CHAPTER FOUR

Register Transfer and Microoperations

IN THIS CHAPTER

4-1 Register Transfer Language

A digital system is an interconnection of digital hardware modules that accomplish a specific information-processing task. Digital systems vary in size and complexity from a few integrated circuits to a complex of interconnected and interacting digital computers. Digital system design invariably uses a modular approach. The modules are constructed from such digital components as registers, decoders, arithmetic elements, and control logic. The various modules are interconnected with common data and control paths to form a digital computer system.

microoperation

Digital modules are best defined by the registers they contain and the operations that are performed on the data stored in them. The operations executed on data stored in registers are called microoperations. A microoperation is an elementary operation performed on the information stored in one or more registers. The result of the operation may replace the previous binary information of a register or may be transferred to another register. Examples of microoperations are shift, count, clear, and load. Some of the digital components introduced in Chap. 2 are registers that implement microoperations. For example, a counter with parallel load is capable of performing the micro-

operations increment and load. A bidirectional shift register is capable of performing the shift right and shift left microoperations.

The internal hardware organization of a digital computer is best defined by specifying:

1. The set of registers it contains and their function.
2. The sequence of microoperations performed on the binary information stored in the registers.
3. The control that initiates the sequence of microoperations.

It is possible to specify the sequence of microoperations in a computer by explaining every operation in words, but this procedure usually involves a lengthy descriptive explanation. It is more convenient to adopt a suitable symbology to describe the sequence of transfers between registers and the various arithmetic and logic microoperations associated with the transfers. The use of symbols instead of a narrative explanation provides an organized and concise manner for listing the microoperation sequences in registers and the control functions that initiate them.

register transfer language

The symbolic notation used to describe the microoperation transfers among registers is called a register transfer language. The term "register transfer" implies the availability of hardware logic circuits that can perform a stated microoperation and transfer the result of the operation to the same or another register. The word "language" is borrowed from programmers, who apply this term to programming languages. A programming language is a procedure for writing symbols to specify a given computational process. Similarly, a natural language such as English is a system for writing symbols and combining them into words and sentences for the purpose of communication between people. A register transfer language is a system for expressing in symbolic form the microoperation sequences among the registers of a digital module. It is a convenient tool for describing the internal organization of digital computers in concise and precise manner. It can also be used to facilitate the design process of digital systems.

The register transfer language adopted here is believed to be as simple as possible, so it should not take very long to memorize. We will proceed to define symbols for various types of microoperations, and at the same time, describe associated hardware that can implement the stated microoperations. The symbolic designation introduced in this chapter will be utilized in subsequent chapters to specify the register transfers, the microoperations, and the control functions that describe the internal hardware organization of digital computers. Other symbology in use can easily be learned once this language has become familiar, for most of the differences between register transfer languages consist of variations in detail rather than in overall purpose.

4-2 Register Transfer

registers

Computer registers are designated by capital letters (sometimes followed by numerals) to denote the function of the register. For example, the register that holds an address for the memory unit is usually called a memory address register and is designated by the name *MAR*. Other designations for registers are *PC* (for program counter), *IR* (for instruction register, and *R*1 (for processor register). The individual flip-flops in an n-bit register are numbered in sequence from 0 through $n - 1$, starting from 0 in the rightmost position and increasing the numbers toward the left. Figure 4-1 shows the representation of registers in block diagram form. The most common way to represent a register is by a rectangular box with the name of the register inside, as in Fig. 4-1(a). The individual bits can be distinguished as in (b). The numbering of bits in a 16-bit register can be marked on top of the box as shown in (c). A 16-bit register is partitioned into two parts in (d). Bits 0 through 7 are assigned the symbol L (for low byte) and bits 8 through 15 are assigned the symbol H (for high byte). The name of the 16-bit register is *PC*. The symbol *PC*(0–7) or *PC*(*L*) refers to the low-order byte and *PC*(8–15) or *PC*(*H*) to the high-order byte.

register transfer

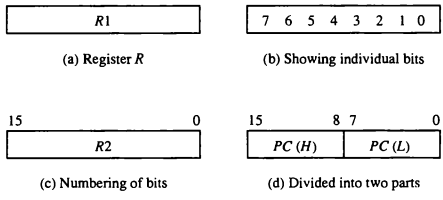
Information transfer from one register to another is designated in symbolic form by means of a replacement operator. The statement

$$R2 \leftarrow R1$$

denotes a transfer of the content of register *R*1 into register *R*2. It designates a replacement of the content of *R*2 by the content of *R*1. By definition, the content of the source register *R*1 does not change after the transfer.

A statement that specifies a register transfer implies that circuits are available from the outputs of the source register to the inputs of the destination register and that the destination register has a parallel load capability. Nor-

Figure 4-1 Block diagram of register.



mally, we want the transfer to occur only under a predetermined control condition. This can be shown by means of an *if-then* statement.

$$If\ (P = 1)\ then\ (R2 \leftarrow R1)$$

where P is a control signal generated in the control section. It is sometimes convenient to separate the control variables from the register transfer operation by specifying a *control function*. A control function is a Boolean variable that is equal to 1 or 0. The control function is included in the statement as follows:

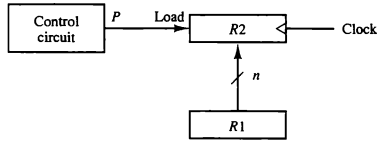
control function

$$P:\ \ R2 \leftarrow R1$$

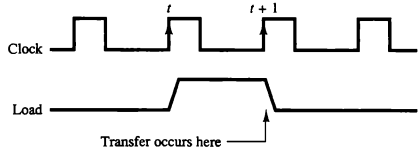
The control condition is terminated with a colon. It symbolizes the requirement that the transfer operation be executed by the hardware only if $P = 1$.

Every statement written in a register transfer notation implies a hardware construction for implementing the transfer. Figure 4-2 shows the block diagram that depicts the transfer from $R1$ to $R2$. The n outputs of register $R1$ are connected to the n inputs of register $R2$. The letter n will be used to indicate any number of bits for the register. It will be replaced by an actual number when the length of the register is known. Register $R2$ has a load input that is activated by the control variable P. It is assumed that the control variable is synchronized with the same clock as the one applied to the register. As shown

Figure 4-2 Transfer from $R1$ to $R2$ when $P = 1$.



(a) Block diagram



(b) Timing diagram

in the timing diagram, P is activated in the control section by the rising edge of a clock pulse at time t. The next positive transition of the clock at time $t + 1$ finds the load input active and the data inputs of $R2$ are then loaded into the register in parallel. P may go back to 0 at time $t + 1$; otherwise, the transfer will occur with every clock pulse transition while P remains active.

Note that the clock is not included as a variable in the register transfer statements. It is assumed that all transfers occur during a clock edge transition. Even though the control condition such as P becomes active just after time t, the actual transfer does not occur until the register is triggered by the next positive transition of the clock at time $t + 1$.

The basic symbols of the register transfer notation are listed in Table 4-1. Registers are denoted by capital letters, and numerals may follow the letters. Parentheses are used to denote a part of a register by specifying the range of bits or by giving a symbol name to a portion of a register. The arrow denotes a transfer of information and the direction of transfer. A comma is used to separate two or more operations that are executed at the same time. The statement

$$T: \quad R2 \leftarrow R1, \quad R1 \leftarrow R2$$

denotes an operation that exchanges the contents of two registers during one common clock pulse provided that $T = 1$. This simultaneous operation is possible with registers that have edge-triggered flip-flops.

TABLE 4-1 Basic Symbols for Register Transfers

Symbol	Description	Examples
Letters (and numerals)	Denotes a register	*MAR*, *R2*
Parentheses ( )	Denotes a part of a register	*R2*(0–7), *R2*(*L*)
Arrow ←	Denotes transfer of information	*R2* ← *R1*
Comma ,	Separates two microoperations	*R2* ← *R1*, *R1* ← *R2*

4-3 Bus and Memory Transfers

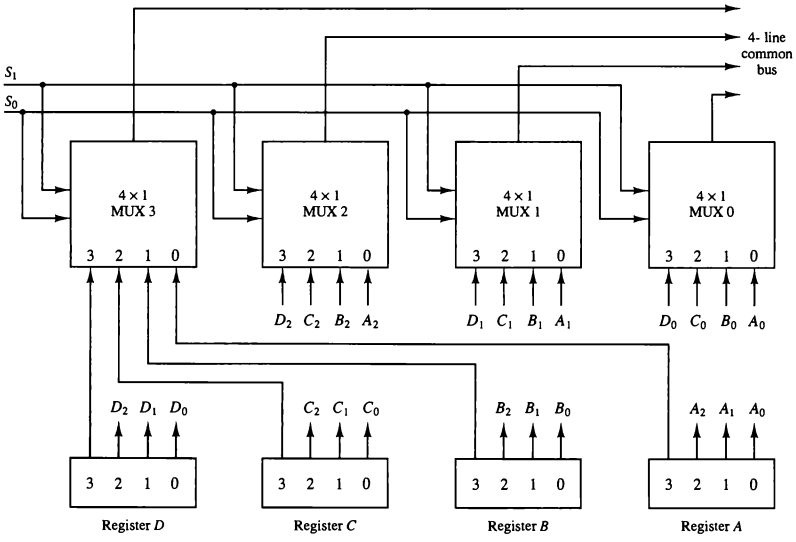
A typical digital computer has many registers, and paths must be provided to transfer information from one register to another. The number of wires will be excessive if separate lines are used between each register and all other registers in the system. A more efficient scheme for transferring information between registers in a multiple-register configuration is a common bus system. A bus structure consists of a set of common lines, one for each bit of a register, through which binary information is transferred one at a time. Control signals

common bus

determine which register is selected by the bus during each particular register transfer.

One way of constructing a common bus system is with multiplexers. The multiplexers select the source register whose binary information is then placed on the bus. The construction of a bus system for four registers is shown in Fig. 4-3. Each register has four bits, numbered 0 through 3. The bus consists of four 4 × 1 multiplexers each having four data inputs, 0 through 3, and two selection inputs, S_1 and S_0. In order not to complicate the diagram with 16 lines crossing each other, we use labels to show the connections from the outputs of the registers to the inputs of the multiplexers. For example, output 1 of register A is connected to input 0 of MUX 1 because this input is labeled A_1. The diagram shows that the bits in the same significant position in each register are connected to the data inputs of one multiplexer to form one line of the bus. Thus MUX 0 multiplexes the four 0 bits of the registers, MUX 1 multiplexes the four 1 bits of the registers, and similarly for the other two bits.

Figure 4-3 Bus system for four registers.



bus selection

The two selection lines S_1 and S_0 are connected to the selection inputs of all four multiplexers. The selection lines choose the four bits of one register and transfer them into the four-line common bus. When $S_1S_0 = 00$, the 0 data inputs of all four multiplexers are selected and applied to the outputs that form the bus. This causes the bus lines to receive the content of register A since the outputs of this register are connected to the 0 data inputs of the multiplexers. Similarly, register B is selected if $S_1S_0 = 01$, and so on. Table 4-2 shows the register that is selected by the bus for each of the four possible binary value of the selection lines.

TABLE 4-2 Function Table for Bus of Fig. 4-3

S_1	S_0	Register selected
0	0	A
0	1	B
1	0	C
1	1	D

In general, a bus system will multiplex k registers of n bits each to produce an n-line common bus. The number of multiplexers needed to construct the bus is equal to n, the number of bits in each register. The size of each multiplexer must be $k \times 1$ since it multiplexes k data lines. For example, a common bus for eight registers of 16 bits each requires 16 multiplexers, one for each line in the bus. Each multiplexer must have eight data input lines and three selection lines to multiplex one significant bit in the eight registers.

The transfer of information from a bus into one of many destination registers can be accomplished by connecting the bus lines to the inputs of all destination registers and activating the load control of the particular destination register selected. The symbolic statement for a bus transfer may mention the bus or its presence may be implied in the statement. When the bus is includes in the statement, the register transfer is symbolized as follows:

$$BUS \leftarrow C, \qquad R1 \leftarrow BUS$$

The content of register C is placed on the bus, and the content of the bus is loaded into register $R1$ by activating its load control input. If the bus is known to exist in the system, it may be convenient just to show the direct transfer.

$$R1 \leftarrow C$$

From this statement the designer knows which control signals must be activated to produce the transfer through the bus.

Three-State Bus Buffers

three-state gate

A bus system can be constructed with three-state gates instead of multiplexers. A three-state gate is a digital circuit that exhibits three states. Two of the states are signals equivalent to logic 1 and 0 as in a conventional gate. The third state is a *high-impedance state*. The high-impedance state behaves like an open circuit, which means that the output is disconnected and does not have a logic significance. Three-state gates may perform any conventional logic, such as AND or NAND. However, the one most commonly used in the design of a bus system is the buffer gate.

high-impedance

buffer

The graphic symbol of a three-state buffer gate is shown in Fig. 4-4. It is distinguished from a normal buffer by having both a normal input and a control input. The control input determines the output state. When the control input is equal to 1, the output is enabled and the gate behaves like any conventional buffer, with the output equal to the normal input. When the control input is 0, the output is disabled and the gate goes to a high-impedance state, regardless of the value in the normal input. The high-impedance state of a three-state gate provides a special feature not available in other gates. Because of this feature, a large number of three-state gate outputs can be connected with wires to form a common bus line without endangering loading effects.

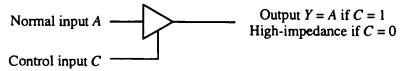
bus system

The construction of a bus system with three-state buffers is demonstrated in Fig. 4-5. The outputs of four buffers are connected together to form a single bus line. (It must be realized that this type of connection cannot be done with gates that do not have three-state outputs.) The control inputs to the buffers determine which of the four normal inputs will communicate with the bus line. No more than one buffer may be in the active state at any given time. The connected buffers must be controlled so that only one three-state buffer has access to the bus line while all other buffers are maintained in a high-impedance state.

One way to ensure that no more than one control input is active at any given time is to use a decoder, as shown in the diagram. When the enable input of the decoder is 0, all of its four outputs are 0, and the bus line is in a high-impedance state because all four buffers are disabled. When the enable input is active, one of the three-state buffers will be active, depending on the binary value in the select inputs of the decoder. Careful investigation will reveal that Fig. 4-5 is another way of constructing a 4×1 multiplexer since the circuit can replace the multiplexer in Fig. 4-3.

To construct a common bus for four registers of n bits each using three-

Figure 4-4 Graphic symbols for three-state buffer.



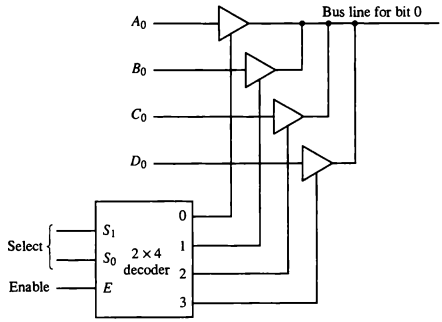


Figure 4-5 Bus line with three state-buffers.

state buffers, we need n circuits with four buffers in each as shown in Fig. 4-5. Each group of four buffers receives one significant bit from the four registers. Each common output produces one of the lines for the common bus for a total of n lines. Only one decoder is necessary to select between the four registers.

Memory Transfer

The operation of a memory unit was described in Sec. 2-7. The transfer of information from a memory word to the outside environment is called a *read* operation. The transfer of new information to be stored into the memory is called a *write* operation. A memory word will be symbolized by the letter M. The particular memory word among the many available is selected by the memory address during the transfer. It is necessary to specify the address of M when writing memory transfer operations. This will be done by enclosing the address in square brackets following the letter M.

Consider a memory unit that receives the address from a register, called the address register, symbolized by AR. The data are transferred to another register, called the data register, symbolized by DR. The read operation can be stated as follows:

memory read

$$\text{Read:} \quad DR \leftarrow M[AR]$$

This causes a transfer of information into DR from the memory word M selected by the address in AR.

memory write

The write operation transfers the content of a data register to a memory word M selected by the address. Assume that the input data are in register $R1$

$$R3 \leftarrow R1 + \overline{R2} + 1$$

$\overline{R2}$ is the symbol for the 1's complement of $R2$. Adding 1 to the 1's complement produces the 2's complement. Adding the contents of $R1$ to the 2's complement of $R2$ is equivalent to $R1 - R2$.

TABLE 4-3 Arithmetic Microoperations

Symbolic designation	Description
$R3 \leftarrow R1 + R2$	Contents of $R1$ plus $R2$ transferred to $R3$
$R3 \leftarrow R1 - R2$	Contents of $R1$ minus $R2$ transferred to $R3$
$R2 \leftarrow \overline{R2}$	Complement the contents of $R2$ (1's complement)
$R2 \leftarrow \overline{R2} + 1$	2's complement the contents of $R2$ (negate)
$R3 \leftarrow R1 + \overline{R2} + 1$	$R1$ plus the 2's complement of $R2$ (subtraction)
$R1 \leftarrow R1 + 1$	Increment the contents of $R1$ by one
$R1 \leftarrow R1 - 1$	Decrement the contents of $R1$ by one

The increment and decrement microoperations are symbolized by plus-one and minus-one operations, respectively. These microoperations are implemented with a combinational circuit or with a binary up-down counter.

The arithmetic operations of multiply and divide are not listed in Table 4-3. These two operations are valid arithmetic operations but are not included in the basic set of microoperations. The only place where these operations can be considered as microoperations is in a digital system, where they are implemented by means of a combinational circuit. In such a case, the signals that perform these operations propagate through gates, and the result of the operation can be transferred into a destination register by a clock pulse as soon as the output signal propagates through the combinational circuit. In most computers, the multiplication operation is implemented with a sequence of add and shift microoperations. Division is implemented with a sequence of subtract and shift microoperations. To specify the hardware in such a case requires a list of statements that use the basic microoperations of add, subtract, and shift (see Chapter 10).

Binary Adder

To implement the add microoperation with hardware, we need the registers that hold the data and the digital component that performs the arithmetic addition. The digital circuit that forms the arithmetic sum of two bits and a previous carry is called a full-adder (see Fig. 1-17). The digital circuit that generates the arithmetic sum of two binary numbers of any length is called a binary adder. The binary adder is constructed with full-adder circuits con-

binary adder

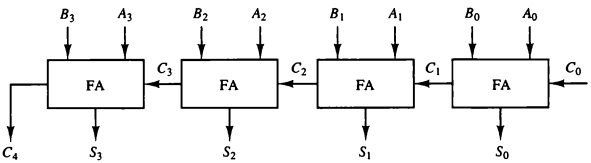


Figure 4-6 4-bit binary adder.

nected in cascade, with the output carry from one full-adder connected to the input carry of the next full-adder. Figure 4-6 shows the interconnections of four full-adders (FA) to provide a 4-bit binary adder. The augend bits of A and the addend bits of B are designated by subscript numbers from right to left, with subscript 0 denoting the low-order bit. The carries are connected in a chain through the full-adders. The input carry to the binary adder is C_0 and the output carry is C_4. The S outputs of the full-adders generate the required sum bits.

full-adder

An n-bit binary adder requires n full-adders. The output carry from each full-adder is connected to the input carry of the next-high-order full-adder. The n data bits for the A inputs come from one register (such as $R1$), and the n data bits for the B inputs come from another register (such as $R2$). The sum can be transferred to a third register or to one of the source registers ($R1$ or $R2$), replacing its previous content.

Binary Adder-Subtractor

The subtraction of binary numbers can be done most conveniently by means of complements as discussed in Sec. 3-2. Remember that the subtraction $A - B$ can be done by taking the 2's complement of B and adding it to A. The 2's complement can be obtained by taking the 1's complement and adding one to the least significant pair of bits. The 1's complement can be implemented with inverters and a one can be added to the sum through the input carry.

The addition and subtraction operations can be combined into one common circuit by including an exclusive-OR gate with each full-adder. A 4-bit adder-subtractor circuit is shown in Fig. 4-7. The mode input M controls the operation. When $M = 0$ the circuit is an adder and when $M = 1$ the circuit becomes a subtractor. Each exclusive-OR gate receives input M and one of the inputs of B. When $M = 0$, we have $B \oplus 0 = B$. The full-adders receive the value of B, the input carry is 0, and the circuit performs A plus B. When $M = 1$, we have $B \oplus 1 = B'$ and $C_0 = 1$. The B inputs are all complemented and a 1 is added through the input carry. The circuit performs the operation A plus the

adder-subtractor

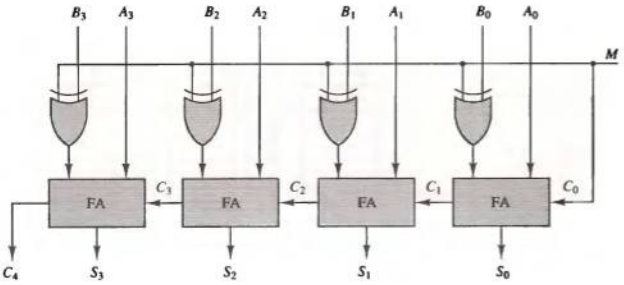


Figure 4-7 4-bit adder-subtractor.

2's complement of B. For unsigned numbers, this gives $A - B$ if $A \geq B$ or the 2's complement of $(B - A)$ if $A < B$. For signed numbers, the result is $A - B$ provided that there is no overflow.

Binary Incrementer

The increment microoperation adds one to a number in a register. For example, if a 4-bit register has a binary value 0110, it will go to 0111 after it is incremented. This microoperation is easily implemented with a binary counter (see Fig. 2-10). Every time the count enable is active, the clock pulse transition increments the content of the register by one. There may be occasions when the increment microoperation must be done with a combinational circuit independent of a particular register. This can be accomplished by means of half-adders (see Fig. 1-16) connected in cascade.

incrementer

The diagram of a 4-bit combinational circuit incrementer is shown in Fig. 4-8. One of the inputs to the least significant half-adder (HA) is connected to logic-1 and the other input is connected to the least significant bit of the number to be incremented. The output carry from one half-adder is connected to one of the inputs of the next-higher-order half-adder. The circuit receives the four bits from A_0 through A_3, adds one to it, and generates the incremented output in S_0 through S_3. The output carry C_4 will be 1 only after incrementing binary 1111. This also causes outputs S_0 through S_3 to go to 0.

The circuit of Fig. 4-8 can be extended to an n-bit binary incrementer by extending the diagram to include n half-adders. The least significant bit must have one input connected to logic-1. The other inputs receive the number to be incremented or the carry from the previous stage.

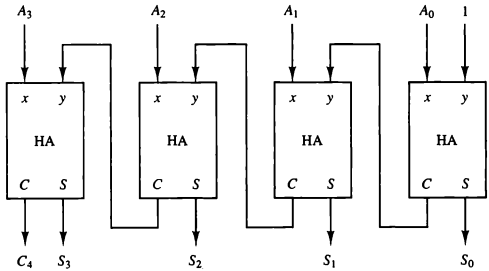


Figure 4-8 4-bit binary incrementer.

Arithmetic Circuit

arithmetic circuit

The arithmetic microoperations listed in Table 4-3 can be implemented in one composite arithmetic circuit. The basic component of an arithmetic circuit is the parallel adder. By controlling the data inputs to the adder, it is possible to obtain different types of arithmetic operations.

The diagram of a 4-bit arithmetic circuit is shown in Fig. 4-9. It has four full-adder circuits that constitute the 4-bit adder and four multiplexers for choosing different operations. There are two 4-bit inputs A and B and a 4-bit output D. The four inputs from A go directly to the X inputs of the binary adder. Each of the four inputs from B are connected to the data inputs of the multiplexers. The multiplexers data inputs also receive the complement of B. The other two data inputs are connected to logic-0 and logic-1. Logic-0 is a fixed voltage value (0 volts for TTL integrated circuits) and the logic-1 signal can be generated through an inverter whose input is 0. The four multiplexers are controlled by two selection inputs, S_1 and S_0. The input carry C_{in} goes to the carry input of the FA in the least significant position. The other carries are connected from one stage to the next.

input carry

The output of the binary adder is calculated from the following arithmetic sum:

$$D = A + Y + C_{in}$$

where A is the 4-bit binary number at the X inputs and Y is the 4-bit binary number at the Y inputs of the binary adder. C_{in} is the input carry, which can be equal to 0 or 1. Note that the symbol + in the equation above denotes an arithmetic plus. By controlling the value of Y with the two selection inputs S_1 and S_0 and making C_{in} equal to 0 or 1, it is possible to generate the eight arithmetic microoperations listed in Table 4-4.

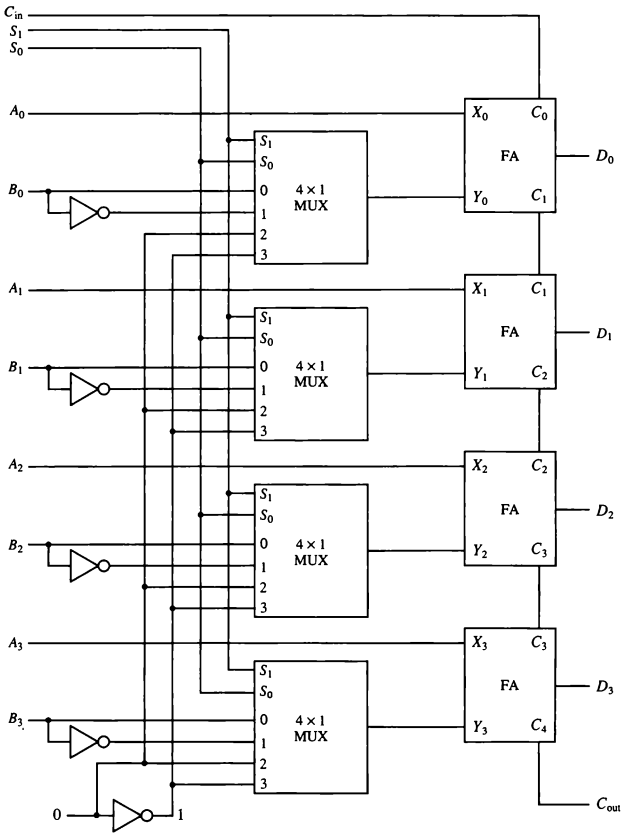


Figure 4-9 4-bit arithmetic circuit.

TABLE 4-4 Arithmetic Circuit Function Table

Select			Input	Output	
S_1	S_0	C_{in}	Y	$D = A + Y + C_{in}$	Microoperation
0	0	0	B	$D = A + B$	Add
0	0	1	B	$D = A + B + 1$	Add with carry
0	1	0	$\overline{B}$	$D = A + \overline{B}$	Subtract with borrow
0	1	1	$\overline{B}$	$D = A + \overline{B} + 1$	Subtract
1	0	0	0	$D = A$	Transfer A
1	0	1	0	$D = A + 1$	Increment A
1	1	0	1	$D = A - 1$	Decrement A
1	1	1	1	$D = A$	Transfer A

When $S_1S_0 = 00$, the value of B is applied to the Y inputs of the adder. If $C_{in} = 0$, the output $D = A + B$. If $C_{in} = 1$, output $D = A + B + 1$. Both cases perform the add microoperation with or without adding the input carry.

addition

When $S_1S_0 = 01$, the complement of B is applied to the Y inputs of the adder. If $C_{in} = 1$, then $D = A + \overline{B} + 1$. This produces A plus the 2's complement of B, which is equivalent to a subtraction of $A - B$. When $C_{in} = 0$, then $D = A + \overline{B}$. This is equivalent to a subtract with borrow, that is, $A - B - 1$.

subtraction

When $S_1S_0 = 10$, the inputs from B are neglected, and instead, all 0's are inserted into the Y inputs. The output becomes $D = A + 0 + C_{in}$. This gives $D = A$ when $C_{in} = 0$ and $D = A + 1$ when $C_{in} = 1$. In the first case we have a direct transfer from input A to output D. In the second case, the value of A is incremented by 1.

increment

When $S_1S_0 = 11$, all 1's are inserted into the Y inputs of the adder to produce the decrement operation $D = A - 1$ when $C_{in} = 0$. This is because a number with all 1's is equal to the 2's complement of 1 (the 2's complement of binary 0001 is 1111). Adding a number A to the 2's complement of 1 produces $F = A$ + 2's complement of $1 = A - 1$. When $C_{in} = 1$, then $D = A - 1 + 1 = A$, which causes a direct transfer from input A to output D. Note that the microoperation $D = A$ is generated twice, so there are only seven distinct microoperations in the arithmetic circuit.

decrement

4-5 Logic Microoperations

Logic microoperations specify binary operations for strings of bits stored in registers. These operations consider each bit of the register separately and treat them as binary variables. For example, the exclusive-OR microoperation with the contents of two registers $R1$ and $R2$ is symbolized by the statement

$$P: \quad R1 \leftarrow R1 \oplus R2$$

It specifies a logic microoperation to be executed on the individual bits of the registers provided that the control variable $P = 1$. As a numerical example, assume that each register has four bits. Let the content of $R1$ be 1010 and the content of $R2$ be 1100. The exclusive-OR microoperation stated above symbolizes the following logic computation:

$$\begin{array}{ll} 1010 & \text{Content of } R1 \\ \underline{1100} & \text{Content of } R2 \\ 0110 & \text{Content of } R1 \text{ after } P = 1 \end{array}$$

The content of $R1$, after the execution of the microoperation, is equal to the bit-by-bit exclusive-OR operation on pairs of bits in $R2$ and previous values of $R1$. The logic microoperations are seldom used in scientific computations, but they are very useful for bit manipulation of binary data and for making logical decisions.

special symbols

Special symbols will be adopted for the logic microoperations OR, AND, and complement, to distinguish them from the corresponding symbols used to express Boolean functions. The symbol $\vee$ will be used to denote an OR microoperation and the symbol $\wedge$ to denote an AND microoperation. The complement microoperation is the same as the 1's complement and uses a bar on top of the symbol that denotes the register name. By using different symbols, it will be possible to differentiate between a logic microoperation and a control (or Boolean) function. Another reason for adopting two sets of symbols is to be able to distinguish the symbol + , when used to symbolize an arithmetic plus, from a logic OR operation. Although the + symbol has two meanings, it will be possible to distinguish between them by noting where the symbol occurs. When the symbol + occurs in a microoperation, it will denote an arithmetic plus. When it occurs in a control (or Boolean) function, it will denote an OR operation. We will never use it to symbolize an OR microoperation. For example, in the statement

$$P + Q: \quad R1 \leftarrow R2 + R3, \quad R4 \leftarrow R5 \vee R6$$

the + between P and Q is an OR operation between two binary variables of a control function. The + between $R2$ and $R3$ specifies an add microoperation. The OR microoperation is designated by the symbol $\vee$ between registers $R5$ and $R6$.

List of Logic Microoperations

There are 16 different logic operations that can be performed with two binary variables. They can be determined from all possible truth tables obtained with two binary variables as shown in Table 4-5. In this table, each of the 16 columns F_0 through F_{15} represents a truth table of one possible Boolean function for the

TABLE 4-5 Truth Tables for 16 Functions of Two Variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

two variables x and y. Note that the functions are determined from the 16 binary combinations that can be assigned to F.

The 16 Boolean functions of two variables x and y are expressed in algebraic form in the first column of Table 4-6. The 16 logic microoperations are derived from these functions by replacing variable x by the binary content of register A and variable y by the binary content of register B. It is important to realize that the Boolean functions listed in the first column of Table 4-6 represent a relationship between two binary variables x and y. The logic microoperations listed in the second column represent a relationship between the binary content of two registers A and B. Each bit of the register is treated as a binary variable and the microoperation is performed on the string of bits stored in the registers.

TABLE 4-6 Sixteen Logic Microoperations

Boolean function	Microoperation	Name
$F_0 = 0$	$F \leftarrow 0$	Clear
$F_1 = xy$	$F \leftarrow A \wedge B$	AND
$F_2 = xy'$	$F \leftarrow A \wedge \overline{B}$	
$F_3 = x$	$F \leftarrow A$	Transfer A
$F_4 = x'y$	$F \leftarrow \overline{A} \wedge B$	
$F_5 = y$	$F \leftarrow B$	Transfer B
$F_6 = x \oplus y$	$F \leftarrow A \oplus B$	Exclusive-OR
$F_7 = x + y$	$F \leftarrow A \vee B$	OR
$F_8 = (x + y)'$	$F \leftarrow \overline{A \vee B}$	NOR
$F_9 = (x \oplus y)'$	$F \leftarrow \overline{A \oplus B}$	Exclusive-NOR
$F_{10} = y'$	$F \leftarrow \overline{B}$	Complement B
$F_{11} = x + y'$	$F \leftarrow A \vee \overline{B}$	
$F_{12} = x'$	$F \leftarrow \overline{A}$	Complement A
$F_{13} = x' + y$	$F \leftarrow \overline{A} \vee B$	
$F_{14} = (xy)'$	$F \leftarrow \overline{A \wedge B}$	NAND
$F_{15} = 1$	$F \leftarrow$ all 1's	Set to all 1's

Hardware Implementation

The hardware implementation of logic microoperations requires that logic gates be inserted for each bit or pair of bits in the registers to perform the required logic function. Although there are 16 logic microoperations, most computers use only four—AND, OR, XOR (exclusive-OR), and complement—from which all others can be derived.

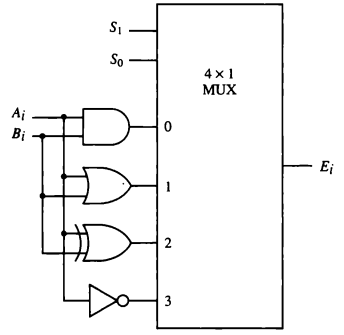
logic circuit

Figure 4-10 shows one stage of a circuit that generates the four basic logic microoperations. It consists of four gates and a multiplexer. Each of the four logic operations is generated through a gate that performs the required logic. The outputs of the gates are applied to the data inputs of the multiplexer. The two selection inputs S_1 and S_0 choose one of the data inputs of the multiplexer and direct its value to the output. The diagram shows one typical stage with subscript i. For a logic circuit with n bits, the diagram must be repeated n times for $i = 0, 1, 2, \ldots, n - 1$. The selection variables are applied to all stages. The function table in Fig. 4-10(b) lists the logic microoperations obtained for each combination of the selection variables.

Some Applications

Logic microoperations are very useful for manipulating individual bits or a portion of a word stored in a register. They can be used to change bit values, delete a group of bits, or insert new bit values into a register. The following examples show how the bits of one register (designated by A) are manipulated

Figure 4-10 One stage of logic circuit.



(a) Logic diagram

S_1	S_0	Output	Operation
0	0	$E = A \wedge B$	AND
0	1	$E = A \vee B$	OR
1	0	$E = A \oplus B$	XOR
1	1	$E = \overline{A}$	Complement

(b) Function table

by logic microoperations as a function of the bits of another register (designated by B). In a typical application, register A is a processor register and the bits of register B constitute a logic operand extracted from memory and placed in register B.

selective-set

The *selective-set* operation sets to 1 the bits in register A where there are corresponding 1's in register B. It does not affect bit positions that have 0's in B. The following numerical example clarifies this operation:

```
1010    A before
1100    B (logic operand)
1110    A after
```

The two leftmost bits of B are 1's, so the corresponding bits of A are set to 1. One of these two bits was already set and the other has been changed from 0 to 1. The two bits of A with corresponding 0's in B remain unchanged. The example above serves as a truth table since it has all four possible combinations of two binary variables. From the truth table we note that the bits of A after the operation are obtained from the logic-OR operation of bits in B and previous values of A. Therefore, the OR microoperation can be used to selectively set bits of a register.

selective-complement

The *selective-complement* operation complements bits in A where there are corresponding 1's in B. It does not affect bit positions that have 0's in B. For example:

```
1010    A before
1100    B (logic operand)
0110    A after
```

Again the two leftmost bits of B are 1's, so the corresponding bits of A are complemented. This example again can serve as a truth table from which one can deduce that the selective-complement operation is just an exclusive-OR microoperation. Therefore, the exclusive-OR microoperation can be used to selectively complement bits of a register.

selective-clear

The *selective-clear* operation clears to 0 the bits in A only where there are corresponding 1's in B. For example:

```
1010    A before
1100    B (logic operand)
0010    A after
```

Again the two leftmost bits of B are 1's, so the corresponding bits of A are cleared to 0. One can deduce that the Boolean operation performed on the individual bits is AB'. The corresponding logic microoperation is

$$A \leftarrow A \wedge \overline{B}$$

The *mask* operation is similar to the selective-clear operation except that the bits of A are cleared only where there are corresponding 0's in B. The mask operation is an AND micro operation as seen from the following numerical example:

1010	A before
1100	B (logic operand)
1000	A after masking

The two rightmost bits of A are cleared because the corresponding bits of B are 0's. The two leftmost bits are left unchanged because the corresponding bits of B are 1's. The mask operation is more convenient to use than the selective-clear operation because most computers provide an AND instruction, and few provide an instruction that executes the microoperation for selective-clear.

The *insert* operation inserts a new value into a group of bits. This is done by first masking the bits and then ORing them with the required value. For example, suppose that an A register contains eight bits, 0110 1010. To replace the four leftmost bits by the value 1001 we first mask the four unwanted bits:

0110 1010	A before
0000 1111	B (mask)
0000 1010	A after masking

and then insert the new value:

0000 1010	A before
1001 0000	B (insert)
1001 1010	A after insertion

The mask operation is an AND microoperation and the insert operation is an OR microoperation.

The *clear* operation compares the words in A and B and produces an all 0's result if the two numbers are equal. This operation is achieved by an exclusive-OR microoperation as shown by the following example:

1010	A
1010	B
0000	$A \leftarrow A \oplus B$

When A and B are equal, the two corresponding bits are either both 0 or both 1. In either case the exclusive-OR operation produces a 0. The all-0's result is then checked to determine if the two numbers were equal.

4-6 Shift Microoperations

Shift microoperations are used for serial transfer of data. They are also used in conjunction with arithmetic, logic, and other data-processing operations. The contents of a register can be shifted to the left or the right. At the same time that the bits are shifted, the first flip-flop receives its binary information from the serial input. During a shift-left operation the serial input transfers a bit into the rightmost position. During a shift-right operation the serial input transfers a bit into the leftmost position. The information transferred through the serial input determines the type of shift. There are three types of shifts: logical, circular, and arithmetic.

logical shift

A *logical* shift is one that transfers 0 through the serial input. We will adopt the symbols *shl* and *shr* for logical shift-left and shift-right microoperations. For example:

$$R1 \leftarrow \text{shl } R1$$

$$R2 \leftarrow \text{shr } R2$$

are two microoperations that specify a 1-bit shift to the left of the content of register $R1$ and a 1-bit shift to the right of the content of register $R2$. The register symbol must be the same on both sides of the arrow. The bit transferred to the end position through the serial input is assumed to be 0 during a logical shift.

circular shift

The *circular* shift (also known as a *rotate* operation) circulates the bits of the register around the two ends without loss of information. This is accomplished by connecting the serial output of the shift register to its serial input. We will use the symbols *cil* and *cir* for the circular shift left and right, respectively. The symbolic notation for the shift microoperations is shown in Table 4-7.

TABLE 4-7 Shift Microoperations

Symbolic designation	Description
$R \leftarrow \text{shl } R$	Shift-left register R
$R \leftarrow \text{shr } R$	Shift-right register R
$R \leftarrow \text{cil } R$	Circular shift-left register R
$R \leftarrow \text{cir } R$	Circular shift-right register R
$R \leftarrow \text{ashl } R$	Arithmetic shift-left R
$R \leftarrow \text{ashr } R$	Arithmetic shift-right R

arithmetic shift

An *arithmetic* shift is a microoperation that shifts a signed binary number to the left or right. An arithmetic shift-left multiplies a signed binary number by 2. An arithmetic shift-right divides the number by 2. Arithmetic shifts must leave the sign bit unchanged because the sign of the number remains the same

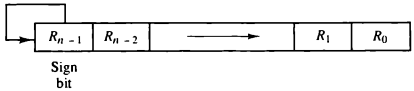


Figure 4-11 Arithmetic shift right.

when it is multiplied or divided by 2. The leftmost bit in a register holds the sign bit, and the remaining bits hold the number. The sign bit is 0 for positive and 1 for negative. Negative numbers are in 2's complement form. Figure 4-11 shows a typical register of n bits. Bit R_{n-1} in the leftmost position holds the sign bit. R_{n-2} is the most significant bit of the number and R_0 is the least significant bit. The arithmetic shift-right leaves the sign bit unchanged and shifts the number (including the sign bit) to the right. Thus R_{n-1} remains the same, R_{n-2} receives the bit from R_{n-1}, and so on for the other bits in the register. The bit in R_0 is lost.

The arithmetic shift-left inserts a 0 into R_0, and shifts all other bits to the left. The initial bit of R_{n-1} is lost and replaced by the bit from R_{n-2}. A sign reversal occurs if the bit in R_{n-1} changes in value after the shift. This happens if the multiplication by 2 causes an overflow. An overflow occurs after an arithmetic shift left if initially, before the shift, R_{n-1} is not equal to R_{n-2}. An overflow flip-flop V_s can be used to detect an arithmetic shift-left overflow.

$$V_s = R_{n-1} \oplus R_{n-2}$$

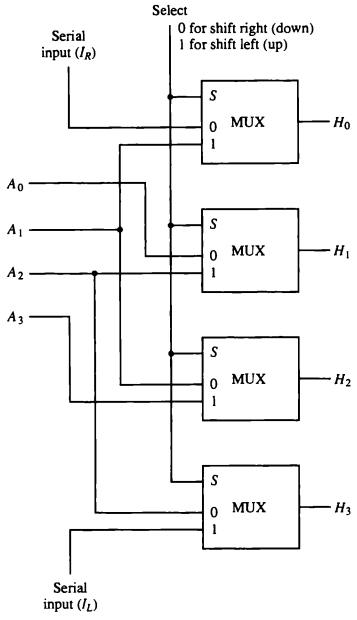
If $V_s = 0$, there is no overflow, but if $V_s = 1$, there is an overflow and a sign reversal after the shift. V_s must be transferred into the overflow flip-flop with the same clock pulse that shifts the register.

Hardware Implementation

A possible choice for a shift unit would be a bidirectional shift register with parallel load (see Fig. 2-9). Information can be transferred to the register in parallel and then shifted to the right or left. In this type of configuration, a clock pulse is needed for loading the data into the register, and another pulse is needed to initiate the shift. In a processor unit with many registers it is more efficient to implement the shift operation with a combinational circuit. In this way the content of a register that has to be shifted is first placed onto a common bus whose output is connected to the combinational shifter, and the shifted number is then loaded back into the register. This requires only one clock pulse for loading the shifted value into the register.

shifter

A combinational circuit shifter can be constructed with multiplexers as shown in Fig. 4-12. The 4-bit shifter has four data inputs, A_0 through A_3, and four data outputs, H_0 through H_3. There are two serial inputs, one for shift left



Function table

Select	Output			
S	H_0	H_1	H_2	H_3
0	I_R	A_0	A_1	A_2
1	A_1	A_2	A_3	I_L

Figure 4-12 4-bit combinational circuit shifter.

(I_L) and the other for shift right (I_L). When the selection input $S = 0$, the input data are shifted right (down in the diagram). When $S = 1$, the input data are shifted left (up in the diagram). The function table in Fig. 4-12 shows which input goes to each output after the shift. A shifter with n data inputs and outputs requires n multiplexers. The two serial inputs can be controlled by another multiplexer to provide the three possible types of shifts.

4-7 Arithmetic Logic Shift Unit

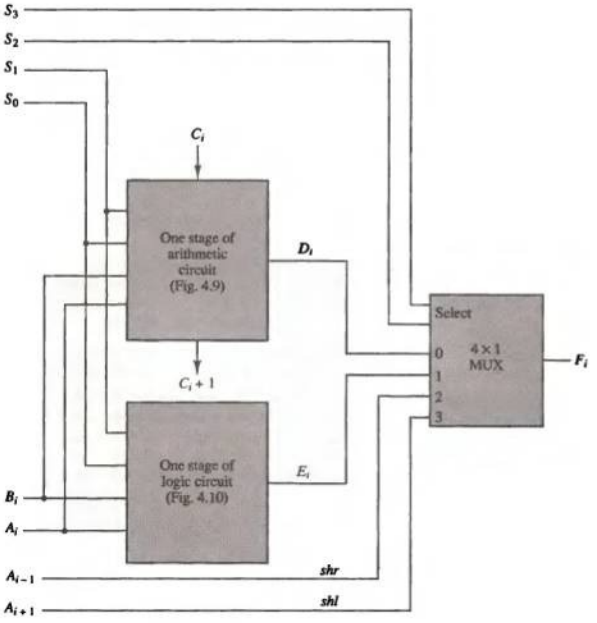
Instead of having individual registers performing the microoperations directly, computer systems employ a number of storage registers connected to a common operational unit called an arithmetic logic unit, abbreviated ALU. To

ALU

perform a microoperation, the contents of specified registers are placed in the inputs of the common ALU. The ALU performs an operation and the result of the operation is then transferred to a destination register. The ALU is a combinational circuit so that the entire register transfer operation from the source registers through the ALU and into the destination register can be performed during one clock pulse period. The shift microoperations are often performed in a separate unit, but sometimes the shift unit is made part of the overall ALU.

The arithmetic, logic, and shift circuits introduced in previous sections can be combined into one ALU with common selection variables. One stage of an arithmetic logic shift unit is shown in Fig. 4-13. The subscript i designates a typical stage. Inputs A_i and B_i are applied to both the arithmetic and logic

Figure 4-13 One stage of arithmetic logic shift unit.



units. A particular microoperation is selected with inputs S_1 and S_0. A 4×1 multiplexer at the output chooses between an arithmetic output in E_i and a logic output in H_i. The data in the multiplexer are selected with inputs S_3 and S_2. The other two data inputs to the multiplexer receive inputs A_{i-1} for the shift-right operation and A_{i+1} for the shift-left operation. Note that the diagram shows just one typical stage. The circuit of Fig. 4-13 must be repeated n times for an n-bit ALU. The output carry C_{i+1} of a given arithmetic stage must be connected to the input carry C_i of the next stage in sequence. The input carry to the first stage is the input carry C_{in}, which provides a selection variable for the arithmetic operations.

The circuit whose one stage is specified in Fig. 4-13 provides eight arithmetic operation, four logic operations, and two shift operations. Each operation is selected with the five variables S_3, S_2, S_1, S_0, and C_{in}. The input carry C_{in} is used for selecting an arithmetic operation only.

Table 4-8 lists the 14 operations of the ALU. The first eight are arithmetic operations (see Table 4-4) and are selected with $S_3S_2 = 00$. The next four are logic operations (see Fig. 4-10) and are selected with $S_3S_2 = 01$. The input carry has no effect during the logic operations and is marked with don't-care $\times$'s. The last two operations are shift operations and are selected with $S_3S_2 = 10$ and 11. The other three selection inputs have no effect on the shift.

TABLE 4-8 Function Table for Arithmetic Logic Shift Unit

Operation select						
S_3	S_2	S_1	S_0	C_{in}	Operation	Function
0	0	0	0	0	$F = A$	Transfer A
0	0	0	0	1	$F = A + 1$	Increment A
0	0	0	1	0	$F = A + B$	Addition
0	0	0	1	1	$F = A + B + 1$	Add with carry
0	0	1	0	0	$F = A + \overline{B}$	Subtract with borrow
0	0	1	0	1	$F = A + \overline{B} + 1$	Subtraction
0	0	1	1	0	$F = A - 1$	Decrement A
0	0	1	1	1	$F = A$	Transfer A
0	1	0	0	$\times$	$F = A \wedge B$	AND
0	1	0	1	$\times$	$F = A \vee B$	OR
0	1	1	0	$\times$	$F = A \oplus B$	XOR
0	1	1	1	$\times$	$F = \overline{A}$	Complement A
1	0	$\times$	$\times$	$\times$	$F =$ shr A	Shift right A into F
1	1	$\times$	$\times$	$\times$	$F =$ shl A	Shift left A into F

PROBLEMS

4-1. Show the block diagram of the hardware (similar to Fig. 4-2a) that implements the following register transfer statement:

$$yT_2: \quad R2 \leftarrow R1, \quad R1 \leftarrow R2$$

4-2. The outputs of four registers, $R0$, $R1$, $R2$, and $R3$, are connected through 4-to-1-line multiplexers to the inputs of a fifth register, $R5$. Each register is eight bits long. The required transfers are dictated by four timing variables T_0 through T_3 as follows:

$$\begin{aligned} T_0&: \quad R5 \leftarrow R0 \\ T_1&: \quad R5 \leftarrow R1 \\ T_2&: \quad R5 \leftarrow R2 \\ T_3&: \quad R5 \leftarrow R3 \end{aligned}$$

The timing variables are mutually exclusive, which means that only one variable is equal to 1 at any given time, while the other three are equal to 0. Draw a block diagram showing the hardware implementation of the register transfers. Include the connections necessary from the four timing variables to the selection inputs of the multiplexers and to the load input of register $R5$.

4-3. Represent the following conditional control statement by two register transfer statements with control functions.

$$\text{If } (P = 1) \text{ then } (R1 \leftarrow R2) \text{ else if } (Q = 1) \text{ then } (R1 \leftarrow R3)$$

4-4. What has to be done to the bus system of Fig. 4-3 to be able to transfer information from any register to any other register? Specifically, show the connections that must be included to provide a path from the outputs of register C to the inputs of register A.

4-5. Draw a diagram of a bus system similar to the one shown in Fig. 4-3, but use three-state buffers and a decoder instead of the multiplexers.

4-6. A digital computer has a common bus system for 16 registers of 32 bits each. The bus is constructed with multiplexers.

a. How many selection inputs are there in each multiplexer?
b. What size of multiplexers are needed?
c. How many multiplexers are there in the bus?

4-7. The following transfer statements specify a memory. Explain the memory operation in each case.

a. $R2 \leftarrow M[AR]$
b. $M[AR] \leftarrow R3$
c. $R5 \leftarrow M[R5]$

4-8. Draw the block diagram for the hardware that implements the following statements:

$$x + yz: \quad AR \leftarrow AR + BR$$

where AR and BR are two n-bit registers and x, y, and z are control variables. Include the logic gates for the control function. (Remember that the symbol + designates an OR operation in a control or Boolean function but that it represents an arithmetic plus in a microoperation.)

4-9. Show the hardware that implements the following statement. Include the logic gates for the control function and a block diagram for the binary counter with a count enable input.

$$xyT_0 + T_1 + y'T_2: \quad AR \leftarrow AR + 1$$

4-10. Consider the following register transfer statements for two 4-bit registers $R1$ and $R2$.

$$\begin{aligned} xT: &\quad R1 \leftarrow R1 + R2 \\ x'T: &\quad R1 \leftarrow R2 \end{aligned}$$

Every time that variable $T = 1$, either the content of $R2$ is added to the content of $R1$ if $x = 1$, or the content of $R2$ is transferred to $R1$ if $x = 0$. Draw a diagram showing the hardware implementation of the two statements. Use block diagrams for the two 4-bit registers, a 4-bit adder, and a quadruple 2-to-1-line multiplexer that selects the inputs to $R1$. In the diagram, show how the control variables x and T select the inputs of the multiplexer and the load input of register $R1$.

4-11. Using a 4-bit counter with parallel load as in Fig. 2-11 and a 4-bit adder as in Fig. 4-6, draw a block diagram that shows how to implement the following statements:

$$\begin{aligned} x: &\quad R1 \leftarrow R1 + R2 \quad \text{Add } R2 \text{ to } R1 \\ x'y: &\quad R1 \leftarrow R1 + 1 \quad \text{Increment } R1 \end{aligned}$$

where $R1$ is a counter with parallel load and $R2$ is a 4-bit register.

4-12. The adder-subtractor circuit of Fig. 4-7 has the following values for input mode M and data inputs A and B. In each case, determine the values of the outputs: S_3, S_2, S_1, S_0, and C_4.

	M	A	B
a.	0	0111	0110
b.	0	1000	1001
c.	1	1100	1000
d.	1	0101	1010
e.	1	0000	0001

4-13. Design a 4-bit combinational circuit decrementer using four full-adder circuits.

4-14. Assume that the 4-bit arithmetic circuit of Fig. 4-9 is enclosed in one IC package. Show the connections among two such ICs to form an 8-bit arithmetic circuit.

4-15. Design an arithmetic circuit with one selection variable S and two n-bit data inputs A and B. The circuit generates the following four arithmetic operations in conjunction with the input carry C_{in}. Draw the logic diagram for the first two stages.

S	$C_{in} = 0$	$C_{in} = 1$
0	$D = A + B$ (add)	$D = A + 1$ (increment)
1	$D = A - 1$ (decrement)	$D = A + \overline{B} + 1$ (subtract)

4-16. Derive a combinational circuit that selects and generates any of the 16 logic functions listed in Table 4-5.

4-17. Design a digital circuit that performs the four logic operations of exclusive-OR, exclusive-NOR, NOR, and NAND. Use two selection variables. Show the logic diagram of one typical stage.

4-18. Register A holds the 8-bit binary 11011001. Determine the B operand and the logic microoperation to be performed in order to change the value in A to:
a. 01101101
b. 11111101

4-19. The 8-bit registers AR, BR, CR, and DR initially have the following values:

$$\begin{aligned} AR &= 11110010 \\ BR &= 11111111 \\ CR &= 10111001 \\ DR &= 11101010 \end{aligned}$$

Determine the 8-bit values in each register after the execution of the following sequence of microoperations.

$AR \leftarrow AR + BR$	Add BR to AR
$CR \leftarrow CR \wedge DR$, $BR \leftarrow BR + 1$	AND DR to CR, increment BR
$AR \leftarrow AR - CR$	Subtract CR from AR

4-20. An 8-bit register contains the binary value 10011100. What is the register value after an arithmetic shift right? Starting from the initial number 10011100, determine the register value after an arithmetic shift left, and state whether there is an overflow.

4-21. Starting from an initial value of R = 11011101, determine the sequence of binary values in R after a logical shift-left, followed by a circular shift-right, followed by a logical shift-right and a circular shift-left.

4-22. What is the value of output H in Fig. 4-12 if input A is 1001, $S = 1$, $I_R = 1$, and $I_L = 0$?

4-23. What is wrong with the following register transfer statements?

a. xT: $AR \leftarrow \overline{AR}$, $AR \leftarrow 0$

b. yT: $R1 \leftarrow R2$, $R1 \leftarrow R3$

c. zT: $PC \leftarrow AR$, $PC \leftarrow PC + 1$

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