1. **[15 points]** Determine whether the following problems are recursive problems.
   (a) $R(0) = 1$, $R(1) = 1$, $R(n) = 2R(n-1) + R(n-2)$
   (b) $R(1) = 0$, $R(n) = 5R(n-1)$
   (c) $R(0) = 0$, $R(n) = n$
   (d) $R(1) = 1$, $R(n) = 2R(\frac{n}{2}) + 5$
   (e) $R(1) = 1$, $R(n) = R(1)$

2. Solve the following recurrence relations:
   (a) **[5 points]** $R(0) = 3$, $R(n) = 3R(n-1)$
   (b) **[10 points]** $R(0) = k$, $R(n) = bR(n-1)$, where b and k are integers.
   (c) **[5 points]** $R(1) = 0$, $R(n) = R(\frac{n}{5}) + 1$, where n is always of the form $n = 5^k$
   (d) **[10 points]** $R(1) = 0$, $R(n) = R(\frac{n}{b}) + c$, where n is always of the form $n = b^k$, and b, c are integers.

3. **[35 points]** Show that the following recursive relations have the closed forms given.
   (a) $R(1) = 1$, $R(n) = R(n-1) + 2n - 1$. Closed form: n^2
   (b) $R(1) = 1$, $R(n) = \frac{n}{n-1}R(n-1)$. Closed form: n
   (c) $R(1) = 1$, $R(n) = R(n-1) + n$. Closed form: $\frac{n(n+1)}{2}$
   (d) $R(0) = 1$, $R(1) = 1$, $R(n) = 2R(n-1) - R(n-2)$. Closed form: 1
   (e) $R(1) = 5$, $R(n) = 5R(\frac{n}{2})$, where $n = 2^k$. Closed form: 5^{k+1}
   (f) $R(1) = 1$, $R(n) = R(\frac{n}{2}) + n$, where $n = 2^k$. Closed form: $2n - 1$
   (g) $R(1) = 0$, $R(n) = R(\frac{n}{2}) + \log_2(n)$, where $n = 2^k$. Closed form: $\frac{k(k+1)}{2}$

4. **[10 points]** Write a recursive function, then write a recurrence relation to represent the function. Include a screenshot or a recreation of your code for this question.