Support Vector Classifier

In linear discrimination, we seek an affine function that classifies the points. i.e.

$$\begin{cases} a^T x_i - b > 0 & (i = 1, \cdots, N) \\ a^T y_i - b < 0 & (i = 1, \cdots, M) \end{cases} \qquad (8.20)$$

Since the inequalities above are homogeneous in a and b, (8.20) is feasible if and only if

$$\begin{cases} a^T x_i - b \geq 1 & (i = 1, \cdots, N) \\ a^T y_i - b \leq -1 & (i = 1, \cdots, M) \end{cases}$$

is feasible.

$$\Big\downarrow \text{ relax by introducing nonnegative variables } u_i,\ v_i$$

$$a^T x_i - b \geq 1 - u_i \qquad (i = 1, \cdots, N)$$

$$a^T y_i - b \leq -1 + v_i \quad (i = 1, \dots, M)$$

$$\boxed{\begin{aligned} \text{min.} \quad & \mathbb{1}^T u + \mathbb{1}^T v \\ \text{s.t.} \quad & a^T x_i - b \geq 1 - u_i \quad (i = 1, \dots, N) \\ & a^T y_i - b \leq -1 + v_i \quad (i = 1, \dots, M) \\ & u \succeq 0 \qquad v \succeq 0 \end{aligned}}$$

Linear Program