Assignment 4

Q1. Direct Proofs

Assume you are trying to prove a statement $$P\Rightarrow Q$$ using the direct proof method.

Which of the following apply:

Q2. Proofs by contradiction

Assume you are trying to prove a statement $$P\Rightarrow Q$$ using the proof by contradiction method.

Which of the following apply:

Q3. Proofs by contrapositive

Assume you are trying to prove a statement $$P\Rightarrow Q$$ using the proof by contrapositive method.

Which of the following apply:

Q4. Negations

Determine if proposition A is the negation of proposition B

Q4.1.

$$A = \forall x \exists y \neg P(x,y)$$
$$B = \exists x \forall y P(x,y)$$

A. $$B\equiv \neg A$$

B. $$B\not\equiv \neg A$$

Q4.2.

$$A = \forall x (P(x)\Rightarrow Q(x))$$
$$B = \exists x (P(x)\wedge \neg Q(x))$$

A. $$B\equiv \neg A$$

B. $$B\not\equiv \neg A$$

Q4.3.

$$A = \exists x (P(x) \vee \neg Q(x))$$
$$B = \forall x (P(x) \wedge \neg Q(x))$$

A. $$B\equiv \neg A$$

B. $$B\not\equiv \neg A$$

Q5. Error in proof

Consider the following proof of 2=0:
(i) Assume $$a = b= 1$$
(ii) $$ab +ab = b^2 +b^2$$
(iii) $$a^2 + 2ab = a^2 +b^2 + b^2$$
(iii) $$a^2 - b^2 = a^2 - 2ab + b^2$$
(iv) $$(a+b)(a-b)=(a-b)(a-b)$$
(v) $$(a+b) = (a-b)$$
(vi) $$2 = 0$$

What lines are introduced in error?

Q6. True or False

Determine if the following statements are always true or if there is an instance in which they are false.

Q6.1. Proving existential statements

In order to prove a statement of the form $$\exists x P(x)$$ it suffices to find a concrete example let us say $$a$$ such that $$P(a)$$ is true.

A. True

B. False

Q6.2. Proving universal statements

In order to prove a statement of the form $$\forall x P(x)$$ it suffices to find a concrete example let us say $$a$$ such that $$P(a)$$ is true.

A. True

B. False

Q6.3. Disproving existential statements

In order to disprove a statement of the form $$\exists x P(x)$$ it suffices to find a concrete example let us say $$a$$ such that $$\neg P(a)$$ is true.

A. True

B. False

Q6.4. Disproving universal statements

In order to disprove a statement of the form $$\forall x P(x)$$ it suffices to find a concrete example let us say $$a$$ such that $$\neg P(a)$$ is true.

A. True

B. False