

Full-Time-Pad Symmetric Stream Cipher

Improved One-Time-Pad Encryption Scheme

Taha Canturk
kibnakanoto@protonmail.com

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Abstract

One-Time-Pad Encryption Scheme is a secure algorithm but there are 2 main security risks. One, a key cannot be reused. Two, plaintext length equals key length which is very inefficient when dealing with long plaintexts. These 2 security risks only exist due to a lack of confusion and diffusion per ciphertext. As denoted by Claude Shannon in the report he published in 1945, A Mathematical Theory of Cryptography, A secure cryptographic algorithm requires confusion and diffusion. The **Full-Time-Pad** symmetric stream cipher is developed based on the **One-Time-Pad** with solutions to the security risks while maintaining high speed computation. To achieve diffusion, the key is permuted in it's byte array form using a constant permutation matrix. To achieve the confusion, the key is manipulated in it's 32-bit integer representation using Modular **Addition** in F_p , Bitwise **Rotations**, and **Xor** (**ARX**). The permutation guarantees that every time there is a manipulation, each 32-bit number is made up of a different byte order. This algorithm is not necessarily a substitute for existing algorithms. This version of the algorithm is better suited towards cases where performance can be sacrificed, ChaCha20 is objectively faster, AES with AES-NI instruction set is faster, but algorithms that are a little slower can be better at limiting brute-force attacks.

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1 Introduction

1.1 Pre-requisite Terminology

Key	32-byte random array that's transformed, then hashed before XORed with the plaintext to encrypt
Symmetric	Same key is used for encryption and decryption
Stream	Plaintext is encrypted without separating it into blocks
Plaintext	Plain data before encryption
Ciphertext	Encrypted plaintext
Cipher	Encryption algorithm. Plaintext is transformed into a ciphertext that can only be reversed with a key
Diffusion	plaintext/key is spread out in the ciphertext
Confusion	The ciphertext has no possible statistical analysis, or cryptanalysis to determine the plaintext
Bit	0 or 1. Smallest discrete unit for computation
Byte	8-bit number
Galois Field	Finite Field where there are only limited number of numbers. Only prime galois fields (F_p) are used where size of the field is denoted by prime number p
Avalanche Effect	An aspect of diffusion. If smallest unit (1 bit) of data is changed, the ciphertext changes in an unrecognizable way.
Permutation	transposing (moving) elements of a vector without actually changing their values. A reversible operation

1.2 Applications

This encryption algorithm is fast yet reliable as the avalanche effect is strong and the confusion operations used have proved themselves in cryptography. If the generated key is potentially found due to a side-channel attack or any other method not relating to the specifics of this algorithm, then all plaintexts can be found. However this isn't a pressing issue as all encryption algorithms offer this flaw. In contrast, without this flaw, a new key would have to be generated per plaintext like in One-Time-Pad which is inefficient.

1.3 Key Generation

The 32-byte key should be generated using a cryptographically secure method, including but not limited to cryptographic random number generators and Elliptic Cryptography Diffie Hellman (ECDH) protocol with Hash-based Key Derivation Function (HKDF).

1.4 Prerequisite Mathematics

1.4.1 Galois Field Addition

In a prime galois field, where two variables (x, y) are added, $x, y \in \mathbb{F}_p$ where \mathbb{F}_p denotes the finite field with a prime field size. There are finite number of elements in \mathbb{F}_p which means that $x, y < p$.

The expression:

$$\begin{aligned}x + y &= z \\x, y, z &\in \mathbb{F}_7\end{aligned}$$

Implicitly denotes

$$x + y = z \pmod{7}$$

So all addition operations require modulo p .

In the context of this encryption algorithm, it's used in the tranformation algorithm as one of three operations (ARX).

1.4.2 Vector Permutation

Permutating a vector refers to transposing the elements in a vector. It is a reversible operation. For example, if I have the vector: V

$$V = \begin{pmatrix} 0 & 1 & 2 & 3 \end{pmatrix}$$

and have the permutation vector P :

$$P = \begin{pmatrix} 3 & 2 & 1 & 0 \end{pmatrix}$$

then the resulting matrix would be

$$V' = \begin{pmatrix} 3 & 2 & 1 & 0 \end{pmatrix}$$

If we look at V_0 (**0**), according to P_0 , it should be replaced with the 3rd value in P . so then $V'_0 = 3$. This doesn't mean that $V'_3 = 0$. This algorithm is demonstrated in dynamic_permutation section (5.3). . The permutation in this algorithm is used for the bytearray **key** and the 32-bit segmented **k**. In order to confuse (ARX) and diffuse (permute), **k** and **key** needs to point to the same memory address. Most efficient way to do this, would be using `reinterpret_cast` in C++ or using `unions` in C. Since these are

low level approaches, endiannes of the system will affect the byte order. In a little-endian system, the data requires the following permutation before the transformation iterations:

$$V' = \left(3 \ 2 \ 1 \ 0 \ 7 \ 6 \ 5 \ 4 \ 11 \ 10 \ 9 \ 8 \ 15 \ 14 \ 13 \ 12 \ 19 \ 18 \ 17 \ 16 \ 23 \ 22 \ 21 \ 20 \ 27 \ 26 \ 25 \ 24 \ 31 \ 30 \ 29 \ 28 \right)$$

This means that for little endian systems, the permutation matrix also needs to change. The little-endian permutation matrix derivation is given in the `test/perm.py`.

2 Security

In One-Time-Pad, key isn't reusable. Here is the proof:

```
let m1, m2 be 2 plaintexts
let k be the key
let c1 = m1 ⊕ k
let c2 = m2 ⊕ k
c1 ⊕ c2 = (m1 ⊕ k) ⊕ (m2 ⊕ k)
c1 ⊕ c2 = m1 ⊕ m2
```

Since the key is reused, the 2 ciphertext's XORed factor out the key since $k \oplus k = 0$. Using cryptanalysis, the 2 plaintexts can be found.

For $c_1 \oplus c_2 = m_1 \oplus m_2$ to not hold true, for each encryption, the key needs to be different. If k is transformed each time so that it has an avalanche effect. Even with no confusion, it would still be secure since $k' \oplus k \neq 0$ where k' is transformed key.

But there is another concern,

What if the plaintext and ciphertext are known, then it is possible to find k so don't use k without transformation, since $\text{plaintext} \oplus \text{ciphertext} = \text{key}$. So for each plaintext, key needs to be transformed irreversibly and it also requires confusion since if k' is found, k is still unknown but if k is found, then all instances of k'_n are known, which means that:

```
k'1 = hash(k + 1) where hash() is an irreversible transformation
k'2 = hash(k + 2)
c1 ⊕ c2 = (m1 ⊕ k'1) ⊕ (m2 ⊕ k'2)
c1 ⊕ c2 ≠ m1 ⊕ m2
m1 ⊕ c1 = k'1
m2 ⊕ c2 = k'2
k'1, k'2 are calculated using an irreversible hashing algorithm
```

∴ the Full-Time-Pad Cipher requires both diffusion and confusion

2.1 Brute-Force

Due to the use of a galois field. The total number of combinations per 256-bit key isn't $a = 2^{256}$, but rather $b = 4294967291^8$ where $p = 4294967291$ for arithmetic in F_p and there are 8 32-bit numbers in a 256-bit key.

```

a = 11579208923731619542357098500868790785326998466564056403945758400791312963993610
b = 11579208815891833313151659776217239262857046546585679399233288413030729265712110
let Δ = a - b
Δ = 107839786229205438724651551522469951919978377004712469987760583698281510

```

So the difference Δ is a somewhat large integer. The number of combinations with a galois field is lower than without a galois field ($b < a$). This isn't a big concern as their difference measured exponentially is only around $2^{\log_2 \Delta} \approx 2^{229}$ which means that their difference is around 2^{229} , this is a negligible difference as the difference between 2^{230} and 2^{229} is also huge.

∴ Using a galois field doesn't negatively impact number of combinations in terms of brute force as the total number of combinations when using a galois field vs not is a negligible amount

2.1.1 Birthday Problem

The birthday problem is a paradox. It goes as follows: how many people are required so that there is more than 50% chance that at least 2 people have the same birthday.

The answer is an unexpected 23 people.

In the context of this encryption algorithm, it might be a concern, as number of key reused (with transformation) increase, the chances of finding the key increase:

```

let  $V_c$  be the number of combinations per key without order and repetitions
let  $k$  be the number of keys needed for hash(key) to have a 50% chance to equal
another hash(key)
let  $V_t$  be the number of combinations per key with order and repetitions

```

$$V_c = \frac{b!}{(b-k)!} = \frac{4294967291^8!}{(4294967291^8 - k)!}$$

$$V_t = b^k = 4294967291^{8^k}$$

$$P(A) = \frac{V_c}{V_t}$$

$$P(A) = \frac{\frac{b!}{(b-k)!}}{b^k}$$

$$P(B) = 1 - P(A) = 50\%$$

$$P(A) = 1 - 50\%$$

$$1 - 50\% = \frac{\frac{b!}{(b-k)!}}{b^k}$$

$$\frac{1}{2}b^k = \frac{b!}{(b-k)!}$$

$$\text{since } 50\% = \frac{1}{2}$$

$$\log_b \frac{1}{2}b^k = \log_b \frac{b!}{(b-k)!}$$

$$\log_b \frac{1}{2} + \log_b b^k = \log_b b! - \log_b (b-k)!$$

$$0 = \log_b b! - \log_b (b-k)! - \log_b \frac{1}{2} - k$$

$$\text{since } \log_b b^k = k$$

According to Ramanujan's Approximation:

$$\log_b b! \approx \frac{b \ln b - b + \frac{\ln \left[\frac{1}{\pi^3} + b(1+4b(1+2b)) \right]}{6}}{\ln b} + \frac{\ln \pi}{2}$$

And

$$\log_b(b-k)! \approx \frac{(b-k) \ln(b-k) - (b-k) + \frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6}}{\ln b} + \frac{\ln \pi}{2}$$

Recall:

$$0 = \log_b b! - \log_b(b-k)! - \log_b \frac{1}{2} - k \quad \text{isolate } \log_b(b-k)!$$

$$\log_b(b-k)! = \log_b b! - \log_b \frac{1}{2} - k$$

Combine both equations for $\log_b(b-k)!$:

$$\log_b b! - \log_b \frac{1}{2} - k \approx \frac{(b-k) \ln(b-k) - (b-k) + \frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6}}{\ln b} + \frac{\ln \pi}{2}$$

$$\begin{aligned} \frac{b \ln b - b + \frac{\ln \left[\frac{1}{\pi^3} + b(1+4b(1+2b)) \right]}{6}}{\ln b} + \frac{\ln \pi}{2} - \log_b \frac{1}{2} - k &\approx \frac{(b-k) \ln(b-k) - (b-k)}{\ln b} + \\ &+ \frac{\frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6}}{\ln b} + \frac{\ln \pi}{2} \end{aligned}$$

$$\begin{aligned}
& \frac{b \log b + \frac{\ln \left[\frac{1}{\pi^3} + b(1+4b(1+2b)) \right]}{6} + \frac{\ln \pi}{2} - \ln b \log_b \frac{1}{2} - \ln bk}{\ln b} \approx \frac{(b-k) \ln(b-k) + k}{\ln b} + \\
& \quad + \frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6} + \frac{\ln \pi}{2} \\
\text{let } C &= b \ln b + \frac{\ln \left[\frac{1}{\pi^3} + b(1+4b(1+2b)) \right]}{6} - \ln b \log_b \frac{1}{2} \approx (b-k) \ln(b-k) + k + \ln bk \\
& \quad + \frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6} \\
\text{let } f(k) &= (b-k) \ln(b-k) + k + \ln bk + \frac{\ln \left[\frac{1}{\pi^3} + (b-k)(1+4(b-k)(1+2(b-k))) \right]}{6} - C = 0
\end{aligned}$$

$\therefore f(k)$ can be used to evaluate how many keys it would take so that 2 hashes have a 50% chance of being equal. $f(k)$ can be evaluated using the secant algorithm

After running `test/secant.py`, given the parameters:

Based on Wikipedia Article: Birthday Attack, we can approximate x_0 and x_1

$$x_0 = \frac{1}{2} + \sqrt{\frac{1}{4} + 2 \times \ln(2) \times b} \text{ (due to Approximation of number of people)}$$

$$x_1 = \sqrt{b} \text{ (due to square approximation)}$$

error tolerance: $e = 1 \times 10^{-200}$

for $b = 4294967291^8$, we get $k_1 = 400651867432320527534628274526034254879$ for the root.

And for $b = 2^{256}$, we get $k_2 = 400651869298001176472314306405665023048$ for the root

So then $\Delta k = k_2 - k_1 = 1865680648937686031879630768169 \approx 2^{101}$ Since the difference between k_1 and k_2 is negligible (2^{101} isn't big considering the magnitude of b). We can conclude that using a galois field doesn't increase risk of birthday attacks which justifies the use of Galois fields to increase avalanche effect.

2.1.2 Collision Attack

Most denial of service attacks related to encryption algorithms are based on brute-force methods. To see if this algorithm has a potential collision attack:

$$\text{transform}(key_1) = \text{transform}(key_2)$$

For example: $x + y = 16$

$$x, y \in \mathbb{Z}, 0 \leq x, y < 256$$

there are 17-combinations for x to satisfy this equation, and simultaneously, there are 17 combinations for y to satisfy the equation, so a total of 17 combinations.

But for $x + y = z$, there are 257 combinations to try. if the result of an arithmetic operation

is known, there may be ways to get the same end-result with less combinations to brute-force. Knowing the value of z reduced the number of combinations by 15 times.

This means that the calculation done on 2.1.1 for the birthday problem would be irrelevant because there is a better algorithm than random brute forcing (to find collisions for `transform(key)`)

\therefore if there is an operation that can provide the same output for a wide range of inputs, there can be a collision attack. Collision attacks can be used to derive the same transformed key using a different input key and decrypt the plaintext without actually having the original key. In the context of this encryption algorithm (using addition as an example): keysize is 32-bytes so for byte n : $x_n + y_n = z_n$
 $x, y, z \in \mathbb{Z}, 0 \leq x, y, z < 256$

Number of combinations can be represented by

$$\prod_{n=0}^{32-1} (z_n + 1)$$

so the number of combinations would be between a minimum of 32 combinations ($z_n = 0$ for all 32-bytes) up to a maximum of 2^{256} combinations ($z_n = 255$ for all 32-bytes) which can be brute forced for small z_n . So a simple addition is prone to collision attacks for $x_n + y_n = z_n$, where x, y are unknown. The use of galois field makes z_n even smaller. So even less combinations. Solution is to use operations that cannot be represented differently. e.g.

$$\sum_{i=0}^{z_n} x_n + y_n = z_n \implies \sum_{i=0}^{z_n} (z_n - i) + (i) = z_n$$

solves for all possible x, y values for each z_n . An addition operation can be represented differently to solve for 2 unknowns, while a good mix of ARX operations cannot be reverse engineered. This is also the reason why pre-manipulating the key (using addition) before `transform()` isn't a good option. Since it provides a very obvious collision attack which makes it invalid even though pre-manipulation will provide a good avalanche effect for every single byte of the key (if 1-bit of any byte is changed, ciphertext changes completely).

So the final solution is to calculate sum of each 32-bit segment of the key (represented by k_i) in order to interlink them to make sure that every byte of the key offers the same avalanche effect:

$$\sum_{i=0}^7 k_i$$

To test if this offers enough collision resistance: think of this problem as an example:

1. $x + y = 16$ offers n=17 combinations
 2. $x + y + z = 16$ offers n=153 combinations (determined experimentally)
 3. $x + y + z + v = 16$ offers n=969 combinations (determined experimentally)
- $x, y, z, v \in \mathbb{Z}, 0 \leq x, y, z, v < 256$

So there has to be an equation or algorithm to summarize the relationship between number of variables (l) and the sum of the addition operation (16);

Knowing that equation 1. is the simplest equation and it offers 17 combinations. Then if the rest of the equations are represented in 2-variable fashion. we can find number of combinations n :

For equation 2.: There are 3 ways to represent as 2-variable equation

$$x + y \quad x + z \quad y + z$$

For equation **3.**: There are 6 ways to represent as 2-variable equation

$$x + y \quad x + z \quad x + v$$

$$y + z \quad y + v \quad z + v$$

The number of ways a multi-variable equation can be represented as a 2 variable equation can be summarized by the following:

$$\sum_{i=1}^{l-1} i$$

Using some number crunching and logic, I found that there is a direct corrolation between the number of combinations and the ratio between the current number of ways to represent as 2-variable equation over the previous number of ways to represent as a 2-variable equation:

$$n_l \propto \frac{\sum_{i=1}^{l-1} i}{\sum_{i=1}^{l-2} i}$$

Using more number crunching: I found the following recursive formula that finds the number of combinations that satisfies $x+y+\dots = 16$:

$$n_l = (n_{l-1} \frac{\sum_{i=1}^x i}{\sum_{i=1}^{x-1} i} + 17) \times 3 - 17 \times 3((x+1) \mod 2)$$

where $x = l - 1$ and x should be incremented until correct answer is reached for $l \geq 6$ and n_{l-1} is previous number of combinations. This formula doesn't translate to cases where the 2-variable equation doesn't have 17 combinations.

Simply put this equation couldn't be used accurately, it can only be an approximation. But upon further number crunching, I derived the following equation that satisfies all cases:

$$\prod_{i=0}^{l-1} \frac{(x+i)}{1+i}$$

where x is the number of combinations for 2 variable equations. e.g. for $a+b = 16$, $x = 16+1 = 17$.

Using this equation for the context of this encryption algorithm:
Recall:

$$c = \sum_{i=1}^{l-1} i$$

where $l = 8$ since 8 32-bit segments to the 256-bit key then, $x = c + 1$

the total number of combinations according to the equation is between 1 and
 $2871827628774669857283799072180574717903946432793745331030345747716374528 \approx 2.9 \times 10^{72}$

Which isn't possible to brute force provided key is random and not chosen to be a small value.

2.2 Joux's Attack: Multicollisions in Iterated Hash Functions

This attack requires knowing a collision: `transform(k1) = transform(k2)`. Given a secure hashing algorithm, this isn't possible. Therefore, the Joux's attack isn't possible. The only way to evaluate k_1 and k_2 would be using brute-force. That is mathematically proven in sections 2.1.1 and 2.1.2. This is also proven experimentally in `test/collision.cpp`. The values in `test/collision_report` show that over lots of iterations, getting the same hash is still impossible.

2.3 Related Key Attack

Related key attacks isn't possible considering that it requires that `transform(k1)` is similar to `transform(k2)` where k_1 and k_2 have a predictable difference (incremented by encryption index). But this is only under the assumption that the `transform` function is an irreversible hashing algorithm.

2.4 Time Based Attacks

Start from 32 zeros for a key, then incremented each byte by 1 until there are 32 255s and test again. This is tested in `test/benchmark` in function `benchmark_hash_time_attack_v()`. This test passed. The keys with small numbers have the same speed as keys with large numbers (This was never in doubt), therefore algorithm is proven to be constant time.

2.5 Linear Cryptoanalysis

Linear Cryptoanalysis is used to find linear connections between ciphertext and plaintexts. Given that running `test/significant_perm_byte -r` provides that all bytes of the key provide approximately the same collision resistance and shows that there is no pattern. This means that modifying a certain byte isn't only affected some bytes but all of them without a pattern, therefore there are no significant bytes. And all bytes of the key affect the result equally. This is mainly due to the use of sum of k_i in the transformation algorithm.

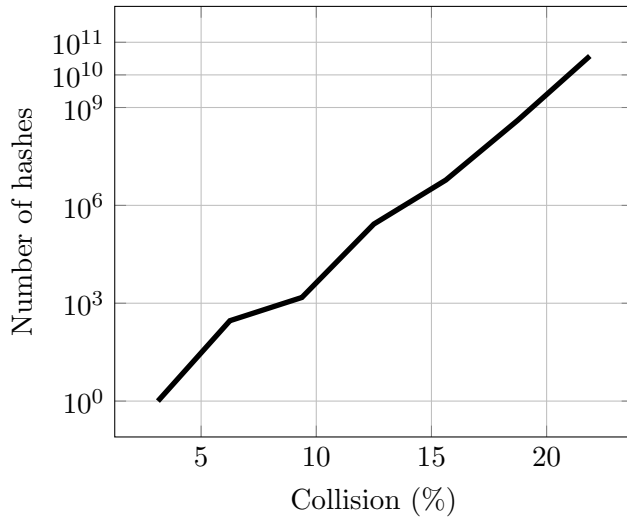
Here is data for SHA-256 hashing algorithm, what percentage of bytes can be equal, if this number gets to 100%, this means that there is a successful collision. i is the number of hashes that are generated to get $n\%$ equality.

This data fluctuates a lot and therefore doesn't actually have any visible pattern, The numbers being similar to SHA-256 would be good news to suggest that the avalanche effect is real and that the math checks out for the birthday problem, it's not possible to find a collision.

%	Full-Time-Pad -(i)	SHA-256 (i)
3.125	1	2
6.25	291	6
9.375	1501	2648
12.5	267184	36595
15.625	5981439	6167198
18.75	408355721	530210645

Table 1: collision (%) VS. number of hashes

Collision (%) VS. Number of Hashes for Full-Time-Pad



2.6 Differential Cryptanalysis

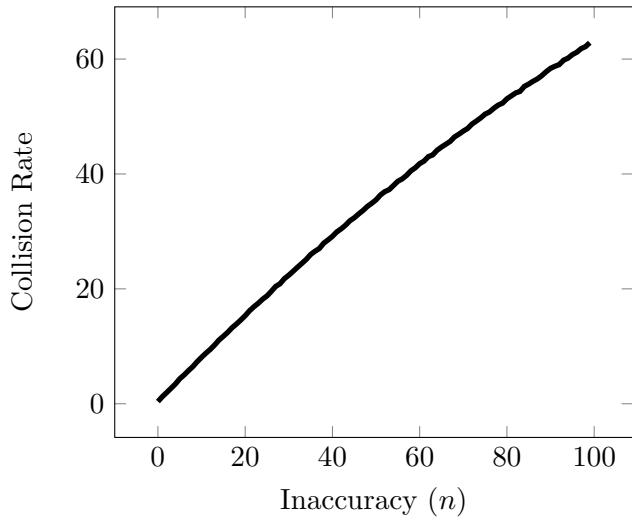
In `test/significant_perm_byte.cpp`, instead of comparing if values are equal, but rather approximately equal with accuracy of $\pm n$. This gave the same results as sha-256. And no specific bytes seemed to have followed a specific pattern. This means that there isn't a differential statistical pattern.

Results of collision rates are approximately equal given lower accuracy ($\pm n$). This suggests that there isn't any cryptoanalysis pattern. These numbers are average of 255 tries. Each try also yields similar results which means that there isn't a differential cryptoanalysis exploit regarding accuracy of comparison.

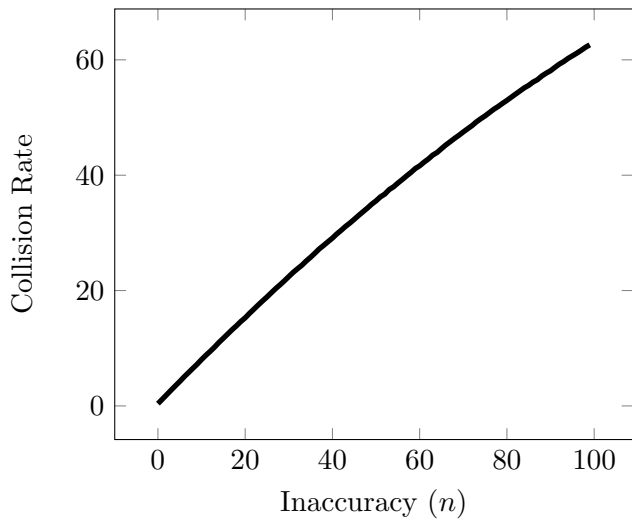
n	Full-Time-Pad (%)	SHA-256 (%)
0	0.3883	0.3923
5	4.2678	4.224
10	8.0342	8.0322
15	11.7421	11.7403
20	15.4626	15.4287
25	18.8071	18.8382

Table 2: Comparision Accuracy VS. Collision Rate

Collision Rate (%) vs. Inaccuracy (n) - Full-Time-Pad



Collision Rate (%) vs. Inaccuracy (n) - SHA-256



This graph demonstrates how lower accuracy of comparison results in higher collision rate (For both SHA-256 and Full-Time-Pad). The points for this graph is evaluated by running `test/significant_perm_byte -d`. The consistent non-volatile increase in the numbers are a good thing.

2.7 Reverse Engineering the Transformation

this is demonstrated in the `test/reverse.cpp` code. The reversed key and key are very different.

2.8 Collision-Resistance

Collision resistance has been proven in the birthday problem section and the collision attack section. The collision resistance is on average about as low as the numbers in sha256. and don't have any particular patterns as proven in `test/significant_perm_byte.cpp`. This test file shows the collision rate as any byte of the 32-byte key is modified by 1-bit.

$$\text{collision_rate} = \frac{\text{\#collisions}}{32}$$

where `\#collisions` is the number of bytes where `previous keyi` and `modified keyi` are equal

2.8.1 Different Permutation Matrices

tried permutation matrices that followed logic or randomness. But they didn't offer the proper diffusion and collision resistance required to make a secure algorithm. The permutation matrix needs to be perfect so that the chances of collision (tested in `test/significant_perm_byte.cpp`) for every byte of the key should be around the same.

2.8.2 Number of Rounds

Number of rounds is determined by the number of rows in the permutation matrix. as using a permutation twice will cancel out.

2.8.3 Constant - F_p - Prime Galois Field Size

every addition operation is done in a prime galois field as every $k_i \in \mathbb{F}_p$ where k_i is the i'th index of the 32-bit segmented key $p = 0xffffffffb$. p is the largest unsigned 32-bit prime number. It was evaluated using Fermat's Little Theorem:

Algorithm 1 Find the largest 32-bit prime number pseudo-code

```
1: Output: Biggest unsigned 32-bit prime number ( $p$ )
2: Begin
3:  $p = 0$ 
4: for  $i = 0\text{xffffffff}$  to 0 do
5:   if  $2^{i-1} \bmod i = 1$  then
6:      $p = i$ 
7:     break
8:   end if
9: end for
10: Return  $p$ 
```

2.8.4 Constant - r - Dynamic Rotation Constant

Cicadas only surface once every 17 years. Assuming that their predator only surfaces once every 2 years, they will only meet each other once every 34 years. Since 17 is a prime number, they will barely ever meet. However, if they surfaced once every 18 years, while their predator once every 2 years. Then every 18 years they would meet. This would also make it easier for the predator to synchronize their emergence timing.

let x be the time interval for when the cicadas surface

let t be the time interval for when the predators surface

We can conclude:

$GCD(x, y) = 1$, then they meet up every $x \times y$ years.

Whereas if $GCD(x, y) \neq 1$, then they meet up every x years (assuming $x > y$).

Bitwise rotation operation should also be done with either a prime number or a number where $GCD(x, y) = 1$ to maximize the collision resistance.

If not, then for example:

let w be 32-bit number

let $q = 8$ be rotation count

rotation can be represented by: $w = w \lll q$

If this rotation was repeated for 4 iterations, then the rotations would cancel out and rotated w (w') and w would be equal. Make sure that $GCD(q, 32) = 1$.

If there's only one rotation number, the likelihood of finding a pattern increases. Imagine a card dealer shuffling cards with two stacks, ensuring that one card from each stack is placed on top of the other in a consistent pattern. In this case, you could easily "unshuffle" the cards by simply separating them into two stacks, putting one card on one side and one card on the other. Using a single rotation like this is too simple, but if the cards were shuffled using a more complex method, the process becomes harder to reverse. Even though each method is reversible on its own, combining them makes it significantly more difficult to undo. The use of bitwise rotations on 32-bit segments combined with byte-level permutations creates excellent diffusion. The only potential improvement would be to ensure that all bytes are interlinked solely through the bitwise rotations and permutations, rather than relying on the sum, as seen in the current transformation algorithm.

Rotation numbers that are selected are represented by r :

$$r = \begin{pmatrix} 23 & 5 & 17 & 31 & 13 \end{pmatrix}$$

There are 5 numbers for r , this is because $16 \bmod 5 \neq 0$, after 16 iterations in the transformation algorithm, all values of k aren't rotated the same number of times.

If you were to rotate 32-bit unsigned integer (x)

$$\sum_{i=0}^4 r_i = 89$$

number of times, the net rotation would be 89 (a prime number). $x \ggg 89$ would produce a net rotation of $x \ggg 25$, since $89 \bmod 32 = 25$. Knowing that $GCD(25, 32) = 1$, the minimum number of times where the expression $x \ggg 89$ is repeated till rotated x equals x is $89 \times 32 = 2848$. This means that only after 2848 iterations would rotated x equal x . Whereas if there was a singular number for r , the number of rotations for rotated x to equal x would be $r \times 32$ (e.g. $r = 23, 23 \times 32 = 736$).

\therefore there needs to be multiple numbers for r .

It is designed so that $|r_i - r_{i-1}|$ results in a different magnitude each time so that the net rotation is a bigger number (more rotation means better diffusion/confusion).

For the derivation, the numbers started with prime numbers, all prime numbers from 0-32, but most were eliminated due to similar values for $|r_i - r_{i-1}|$ results (e.g. $23 - 5 = 18, |5 - 17| = 12$ aren't too close). A good net rotation would be prime and without similarities to net rotations. In the transformation algorithm, There are bitwise right rotate (\ggg) and bitwise left rotate (\lll), this is done to properly mix the results.

3 Hashing

The 'hashing' algorithm is used for manipulating the key each time with proper confusion and avalanche effect. The main reason why the key can be reused is due to the algorithm's strong avalanche effect as well as the lack of statistical patterns with collision rates as proven in the unit tests, `test/significant_perm_byte.cpp` which shows if some bytes have more or less collision resistance.

To maximize the effects of confusion using the current logic, 4 out of 8 values k (32-bit segmented key) should be modified per iteration (half of key). So each byte of the key is transformed a total of 8 times ($\frac{16 \text{ iterations}}{8 \div 4}$). since $16 \bmod 4 = 0$, all bytes are transformed the same number of times which would also tarnish the possibility of a statistical exploit that could target less transformed bytes of the key.

Half of the key should be modified per iteration because classical cryptographic algorithms (non-quantum) rely on computational complexity for their security. So more transformation the better. But the most minimum work to get a secure transformed key is enough, therefore it's the goal. this was determined experimentally; testing collision resistance of each byte upon modifying the transformation algorithm.

I would represent the transformation algorithm with a game of cards. Imagine that 8 cards are randomly picked in a pile of 2^{256} cards, each card is cut up into 4 equal pieces, the cut up pieces are shuffled using a permutation algorithm while also replaced with other cut up values from the pile of cards and simultaneously, some pieces are cut up to 2 pieces with magnitude of a prime number (e.g. 5mm, other pieces being 32-5mm if card is 32mm). If this process repeated for 16 iterations, could you recover the original cards? Probably not.

3.1 Dynamic Matrix Permutation

Considering the use of `reinterpret_cast` for representing the bytearray (key) as 32-bit segments (for confusion operations), the endiannes of the computer will affect the matrix. the matrix given in this documentation is the big endian version as the algorithm is designed to be big-endian. The little endian version requires a new permutation matrix where the byte order every 32-bits is reversed. This is in the C++ implementation of **Full-Time-Pad** and the python Deravation program mentioned below.

3.1.1 Deravation

Algorithm 2 Dynamic Permutation Matrix Deravation Pseudo-code

```
1: Input: an array of incrementing numbers (0-31)  $A$ 
2: Output: Most Efficient Permutation Matrix  $V$  ( $16 \times 32$ )
3: Begin
4:  $P \leftarrow \text{copy of } A$ 
5: for  $k = 0$  to 4 do
6:   for  $i = 0$  to 8 do
7:      $P_i \leftarrow A_{i \times 4}$ 
8:      $P_{i+8} \leftarrow A_{i \times 4 + 1}$ 
9:      $P_{i+16} \leftarrow A_{i \times 4 + 2}$ 
10:     $P_{i+24} \leftarrow A_{i \times 4 + 3}$ 
11:   end for
12:    $A \leftarrow \text{copy of } P$ 
13:    $V.append(P)$ 
14:    $C \leftarrow \text{copy of } P$ 
15:   for  $m = 0$  to 3 do
16:     for  $i = 0$  to 8 do
17:       for  $n = 0$  to 4 do
18:          $P_{i \times 4 + n} \leftarrow C_{(1+n+m) \bmod 4 + i \times 4}$ 
19:       end for
20:     end for
21:      $V.append(P)$ 
22:   end for
23:    $A \leftarrow \text{copy of } P$ 
24: end for
25: Return  $V$ 
```

Python code is in the test/perm.py

3.1.2 Dynamic Permutation Matrix Values

0	4	8	12	16	20	24	28	1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	31
4	8	12	0	20	24	28	16	5	9	13	1	21	25	29	17	6	10	14	2	22	26	30	18	7	11	15	3	23	27	31	19
8	12	0	4	24	28	16	20	9	13	1	5	25	29	17	21	10	14	2	6	26	30	18	22	11	15	3	7	27	31	19	23
12	0	4	8	28	16	20	24	13	1	5	9	29	17	21	25	14	2	6	10	30	18	22	26	15	3	7	11	31	19	23	27
12	28	13	29	14	30	15	31	0	16	1	17	2	18	3	19	4	20	5	21	6	22	7	23	8	24	9	25	10	26	11	27
28	13	29	12	30	15	31	14	16	1	17	0	18	3	19	2	20	5	21	4	22	7	23	6	24	9	25	8	26	11	27	10
13	29	12	28	15	31	14	30	1	17	0	16	3	19	2	18	5	21	4	20	7	23	6	22	9	25	8	24	11	27	10	26
29	12	28	13	31	14	30	15	17	0	16	1	19	2	18	3	21	4	20	5	23	6	22	7	25	8	24	9	27	10	26	11
29	31	17	19	21	23	25	27	12	14	0	2	4	6	8	10	28	30	16	18	20	22	24	26	13	15	1	3	5	7	9	11
31	17	19	29	23	25	27	21	14	0	2	12	6	8	10	4	30	16	18	28	22	24	26	20	15	1	3	13	7	9	11	5
17	19	29	31	25	27	21	23	0	2	12	14	8	10	4	6	16	18	28	30	24	26	20	22	1	3	13	15	9	11	5	7
19	29	31	17	27	21	23	25	2	12	14	0	10	4	6	8	18	28	30	16	26	20	22	24	3	13	15	1	11	5	7	9
19	27	2	10	18	26	3	11	29	21	12	4	28	20	13	5	31	23	14	6	30	22	15	7	17	25	0	8	16	24	1	9
27	2	10	19	26	3	11	18	21	12	4	29	20	13	5	28	23	14	6	31	22	15	7	30	25	0	8	17	24	1	9	16
2	10	19	27	3	11	18	26	12	4	29	21	13	5	28	20	14	6	31	23	15	7	30	22	0	8	17	25	1	9	16	24
10	19	27	2	11	18	26	3	4	29	21	12	5	28	20	13	6	31	23	14	7	30	22	15	8	17	25	0	9	16	24	1

3.2 Confusion - ARX

For the confusion aspect of the key transformation algorithm, ARX operations are used.

3.2.1 A - Modular Addition

For the modular addition, a constant array of 8 values is used (**A**):

```

encryption index >> 32
encryption index & 0xffffffff
0x119f904f
0x73d44db5
0x3918fa83
0x5546b403
0x216c46df
0x64997dfd

```

A is derived as follows:

Start with the message: "**cipher**" (6 digit message with 6 unique characters)

Take every ascii encoding of each character of the message and calculate to the 32nd power and take the first 32-bits:

Example:

"c": 99 in ascii encoding

In python: `hex(9932)[10:] = 0x0x119f904f`

The word **cipher** and **A_i** is arbitrary. In other words, it's "nothing up my sleeve numbers". This is an initial array. In the transformation algorithm, every **A_i** is modified twice (in 16 iterations). This provides the confusion. The reason why the first 2 values are the 64-bit encryption index is because this was the most efficient way to diffuse in the encryption index.

3.2.2 R - Bitwise Rotation

"Using the values determined for *r* (as discussed in Section 2.8.4), rotate one value to the right, another to the left, and then the third to the right again. The reasoning behind alternating the rotation directions goes back to the card analogy. Imagine two stacks of cards. One on

the left and the other on the right. To shuffle them efficiently, a logical approach would be to alternately take a card from the left stack and place a card from the right stack on top. For a "randomized" shuffle, the best strategy would be to take n th card from alternating sides, where n is a prime number, to minimize collisions."

3.2.3 X - XOR

Some of the values are xored because xor operator offers 256 combinations per byte unlike unsigned addition which mostly offers less (section 2.1.2). This will bring more possibilities which in return limits brute-forcing attacks. It is avoided when there is a good possibility of xoring the same 2 values ($x \oplus x$), this is why the sum of k_i is used rather than xor.

4 Performance

5 Cipher

5.1 Transformation

Algorithm 3 The key transformation operation per encryption

```
1: Input: 32-byte bytearray key (key), 64-bit encryption_index
2: Output: Transformed 32-byte bytearray key ( $k$ )
3: Begin
4: // keysize = 32, length of input key is 32-bytes
5: uint32_t *k  $\leftarrow$  endian_8_to_32_arr(key)
6: encryption_index  $\leftarrow$  encryption_index + 1
7: for  $i = 0$  to 16 do
8:   index  $\leftarrow i \ll 2$ 
9:   i1mod  $\leftarrow$  index mod 8
10:  i2mod  $\leftarrow$  (index + 1) mod 8
11:  i3mod  $\leftarrow$  (index + 2) mod 8
12:  i4mod  $\leftarrow$  (index + 3) mod 8
13:  imod8  $\leftarrow i$  mod 8
14:  imod9  $\leftarrow (i + 1)$  mod 8
15:  rmod  $\leftarrow i$  mod 5
16:
17:  // No unwanted overflow, so convert to uint64_t
18:   $k_{i1mod} \leftarrow (\text{uint64\_t})k_{i1mod} + A_{imod8} + k_{i1mod} \ggg r_{rmod} \pmod{p}$ 
19:  uint32_t sum  $\leftarrow \sum_{j=0}^8 k_j \pmod{p}$ 
20:   $A_{imod9} \leftarrow A_{imod9} \oplus \text{sum}$ 
21:   $k_{i2mod} \leftarrow (\text{uint64\_t})k_{i2mod} + A_{imod9} + k_{i2mod} \lll r_{rmod} \pmod{p}$ 
22:   $A_{imod8} \leftarrow A_{imod8} \oplus (\text{uint64\_t})k_{i2mod} + k_{i1mod} \ggg r_{(i+1) \bmod 5} \pmod{p}$ 
23:   $k_{i3mod} \leftarrow (\text{uint64\_t})A_{imod8} \oplus k_{i3mod} + A_{imod9} \oplus k_{i4mod} \pmod{p}$ 
24:   $k_{i4mod} \leftarrow (\text{uint64\_t})A_{imod8} \oplus k_{i4mod} + A_{imod9} \oplus k_{i3mod} \pmod{p}$ 
25:
26:  // Permutate the bytearray key
27:  key  $\leftarrow$  dynamic_permutation(key, i);
28: end for
29: Return key
```

5.2 dynamic_permutation

Algorithm 4 The `dynamic_permutation()` in the transformation function

```
1: Input: 32-byte bytearray ( $B$ ), permutation matrix ( $V$ ), iteration index ( $i$ )
2: Output: 32-byte permuted bytearray ( $B$ )
3: let  $P$  be placeholder vector declared with length of 32-bytes
4: for  $j = 0$  to 32 do
5:      $P_j = B_{V_{ij}}$ 
6: end for
7: Return  $P$ 
```

5.3 endian_8_to_32_arr

Algorithm 5 The `endian_8_to_32_arr()` in the transformation function

```
1: Input: 32-byte bytearray ( $B$ )
2: Output: 32-byte bytearray ( $B$ ) permuted to flip endiannes if necesarry
3: if big endian system then
4:     for  $j = 0$  to 32 do
5:         swap  $B_i$  with  $B_{i+3}$ 
6:         swap  $B_{i+1}$  with  $B_{i+2}$ 
7:     end for
8: end if
9: Return reinterpret_cast <uint32_t*>(B)
```

5.4 Key Reuse

In order to reuse the key, there is an encryption index which is a 64-bit nonce that is incremented per encryption in order to keep the transformed key unique. The encryption index is infused in the encryption algorithm in an efficient manner, the first 2 values is the encryption index. It's the first 2, not the last 2 because indexing in the transformation algorithm uses the first 2 indexes of \mathbf{A} so this would provide more iterations to diffuse in the encryption index. The encryption index could also be randomly generated each time. If it's incremented it's necessary to make sure that the same encryption index is incremented when the plaintext is long (not just incremented per message but per 32-byte segment of every message sent.)

5.5 Long Plaintexts

When there are long plaintexts used, the encryption index should be incremented by 1 the same way as key reuse. It might be logical to randomly generate the encryption index per plaintext so that you don't have to keep track of incrementing the encryption index for all plaintexts but rather per plaintext. This isn't an issue in the algorithm itself but rather a side-channel exploit. My implementation of the algorithm doesn't impose this problem as long as you don't use the same key for 2 `FullTimePad` Objects.

5.6 Encryption/Decryption operation

Encryption operation is the same as decryption