Find the mistake in the following proofs:

1. All odd numbers are prime.

   Proof

   Every prime number, except for 2, are odd.

   Therefore, all odd numbers are prime.

   QED

2. All odd numbers > 2 are prime.

   Proof.

   Every even number greater than 2 is composite (not prime).

   Therefore, all odd numbers greater than 2 are prime.

   QED

3. Every number is not prime.

   Proof.

   Base case: 1 is not prime.

   For every other number, we can determine that $n = 1 \cdot n$. So all other numbers are composite, since they can be written as a multiplication of two numbers

   QED

Prove the following in at least 2 different ways:

- If n^2 is even, n is even.
- The sum of the numbers from 1 to n is $\frac{n(n+1)}{2}$.
- The sum of any two odd numbers is even.

We can prove that $\sqrt{2}$ is irrational via this proof:

Proof. Assume $\sqrt{2}$ is rational.

By definition of rational, $\sqrt{2} = \frac{p}{q}$ where p and q are integers, and p and q share no divisors. (Note: this is so the fraction is in simplest form).

Squaring both sides, we get $2 = \frac{p^2}{q^2}$

Multiplying both sides by q^2: $2q^2 = p^2$.

Since q is an integer, q^2 is an integer, and by definition of even, p^2 is even. Below, you'll prove that if n^2 is even, n is even, so we know p is even.

Let $p = 2k$, where k is an integer

$2q^2 = (2k)^2 = 4k^2$

Dividing by 2: $q^2 = 2k^2$. Since k is an integer, k^2 is an integer, and by definition of even, q is even.

This is a contradiction: If p and q are both even, then the fraction is not in lowest terms, as we required.

Therefore, $\sqrt{2}$ is irrational.

$\square$

What other numbers can you prove are irrational using a similar proof? Complete the proof for one of these numbers.

Ask ChatGPT to prove a mathematical statement or theorem. How did it do? We will discuss where ChatGPT succeeds and fails in class, so try to come up with a unique claim for ChatGPT to prove.

Are there other indicators that were not discussed that could help decide what proof type should be used? What are they?

Of the 4 main proof types discussed this week, which do you find most challenging?