Problem Set 3

Submission

This problem set is due Sunday October 30th at 10pm

- You may collaborate with up your peers. If you do, be sure to mention who you collaborated with explicitly in your submission. For example, write "I worked with Ope and Mehdi on this problem set" at the top.

- Show your work. Direct answers will not be accepted.

Submit your response in Gradescope, either using the app or the website.

Problems

1. Prove the following statements:
    1. Given that $\forall x, F(x) \lor G(x)$ and $\neg\exists x G(x)$, conclude that $\forall x, F(x)$
    2. $\forall x(\neg F(x) \lor G(x)) \to \forall x(F(x) \to G(x))$
    3. $\exists x F(x) \iff \neg\forall x \neg F(x)$

2. Consider the statement: For all integers *n*, if *n* is even then *8n* is even.

    1. Prove the statement. What kind of proof are you using?

    2. What is the converse of the statement?

    3. Prove or disprove the converse.

3. Prove the statement: For all integers *n*, if *5n* is odd then *n* is odd. Clearly state what kind of proof you are using.

4. Prove the statement: For all real numbers x,y, $x = y$ if and only if $xy = \frac{(x+y)^2}{4}$.

   Note that you will need to prove both 'directions' of the implication.

5. Suppose you would like to prove the following implication:

   "For all numbers *n*, if *n* is prime, then *n* is solitary"

   How would you *start* and *end* your argument if you tried to prove the statement...

    1. Directly

    2. By contradiction

3. By contrapositive

You do not have to actually prove the statement, as we haven't covered primes and solitary numbers. Focus on the structure of the argument.

6. A friend shows you the following proof that shows the statement 1 = 3:
    - (i)Assume $1 = 3$
    - (ii)Therefore $1 - 2 = 3 - 2$
    - (iii)Simplifying we get $-1 = 1$
    - (iv)$(-1)^2 = 1^2$
    - (v)$1 = 1$ which is true
    - (vi)Therefore $1 = 3$

Where is the flaw in the argument?

References

These problems were drawn from:

- Discrete mathematics - an open introduction part 3.2

- A Concise introduction to logic part 2