

1. **[10 Points]** Using a combinatorial proof, show that the sum of the  $n$ th row in Pascal's triangle is  $2^n$
2. **[10 Points]** Using a combinatorial proof, show that  $\binom{4n}{2} = 4 \cdot \binom{n}{2} + 6n^2$   
*Hint:  $6 = \binom{4}{2}$  and  $n$  can also be written as  $\binom{n}{1}$*
3. **[30 Points]** Find the probabilities of the following events:
  - (a) A randomly chosen integer number between 1 and 100 contains a 2.
  - (b) Rolling a sum of 3 on two standard, fair, 6-sided die.
  - (c) Rolling a sum of 3 on three standard, fair 6-sided die.
  - (d) Drawing all 4 letters from our Kibo deck in a hand of 5 cards.
  - (e) A randomly chosen real number chosen between 1 and 10 falls in the range from 2 to 4.
  - (f) A point randomly selected in a square of side length 1 also appears in a circle centered on the square with radius  $\frac{1}{2}$ .
4. **[15 Points]** Imagine there's a test for a disease that only 0.5% of people have. The test has a 99.9% sensitivity rate, and a 99% specificity rate.
  - (a) If a person has a positive result, what is the probability they have the disease?
  - (b) If a person has a negative result, what is the probability they do not have the disease?
5. **[20 Points]** Find the expected value of the following events
  - (a) The sum of numbers on 5 cards drawn from our Kibo deck.
  - (b) The sum of numbers on 10 cards drawn from our kibo deck.
  - (c) The outcome of 4 coin flips where heads add 1 and tails subtract 1.
  - (d) The multiplication of two rolls of a standard, fair 6-sided die.
6. **[15 Points]** Draw or graph a probability mass function for the following:
  - (a) The sum of two dice rolls.
  - (b) The results of a single coin flip.
  - (c) The value of a random integer number between 1 and 10.