-1} x_{ijtl}\right] && \text{(4a)}\\
\text{s.t.}\ & \sum_{i\in N} y_{it} \le c_{tl}, \quad \forall t\in T,\ \forall l\in\mathcal{L}, && \text{(4b)}\\
& \sum_{i=j}^{|N|} x_{ijtl} \ge d_{jtl}, \quad \forall j\in N,\ \forall t\in T,\ \forall l\in\mathcal{L}, && \text{(4c)}\\
& \sum_{t'=1}^{t}\sum_{j=1}^{i} x_{ijt'l} \le \sum_{t'=1}^{t} y_{it'}, \quad \forall i\in N,\ \forall t\in T,\ \forall l\in\mathcal{L}, && \text{(4d)}\\
& y_{it} \le M z_{itl}, \quad \forall i\in N,\ \forall t\in T,\ \forall l\in\mathcal{L}, && \text{(4e)}\\
& x_{ijtl} \in \mathbb{Z}_+, \quad \forall i\in N,\ \forall j\in N,\ \forall t\in T,\ \forall l\in\mathcal{L}, && \text{(4f)}\\
& y_{it} \in \mathbb{Z}_+, \quad \forall j\in N,\ \forall t\in T, && \text{(4g)}\\
& z_{itl} \in \{0,1\}, \quad \forall i\in N,\ \forall t\in T,\ \forall l\in\mathcal{L}. && \text{(4h)}
\end{aligned}
$$

6.1.2 Scenario data generation

6.2 `MPTSPs`: Mutli-path Traveling Salesman Problem with Stochastic Travel Times

`MPTSPs` is a variant of the travelling salesman problem (TSP) where a set of paths exists between any two nodes and each path is chracterized by a random travel time. In `SIPLIB`, only limited data (e.g., number of nodes, coordinates of nodes, generated travel times) are provided and no `SMPS` files are available. We mainly refer to [13] for deriving the mathematical formulation. Due to some unclear subtour breaking constraints, we refer to another paper [15]. Combining the two references, we construct the forthcoming single commodity flow-based formulation `MPTSPs` that is used for `SIPLIB 2.0`.

6.2.1 Mathematical formulation

Table 6 Notations for `MPTSPs`

Index sets	
N	index set of nodes ($i, j, l \in N$)
K_{ij}	index set of paths between nodes i and j ($k \in K_{ij}$)
$\mathcal{S}$	index set of scenarios ($s \in \mathcal{S}$)
Parameters	
c_{ij}^{ks}	unit random travel time of path k between nodes i, j under scenario s
$\bar{c}_{ij}$	estimation of the mean unit travel time (expectation of c_{ij}^{ks} over all s and k)
e_{ij}^{ks}	the error on the travel time on estimated for arc (i, j) and path k under scenario s
$\mathbb{P}(s)$	the probability of occurence of scenario s
Variables	
ϕ_{ij} (1st stage)	the nonnegative real-valued flow on arc (i, j)
x_{ij}^{ks} (1st stage)	1 if node j is visited just after node i, 0 otherwise
y_{ij} (2nd stage)	1 if path k between nodes $i, j \in N$ is selected at the second stage, 0 otherwise

$$
\begin{aligned}
(\texttt{MPTSPs})\ \min\ & \sum_{i \in N} \sum_{j \in N} \bar{c}_{ij} y_{ij} + \sum_{s \in \mathcal{S}} \mathbb{P}(s) \sum_{i \in N} \sum_{j \in N} \sum_{k \in K_{ij}} e_{ij}^{ks} x_{ij}^{ks} && \text{(5a)}\\
\text{s.t.}\ & \sum_{j \in N : j \neq i} y_{ij} = 1, \quad \forall i \in N, && \text{(5b)}\\
& \sum_{i \in N : i \neq j} y_{ij} = 1, \quad \forall j \in N, && \text{(5c)}\\
& \sum_{j \in N} \phi_{lj} - \sum_{i \in N : i \neq 1} \phi_{il} = 1, \quad \forall l \in N \backslash \{1\}, && \text{(5d)}\\
& \phi_{ij} \leq (|N| - 1)\, y_{ij}, \quad \forall i \in N \backslash \{1\},\ \forall j \in N, && \text{(5e)}\\
& \sum_{k \in K_{ij}} x_{ij}^{ks} = y_{ij}, \quad \forall i \in N,\ \forall j \in N,\ \forall s \in \mathcal{S}, && \text{(5f)}\\
& x_{ij}^{ks} \in \{0,1\}, \quad \forall i \in N,\ \forall j \in N,\ \forall k \in K_{ij},\ \forall s \in \mathcal{S}, && \text{(5g)}\\
& y_{ij} \in \{0,1\}, \quad \forall i \in N,\ \forall j \in N, && \text{(5h)}\\
& \phi_{ij} \geq 0, \quad \forall i \in N,\ \forall j \in N. && \text{(5i)}
\end{aligned}
$$

6.2.2 Scenario data generation

We follow the scenario generation methods described through the references [12,13,14]. For `MPTSPs`, there are three mainly distinguished characteristics for each instance: the nodes partition strategy ($D \in \{D0, D1, D2, D3\}$, explanation on each strategy is forthcoming), the number of nodes ($|N| \in \{2, 3, \ldots\}$), and the number of scenarios ($|S| \in \{1, 2, \ldots\}$). Another important charicteristic $|K_{ij}| \in \{1, 2, 3, \ldots\}$ is the number of paths for each edge which is fixed by 3 as a default following [14]. Once we decide D, $|N|$, and $|S|$ by $D = D$x, $|N| =$ y, and $|S| =$ z, each instance is named by `MPTSPs_Dx_Ny_Sz`.

The nodes are distributed in a circle with radius equal to r km. We use Cartesian coordinate system where the geometric center of the circle is (r, r). The nodes are distinguished by two subsets: *central* and *suburban*. If the Euclidean distance between a node and the geometric center is less than or equal to the half of the radius ($r/2$), then the node is of *central* type. Otherwise, if the Euclidean distance is greater than the half of the radius, the node is of *suburban* type. Each arc between any two nodes i and j is either *homogeneous* or *heterogeneous*. If the two nodes are of the same type of node, i.e., both are *central* or both are *suburban*, the type of the arc is *homogeneous*. Otherwise, the type of the arc is *heterogeneous*. Later, the travel time of each path between two nodes are affected by the type of arc.

The nodes are generated by one of the following distribution strategies:

- $D0$: All the nodes are *central*.
- $D1$: All the nodes are *suburban*.
- $D2$: 3/4 of the nodes are *central* and the remaining 1/4 are *suburban*.
- $D3$: 1/2 of the nodes are *central* and the remaining 1/2 are *suburban*.

Given $D, |N|$ and $|S|$, the next procedure can be summarized as follows:

1. Generate $|N|$ nodes based on the predetermined strategy D. Then, the nodes are generated by acceptance-rejection procedure with uniform random number generation. Again following [14], we fix $r = 7km$.
2. Calculate Euclidean distances between the nodes (EC_{ij}).
3. We guess and fix the deterministic velocity profile by $40km/h$ for the *central* nodes and $80km/h$ for the *suburban* nodes: $v^c = 40$ and $v^s = 80$.
4. Generate random travel times (c_{ij}^{ks}) for each scenario s.
   - The velocity for traveling arc (i, j) is affected by its arc type.
   - If the arc is *homogeneous*, the random travel time of all the paths are generated only based on the corresponding velocity profile.
   - If the arc is *heterogeneous*, $\left\lceil \frac{|K_{ij}|}{3} \right\rceil$ paths are generated based on $v^c = 40$ and the remaining paths are generated based on $v^s = 80$.
   - The velocities are distributed by $Unif(\frac{v}{2}, 2v)$ for $v = v^c, v^s$.
   - In summary, if the arc (i, j) is *homogeneous*,

$$c_{ij}^{ks} \sim \begin{cases} \frac{EC_{ij}}{Unif(\frac{v^c}{2}, 2v^c)} & \text{if } i, j \text{ are both } \textit{central}, \\ \frac{EC_{ij}}{Unif(\frac{v^s}{2}, 2v^s)} & \text{if } i, j \text{ are both } \textit{suburban} \end{cases} \quad \forall k \in K_{ij}.$$

   - Otherwise, if (i, j) is *heterogeneous*,

$$c_{ij}^{ks} \sim \begin{cases} \frac{EC_{ij}}{Unif(\frac{v^c}{2}, 2v^c)} & \text{for } k \in \left\{1, \ldots, \left\lceil \frac{|K_{ij}|}{3} \right\rceil \right\}, \\ \frac{EC_{ij}}{Unif(\frac{v^s}{2}, 2v^s)} & \text{for } k \in \left\{ \left\lceil \frac{|K_{ij}|}{3} \right\rceil + 1, \ldots, |K_{ij}| \right\}. \end{cases}$$

5. Finally, we multiply 3600 for each component of c_{ij}^{ks} to convert the unit from *hours* to *seconds*.

7 Solution report

8 Concluding remarks

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