**Math 412**            **HW1**

Due Wednesday, January 25, 2023

Solve four of the next five problems. Answers without proofs do not count.

1. # 1.1.20 in the book.

2. Consider the following four families of simple graphs: $A = \{$complete graphs$\}$, $B = \{$cycles$\}$, $C = \{$graphs whose complement has exactly two edges$\}$, $D = \{$graphs with exactly six edges$\}$. For each pair of these families, determine* all graphs that belong to both families in the pair.

3. How many independent sets of size 4 has the Petersen graph?
(Hint: Count to how many independent sets of size 4 belongs a given vertex in the Petersen graph.)

4. Determine* all simple graphs $G$ for which the adjacency matrix of $G$ equals the incidence matrix of $G$ (with an appropriate ordering of vertices and edges).

5. For $n \geq 4$ and every of the three properties below, describe all $n$-vertex simple graphs $G$ satisfying this property:
(a) every induced subgraph $H$ with 3 vertices of $G$ is isomorphic to the path $P_3$ with two edges;
(b) every path contained in $G$ is an induced subgraph of $G$;
(c) every induced subgraph of $G$ with 3 vertices has either 2 or 0 edges.

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* The word **"determine"** means that you need to prove all your claims.

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Problems below review basic concepts and their ideas could be used in the tests/quizzes.
WARMUP PROBLEMS: Section 1.1: # 2, 4, 5, 7, 9. Do not write these up!
OTHER INTERESTING PROBLEMS: Section 1.1: # 13, 18, 19, 21, 22, 24, 26, 29, 37. Do not write these up!