# Math 447 Midterm I

September 29, 2011

1. (20 points) Prove that in a field $\mathbb{F}$ the following hold:

   (a) $a \cdot 0 = 0 \cdot a = 0, \ \forall a \in \mathbb{F}$. Deduce that if $0 = 1$ then the field has only one element.

   (b) $a \cdot b = 0$ if and only if $a = 0$ or $b = 0$.

   (c) $-(-a) = a, \ \forall a \in \mathbb{F}$.

   (d) $(-1) \cdot a = -a, \ \forall a \in \mathbb{F}$.

   **Note:** in this problem you can only use the axioms of a field!

2. (20 points) Show that the axiom of completeness implies the Archimedean property.

3. (20 points) Show that the limit of a sequence of real numbers is unique.

4. (20 points) Compute the following limits:

   (i) $\lim_{n\to\infty} \frac{n}{n+1}$

   (ii) $\lim_{n\to\infty} \frac{n^2+3n+1}{n^3+1}$

   (iii) $\lim_{n\to\infty} \frac{n^4+1}{n^3+1}$

   (iv) $\lim_{n\to\infty} \frac{2^{\frac{1}{n}}}{\sqrt{n}}$

   Justify your answers.

5. (20 points) Find the set of subsequential limits (limit points) for the sequence:

$$s_n = (-1)^n + \frac{1}{n}$$

   What is $\limsup s_n$ and $\liminf s_n$? Justify your answers.