CHAPTER THREE

# Data Representation

**IN THIS CHAPTER**



## 3-1 Data Types

Binary information in digital computers is stored in memory or processor registers. Registers contain either data or control information. Control information is a bit or a group of bits used to specify the sequence of command signals needed for manipulation of the data in other registers. Data are numbers and other binary-coded information that are operated on to achieve required computational results. In this chapter we present the most common types of data found in digital computers and show how the various data types are represented in binary-coded form in computer registers.

The data types found in the registers of digital computers may be classified as being one of the following categories: (1) numbers used in arithmetic computations, (2) letters of the alphabet used in data processing, and (3) other discrete symbols used for specific purposes. All types of data, except binary numbers, are represented in computer registers in binary-coded form. This is because registers are made up of flip-flops and flip-flops are two-state devices that can store only 1's and 0's. The binary number system is the most natural system to use in a digital computer. But sometimes it is convenient to employ different number systems, especially the decimal number system, since it is used by people to perform arithmetic computations.

## Number Systems

*radix*

A number system of *base*, or *radix*, $r$ is a system that uses distinct symbols for $r$ digits. Numbers are represented by a string of digit symbols. To determine the quantity that the number represents, it is necessary to multiply each digit by an integer power of $r$ and then form the sum of all weighted digits. For example, the decimal number system in everyday use employs the radix 10 system. The 10 symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The string of digits 724.5 is interpreted to represent the quantity

*decimal*

$$7 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1}$$

that is, 7 hundreds, plus 2 tens, plus 4 units, plus 5 tenths. Every decimal number can be similarly interpreted to find the quantity it represents.

*binary*

The *binary* number system uses the radix 2. The two digit symbols used are 0 and 1. The string of digits 101101 is interpreted to represent the quantity

$$1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 45$$

To distinguish between different radix numbers, the digits will be enclosed in parentheses and the radix of the number inserted as a subscript. For example, to show the equality between decimal and binary forty-five we will write $(101101)_2 = (45)_{10}$.

*octal*
*hexademical*

Besides the decimal and binary number systems, the *octal* (radix 8) and *hexadecimal* (radix 16) are important in digital computer work. The eight symbols of the octal system are 0, 1, 2, 3, 4, 5, 6, and 7. The 16 symbols of the hexadecimal system are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. The last six symbols are, unfortunately, identical to the letters of the alphabet and can cause confusion at times. However, this is the convention that has been adopted. When used to represent hexadecimal digits, the symbols A, B, C, D, E, F correspond to the decimal numbers 10, 11, 12, 13, 14, 15, respectively.

A number in radix $r$ can be converted to the familiar decimal system by forming the sum of the weighted digits. For example, octal 736.4 is converted to decimal as follows:

$$\begin{aligned}(736.4)_8 &= 7 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 + 4 \times 8^{-1} \\ &= 7 \times 64 + 3 \times 8 + 6 \times 1 + 4/8 = (478.5)_{10}\end{aligned}$$

The equivalent decimal number of hexadecimal F3 is obtained from the following calculation:

$$(\mathrm{F3})_{16} = \mathrm{F} \times 16 + 3 = 15 \times 16 + 3 = (243)_{10}$$

*conversion*

Conversion from decimal to its equivalent representation in the radix $r$ system is carried out by separating the number into its *integer* and *fraction* parts and converting each part separately. The conversion of a decimal integer into a base $r$ representation is done by successive divisions by $r$ and accumulation of the remainders. The conversion of a decimal fraction to radix $r$ representation is accomplished by successive multiplications by $r$ and accumulation of the integer digits so obtained. Figure 3-1 demonstrates these procedures.

The conversion of decimal 41.6875 into binary is done by first separating the number into its integer part 41 and fraction part .6875. The integer part is converted by dividing 41 by $r = 2$ to give an integer quotient of 20 and a remainder of 1. The quotient is again divided by 2 to give a new quotient and remainder. This process is repeated until the integer quotient becomes 0. The coefficients of the binary number are obtained from the remainders with the first remainder giving the low-order bit of the converted binary number.

The fraction part is converted by multiplying it by $r = 2$ to give an integer and a fraction. The new fraction (*without* the integer) is multiplied again by 2 to give a new integer and a new fraction. This process is repeated until the fraction part becomes zero or until the number of digits obtained gives the required accuracy. The coefficients of the binary fraction are obtained from the integer digits with the first integer computed being the digit to be placed next to the binary point. Finally, the two parts are combined to give the total required conversion.

## Octal and Hexadecimal Numbers

The conversion from and to binary, octal, and hexadecimal representation plays an important part in digital computers. Since $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits. The conversion from binary to octal is easily accomplished by partitioning the binary number into groups of three bits each. The corresponding octal digit is then assigned to each group of bits and the string of digits so obtained gives the octal equivalent of the binary number. Consider, for example, a 16-bit register. Physically, one may think of the

**Figure 3-1** Conversion of decimal 41.6875 into binary.

Integer = 41

| | |
|---|---|
| 41 | |
| 20 | 1 |
| 10 | 0 |
| 5 | 0 |
| 2 | 1 |
| 1 | 0 |
| 0 | 1 |

$(41)_{10} = (101001)_2$

Fraction = 0.6875

$$\begin{array}{r} 0.6875 \\ \underline{2} \\ 1.3750 \\ \underline{\text{x } 2} \\ 0.7500 \\ \underline{\text{x } 2} \\ 1.5000 \\ \underline{\text{x } 2} \\ 1.0000 \end{array}$$

$(0.6875)_{10} = (0.1011)_2$

$(41.6875)_{10} = (101001.1011)_2$

| | | |
|---|---|---|
| 1 2 7 5 4 3 | Octal | |
| 1 0 1 0 1 1 1 1 0 1 1 0 0 0 1 1 | Binary | |
| A F 6 3 | Hexadecimal | |

**Figure 3-2** Binary, octal, and hexadecimal conversion.

register as composed of 16 binary storage cells, with each cell capable of holding either a 1 or a 0. Suppose that the bit configuration stored in the register is as shown in Fig. 3-2. Since a binary number consists of a string of 1's and 0's, the 16-bit register can be used to store any binary number from 0 to $2^{16} - 1$. For the particular example shown, the binary number stored in the register is the equivalent of decimal 44899. Starting from the low-order bit, we partition the register into groups of three bits each (the sixteenth bit remains in a group by itself). Each group of three bits is assigned its octal equivalent and placed on top of the register. The string of octal digits so obtained represents the octal equivalent of the binary number.

Conversion from binary to hexadecimal is similar except that the bits are divided into groups of four. The corresponding hexadecimal digit for each group of four bits is written as shown below the register of Fig. 3-2. The string of hexadecimal digits so obtained represents the hexadecimal equivalent of the binary number. The corresponding octal digit for each group of three bits is easily remembered after studying the first eight entries listed in Table 3-1. The correspondence between a hexadecimal digit and its equivalent 4-bit code can be found in the first 16 entries of Table 3-2.

**TABLE 3-1** Binary-Coded Octal Numbers

| Octal number | Binary-coded octal | Decimal equivalent | |
|---|---|---|---|
| 0 | 000 | 0 | ↑ |
| 1 | 001 | 1 | |
| 2 | 010 | 2 | Code |
| 3 | 011 | 3 | for one |
| 4 | 100 | 4 | octal |
| 5 | 101 | 5 | digit |
| 6 | 110 | 6 | |
| 7 | 111 | 7 | ↓ |
| 10 | 001 000 | 8 | |
| 11 | 001 001 | 9 | |
| 12 | 001 010 | 10 | |
| 24 | 010 100 | 20 | |
| 62 | 110 010 | 50 | |
| 143 | 001 100 011 | 99 | |
| 370 | 011 111 000 | 248 | |

Table 3-1 lists a few octal numbers and their representation in registers in binary-coded form. The binary code is obtained by the procedure explained above. Each octal digit is assigned a 3-bit code as specified by the entries of the first eight digits in the table. Similarly, Table 3-2 lists a few hexadecimal numbers and their representation in registers in binary-coded form. Here the binary code is obtained by assigning to each hexadecimal digit the 4-bit code listed in the first 16 entries of the table.

Comparing the binary-coded octal and hexadecimal numbers with their binary number equivalent we find that the bit combination in all three representations is exactly the same. For example, decimal 99, when converted to binary, becomes 1100011. The binary-coded octal equivalent of decimal 99 is 001 100 011 and the binary-coded hexadecimal of decimal 99 is 0110 0011. If we neglect the leading zeros in these three binary representations, we find that their bit combination is identical. This should be so because of the straightforward conversion that exists between binary numbers and octal or hexadecimal. The point of all this is that a string of 1's and 0's stored in a register could represent a binary number, but this same string of bits may be interpreted as holding an octal number in binary-coded form (if we divide the bits in groups of three) or as holding a hexadecimal number in binary-coded form (if we divide the bits in groups of four).

**TABLE 3-2** Binary-Coded Hexadecimal Numbers

| Hexadecimal number | Binary-coded hexadecimal | Decimal equivalent | |
|---|---|---|---|
| 0 | 0000 | 0 | |
| 1 | 0001 | 1 | |
| 2 | 0010 | 2 | |
| 3 | 0011 | 3 | |
| 4 | 0100 | 4 | |
| 5 | 0101 | 5 | |
| 6 | 0110 | 6 | Code |
| 7 | 0111 | 7 | for one |
| 8 | 1000 | 8 | hexadecimal |
| 9 | 1001 | 9 | digit |
| A | 1010 | 10 | |
| B | 1011 | 11 | |
| C | 1100 | 12 | |
| D | 1101 | 13 | |
| E | 1110 | 14 | |
| F | 1111 | 15 | |
| 14 | 0001 0100 | 20 | |
| 32 | 0011 0010 | 50 | |
| 63 | 0110 0011 | 99 | |
| F8 | 1111 1000 | 248 | |

The registers in a digital computer contain many bits. Specifying the content of registers by their binary values will require a long string of binary digits. It is more convenient to specify content of registers by their octal or hexadecimal equivalent. The number of digits is reduced by one-third in the octal designation and by one-fourth in the hexadecimal designation. For example, the binary number 1111 1111 1111 has 12 digits. It can be expressed in octals as 7777 (four digits) or in hexadecimal as FFF (three digits). Computer manuals invariably choose either the octal or the hexadecimal designation for specifying contents of registers.

## Decimal Representation

The binary number system is the most natural system for a computer, but people are accustomed to the decimal system. One way to solve this conflict is to convert all input decimal numbers into binary numbers, let the computer perform all arithmetic operations in binary and then convert the binary results back to decimal for the human user to understand. However, it is also possible for the computer to perform arithmetic operations directly with decimal numbers provided they are placed in registers in a coded form. Decimal numbers enter the computer usually as binary-coded alphanumeric characters. These codes, introduced later, may contain from six to eight bits for each decimal digit. When decimal numbers are used for internal arithmetic computations, they are converted to a binary code with four bits per digit.

***binary code***

A binary code is a group of $n$ bits that assume up to $2^n$ distinct combinations of 1's and 0's with each combination representing one element of the set that is being coded. For example, a set of four elements can be coded by a 2-bit code with each element assigned one of the following bit combinations; 00, 01, 10, or 11. A set of eight elements requires a 3-bit code, a set of 16 elements requires a 4-bit code, and so on. A binary code will have some unassigned bit combinations if the number of elements in the set is not a multiple power of 2. The 10 decimal digits form such a set. A binary code that distinguishes among 10 elements must contain at least four bits, but six combinations will remain unassigned. Numerous different codes can be obtained by arranging four bits in 10 distinct combinations. The bit assignment most commonly used for the decimal digits is the straight binary assignment listed in the first 10 entries of Table 3-3. This particular code is called *binary-coded decimal* and is commonly referred to by its abbreviation BCD. Other decimal codes are sometimes used and a few of them are given in Sec. 3-5.

***BCD***

It is very important to understand the difference between the *conversion* of decimal numbers into binary and the *binary coding* of decimal numbers. For example, when *converted* to a binary number, the decimal number 99 is represented by the string of bits 1100011, but when represented in BCD, it becomes 1001 1001. The *only* difference between a decimal number represented by the familiar digit symbols 0, 1, 2, . . . , 9 and the BCD symbols 0001, 0010, . . . , 1001 is in the symbols used to represent the digits—the number itself is exactly the

TABLE 3-3 Binary-Coded Decimal (BCD) Numbers

| Decimal number | Binary-coded decimal (BCD) number | |
|---|---|---|
| 0 | 0000 | |
| 1 | 0001 | |
| 2 | 0010 | |
| 3 | 0011 | Code |
| 4 | 0100 | for one |
| 5 | 0101 | decimal |
| 6 | 0110 | digit |
| 7 | 0111 | |
| 8 | 1000 | |
| 9 | 1001 | |
| 10 | 0001 0000 | |
| 20 | 0010 0000 | |
| 50 | 0101 0000 | |
| 99 | 1001 1001 | |
| 248 | 0010 0100 1000 | |

same. A few decimal numbers and their representation in BCD are listed in Table 3-3.

## Alphanumeric Representation

Many applications of digital computers require the handling of data that consist not only of numbers, but also of the letters of the alphabet and certain special characters. An *alphanumeric character set* is a set of elements that includes the 10 decimal digits, the 26 letters of the alphabet and a number of special characters, such as \$, +, and =. Such a set contains between 32 and 64 elements (if only uppercase letters are included) or between 64 and 128 (if both uppercase and lowercase letters are included). In the first case, the binary code will require six bits and in the second case, seven bits. The standard alphanumeric binary code is the ASCII (American Standard Code for Information Interchange), which uses seven bits to code 128 characters. The binary code for the uppercase letters, the decimal digits, and a few special characters is listed in Table 3-4. Note that the decimal digits in ASCII can be converted to BCD by removing the three high-order bits, 011. A complete list of ASCII characters is provided in Table 11-1.

*character*

*ASCII*

Binary codes play an important part in digital computer operations. The codes must be in binary because registers can only hold binary information. One must realize that binary codes merely change the symbols, not the meaning of the discrete elements they represent. The operations specified for digital

**TABLE 3-4** American Standard Code for Information Interchange (ASCII)

| Character | Binary code | Character | Binary code |
|---|---|---|---|
| A | 100 0001 | 0 | 011 0000 |
| B | 100 0010 | 1 | 011 0001 |
| C | 100 0011 | 2 | 011 0010 |
| D | 100 0100 | 3 | 011 0011 |
| E | 100 0101 | 4 | 011 0100 |
| F | 100 0110 | 5 | 011 0101 |
| G | 100 0111 | 6 | 011 0110 |
| H | 100 1000 | 7 | 011 0111 |
| I | 100 1001 | 8 | 011 1000 |
| J | 100 1010 | 9 | 011 1001 |
| K | 100 1011 | | |
| L | 100 1100 | | |
| M | 100 1101 | space | 010 0000 |
| N | 100 1110 | . | 010 1110 |
| O | 100 1111 | ( | 010 1000 |
| P | 101 0000 | + | 010 1011 |
| Q | 101 0001 | $ | 010 0100 |
| R | 101 0010 | * | 010 1010 |
| S | 101 0011 | ) | 010 1001 |
| T | 101 0100 | − | 010 1101 |
| U | 101 0101 | / | 010 1111 |
| V | 101 0110 | , | 010 1100 |
| W | 101 0111 | = | 011 1101 |
| X | 101 1000 | | |
| Y | 101 1001 | | |
| Z | 101 1010 | | |

computers must take into consideration the meaning of the bits stored in registers so that operations are performed on operands of the same type. In inspecting the bits of a computer register at random, one is likely to find that it represents some type of coded information rather than a binary number.

Binary codes can be formulated for any set of discrete elements such as the musical notes and chess pieces and their positions on the chessboard. Binary codes are also used to formulate instructions that specify control information for the computer. This chapter is concerned with *data* representation. Instruction codes are discussed in Chap. 5.

## 3-2 Complements

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation. There are two types of complements for each base $r$ system: the $r$'s complement and the $(r - 1)$'s complement.

When the value of the base $r$ is substituted in the name, the two types are referred to as the 2's and 1's complement for binary numbers and the 10's and 9's complement for decimal numbers.

## $(r - 1)$'s Complement

Given a number $N$ in base $r$ having $n$ digits, the $(r - 1)$'s complement of $N$ is defined as $(r^n - 1) - N$. For decimal numbers $r = 10$ and $r - 1 = 9$, so the 9's complement of $N$ is $(10^n - 1) - N$. Now, $10^n$ represents a number that consists of a single 1 followed by $n$ 0's. $10^n - 1$ is a number represented by $n$ 9's. For example, with $n = 4$ we have $10^4 = 10000$ and $10^4 - 1 = 9999$. It follows that the 9's complement of a decimal number is obtained by subtracting each digit from 9. For example, the 9's complement of 546700 is $999999 - 546700 = 453299$ and the 9's complement of 12389 is $99999 - 12389 = 87610$.

***9's complement***

***1's complement***

For binary numbers, $r = 2$ and $r - 1 = 1$, so the 1's complement of $N$ is $(2^n - 1) - N$. Again, $2^n$ is represented by a binary number that consists of a 1 followed by $n$ 0's. $2^n - 1$ is a binary number represented by $n$ 1's. For example, with $n = 4$, we have $2^4 = (10000)_2$ and $2^4 - 1 = (1111)_2$. Thus the 1's complement of a binary number is obtained by subtracting each digit from 1. However, the subtraction of a binary digit from 1 causes the bit to change from 0 to 1 or from 1 to 0. Therefore, the 1's complement of a binary number is formed by changing 1's into 0's and 0's into 1's. For example, the 1's complement of 1011001 is 0100110 and the 1's complement of 0001111 is 1110000.

The $(r - 1)$'s complement of octal or hexadecimal numbers are obtained by subtracting each digit from 7 or F (decimal 15) respectively.

## $(r$'s$)$ Complement

The $r$'s complement of an $n$-digit number $N$ in base $r$ is defined as $r^n - N$ for $N \neq 0$ and 0 for $N = 0$. Comparing with the $(r - 1)$'s complement, we note that the $r$'s complement is obtained by adding 1 to the $(r - 1)$'s complement since $r^n - N = [(r^n - 1) - N] + 1$. Thus the 10's complement of the decimal 2389 is $7610 + 1 = 7611$ and is obtained by adding 1 to the 9's complement value. The 2's complement of binary 101100 is $010011 + 1 = 010100$ and is obtained by adding 1 to the 1's complement value.

***10's complement***

Since $10^n$ is a number represented by a 1 followed by $n$ 0's, then $10^n - N$, which is the 10's complement of $N$, can be formed also be leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and then subtracting all higher significant digits from 9. The 10's complement of 246700 is 753300 and is obtained by leaving the two zeros unchanged, subtracting 7 from 10, and subtracting the other three digits from 9. Similarly, the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged, and then replacing 1's by 0's and 0's by 1's in all other higher significant bits. The 2's complement of 1101100 is 0010100 and is obtained by leaving the two low-order 0's and the first 1 unchanged, and then replacing 1's by 0's and 0's by 1's in the other four most significant bits.

***2's complement***

In the definitions above it was assumed that the numbers do not have a radix point. If the original number $N$ contains a radix point, it should be removed temporarily to form the $r$'s or $(r-1)$'s complement. The radix point is then restored to the complemented number in the same relative position. It is also worth mentioning that the complement of the complement restores the number to its original value. The $r$'s complement of $N$ is $r^n - N$. The complement of the complement is $r^n - (r^n - N) = N$ giving back the original number.

### Subtraction of Unsigned Numbers

The direct method of subtraction taught in elementary schools uses the borrow concept. In this method we borrow a 1 from a higher significant position when the minuend digit is smaller than the corresponding subtrahend digit. This seems to be easiest when people perform subtraction with paper and pencil. When subtraction is implemented with digital hardware, this method is found to be less efficient than the method that uses complements.

*subtraction*

The subtraction of two $n$-digit unsigned numbers $M - N\ (N \neq 0)$ in base $r$ can be done as follows:

1. Add the minuend $M$ to the $r$'s complement of the subtrahend $N$. This performs $M + (r^n - N) = M - N + r^n$.
2. If $M \geq N$, the sum will produce an end carry $r^n$ which is discarded, and what is left is the result $M - N$.
3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the $r$'s complement of $(N - M)$. To obtain the answer in a familiar form, take the $r$'s complement of the sum and place a negative sign in front.

*end carry*

Consider, for example, the subtraction $72532 - 13250 = 59282$. The 10's complement of 13250 is 86750. Therefore:

$$\begin{aligned} M &= \phantom{+}72532 \\ \text{10's complement of } N &= +\underline{86750} \\ \text{Sum} &= \phantom{+}159282 \\ \text{Discard end carry } 10^5 &= -\underline{100000} \\ \text{Answer} &= \phantom{+}59282 \end{aligned}$$

Now consider an example with $M < N$. The subtraction $13250 - 72532$ produces negative 59282. Using the procedure with complements, we have

$$\begin{aligned} M &= \phantom{+}13250 \\ \text{10's complement of } N &= +\underline{27468} \\ \text{Sum} &= \phantom{+}40718 \end{aligned}$$

There is no end carry

Answer is negative 59282 = 10's complement of 40718

Since we are dealing with unsigned numbers, there is really no way to get an unsigned result for the second example. When working with paper and pencil, we recognize that the answer must be changed to a signed negative number. When subtracting with complements, the negative answer is recognized by the absence of the end carry and the complemented result.

Subtraction with complements is done with binary numbers in a similar manner using the same procedure outlined above. Using the two binary numbers $X = 1010100$ and $Y = 1000011$, we perform the subtraction $X - Y$ and $Y - X$ using 2's complements:

$$\begin{array}{rr} X = & 1010100 \\ \text{2's complement of } Y = & +\underline{0111101} \\ \text{Sum} = & 10010001 \\ \text{Discard end carry } 2^7 = & -\underline{10000000} \\ \text{Answer: } X - Y = & 0010001 \end{array}$$

$$\begin{array}{rr} Y = & 1000011 \\ \text{2's complement of } X = & +\underline{0101100} \\ \text{Sum} = & 1101111 \end{array}$$

There is no end carry

Answer is negative 0010001 = 2's complement of 1101111

## 3-3 Fixed-Point Representation

Positive integers, including zero, can be represented as unsigned numbers. However, to represent negative integers, we need a notation for negative values. In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign. Because of hardware limitations, computers must represent everything with 1's and 0's, including the sign of a number. As a consequence, it is customary to represent the sign with a bit placed in the leftmost position of the number. The convention is to make the sign bit equal to 0 for positive and to 1 for negative.

***binary point***

In addition to the sign, a number may have a binary (or decimal) point. The position of the binary point is needed to represent fractions, integers, or mixed integer-fraction numbers. The representation of the binary point in a register is complicated by the fact that it is characterized by a position in the register. There are two ways of specifying the position of the binary point in a register: by giving it a fixed position or by employing a floating-point representation. The fixed-point method assumes that the binary point is always fixed in one position. The two positions most widely used are (1) a binary point in the extreme left of the register to make the stored number a fraction, and (2) a binary point in the extreme right of the register to make the stored number an integer. In either case, the binary point is not actually present, but its presence is assumed from the fact that the number stored in the register is treated as a fraction or as an integer. The floating-point representation uses a second register to store a number that designates the position of the decimal point in the first register. Floating-point representation is discussed further in the next section.

## Integer Representation

*signed numbers*

When an integer binary number is positive, the sign is represented by 0 and the magnitude by a positive binary number. When the number is negative, the sign is represented by 1 but the rest of the number may be represented in one of three possible ways:

1. Signed-magnitude representation
2. Signed-1's complement representation
3. Signed 2's complement representation

The signed-magnitude representation of a negative number consists of the magnitude and a negative sign. In the other two representations, the negative number is represented in either the 1's or 2's complement of its positive value. As an example, consider the signed number 14 stored in an 8-bit register. +14 is represented by a sign bit of 0 in the leftmost position followed by the binary equivalent of 14: 00001110. Note that each of the eight bits of the register must have a value and therefore 0's must be inserted in the most significant positions following the sign bit. Although there is only one way to represent +14, there are three different ways to represent −14 with eight bits.

| | |
|---|---|
| In signed-magnitude representation | 1 0001110 |
| In signed-1's complement representation | 1 1110001 |
| In signed-2's complement representation | 1 1110010 |

The signed-magnitude representation of −14 is obtained from +14 by complementing only the sign bit. The signed-1's complement representation of −14 is obtained by complementing all the bits of +14, including the sign bit. The signed-2's complement representation is obtained by taking the 2's complement of the positive number, including its sign bit.

The signed-magnitude system is used in ordinary arithmetic but is awkward when employed in computer arithmetic. Therefore, the signed-complement is normally used. The 1's complement imposes difficulties because it has two representations of 0 (+0 and −0). It is seldom used for arithmetic operations except in some older computers. The 1's complement is useful as a logical operation since the change of 1 to 0 or 0 to 1 is equivalent to a logical complement operation. The following discussion of signed binary arithmetic deals exclusively with the signed-2's complement representation of negative numbers.

## Arithmetic Addition

The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic. If the signs are the same, we add the two magnitudes and give the sum the common sign. If the signs are different, we subtract the smaller magnitude from the larger and give the result the sign of the larger magnitude. For example, $(+25) + (-37) = -(37 - 25) = -12$ and is done by subtracting the smaller magnitude 25 from the larger magnitude 37 and using the sign of 37 for the sign of the result. This is a process that requires the comparison of the signs and the magnitudes and then performing either addition or subtraction. (The procedure for adding binary numbers in signed-magnitude representation is described in Sec. 10-2.) By contrast, the rule for adding numbers in the signed-2's complement system does not require a comparison or subtraction, only addition and complementation. The procedure is very simple and can be stated as follows: Add the two numbers, including their sign bits, and discard any carry out of the sign (leftmost) bit position. Numerical examples for addition are shown below. Note that negative numbers must initially be in 2's complement and that if the sum obtained after the addition is negative, it is in 2's complement form.

***2's complement addition***

| | | | |
|---|---|---|---|
| +6 | 00000110 | −6 | 11111010 |
| +13 | 00001101 | +13 | 00001101 |
| +19 | 00010011 | +7 | 00000111 |
| | | | |
| +6 | 00000110 | −6 | 11111010 |
| −13 | 11110011 | −13 | 11110011 |
| −7 | 11111001 | −19 | 11101101 |

In each of the four cases, the operation performed is always addition, including the sign bits. Any carry out of the sign bit position is discarded, and negative results are automatically in 2's complement form.

The complement form of representing negative numbers is unfamiliar to people used to the signed-magnitude system. To determine the value of a negative number when in signed-2's complement, it is necessary to convert it to a positive number to place it in a more familiar form. For example, the signed binary number 11111001 is negative because the leftmost bit is 1. Its 2's complement is 00000111, which is the binary equivalent of +7. We therefore recognize the original negative number to be equal to −7.

## Arithmetic Subtraction

*2's complement subtraction*

Subtraction of two signed binary numbers when negative numbers are in 2's complement form is very simple and can be stated as follows: Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit). A carry out of the sign bit position is discarded.

This procedure stems from the fact that a subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed. This is demonstrated by the following relationship:

$$(\pm A) - (+B) = (\pm A) + (-B)$$
$$(\pm A) - (-B) = (\pm A) + (+B)$$

But changing a positive number to a negative number is easily done by taking its 2's complement. The reverse is also true because the complement of a negative number in complement form produces the equivalent positive number. Consider the subtraction of $(-6) - (-13) = +7$. In binary with eight bits this is written as 11111010 − 11110011. The subtraction is changed to addition by taking the 2's complement of the subtrahend $(-13)$ to give $(+13)$. In binary this is 11111010 + 00001101 = 100000111. Removing the end carry, we obtain the correct answer 00000111 $(+7)$.

It is worth noting that binary numbers in the signed-2's complement system are added and subtracted by the same basic addition and subtraction rules as unsigned numbers. Therefore, computers need only one common hardware circuit to handle both types of arithmetic. The user or programmer must interpret the results of such addition or subtraction differently depending on whether it is assumed that the numbers are signed or unsigned.

## Overflow

*overflow*

When two numbers of $n$ digits each are added and the sum occupies $n + 1$ digits, we say that an overflow occurred. When the addition is performed with paper and pencil, an overflow is not a problem since there is no limit to the width of the page to write down the sum. An overflow is a problem in digital computers because the width of registers is finite. A result that contains $n + 1$ bits cannot be accommodated in a register with a standard length of $n$ bits. For this reason, many computers detect the occurrence of an overflow, and when it occurs, a corresponding flip-flop is set which can then be checked by the user.

The detection of an overflow after the addition of two binary numbers depends on whether the numbers are considered to be signed or unsigned. When two unsigned numbers are added, an overflow is detected from the end carry out of the most significant position. In the case of signed numbers, the leftmost bit always represents the sign, and negative numbers are in 2's complement form. When two signed numbers are added, the sign bit is treated as part of the number and the end carry does not indicate an overflow.

An overflow cannot occur after an addition if one number is positive and the other is negative, since adding a positive number to a negative number produces a result that is smaller than the larger of the two original numbers. An overflow may occur if the two numbers added are both positive or both negative. To see how this can happen, consider the following example. Two signed binary numbers, +70 and +80, are stored in two 8-bit registers. The range of numbers that each register can accommodate is from binary +127 to binary −128. Since the sum of the two numbers is +150, it exceeds the capacity of the 8-bit register. This is true if the numbers are both positive or both negative. The two additions in binary are shown below together with the last two carries.

| | | | |
|---|---|---|---|
| carries: | 0 1 | carries: | 1 0 |
| +70 | 0 1000110 | −70 | 1 0111010 |
| +80 | 0 1010000 | −80 | 1 0110000 |
| +150 | 1 0010110 | −150 | 0 1101010 |

Note that the 8-bit result that should have been positive has a negative sign bit and the 8-bit result that should have been negative has a positive sign bit. If, however, the carry out of the sign bit position is taken as the sign bit of the result, the 9-bit answer so obtained will be correct. Since the answer cannot be accommodated within 8 bits, we say that an overflow occurred.

*flow detection*

An overflow condition can be detected by observing the carry into the sign bit position and the carry out of the sign bit position. If these two carries are not equal, an overflow condition is produced. This is indicated in the examples where the two carries are explicitly shown. If the two carries are applied to an exclusive-OR gate, an overflow will be detected when the output of the gate is equal to 1.

## Decimal Fixed-Point Representation

The representation of decimal numbers in registers is a function of the binary code used to represent a decimal digit. A 4-bit decimal code requires four flip-flops for each decimal digit. The representation of 4385 in BCD requires 16 flip-flops, four flip-flops for each digit. The number will be represented in a register with 16 flip-flops as follows:

0100 0011 1000 0101

By representing numbers in decimal we are wasting a considerable amount of storage space since the number of bits needed to store a decimal number in a binary code is greater than the number of bits needed for its equivalent binary representation. Also, the circuits required to perform decimal arithmetic are more complex. However, there are some advantages in the use of decimal representation because computer input and output data are generated by people who use the decimal system. Some applications, such as business data processing, require small amounts of arithmetic computations compared to the amount required for input and output of decimal data. For this reason, some computers and all electronic calculators perform arithmetic operations directly with the decimal data (in a binary code) and thus eliminate the need for conversion to binary and back to decimal. Some computer systems have hardware for arithmetic calculations with both binary and decimal data.

The representation of signed decimal numbers in BCD is similar to the representation of signed numbers in binary. We can either use the familiar signed-magnitude system or the signed-complement system. The sign of a decimal number is usually represented with four bits to conform with the 4-bit code of the decimal digits. It is customary to designate a plus with four 0's and a minus with the BCD equivalent of 9, which is 1001.

The signed-magnitude system is difficult to use with computers. The signed-complement system can be either the 9's or the 10's complement, but the 10's complement is the one most often used. To obtain the 10's complement of a BCD number, we first take the 9's complement and then add one to the least significant digit. The 9's complement is calculated from the subtraction of each digit from 9.

The procedures developed for the signed-2's complement system apply also to the signed-10's complement system for decimal numbers. Addition is done by adding all digits, including the sign digit, and discarding the end carry. Obviously, this assumes that all negative numbers are in 10's complement form. Consider the addition (+375) + (−240) = +135 done in the signed-10's complement system.

$$
\begin{array}{rl}
0\ 375 & (0000\ 0011\ 0111\ 0101)_{\text{BCD}} \\
+9\ 760 & (1001\ 0111\ 0110\ 0000)_{\text{BCD}} \\
\hline
0\ 135 & (0000\ 0001\ 0011\ 0101)_{\text{BCD}}
\end{array}
$$

The 9 in the leftmost position of the second number indicates that the number is negative. 9760 is the 10's complement of 0240. The two numbers are added and the end carry is discarded to obtain +135. Of course, the decimal numbers inside the computer must be in BCD, including the sign digits. The addition is done with BCD adders (see Fig. 10-18).

The subtraction of decimal numbers either unsigned or in the signed-10's complement system is the same as in the binary case. Take the 10's complement of the subtrahend and add it to the minuend. Many computers have special hardware to perform arithmetic calculations directly with decimal numbers in BCD. The user of the computer can specify by programmed instructions that the arithmetic operations be performed with decimal numbers directly without having to convert them to binary.

## 3-4 Floating-Point Representation

*mantissa*

*exponent*

The floating-point representation of a number has two parts. The first part represents a signed, fixed-point number called the mantissa. The second part designates the position of the decimal (or binary) point and is called the exponent. The fixed-point mantissa may be a fraction or an integer. For example, the decimal number +6132.789 is represented in floating-point with a fraction and an exponent as follows:

| *Fraction* | *Exponent* |
|---|---|
| +0.6132789 | +04 |

The value of the exponent indicates that the actual position of the decimal point is four positions to the right of the indicated decimal point in the fraction. This representation is equivalent to the scientific notation $+0.6132789 \times 10^{+4}$.

Floating-point is always interpreted to represent a number in the following form:

$$m \times r^e$$

Only the mantissa $m$ and the exponent $e$ are physically represented in the register (including their signs). The radix $r$ and the radix-point position of the mantissa are always assumed. The circuits that manipulate the floating-point numbers in registers conform with these two assumptions in order to provide the correct computational results.

A floating-point binary number is represented in a similar manner except that it uses base 2 for the exponent. For example, the binary number +1001.11 is represented with an 8-bit fraction and 6-bit exponent as follows:

| *Fraction* | *Exponent* |
|---|---|
| 01001110 | 000100 |

*fraction*

The fraction has a 0 in the leftmost position to denote positive. The binary point of the fraction follows the sign bit but is not shown in the register. The exponent has the equivalent binary number +4. The floating-point number is equivalent to

$$m \times 2^e = +(.1001110)_2 \times 2^{+4}$$

*normalization*

A floating-point number is said to be *normalized* if the most significant digit of the mantissa is nonzero. For example, the decimal number 350 is normalized but 00035 is not. Regardless of where the position of the radix point is assumed to be in the mantissa, the number is normalized only if its leftmost digit is nonzero. For example, the 8-bit binary number 00011010 is not normalized because of the three leading 0's. The number can be normalized by shifting it three positions to the left and discarding the leading 0's to obtain 11010000. The three shifts multiply the number by $2^3 = 8$. To keep the same value for the floating-point number, the exponent must be subtracted by 3. Normalized numbers provide the maximum possible precision for the floating-point number. A zero cannot be normalized because it does not have a nonzero digit. It is usually represented in floating-point by all 0's in the mantissa and exponent.

Arithmetic operations with floating-point numbers are more complicated than arithmetic operations with fixed-point numbers and their execution takes longer and requires more complex hardware. However, floating-point representation is a must for scientific computations because of the scaling problems involved with fixed-point computations. Many computers and all electronic calculators have the built-in capability of performing floating-point arithmetic operations. Computers that do not have hardware for floating-point computations have a set of subroutines to help the user program scientific problems with floating-point numbers. Arithmetic operations with floating-point numbers are discussed in Sec. 10-5.

## 3-5 Other Binary Codes

In previous sections we introduced the most common types of binary-coded data found in digital computers. Other binary codes for decimal numbers and alphanumeric characters are sometimes used. Digital computers also employ other binary codes for special applications. A few additional binary codes encountered in digital computers are presented in this section.

### Gray Code

*Gray code*

Digital systems can process data in discrete form only. Many physical systems supply continuous output data. The data must be converted into digital form before they can be used by a digital computer. Continuous, or analog, information is converted into digital form by means of an analog-to-digital converter. The reflected binary or *Gray code*, shown in Table 3-5, is sometimes used for the converted digital data. The advantage of the Gray code over straight binary numbers is that the Gray code changes by only one bit as it sequences from one number to the next. In other words, the change from any number to the next in sequence is recognized by a change of only one bit from 0 to 1 or from 1 to 0. A typical application of the Gray code occurs when the analog data are represented by the continuous change of a shaft position. The shaft is partitioned into segments with each segment assigned a number. If adjacent segments are made to correspond to adjacent Gray code numbers, ambiguity is reduced when the shaft position is in the line that separates any two segments.

Gray code counters are sometimes used to provide the timing sequences

TABLE 3-5 4-Bit Gray Code

| Binary code | Decimal equivalent | Binary code | Decimal equivalent |
|---|---|---|---|
| 0000 | 0 | 1100 | 8 |
| 0001 | 1 | 1101 | 9 |
| 0011 | 2 | 1111 | 10 |
| 0010 | 3 | 1110 | 11 |
| 0110 | 4 | 1010 | 12 |
| 0111 | 5 | 1011 | 13 |
| 0101 | 6 | 1001 | 14 |
| 0100 | 7 | 1000 | 15 |

that control the operations in a digital system. A Gray code counter is a counter whose flip-flops go through a sequence of states as specified in Table 3-5. Gray code counters remove the ambiguity during the change from one state of the counter to the next because only one bit can change during the state transition.

## Other Decimal Codes

Binary codes for decimal digits require a minimum of four bits. Numerous different codes can be formulated by arranging four or more bits in 10 distinct possible combinations. A few possibilities are shown in Table 3-6.

TABLE 3-6 Four Different Binary Codes for the Decimal Digit

| Decimal digit | BCD 8421 | 2421 | Excess-3 | Excess-3 gray |
|---|---|---|---|---|
| 0 | 0000 | 0000 | 0011 | 0010 |
| 1 | 0001 | 0001 | 0100 | 0110 |
| 2 | 0010 | 0010 | 0101 | 0111 |
| 3 | 0011 | 0011 | 0110 | 0101 |
| 4 | 0100 | 0100 | 0111 | 0100 |
| 5 | 0101 | 1011 | 1000 | 1100 |
| 6 | 0110 | 1100 | 1001 | 1101 |
| 7 | 0111 | 1101 | 1010 | 1111 |
| 8 | 1000 | 1110 | 1011 | 1110 |
| 9 | 1001 | 1111 | 1100 | 1010 |
| | 1010 | 0101 | 0000 | 0000 |
| Unused | 1011 | 0110 | 0001 | 0001 |
| bit | 1100 | 0111 | 0010 | 0011 |
| combi- | 1101 | 1000 | 1101 | 1000 |
| nations | 1110 | 1001 | 1110 | 1001 |
| | 1111 | 1010 | 1111 | 1011 |

The BCD (binary-coded decimal) has been introduced before. It uses a straight assignment of the binary equivalent of the digit. The six unused bit combinations listed have no meaning when BCD is used, just as the letter H has no meaning when decimal digit symbols are written down. For example, saying that 1001 1110 is a decimal number in BCD is like saying that 9H is a decimal number in the conventional symbol designation. Both cases contain an invalid symbol and therefore designate a meaningless number.

One disadvantage of using BCD is the difficulty encountered when the 9's complement of the number is to be computed. On the other hand, the 9's complement is easily obtained with the 2421 and the excess-3 codes listed in Table 3-6. These two codes have a self-complementing property which means that the 9's complement of a decimal number, when represented in one of these codes, is easily obtained by changing 1's to 0's and 0's to 1's. This property is useful when arithmetic operations are done in signed-complement representation.

***self-complementing***

***weighted code***

The 2421 is an example of a *weighted code*. In a weighted code, the bits are multiplied by the weights indicated and the sum of the weighted bits gives the decimal digit. For example, the bit combination 1101, when weighted by the respective digits 2421, gives the decimal equivalent of $2 \times 1 + 4 \times 1 + 2 \times 0 + 1 + 1 = 7$. The BCD code can be assigned the weights 8421 and for this reason it is sometimes called the 8421 code.

***excess-3 code***

The excess-3 code is a decimal code that has been used in older computers. This is an unweighted code. Its binary code assignment is obtained from the corresponding BCD equivalent binary number after the addition of binary 3 (0011).

From Table 3-5 we note that the Gray code is not suited for a decimal code if we were to choose the first 10 entries in the table. This is because the transition from 9 back to 0 involves a change of three bits (from 1101 to 0000). To overcome this difficulty, we choose the 10 numbers starting from the third entry 0010 up to the twelfth entry 1010. Now the transition from 1010 to 0010 involves a change of only one bit. Since the code has been shifted up three numbers, it is called the excess-3 Gray. This code is listed with the other decimal codes in Table 3-6.

## Other Alphanumeric Codes

The ASCII code (Table 3-4) is the standard code commonly used for the transmission of binary information. Each character is represented by a 7-bit code and usually an eighth bit is inserted for parity (see Sec. 3-6). The code consists of 128 characters. Ninety-five characters represent *graphic symbols* that include upper- and lowercase letters, numerals zero to nine, punctuation marks, and special symbols. Twenty-three characters represent *format effectors*, which are functional characters for controlling the layout of printing or display devices such as carriage return, line feed, horizontal tabulation, and back space. The other 10 characters are used to direct the data communication flow and report its status.

*EBCDIC*

Another alphanumeric (sometimes called *alphameric*) code used in IBM equipment is the EBCDIC (Extended BCD Interchange Code). It uses eight bits for each character (and a ninth bit for parity). EBCDIC has the same character symbols as ASCII but the bit assignment to characters is different.

When alphanumeric characters are used internally in a computer for data processing (not for transmission purposes) it is more convenient to use a 6-bit code to represent 64 characters. A 6-bit code can specify the 26 uppercase letters of the alphabet, numerals zero to nine, and up to 28 special characters. This set of characters is usually sufficient for data-processing purposes. Using fewer bits to code characters has the advantage of reducing the memory space needed to store large quantities of alphanumeric data.

## 3-6 Error Detection Codes

Binary information transmitted through some form of communication medium is subject to external noise that could change bits from 1 to 0, and vice versa. An error detection code is a binary code that detects digital errors during transmission. The detected errors cannot be corrected but their presence is indicated. The usual procedure is to observe the frequency of errors. If errors occur infrequently at random, the particular erroneous information is transmitted again. If the error occurs too often, the system is checked for malfunction.

*parity bit*

The most common error detection code used is the *parity bit*. A parity bit is an extra bit included with a binary message to make the total number of 1's either odd or even. A message of three bits and two possible parity bits is shown in Table 3-7. The $P$(odd) bit is chosen in such a way as to make the sum of 1's (in all four bits) odd. The $P$(even) bit is chosen to make the sum of all 1's even. In either case, the sum is taken over the message and the $P$ bit. In any particular application, one or the other type of parity will be adopted. The even-parity scheme has the disadvantage of having a bit combination of all 0's, while in the odd parity there is always one bit (of the four bits that constitute the message and $P$) that is 1. Note that the $P$(odd) is the complement of the $P$(even).

*parity generator*

*parity checker*

Dunng transfer of information from one location to another, the parity bit is handled as follows. At the sending end, the message (in this case three bits) is applied to a *parity generator*, where the required parity bit is generated. The message, including the parity bit, is transmitted to its destination. At the receiving end, all the incoming bits (in this case, four) are applied to a *parity checker* that checks the proper parity adopted (odd or even). An error is detected if the checked parity does not conform to the adopted parity. The parity method detects the presence of one, three, or any odd number of errors. An even number of errors is not detected.

TABLE 3-7 Parity Bit Generation

| Message xyz | P(odd) | P(even) |
|---|---|---|
| 000 | 1 | 0 |
| 001 | 0 | 1 |
| 010 | 0 | 1 |
| 011 | 1 | 0 |
| 100 | 0 | 1 |
| 101 | 1 | 0 |
| 110 | 1 | 0 |
| 111 | 0 | 1 |

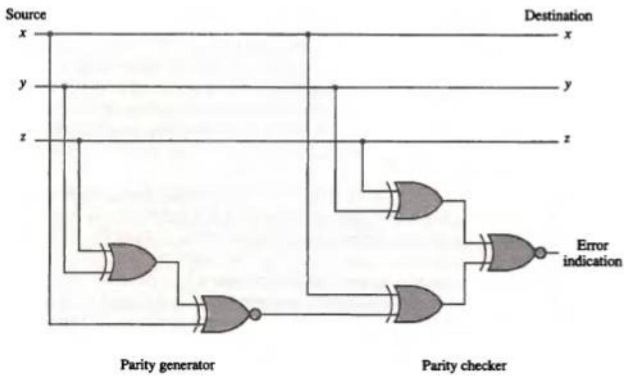
Parity generator and checker networks are logic circuits constructed with exclusive-OR functions. This is because, as mentioned in Sec. 1-2, the exclusive-OR function of three or more variables is by definition an odd function.

***odd function***

An odd function is a logic function whose value is binary 1 if, and only if, an odd number of variables are equal to 1. According to this definition, the *P*(even) function is the exclusive-OR of $x$, $y$, and $z$ because it is equal to 1 when either one or all three of the variables are equal to 1 (Table 3-7). The *P*(odd) function is the complement of the *P*(even) function.

As an example, consider a 3-bit message to be transmitted with an odd parity bit. At the sending end, the odd-parity bit is generated by a parity

Figure 3-3 Error detection with odd parity bit.



generator circuit. As shown in Fig. 3-3, this circuit consists of one exclusive-OR and one exclusive-NOR gate. Since $P$(even) is the exclusive-OR of $x$, $y$, $z$, and $P$(odd) is the complement of $P$(even), it is necessary to employ an exclusive-NOR gate for the needed complementation. The message and the odd-parity bit are transmitted to their destination where they are applied to a parity checker. An error has occurred during transmission if the parity of the four bits received is even, since the binary information transmitted was originally odd. The output of the parity checker would be 1 when an error occurs, that is, when the number of 1's in the four inputs is even. Since the exclusive-OR function of the four inputs is an odd function, we again need to complement the output by using an exclusive-NOR gate..

It is worth noting that the parity generator can use the same circuit as the parity checker if the fourth input is permanently held at a logic-0 value. The advantage of this is that the same circuit can be used for both parity generation and parity checking.

It is evident from the example above that even-parity generators and checkers can be implemented with exclusive-OR functions. Odd-parity networks need an exclusive-NOR at the output to complement the function.

## PROBLEMS

**3-1.** Convert the following binary numbers to decimal: 101110; 1110101; and 110110100.

**3-2.** Convert the following numbers with the indicated bases to decimal: $(12121)_3$; $(4310)_5$; $(50)_7$; and $(198)_{12}$.

**3-3.** Convert the following decimal numbers to binary: 1231; 673; and 1998.

**3-4.** Convert the following decimal numbers to the bases indicated.

a. 7562 to octal
b. 1938 to hexadecimal
c. 175 to binary

**3-5.** Convert the hexadecimal number F3A7C2 to binary and octal.

**3-6.** What is the radix of the numbers if the solution to the quadratic equation $x^2 - 10x + 31 = 0$ is $x = 5$ and $x = 8$?

**3-7.** Show the value of all bits of a 12-bit register that hold the number equivalent to decimal 215 in (a) binary; (b) binary-coded octal; (c) binary-coded hexadecimal; (d) binary-coded decimal (BCD).

**3-8.** Show the bit configuration of a 24-bit register when its content represents the decimal equivalent of 295: (a) in binary; (b) in BCD; (c) in ASCII using eight bits with even parity.

**3-9.** Write your name in ASCII using an 8-bit code with the leftmost bit always 0. Include a space between names and a period after a middle initial.

**3-10.** Decode the following ASCII code:

1001010 1001111 1001000 1001110 0100000 1000100 1001111 1000101

**3-11.** Obtain the 9's complement of the following eight-digit decimal numbers: 12349876; 00980100; 90009951; and 00000000.

**3-12.** Obtain the 10's complement of the following six-digit decimal numbers: 123900; 090657; 100000; and 000000.

**3-13.** Obtain the 1's and 2's complements of the following eight-digit binary numbers: 10101110; 10000001; 10000000; 00000001; and 00000000.

**3-14.** Perform the subtraction with the following unsigned decimal numbers by taking the 10's complement of the subtrahend.

**a.** 5250 − 1321 **b.** 1753 − 8640
**c.** 20 − 100 **d.** 1200 − 250

**3-15.** Perform the subtraction with the following unsigned binary numbers by taking the 2's complement of the subtrahend.

**a.** 11010 − 10000 **b.** 11010 − 1101
**c.** 100 − 110000 **d.** 1010100 − 1010100

**3-16.** Perform the arithmetic operations (+42) + (−13) and (−42) − (−13) in binary using signed-2's complement representation for negative numbers.

**3-17.** Perform the arithmetic operations (+70) + (+80) and (−70) + (−80) with binary numbers in signed-2's complement representation. Use eight bits to accommodate each number together with its sign. Show that overflow occurs in both cases, that the last two carries are unequal, and that there is a sign reversal.

**3-18.** Perform the following arithmetic operations with the decimal numbers using signed-10's complement representation for negative numbers.

**a.** (−638) + (+785)
**b.** (−638) − (+185)

**3-19.** A 36-bit floating-point binary number has eight bits plus sign for the exponent and 26 bits plus sign for the mantissa. The mantissa is a normalized fraction. Numbers in the mantissa and exponent are in signed-magnitude representation. What are the largest and smallest positive quantities that can be represented, excluding zero?

**3-20.** Represent the number $(+46.5)_{10}$ as a floating-point binary number with 24 bits. The normalized fraction mantissa has 16 bits and the exponent has 8 bits.

**3-21.** The Gray code is sometimes called a reflected code because the bit values are reflected on both sides of any $2^n$ value. For example, as shown in Table 3-5, the values of the three low-order bits are reflected over a line drawn between 7 and 8. Using this property of the Gray code, obtain:

**a.** The Gray code numbers for 16 through 31 as a continuation of Table 3-5.
**b.** The excess-3 Gray code for decimals 10 to 19 as a continuation of the list in Table 3-6.

**3-22.** Represent decimal number 8620 in (a) BCD; (b) excess-3 code; (c) 2421 code; (d) as a binary number.

**3-23.** List the 10 BCD digits with an even parity in the leftmost position (total of five bits per digit). Repeat with an odd-parity bit.

**3-24.** Represent decimal 3984 in the 2421 code of Table 3-6. Complement all bits of the coded number and show that the result is the 9's complement of 3984 in the 2421 code.

**3-25.** Show that the exclusive-OR function $x = A \oplus B \oplus C \oplus D$ is an odd function. One way to show this is to obtain the truth table for $y = A \oplus B$ and for $z = C \oplus D$ and then formulate the truth table for $x = y \oplus z$. Show that $x = 1$ only when the total number of 1's in $A$, $B$, $C$, and $D$ is odd.

**3-26.** Derive the circuits for a 3-bit parity generator and 4-bit parity checker using an even-parity bit. (The circuits of Fig. 3-3 use odd parity.)

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