22/8/2019

1. INTERFERENCE

Interference:

Interference can be defined as the interaction between two or more waves of the same or very close frequencies emitted from coherent sources, where the wavefronts are combined according to the principle of superposition.

In optics, the interference means the superposition of two or more waves which results in a new wave pattern.

The resulting variation in the disturbances produced by the waves is called the interference pattern.

Superposition:

Superposition is the vectorial addition of waves. (electric field wave).

Coherent Sources:

Interference pattern will be observed only if the sources are coherent. Two sources of light are said to be coherent, if they emit waves of the same frequency (or wavelength), nearly the same amplitude and maintain a constant phase difference between them. No source is perfectly coherent.

Examples of coherent source: Lasers, Na lamp.....

We cannot have perfectly coherent waves because of the following reasons:

- Uncertainty principle.

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta E \cdot \Delta t \geq \frac{h}{2} \quad (E - \text{Energy})$$

$$h\Delta\nu \geq \frac{h}{2\Delta t}$$

- Maintaining a constant phase difference.

Maintaining a constant phase difference is difficult because ~~we~~ ~~never~~ wave is not continuous.

<u>Coherence</u>

Coherence is a property of waves that helps in getting stationary interference, i.e., the interference which is temporally and spatially constant. The coherence of a wave depends on the characteristics of its source.

1. <u>Temporal coherence (related to time)</u>

- A measure of the correlation between the phase of a wave (light) at different points along the direction of wave propagation.

- If the phase difference of the wave crossing the two points lying along the direction of wave propagation is independent of time, then the wave is said to have temporal coherence.

- Also known as longitudinal coherence.
- screen should be placed at a distance less than coherent plane.

2. Spatial Coherence (related to space)

- A measure of the correlation between the phases of a wave (light) at different points transverse to the direction of propagation.
- If the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of wave propagation is independent of time, then the wave is said to have spatial coherence.
- Also known as lateral coherence.

Coherence Time and Coherence Length

For a single frequency wave, the time interval over which the phase remains constant is called the coherence time. It is generally represented by Δt.

The distance travelled by the light pulses during the coherence time is known as coherence length. It is represented by ΔL. The coherence length is also called the spatial interval, which is the length over which the phase of the wave remains constant.

$$\Delta L = c \Delta t$$

Constructive Interference:

When two waves are not displaced with respect to each other or when they are displaced through an integral number of wavelengths, constructive interference takes place. Bright bands are observed at those points.

Path difference, $x = n\lambda$

Destructive Interference

When two waves are displaced with respect to each other by an odd number of half-wavelengths, destructive interference results. Dark bands of light are observed at those points.

Path difference, $x = (2n+1)\frac{\lambda}{2}$

Partially destructive or Constructive Interference

Partial interference occurs when two waves have the same frequency and wavelength and are added together but the waves are added so the crests and troughs don't line up. This is between constructive and destructive interference.

Note:

There is no production of energy only redistribution of energy.

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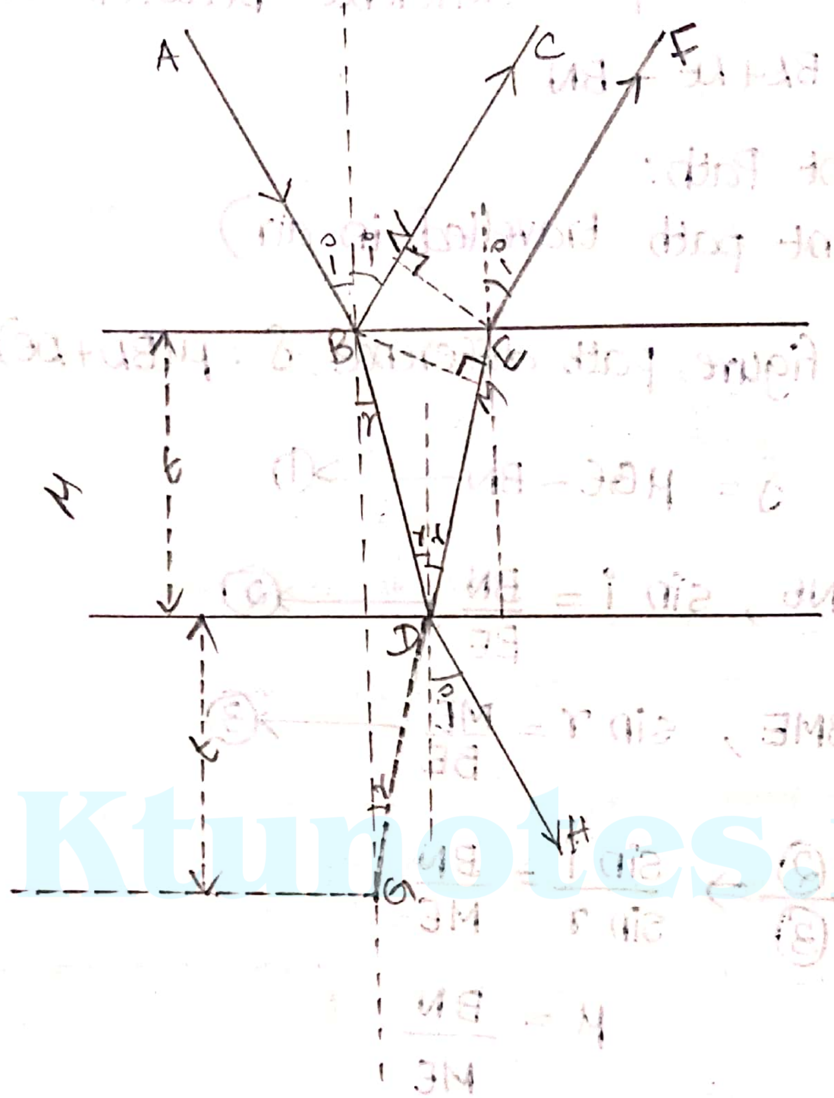
Interference In Thin Films - Reflected System

Consider a transparent thin film of refractive index 'μ' and thickness 't'.



Consider a ray of light AB falling on the upper surface of the film at an angle 'i'. A part of the ~~gay~~ ray gets reflected and the other part gets refracted along BD at an angle 'r'. At point D, the wave BD is again partly reflected from the second surface along DE and partly emerges out along DH. Thus, interference occurs between reflected waves ~~BD~~ BC and EF. ~~and also between the~~

~~transmitted waves~~

Geometrical Path difference:

The geometric path difference between the two rays is BD+DE − BN.

Optical Equivalent Path:

(Equivalent path travelled in air)

From the figure, path difference, $\delta = \mu(BD+DE) - BN$

$$\delta = \mu GE - BN \longrightarrow (1)$$

From ΔBNE, $\sin i = \frac{BN}{BE} \longrightarrow (2)$

From ΔBME, $\sin r = \frac{ME}{BE} \longrightarrow (3)$

$$\frac{(2)}{(3)} \Rightarrow \frac{\sin i}{\sin r} = \frac{BN}{ME}$$

$$\mu = \frac{BN}{ME}$$

$$BN = \mu ME \longrightarrow (4)$$

Sub. (4) in (1),

$$\delta = \mu GE - \mu ME$$

$$= \mu(GE - ME)$$

$$\delta = \mu GM \longrightarrow (5)$$

From ΔBMG, $\cos r = \frac{GM}{BG} = \frac{GM}{2t}$

$$GM = 2t\cos r \longrightarrow (6)$$

sub. (6) in (5) $\Rightarrow \delta = \mu(2t\cos r)$.

$$\boxed{\delta = 2\mu t\cos r}$$

This is called cosine law.

According to electromagnetic theory, a phase ~~ver~~ reversal will occur when light gets reflected from a denser medium.

$\therefore$ BN has to be replaced with $BN + \frac{\lambda}{2}$.

$\therefore$ The actual path difference is

$$\boxed{\delta = 2\mu t\cos r - \frac{\lambda}{2}}$$

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If $\delta = n\lambda$, constructive interference will take place and the film will appear bright.

i.e., $2\mu t\cos r - \frac{\lambda}{2} = n\lambda$

$$2\mu t\cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}$$

(condition for bright)

When $\delta = (2n+1)\frac{\lambda}{2}$, destructive interference will take place and the film will appear dark.

$$2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\cos r = n\lambda + \frac{\lambda}{2} + \frac{\lambda}{2}$$

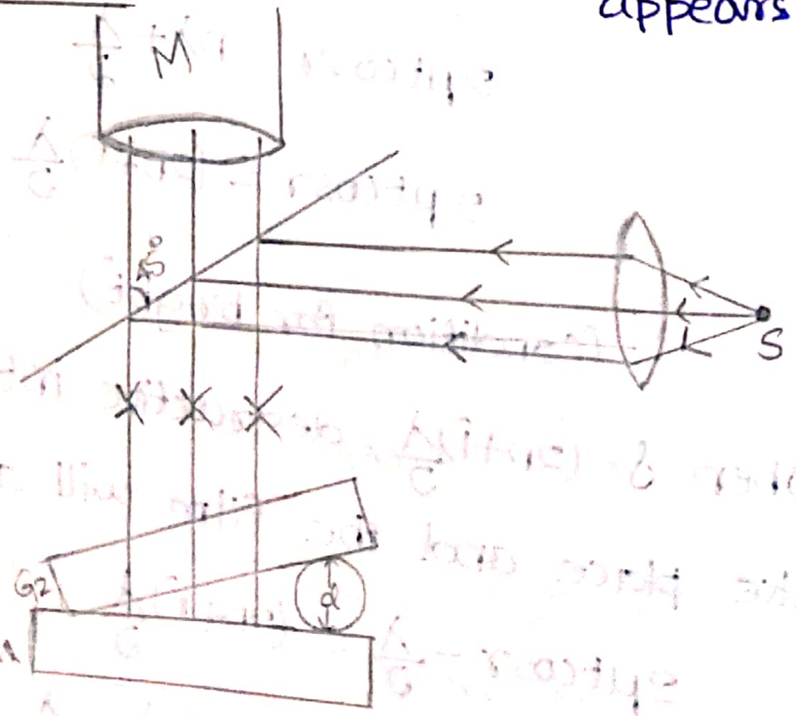
$$2\mu t \cos r = (n+1)\lambda$$

When $2\mu t \cos r$ is an integral multiple of λ, the film will appear dark.

Q: Why do soap bubbles and oil slicks appear coloured?

The condition for destructive interference is $2\mu t \cos r = n\lambda$. So, the thickness of soap bubble may vary from point to point. Wavelength λ and angle r also changes. White light consists of a range of wavelengths and for specific values of 't' and 'r', waves of only certain wavelengths (colours) constructively interfere. Therefore, only those colours are present in the reflected light. Other wavelengths interfere destructively and hence absent. Hence, the film at a particular point appears coloured.

AIR WEDGE:



For dark band, $2\mu t\cos r = n\lambda$

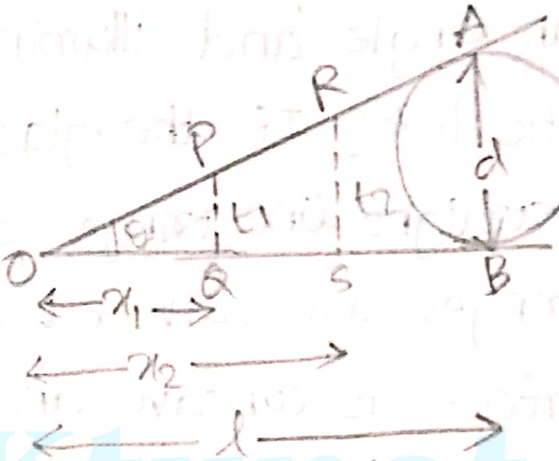
When incident normally, $i=0$ and 'r' tents to zero

$$2\mu t = n\lambda$$

For air, $\mu = 1$

$$\underline{2t = n\lambda}$$

$$2t = \delta \longrightarrow ①$$



Let PQ be the position of m^{th} dark fringe and RS corresponds to n^{th} dark fringe.

$$t_1 = x_1 \tan\theta$$

$$t_2 = x_2 \tan\theta$$

$$t_2 - t_1 = (x_2 - x_1)\tan\theta$$

$$\tan\theta = \frac{t_2 - t_1}{x_2 - x_1} = \frac{n\lambda - m\lambda}{2(x_2 - x_1)} = \frac{(n-m)\lambda}{(x_2 - x_1)2}$$

$$\frac{x_2 - x_1}{n-m} = \beta \text{ (bandwidth)}$$

$$\therefore \tan\theta = \frac{\lambda}{2\beta}$$

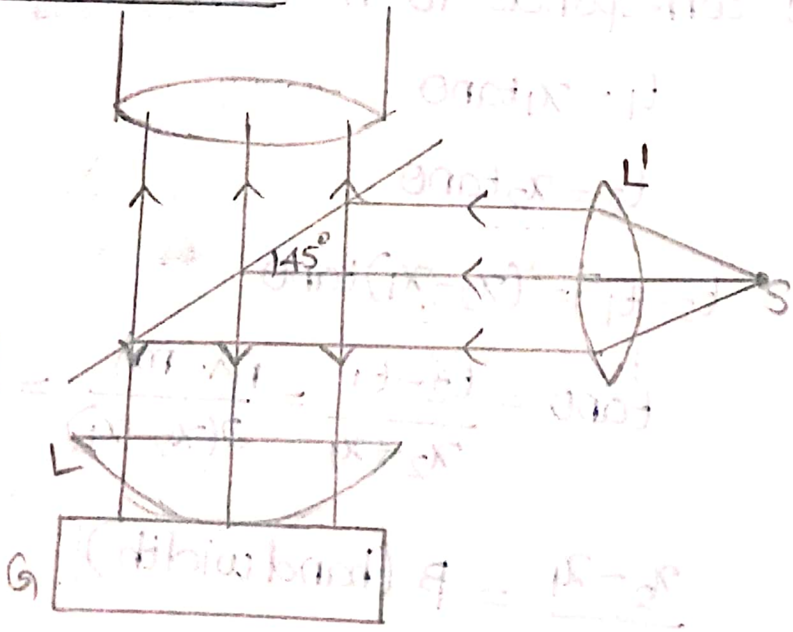
$$\boxed{d = l\tan\theta = \frac{\lambda l}{2\beta}}$$

Q: How will you check the planeness of a glass plate using interferometric technique?

The planeness of a surface can be quickly inspected visually by keeping an optical flat on the component at an angle and illuminating it with a monochromatic light. If the glass plate is regular, parallel straight line fringes are obtained. If the fringes are curved towards the contact edge, the surface is concave and if the fringes curve away, it is convex.

19/9/2019

NEWTON'S RINGS



When a plano-convex lens is placed over a flat glass plate, then a thin air layer is formed between glass plate and a convex lens.

Interference is obtained from bottom surface of lens and top surface of glass.

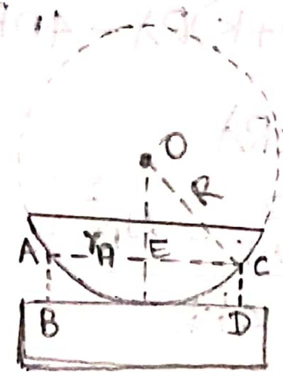
For destructive superposition,

$$2\mu t\cos r = n\lambda$$

$$r \to 0,\ \cos r = 1,\ \mu = 1$$

$$2t = n\lambda$$

As we move to centre, the rings will appear closer and closer. This is because the thickness is not uniform throughout.



Let $AB = t$

Consider ΔOEC,

$$OE^2 + EC^2 = OC^2$$

$$(R-t)^2 + r_n^2 = R^2$$

$$R^2 - 2Rt + t^2 + r_n^2 = R^2$$

$$t^2 + r_n^2 = 2Rt$$

$\because$ t is very small, t^2 can be neglected.

$$r_n^2 = 2Rt$$

$$2t = \frac{r_n^2}{R}$$

$$r_n^2 = nR\lambda$$

$$\boxed{r_n = \sqrt{nR\lambda}}$$

Equation to Find λ in Lab

$$D_n = 2\sqrt{nR\lambda}$$

$$D_n^2 = 4nR\lambda$$

$$D_{n+k} = 2\sqrt{(n+k)R\lambda}$$

$$D_{n+k}^2 - D_n^2 = 4(n+k)R\lambda - 4nR\lambda$$

$$D_{n+k}^2 - D_n^2 = 4kR\lambda$$

$$\boxed{\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}}$$

Q: Explain how will you find refractive index of a transparent liquid using Newton's rings experiment.

Refractive index, $\mu = \dfrac{\text{Velocity of light in air}}{\text{Velocity of light in medium}}$

Let λ be wavelength in air and λ' be wavelength in medium.

$$\mu = \frac{\nu\lambda}{\nu\lambda'}$$

$$\mu = \frac{\lambda}{\lambda'}$$

$$\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}$$

$$\lambda' = \frac{\lambda}{\mu}$$

$$\lambda' = \frac{D'^2_{n+k} - D'^2_n}{4kR}$$

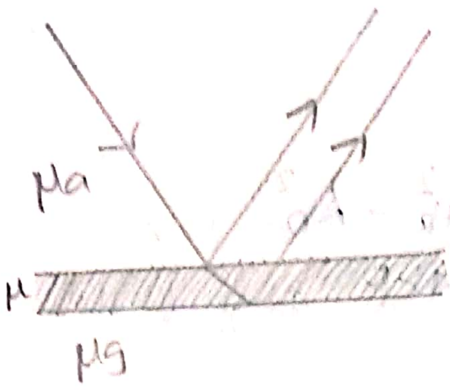
$$\frac{\lambda}{\mu} = \frac{D'^2_{n+k} - D'^2_n}{4kR}$$

$$\frac{D_{n+k}^2 - D_n^2}{4kR\mu} = \frac{D'^2_{n+k} - D'^2_n}{4kR}$$

$$\mu = \frac{D_{n+k}^2 - D_n^2}{D'^2_{n+k} - D'^2_n}$$

ANTIREFLECTION COATINGS

Antireflection coatings are thin transparent coatings of optical thickness of one-quarter wavelength given on a surface in order to suppress reflections from the surface.



Let 't' be the thickness of the film and 'μ' be the refractive index of the film material.

The two waves interfere destructively,

$$\therefore 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

Since, the thickness of the film is minimum,

$$n=0$$

$$2\mu t = \frac{\lambda}{2}$$

$$\mu = \frac{\lambda}{4t}$$

$$t = \frac{\lambda}{4\mu}$$

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DIFFRACTION

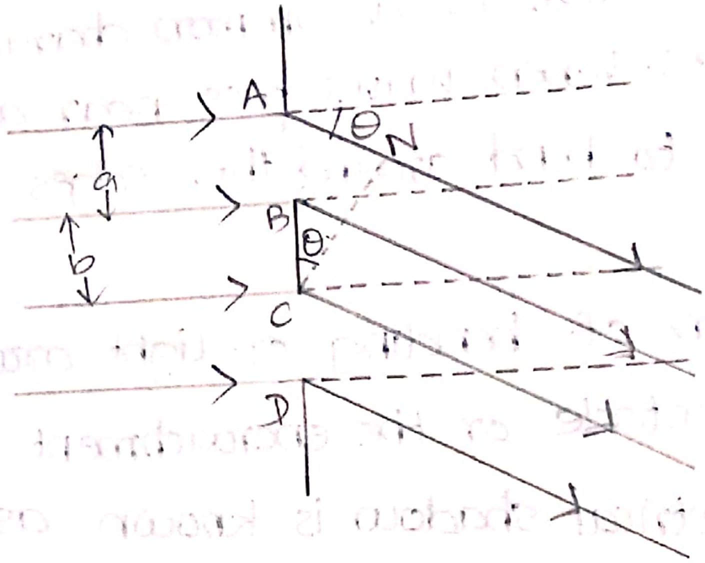
Diffraction phenomenon is a common characteristic of all kinds of waves. When waves pass near an obstacle, they tend to bend around the edges of the obstacle.

The phenomenon of bending of light around the edges of an obstacle or the encroachment of light within the geometrical shadow is known as diffraction.

Fresnel and Fraunhoffer Diffraction.

Fresnel Diffraction	Fraunhoffer Diffraction
• Source of light and the screen are at finite distance from the aperture.	• The source of light and the screen are at infinite distances from the aperture.
• Rays are not parallel.	• Rays are parallel.
• Incident wave front is not planar.	• Incident wave front as such is plane.
• The phase of secondary wavelets is not the same at all points in the plane of the obstacle.	• The secondary wavelets, which originate from the unblocked portions of the wavefront, are in same phase.
• It is expeimentally simple but the analysis proves to be very complex.	• This is simple to handle mathematically because the rays are parallel.

GRATING EQUATION:



$$AN = (a+b)\sin\theta$$

$\boxed{(a+b)\sin\theta = n\lambda}$ (constructive interference)

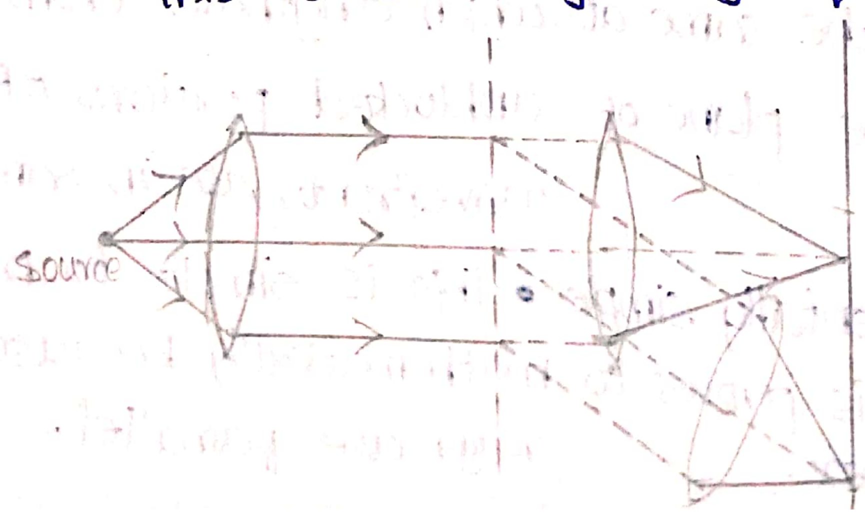
$(a+b)$ is called grating element.

$$\sin\theta = \frac{1}{(a+b)} n\lambda$$

$\frac{1}{a+b} = N =$ no. of grating per unit (lines)

$$\boxed{\sin\theta = Nn\lambda}$$

This is called grating equation.



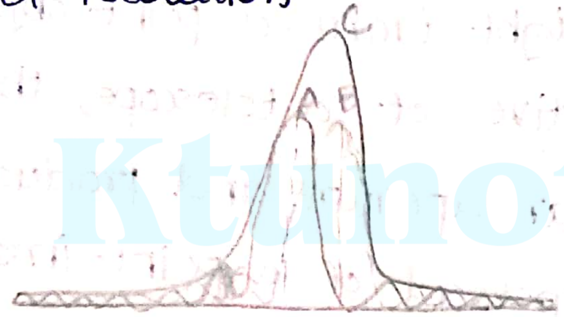
RESOLVING POWER:

Resolving power is the capability of an optical instrument to produce clearly separate images of two objects situated very close to each other.

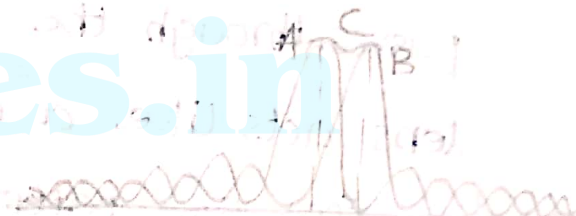
Rayleigh's Criterion:

The theory of optical instruments is based on the laws of geometrical optics and rectilinear propagation of light. These laws are only approximately true. While a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is known as Airy's disc. If there are two close point objects then two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them as separate. To obtain the measure of the resolving power of an objective lens, Rayleigh suggested a criterion known as Rayleigh criterion.

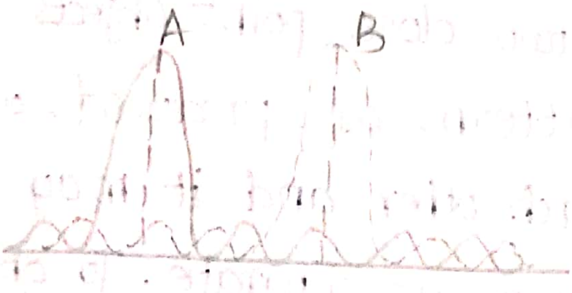
According to Rayleigh criterion, inorder to see two spectral lines as separathe, the n^{th} principal maximum of the higher wavelength should fall atleast on the first secondary minimum after the n^{th} principal maximum of the lower wavelength. This is equivalent to the condition that the distance between the centers of the patterns shall be equal to the radius of the Airy's disc. This is called as Rayleigh's limit of resolution.



Unresolved.



Just resolved



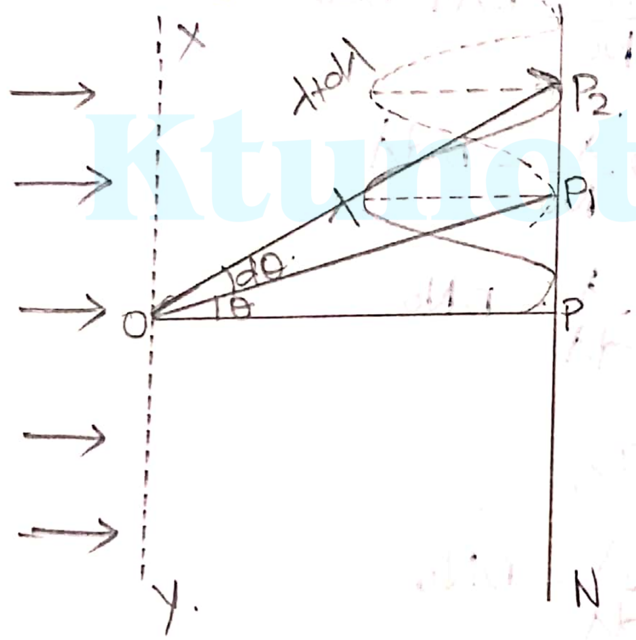
Well resolved

Resolving Power of Grating

The spectral resolving power of a grating is the smallest wavelength interval ($d\lambda$) that can be detected by it. It is the ability of grating to distinctly separate two very close spectral lines.

$$\text{Resolving power, } R = \frac{\lambda}{d\lambda},$$

where $d\lambda$ is the difference of two wavelengths which a grating can resolve.



Consider a parallel beam of light of wavelengths λ and $\lambda+d\lambda$ incident normally on the plane transmission grating having grating element $(a+b)$ and total number of lines 'N'.

The two wavelengths λ and $(\lambda+d\lambda)$ will be just resolved if the following two equations are satisfied.

For the principal maximum,

$$(a+b)\sin\theta = n(\lambda+d\lambda)$$

For secondary minimum,

$$(a+b)\sin\theta = n\lambda + \frac{\lambda}{N_0}$$

where, $\frac{\lambda}{N_0}$ – path difference between secondary wavelets from two successive slits.

$$\therefore n\lambda + \frac{\lambda}{N_0} = n\lambda + n d\lambda$$

$$\frac{\lambda}{N_0} = n d\lambda$$

$$\frac{\lambda}{d\lambda} = nN_0$$

Resolving power,

$$\boxed{R = \frac{\lambda}{d\lambda} = nN_0}$$

Thus, the resolving power of a grating depends on the total number of lines and the order of the spectrum.

Dispersive Power of Grating.

Dispersive power is the change in the angle of diffraction per unit change in wavelength.

The diffraction of the n^{th} order principal maximum,

$$(a+b)\sin\theta = n\lambda$$

Differentiating,

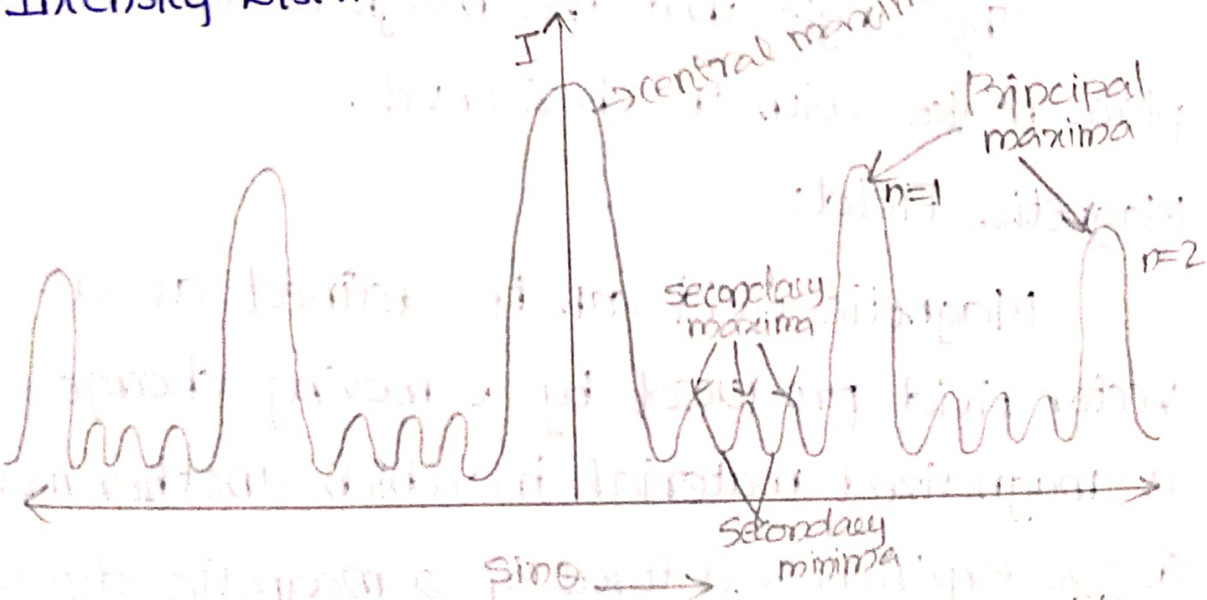
$$(a+b)\cos\theta = n\frac{d\lambda}{d\theta}$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$\boxed{\frac{d\theta}{d\lambda} = \frac{Nn}{\cos\theta}}$$

where, $N = \frac{1}{a+b}$

Intensity Distribution Due to N slits.



N_0 - total no. of slits

$(N_0 - 1) \Rightarrow$ minima

$(N_0 - 2) \Rightarrow$ maxima

MAGNETISM AND ELECTROMAGNETIC THEORY

Magnetism

Fundamental forces of nature

- Gravitational force
- Weak nuclear force
- Electromagnetic force
  - Electric force
  - Magnetic force.
- strong nuclear force

Magnetic declination:

Angle between magnetic and geometric axes which varies with places.

Magnetic inclination:

The angle that the magnetic field at a place makes with the horizontal.

Magnetic Field:

Magnetic field can be defined as a vector field produced by a moving charge or a magnetized material in which another moving charge experiences a force or a magnetic dipole experiences a torque.

The strength of magnetic field is quantitatively expressed by the vector quantity, magnetic induction (Magnetic Flux Density), B.

Magnetic Flux Density/Magnetic Induction

Magnetic induction at a point is defined as the flux passing through unit area around the point. Its unit is Wb/m^2 or Tesla.

Magnetizing field (H):

The magnetic field in which a material is kept is called magnetizing field, H. It represents the strength of the magnetic field. Units of H are A/m in SI system. $\boxed{B = \mu_0 H}$

Magnetization, M:

It is defined as the magnetic dipole moment per unit volume developed inside a solid. Its unit is A/m.

$$M \propto H$$

$$M = \chi H$$

where, χ is the proportionality constant and is known as magnetic susceptibility.

Magnetic susceptibility (χ)

It is the ratio of magnetization M to the magnetizing field H.

$$\chi = \frac{M}{H}$$

Magnetic Permeability

The ability of material to support the formation of magnetic field within it is given by permeability (μ). Its value denotes the ease with which the magnetic field line pass through the material.

Magnetic permeability, μ of a material is defined as the ratio of resultant magnetic field 'B' inside the material to the magnetizing field 'H'.

$$\mu = \frac{B}{H}$$

Its unit is henry/metre (H/m).

Relative permeability, $\mu_r = \frac{\mu}{\mu_0}$

$B_0 \propto H$

$B_0 = \mu_0 H$

$B \propto H$

$B = \mu H$

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r$$

Relation connecting B, H and M

$$B = \mu_0 \mu_r H$$

$$B = \mu_0 (H + M)$$

$$B = \mu_0 H \left(1 + \frac{M}{H}\right)$$

$$B = \mu_0 H (1 + \chi)$$

$$\chi = \mu_r - 1$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

GAUSS' LAW FOR MAGNETIC FLUX DENSITY

Magnetic flux through a small elemental area, $d\phi_B = \vec{B} \cdot \vec{dS}$

Total magnetic flux, $\phi_B = \int \vec{B} \cdot \vec{dS}$

Magnetic flux is scalar quantity and its unit is weber (Wb).

$$1\ Wb = 1\ Tm^2 = 1\ Nm/A$$

Gauss' law in magnetism states that the net magnetic flux through any closed surface is zero.

$$\phi_B = \oint \vec{B} \cdot \vec{dS} = 0$$

This means it is impossible to isolate the magnetic poles and the simplest magnetic structure that can exist is a magnetic dipole.

From the equation, it is clear that the magnetic field lines always form closed loops

DIFFERENTIATION OF SCALARS AND VECTORS.

Sir Hamilton introduced a differential operator ∇ (del) which is capable of differentiating both scalar and vector at equal ease. It is symbolically represented as.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

Three similar operations - gradient, divergence and curl - can be ~~operat~~ performed using ∇ operator. The differentiation of a scalar function appears in the form of gradient and that of vector function as divergence and curl.

1. GRADIENT :

If $\phi(x,y,z)$ is a scalar function, then the product of ∇ and ϕ is called gradient of scalar function.

$$\text{grad}\,\phi = \nabla\cdot\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

- $\nabla\cdot\phi$ is a vector ~~quantity~~ function.
- Gradient of a scalar function gives the unique direction in which the function changes most rapidly.

e.g., If ϕ is temperature, then gradient of ϕ represents the temperature gradient. It gives the net rate of change of temperature with position coordinates and its direction gives the direction in which this change is maximum.

Physical Significance:

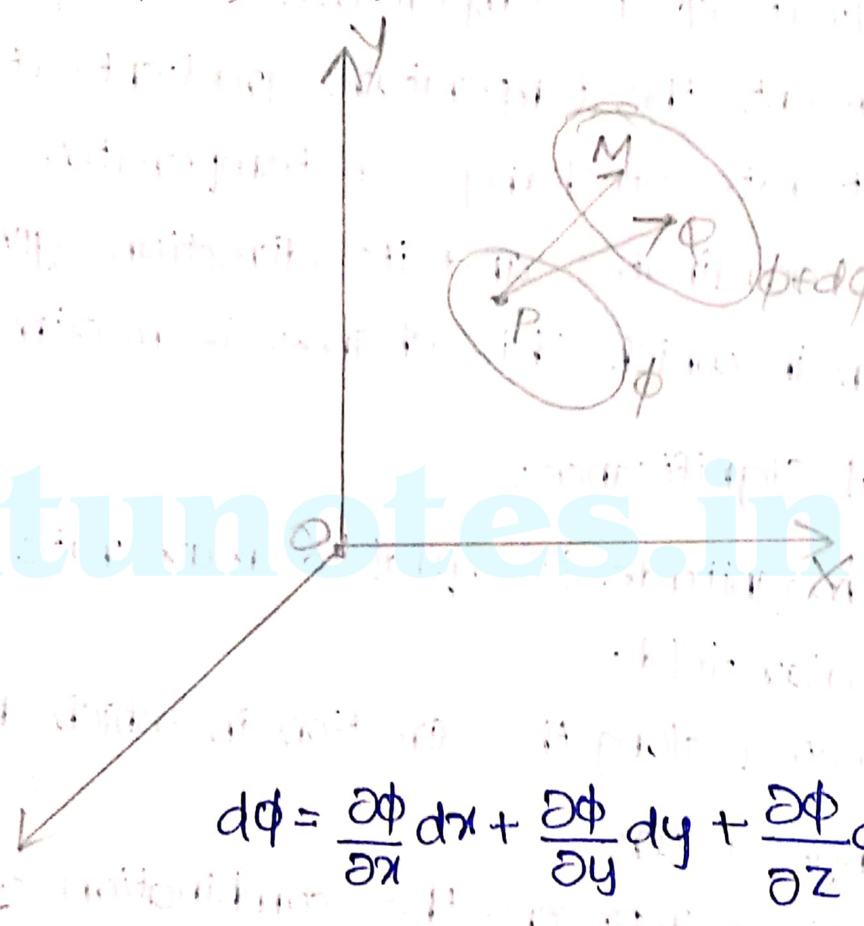
- Its magnitude is equal to max. rate of rise of scalar field.
- Directed along the direction in which max. change occurs.
- Gradient represent the combination of rate of change in x, y, z at a given point in three or higher dimensional curve.

Consider a scalar function $\phi(\vec{r})$,

where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

If a surface $\phi(x,y,z)=c$ drawn through any point such that the function has same value everywhere on the surface, it is called a level surface of the function ϕ.

Let $P(\vec{r})$ be a point on one level surface and $Q(\vec{r}+d\vec{r})$ a point on neighbouring level surface.



$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$

$$\nabla\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

$$d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\vec{\nabla}\phi \cdot d\vec{r} = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$

$\nabla\phi . dr = d\phi$

* If Q is a point on the same level surface,

$d\phi = 0$.

$\nabla\phi . dr = 0$

- $\Rightarrow$ $\nabla\phi$ is $\perp^{lar}$ to level surface.

Consider a line PM $\perp$ level surface.

$$d\phi = \nabla\phi . ds = |\nabla\phi| ds \cos\theta$$

This shows that $\nabla\phi$ is along ds or along PM (outward normal to surface).

- $\therefore$ $\nabla\phi$ is the most rapid rate of variation of ϕ in magnitude and direction (along normal)

- $$\int_A^B \vec{V} . d\vec{r} = \int_A^B \nabla\phi . d\vec{r} = \int_A^B d\phi = \phi_B - \phi_A$$

(independent of path).

$\therefore$ closed line integral of the gradient of a scalar field is zero.

$$\oint \nabla\phi . dr = \oint d\phi = 0$$

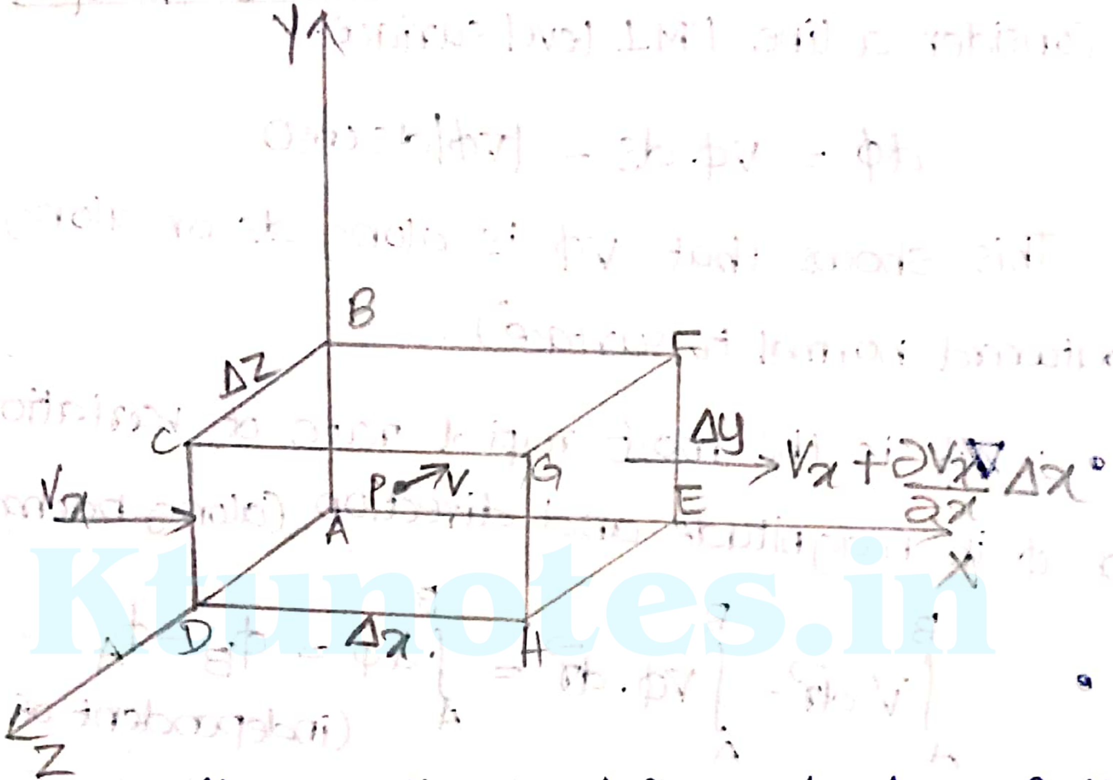
2. DIVERGENCE.

The dot product of vector differential operator ∇ with a vector function is known as the divergence of the vector function.

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

$$= \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$$



Along x-direction, influx $= V_x \times$ Area of ABCD

$$= V_x \Delta y \Delta z$$

$$\text{Outflux} = \left(V_x + \frac{\partial V_x}{\partial x}\Delta x\right)\Delta y \Delta z$$

Net outflow along x-axis.

$$= \text{Outflow} - \text{Inflow}$$

$$= \frac{\partial V_x}{\partial x}\Delta x \Delta y \Delta z$$

Along y-axis,

$$\text{Net outflow} = \frac{\partial V_y}{\partial y} \Delta x \Delta y \Delta z$$

Along z-axis,

$$\text{Net outflow} = \frac{\partial V_z}{\partial z} \Delta x \Delta y \Delta z$$

$$\text{Total outflow} = \Delta x \Delta y \Delta z \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)$$

$$= (\vec{\nabla} \cdot \vec{V}) \Delta x \Delta y \Delta z.$$

Thus, divergence of a vector function at a point is the net outflow per unit volume in unit time at that point.

Physical significance:

- Valid at a point.
- gives the rate per unit volume at which a physical quantity is emanating from that point.
- Divergence of a vector field is:
  - * positive – at source point
  - * negative – at sink point
  - * zero – where there is neither sink nor source.
- In fluid dynamics,

$$\nabla \cdot (\rho V) = -\frac{\partial \rho}{\partial t}$$

* Divergence (+ve) – Fluid expands, density ↓ses.
* Divergence (-ve) – Fluid contracts, density ↑ses.
* Divergence (0) – Incompressible fluid ($\nabla \cdot \vec{V}=0$)

A vector such as $\vec{V}$ whose divergence is zero is called solenoidal.

3. CURL:

The cross product of differential operator ∇ with a vector point function is known as 'curl of the function'.

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

Physical Significance Of Curl

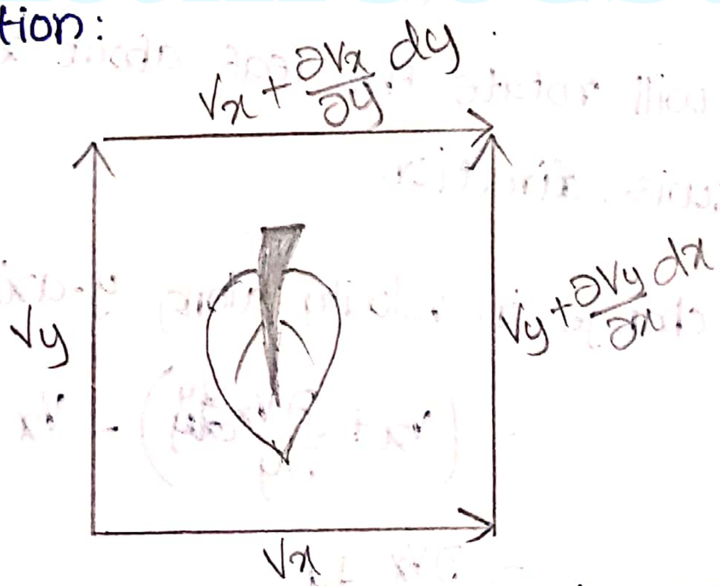
- Measure of circulation.
- There are vector fields which cannot be expressed as the gradient of a scalar point functions. For such fields, Line integral along a

closed path is not zero. The term, 'curl of a vector field' concerns such fields only.

- curl of the vector field: Value of the maximum line integral of a vector field per unit area at a certain point.

- Curl A or rot A is associated with some sort of rotation or circulation.
  - When curl A is non-zero, the vector field must have circulation.
  - When curl ~~$\nabla\times A$~~ $|\nabla\times A|=0$, there will be no circulation and the vector field is called as 'irrotational or non-curl field.

Illustration:



Consider flow of water in a river. Let

width of the river: X-axis

flow of water: Y-axis

depth of river: Z-axis

Let us visualize a floating leaf flowing down the stream.

Rate of ↑se of y-component of velocity in x-dirn.

$$= \frac{\partial V_y}{\partial x} dx$$

Increased velocity ~~in unit length~~ on right side of the leaf $= V_y + \frac{\partial V_y}{\partial x} dx$.

Change in velocity along x-axis

$$= \left(V_y + \frac{\partial V_y}{\partial x} dx\right) - V_y$$

$$= \frac{\partial V_y}{\partial x} dx$$

This will rotate the leaf about z-axis in the clockwise direction.

|||rly, change in velocity along y-axis

$$= \left(V_x + \frac{\partial V_x}{\partial y} dy\right) - V_x$$

$$= \frac{\partial V_x}{\partial y} dy$$

This will rotate the leaf about z-axis in the anticlockwise direction.

Curl

$$= V_x dx + \left(V_y + \frac{\partial V_y}{\partial x} dx\right) dy - V_y dy - \left(V_x + \frac{\partial V_x}{\partial y} dy\right) dx$$

$$= V_x dx + V_y dy + \frac{\partial V_y}{\partial x} dx dy - V_y dy - V_x dx - \frac{\partial V_x}{\partial y} dy dx$$

$$= \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) dy dx.$$

If the curl of a vector is not zero, it is rotational; otherwise irrotational.

Gauss' divergence Theorem

$$\int_S \vec{A} \cdot \vec{dS} = \int_V \vec{\nabla} \cdot \vec{A} dV$$

The surface integral of a vector function A taken over a closed surface S is equal to the volume integral of the divergence of the vector function taken over the volume V bounded by the surface.

Stokes' Theorem:

$$\int_S \text{curl} \vec{A} \cdot \vec{dS} = \oint \vec{A} \cdot \vec{dl}$$

surface integral of the curl of a vector -

~~function A taken over a surf~~

function A taken over a surface is equal to the line integral of the vector function taken over the boundary of the surface.

EQUATION OF CONTINUITY.

Equation of continuity is based on the Principle of conservation of charge.