

1. **[20 Points]** Suppose your teacher has a set of 500 math problems to distribute amongst the class. If there are 50 people in the class, how many ways can these problems be distributed if:
  - (a) There are no other restrictions
  - (b) Each student must have at least 1 problem.
  - (c) The students are in groups of 5, and the problems are distributed to groups.
  - (d) The problems are distributed evenly amongst the students.
2. **[5 Points]** Show that if there are 500 problems and 50 students, at least one student must have 10 or more problems.
3. Imagine you have a set of dice. There are 4 6-sided dice and 4 8-sided dice.
  - (a) **[5 Points]** How would you set up generating functions to solve a problem about the number of ways a specific sum could occur when rolling this set of dice? **Do not attempt to solve this problem, just explain how you would set it up.**
  - (b) **[5 Points]** If you rolled only the 6-sided dice, What are the maximum and minimum sums?
  - (c) **[5 Points]** If you rolled only the 6-sided dice, of all possible sums after the dice are rolled, what sum do you think is most common?
  - (d) **[5 Points]** If you rolled only the 8-sided dice, of all possible sums after the dice are rolled, what sum do you think is most common?
4. If there is a set of the integers from 1 to 9, any subset of size 6 must contain two numbers that sum to ten.
  - (a) **[5 Points]** If you have a set of integers from 1 to 11, what can we say instead? What size subset is needed for two numbers to sum to 12?
  - (b) **[5 Points]** If you have a set of integers from 1 to 13, what can we say now? This time give the size of subset and the sum.
  - (c) **[5 Points]** What would this problem be for a set of integers from 1 to  $n$ ?
5. **[20 Points]** Suppose you are at a deli. There are 5 types of meat, 4 types of cheese, 3 types of bread, and 7 types of toppings.
  - (a) How many ways are there to make a sandwich with one type of each item?
  - (b) How many ways are there to make a sandwich with two types of toppings, and one type of every other item?

- (c) How many ways are there to make a sandwich with any combination of toppings? Assume there is one type of each other item on the sandwich.
- (d) How many ways are there to make a sandwich if you can order double meat or two different kinds of meat? Assume there is one type of each other item on the sandwich.