

Problem Set Week 7

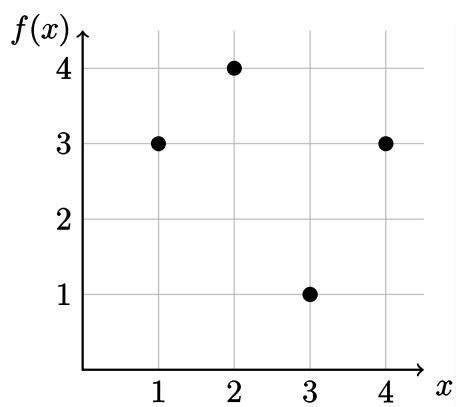
Submission

This problem set is due Sunday November 20th at 10pm

- You may collaborate with up your peers. If you do, be sure to mention who you collaborated with explicitly in your submission. For example, write "I worked with Ope and Mehdi on this problem set" at the top.
- Show your work. Direct answers will not be accepted.

Submit your response in Gradescope, either using the app or the website.

Problems



1. Consider the function $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ represented in the graph above:

1. is f injective? explain

2. is f surjective? explain

2. At the end of the semester a teacher assigns letter grades to each of her students. Argue that this is a function, and describe its domain and codomain.

3. Let $S = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ be the set of fractions. We define a relation R on S such by the formula: $\frac{a}{b} R \frac{c}{d} \iff ad = bc$. Is R an equivalence relation? why or why not?

4. Consider the following function: $f : \mathbb{N} \rightarrow \mathbb{N}$ described as

$$f(n+1) = \begin{cases} \frac{f(n)}{2} & \text{if } f(n) \text{ is even} \\ 3f(n) + 1 & \text{if } f(n) \text{ is odd} \end{cases}$$

Notice that if $f(0) = 1$, we get that: $f(1) = 4$, $f(2) = 2$, $f(3) = 1$, $f(4) = 4$, etc. So we have a cycle.

1. if $f(0) = 5$, can f be injective? Explain why or give a specific example of two elements from the domain with the same image.

2. if $f(0) = 3$, can f be injective? Explain why or give a specific example of two elements from the domain with the same image.

3. show that no matter the initial value, f can not be surjective.

References:

Chapter 0.4 of Discrete Mathematics: An Open Introduction, 3rd edition