

⑩ Penalty function approximation problem

$$\min. \quad \phi(r_1) + \dots + \phi(r_m)$$

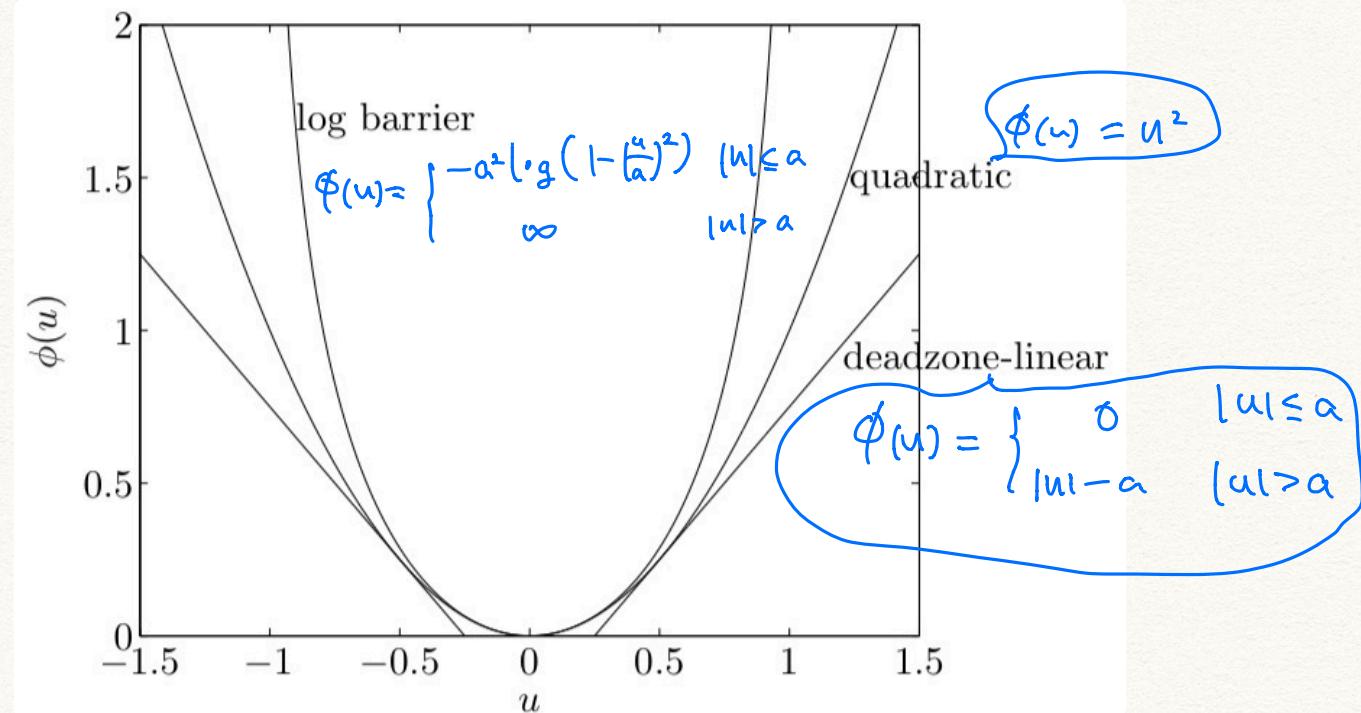
$$\text{s.t.} \quad r = Ax - b$$

If ϕ is convex \rightarrow Convex optimization problem

⑪ Examples of penalty function ϕ

$$\ell_1\text{-norm} \rightarrow \phi(u) = \|u\|_1$$

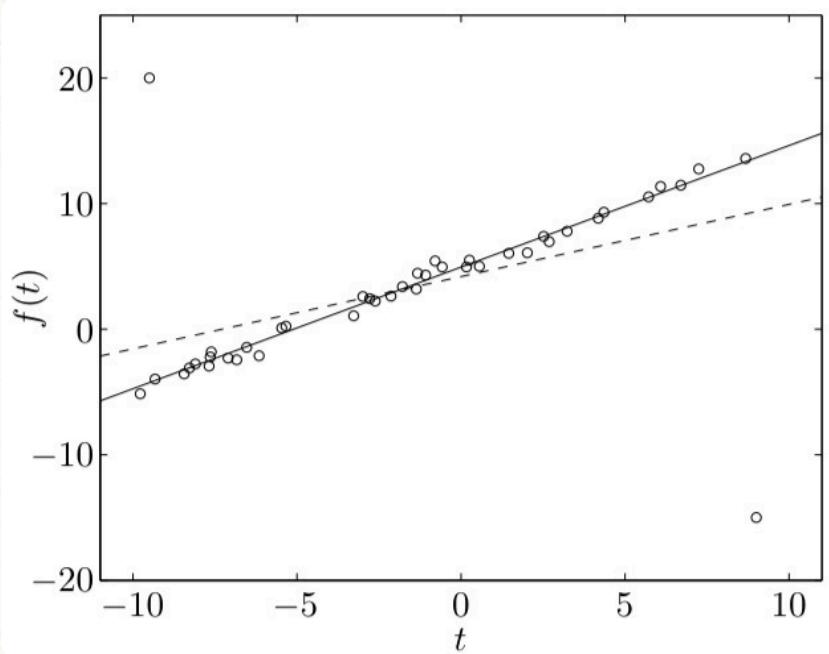
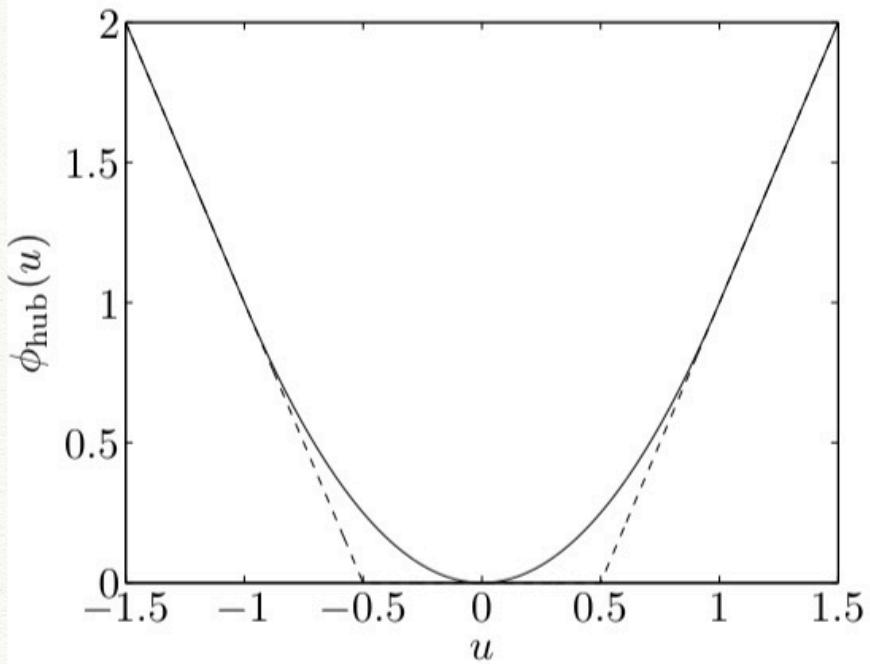
$$\ell_2\text{-norm} \rightarrow \phi(u) = u^2$$



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Robust least-squares

$$\phi(u) = \begin{cases} u^2 & |u| \leq M \\ M(2|u| - M) & |u| > M \end{cases}$$



(11) Least-squares Solution of linear equations

$$\begin{aligned} \text{min. } & \|x\|_2^2 \\ \text{s.t. } & Ax = b \end{aligned}$$

$$g(v) = \inf_x \left(\|x\|_2^2 - v^T(Ax - b) \right)$$

$$= b^T v + \inf_x \left(\|x\|_2^2 - v^T A x \right)$$

$$= b^T v - \frac{1}{4} v^T A A^T v$$

\downarrow dual problem

$$\max. \quad b^T v - \frac{1}{4} v^T A A^T v$$

$$v^* = 2(AA^T)^{-1}b$$

$$\downarrow \quad 2x^* - A^T v = 0$$

$$x^* = \underbrace{A^T (AA^T)^{-1} b}_{\sim}$$

④ Reconstruction problem