Error Model for Scene Reconstruction from Motion and Stereo

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SUMMARY

• Analyze the performance of camera motion and stereo sensing individually.
• Model the uncertainty in the reconstruction of 3D points viewed from two vantage points.
• Express the uncertainty as a function of points' location in the scene and viewpoint change between two images.

INTRODUCTION

• Motion and stereo depth cues combination is similar to human depth perception.
• Focus on the feature measurement noise.
• Cramer-Rao lower bound (CRLB) of the range uncertainty is computed.
• Our contributions are shown as follows:

Limitations	Our contributions
Existing work is mostly algorithm-specific	The CRLB framework is general
Prior error-modeling employs continuous-time (optical flow) formulation is only reasonable for small camera motion	Employs discrete model to make the analysis applicable both to small- and large- baseline problems
Most existing work presents only empirical results	The analytic result and the closed-form solution are given

TECHNICAL APPROACH

• The cameras are well calibrated so that systematic error are negligible.
• Feature correspondences are error-free.
• The relative configuration between cameras (rotation and translation) is known.
• The only source of error is the image measurement noise.

1. Evaluation of the Cramer-Rao Lower Bound

• 3D world coordinates point $P=[X \quad Y \quad Z]^T$ is mapped to image plane point $p=[x \quad y]^T$ by the perspective camera model.
• Measurement equation is given by:

$$M = h(P) + \omega$$

where

$$h(P) = \begin{bmatrix} \frac{X}{Z} & \frac{Y}{Z} & \frac{r_1^T P + t_1}{r_3^T P + t_3} & \frac{r_2^T P + t_2}{r_3^T P + t_3} \end{bmatrix}^T \qquad \omega \sim N(0, \sigma^2 I)$$

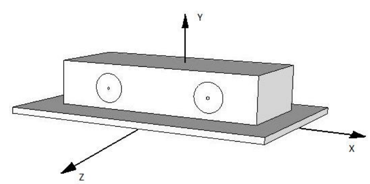
• Maximum-likelihood estimator gives the covariance matrix of the best estimator

$$\Sigma_Q = A\sigma^2 \left(H^T H\right)^{-1} A^T$$

where

$$Q = \begin{bmatrix} \frac{X}{Z} & \frac{Y}{Z} & \sqrt{X^2+Y^2+Z^2} \end{bmatrix} \qquad H = \partial h / \partial P \qquad A = \partial Q / \partial P$$

• The standard deviation of the range estimation error is given by the square root of the (3,3) element of Σ_Q.



Stereo camera model system

2. Special Case: Camera Motion along the Optical Axis

• Assume the camera orientation remains the same. Camera motion is along z axis only by t_3.

$$h_M(P) = \begin{bmatrix} \frac{X}{Z} & \frac{Y}{Z} & \frac{X}{Z+t_3} & \frac{Y}{Z+t_3} \end{bmatrix}^T$$

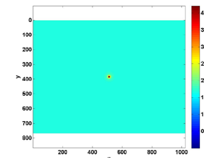
3. Special Case: Stereo reconstruction

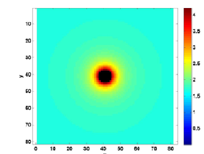
• The camera is along x axis by b.

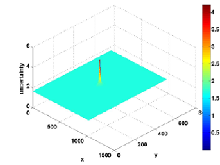
$$h_M(P) = \begin{bmatrix} \frac{X}{Z} & \frac{Y}{Z} & \frac{X+b}{Z} & \frac{Y}{Z} \end{bmatrix}^T$$

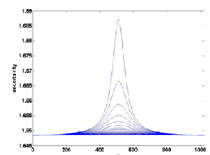
EXPERIMENTAL RESULTS

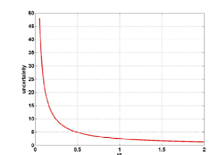
1. Range Uncertainty of Motion along Optical Axis

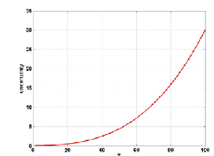

Forward motion: 2D view


Forward motion: center view

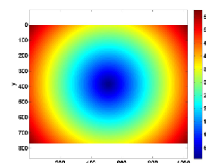

Forward motion: 3D view

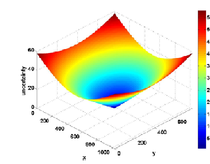

Slice of the 3D view

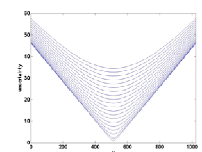

Change with respect to t_3

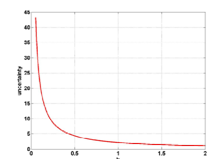

Change with respect to Z

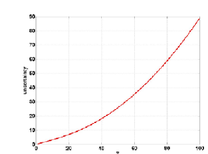
2. Range Uncertainty of Stereo Reconstruction


Stereo : 2D view


Stereo: 3D view


Slice of the 3D view


Change with respect to b


Change with respect to Z

CONCLUSIONS

• For forward-motion based reconstruction, the uncertainty of points along FOE tends to approach infinity.
• For stereo-based reconstruction, the uncertainty is worse close to the image borders.