Assignment 7

Q1. Equivalence Relations

Determine whether the following relations $$R$$ are Equivalence relations or not.

Q1.1. Less than

Set over which the relation is taken: Real numbers
$$aRb$$ if $$a$$ is less than $$b$$

A. $$R$$ is an equivalence relation.

B. $$R$$ is not an equivalence relation.

C. We cannot tell if $$R$$ is an equivalence relation or not with the given information.

Q1.2. Names starting with the same letter

Set over which the relation is taken: Names of Mathematical Thinking Students
$$aRb$$ if $$a$$ and $$b$$ start with the same letter

A. $$R$$ is an equivalence relation.

B. $$R$$ is not an equivalence relation.

C. We cannot tell if $$R$$ is an equivalence relation or not with the given information.

Q1.3. Residue

Set over which the relation is taken: Natural numbers
$$aRb$$ if $$a$$ and $$b$$ have the same residue when divided by the number 10.

A. $$R$$ is an equivalence relation.

B. $$R$$ is not an equivalence relation.

C. We cannot tell if $$R$$ is an equivalence relation or not with the given information.

Q1.4. Shared digits

Set over which the relation is taken: Three digit numbers
$$aRb$$ if $$a$$ and $$b$$ share a digit.

A. $$R$$ is an equivalence relation.

B. $$R$$ is not an equivalence relation.

C. We cannot tell if $$R$$ is an equivalence relation or not with the given information.

Q2. Functions

Determine if the relation $$F$$ is a function

Q2.1. Functions

The domain: $$\mathbb{R}$$
The codomain: $$\mathbb{Z}$$
$$F(a, b)$$ if $$b$$ is the largest integer less than or equal to $$a$$

A. $$F$$ is a function

B. $$F$$ is not a function

C. We cannot tell if $$F$$ is a function or not with the given information

Q2.2. Sum of Squares

The domain: the interval $$[-1,1]$$
The codomain: the interval $$[-1,1]$$
$$F(a, b)$$ if $$a^2 + b^2 \leq 1$$

A. $$F$$ is a function

B. $$F$$ is not a function

C. We cannot tell if $$F$$ is a function or not with the given information

Q2.3. Residues

The domain: Natural numbers ($$\mathbb{N}$$)
The codomain: $$\{0,1,2,3,4,5,6,7,8,9\}$$
$$F(a, b)$$ if $$b$$ is the residue of $$a$$ when divided by 10.

A. $$F$$ is a function

B. $$F$$ is not a function

C. We cannot tell if $$F$$ is a function or not with the given information

Q2.4. Colors

The domain: Objects in your household
The codomain: Lists of colors
$$F(a, b)$$ if $$b$$ is the list of colors present in the object $$a$$

A. $$F$$ is a function

B. $$F$$ is not a function

C. We cannot tell if $$F$$ is a function or not with the given information

Q3. Function properties

Consider the following Functions. Determine if they are injective, surjective, and bijective. Select all that apply.

Q3.1. Squaring a real number

$$F: \mathbb{R}\longrightarrow \mathbb{R}$$
$$x \mapsto x^2$$

Q3.2. Squaring a natural number

$$F: \mathbb{N}\longrightarrow \mathbb{N}$$
$$x \mapsto x^2$$

Q3.3. Grades

$$F: \{x| x \text{ is a possible score on Assignment 1} \}\longrightarrow \{y| y \text{ is a subset of the set of Mathematical Thinking students}\}$$
$$x \mapsto \text{the list of students that got the score } x $$

Q3.4. RGB colors

$$F: \{x| x \text{ is a hexadecimal number with 6 digits}\}\longrightarrow \{y|y \text{ is an RGB color.} \}$$
$$x \mapsto \text{ the RGB color represented by } x $$

Q3.5. Cube plus 1

$$F: \mathbb{R}\longrightarrow \mathbb{R}$$
$$x \mapsto x^3 + 1$$

Q4. Function composition

Determine the result of the composition of $$F$$ and $$G$$.

Q4.1. Adding one and squaring

$$F: \mathbb{N}\longrightarrow \mathbb{N}$$
$$ x\mapsto x+1$$

$$G: \mathbb{N}\longrightarrow \mathbb{N}$$
$$ y\mapsto y^2$$

A. $$x \mapsto x^2 + 1$$

B. $$x \mapsto x^2 + 2x + 1$$

C. $$x \mapsto x^2 + x + 1$$

D. None of the above

Q4.2.

$$F: \mathbb{Q}\longrightarrow \mathbb{Q}$$
$$ x\mapsto 2 x$$

$$G: \mathbb{Q}\longrightarrow \mathbb{Q}$$
$$ y\mapsto \frac{y}{0.5}$$

A. $$x \mapsto 2 x$$

B. $$x \mapsto x$$

C. $$x \mapsto x^2 +1$$

D. None of the above