# Problem Set Week 8

## Submission

This problem set is due Sunday December 4th at 10pm

- You may collaborate with up your peers. If you do, be sure to mention who you collaborated with explicitly in your submission. For example, write "I worked with Ope and Mehdi on this problem set" at the top.

- Show your work. Direct answers will not be accepted.

Submit your response in Gradescope, either using the app or the website.

## Problems

1. Perform the following conversions:
    1. 27 in base 10 to base 2

    2. 1111111 in base 2 to base 10

    3. 101 in base 10 to base 2

2. Compute GCD(1240, 6660)

3. Let *a* be a positive integer. Prove that GCD(*a*, *a*+1) = 1

4. Prove that GCD(*a*, *a*+*2*) = 1 if *a* is odd, and GCD(*a*, *a*+*2*) = 2 if *a* is even

5. Show that if $a \equiv b (\mathrm{mod\ n})$ and if $b \equiv c (\mathrm{mod\ n})$, then $a \equiv c (\mathrm{mod\ n})$

6. Simplify the following congruences:
    1. $15x \equiv 9 \pmod{25}$

    2. $6x \equiv 3 \pmod{9}$

    3. $14x \equiv 42 \pmod{50}$

7. Describe the general solution for *x* and *y*, if it exists: $35x + 47y = 1$

## References:

Problems 1-3 were drawn from:

- chapter 6 of Precalculus, 3rd corrected edition by S&Z

- Chapter 2 and 4 of Number Theory, in context and interactive

- Chapter 5.2 of Discrete Mathematics, an open introduction, 3rd edition