Business Memo for Case Muddy River

Team D: Xuewei Yu, Yuxian Zhang, Xiaoqi Zhang, Kai Mei, Yakun Deng

Executive Summary

In this report, we carry out a real option valuation analysis for Muddy River Power plant. We evaluate its net operating earning for a period of two years, from January 2016 through December 2017. Firstly, we use bootstrap methodology to calibrate the weekly RN power and gas spot price processes. Secondly, we model the plant's operating policy, set optimized on-and-off thresholds using the dynamic programming methodology based on **Bellman Equation** and **Longstaff and Schwartz**.

Contents

1 Calibration methodology and results

Based on power market conditions, the power price will have a spike when the market clears on Power price process consists of two components: the continuous process component CC_t and jump process component JC_t. Gas price is just the continuous process G_t.

$$\begin{cases} dCC_t = \alpha_{CC}[\theta(t) - CC_t)]dt + v(t)CC_t dW_t \\ dJC_t = \alpha_{JC}[0 - JC_t)]dt + m dq_t \\ dG_t = \alpha[\theta(t) - G_t)]dt + \sigma G_t dW_t \end{cases} \tag{1}$$

We need to calibrate $\theta(t)$ for both power prices and gas process and we use bootstrap methodology to choose $\theta(t)$ that minimize the sum of absolute error between the true prices and results of our pricing simulation algorithms. We calibrate in two steps.

1.1 Calculate Power and Gas Prices

We use Monte Carlo simulation to generate Power and Gas prices paths and also the jump process of power prices:

$$\begin{cases} CC_{t+\Delta t} = \alpha_{CC}(\theta_{CC}(t) - CC_t)\Delta t + V(t)CC_t\Delta W_t + CC_t \\ JC_{t+\Delta t} = \alpha_{JC}(0 - JC_t)\Delta t + m\Delta q_t + JC_t \\ P_{t+\Delta t} = CC_{t+\Delta t} + JC_{t+\Delta t} \\ G_{t+\Delta t} = \alpha_G(\theta_G(t) - G_t)\Delta t + \sigma G_t \Delta B_t + G_t \\ Corr(\Delta W_t, \Delta B_t) = \rho \end{cases} \tag{2}$$

$$\text{where } \Delta q_t = \begin{cases} 1, \text{ with probability 0.083} \\ 0, \text{ with probability 0.917} \end{cases}$$

We generate 10000 paths with 96 steps between start and end time, each step represents one week. Our input parameters are as follows given by the case file:

CC_0	JC_0	G_0
35	0	3

1.2 Optimization

Because the power and gas prices are calculated by simulation, the objective function is too irregular for most solvers, thus we use grid search for the optimal parameter. We set the candidate range for each parameter as follow and we take 0.1 as step size for each parameter and find the best one.

$$\begin{cases} \theta_{Power} : [0, 300] \\ \theta_{Gas} : [0, 10] \end{cases} \tag{3}$$

We assume thetas for Power and Gas are step functions, which is consisted of 12 horizontal line segments with monthly jumps. We use the bootstrap methodology to calibrate theta for both Power and Gas. Here is the methodology: In the first month, we find the optimized θ_1 by grid search over all θ_1 points to minimize the absolute error between first week's true price and simulated price. After determining θ_1, we use it to calibrate θ_2. We simulate prices until second months, and find θ_2 which minimizes the sum of absolute error between first two week's true price and simulated price. Using this method, we calculate until month 12.

1.3 Result

We can get theta parameter for Power as follows:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}
40	41.5	41	40.5	37.5	75	55	95	72	44	35.5	28.5

We can get theta parameter for Gas as follows:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}
2.55	2.8	2.65	2.15	5.65	5.2	6.25	7.3	2.55	2.55	2.4	2.05

With calibrated parameters, we simulate Power and Gas prices and compare with real prices in Exhibit1. Here is the our prices result:

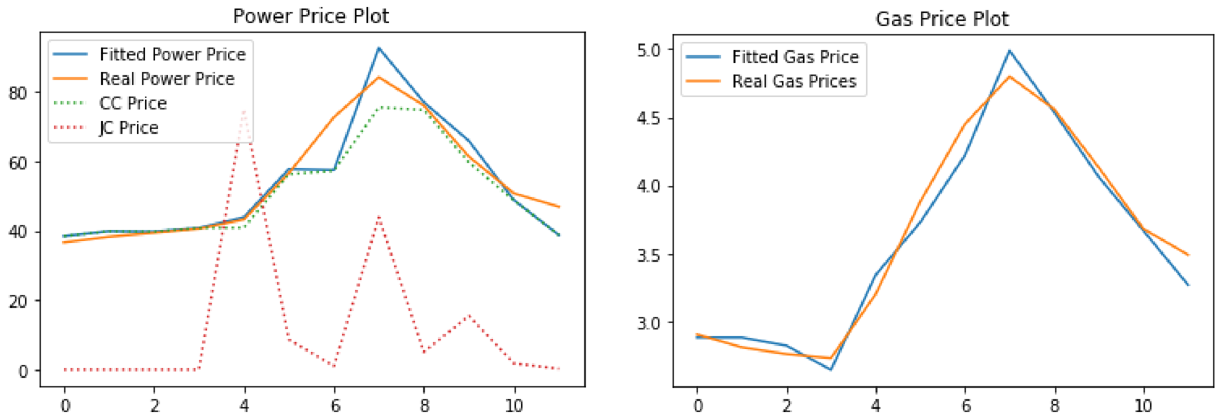
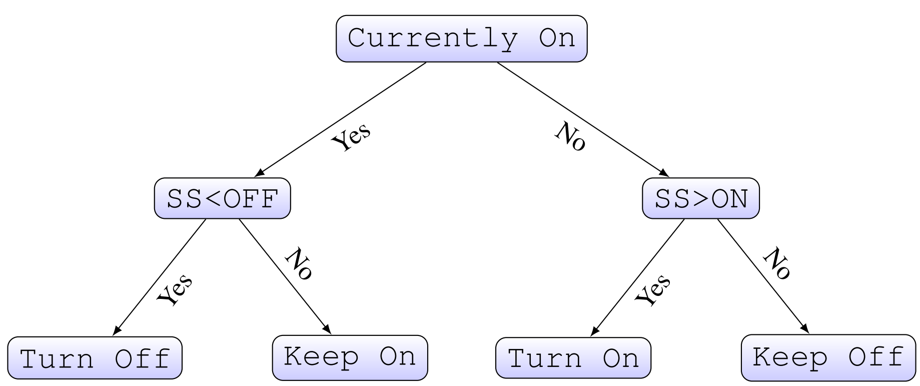


Figure 1: Power and Gas Prices

2 Operating Policy

In this part we deduce Muddy River's operation policy with Bellman Equation and Longstaff Schwartz method. Muddy River's decision is based on spark spread (SS), current state and two thresholds: turn on and turn off threshold, (ON and OFF). Detailed decision rule is shown as below:



We find optimal ON and OFF to maximize the expected revenue. We optimize under a dynamic setting where Muddy River has a different pair of threshold at each week. We use backward induction **(Bellman Equation)** to find optimal threshold from the last week to the first.

Some Notation

s_t : state of the power plants at t (2-dimension) $\{$[1,open], [1,start], [0,close], [0,end]$\}$

ss_t : spark spread at t ($ss_t = P_t - 12G_t$)

a_t : action at stage t based on the on/off thresholds

w_t : random disturbance between stage t and stage t+1

K_{on}/K_{off} : on/off thresholds

C : cost for the first starting week ($C = 3 * 1000 * 16 * 7.6$)

R_t : earning if the plant is working ($R_t = (ss_t - 5) * 1000 * 16 * 7.6$)

$J_t(\cdot)$: maximized value (value-to-go) of the tail problem starting at stage t (including t)

$g_t(\cdot)$: earnings in the current stage

$f_t(\cdot), h_t(\cdot)$: law of motion defined below

Optimality Principle **(Bellman Equation)**

$$\begin{aligned} J_t(s_t) &= \max_{a_t} \mathbb{E}_t[g_t(s_t) + e^{-r\Delta t} J_{t+1}(s_{t+1}, ss_{t+1})] \\ &= \max_{a_t} \mathbb{E}_t[g_t(s_t) + e^{-r\Delta t} J_{t+1}(f_t(s_t, ss_t, a_t, w_t), h_t(ss_t, w_t)))] \end{aligned} \tag{4}$$

$$g_t(s_t) = \begin{cases} -C, s_t = [\ 1,\ \text{start}\ ] \\ 0, s_t = [\ 0,\ \text{close}\ ] \\ R_t, s_t = [\ 1,\ \text{open}\ ] \text{ or } [\ 0,\ \text{end}\ ] \end{cases}$$

Law of Motion

$$s_{t+1} = f_t(s_t, ss_t, a_t, w_t)$$

$$ss_{t+1} = h_t(ss_t, w_t)$$

where:

$$f_t(s_t, ss_t, a_t, w_t) = \begin{cases} [\,1,\ \text{start}\,],\ s_t[0] = 0, a_t = 1\ (ss_t > K_{on}) \\ [\,0,\ \text{close}\,],\ s_t[0] = 0, a_t = 0\ (ss_t \leq K_{on}) \\ [\,1,\ \text{open}\,],\ s_t[0] = 1, a_t = 1\ (ss_t \geq K_{off}) \\ [\,0,\ \text{end}\,],\ s_t[0] = 1, a_t = 0\ (ss_t < K_{off}) \end{cases}$$

$h_t(ss_t, w_t)$: combination of Power process (dP_t) and Gas process (dG_t)

In the equation (4), since the sequential decision problem is stochastic, we need to compute the expectation of the value-to-go (overall net operating earning starting at stage t). To get the expectation, theoretically we need to simulate another n paths at stage t. However, in this step, we adopted the methodology in the paper by **Longstaff and Schwartz**[1]. More specifically, to approximate the expectation, we use regression on the linear combination of basis functions, and use path data to estimate the parameters of those basis functions.

There are many choices for families of sets of basis functions, e.g. polynomials, sin/cos (Fourier representation), Laguerre, etc. In their core methodology, Longstaff and Schwartz use the Laguerre polynomials, and

$$E_t[g_t(s_t) + e^{-r\Delta t} J_{t+1}(s_{t+1}, ss_{t+1})] = \sum_{i=1}^{m} b_i L_i(ss_t)$$

where L_i is the ith Laguerre polynomial and the b_is are the coefficients that are estimated using ordinary least squares.

In the paper where they simulate American Options, Longstaff and Schwartz obtained the estimator based on in-the-money paths. Intuitively this makes sense since we are interested in estimating the expectation conditional on the current state and the event that the option is in the money. If using all paths, and hence, not conditioning on the event that the option is in the money, they proved that estimates of the conditional expectation function have larger standard errors than those obtained by using all of the conditioning information. To build on that, in this paper, we also divided our scenarios into several conditions based on the value of s_t and obtain corresponding conditional expectation by regression. So modifying the above regression equation, we have the following formula.

$$E_t[g_t(s_t) + e^{-r\Delta t} J_{t+1}(s_{t+1}, ss_{t+1})|s_t] = \sum_{i=1}^{m} b_i L_i(ss_t)$$

where $s_t \in$ {[1,open], [1,start], [0,close], [0,end]}

<u>Basis Function</u>

$$\begin{aligned} L_0(X) &= exp(-X/2) \\ L_1(X) &= exp(-X/2)(1 - X) \\ L_2(X) &= exp(-X/2)(1 - 2X + X^2/2) \\ L_n(X) &= exp(-X/2)\frac{e^x}{n!}\frac{d^n}{dX^n}(X^n e^{-X}) \end{aligned} \tag{5}$$

<u>Regression Steps</u>

1. Let $Y = g_t(s_t) + e^{-r\Delta t} J_{t+1}(s_{t+1}, ss_{t+1})$, $X_{list} = [L_0(ss_t), L_1(ss_t).L_2(ss_t), L_n(ss_t)]$
2. Divide Y into 4 groups based on the value of s_t: $Y_{[1,open]}$, $Y_{[1,start]}$, $Y_{[0,close]}$, $Y_{[0,end]}$
3. Linear regression of Y_j on X_{list} to get b_{ji}, $j \in$ {[1,open], [1,start], [0,close], [0,end]}
4. $\hat{Y}_j = E_t[g_t(s_t) + e^{-r\Delta t} J_{t+1}(s_{t+1}, ss_{t+1}|s_t = j)] = \sum_{i=1}^{m} b_{ji} L_{ji}(ss_t)$

   $j \in$ {[1,open], [1,start], [0,close], [0,end]}

2.1 Numerical Result

We applied backward induction to obtain the optimal threshold for each period and calculate the expected profit in 2 years with out of sample gas and power price. In order to eliminate the effect of biased termination value, we optimize over three year horizon and check its profit in the first two years. The threshold and profit is shown as below. The profit of Muddy River is expected to be $104,422,783.

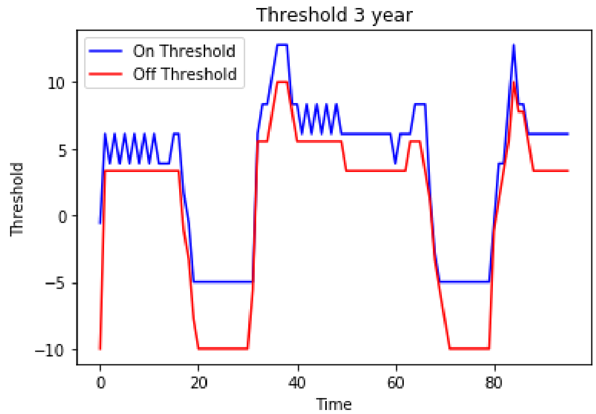


Figure 2: Threshold

The reason to use three year horizon is to avoid the effect of the biased termination value. Termination value affects the strategy in the last few stages but is unknown. We also compared the profit of two, three and four years' horizon. The expected profit is as below:

Horizon (year)	2	3	4
Profit ($m)	104.11	104.42	104.58

Table 1: Horizon and Profit

3 Valuation

In this section, we use forward simulation methodology to calculate the final net operating earnings value. After we determine the optimized on-and-off thresholds, we turn on the plant if the next periods profit is above upper bounds and turn off the plant if the next periods profit is below upper bounds. Then we discount the profits to the initial time point.

4 Numerical Sensitivity Analysis

In this section, we did several sensitivity analysis on our calibration and pricing results. By changing our model's each parameter input, we want to see whether the model have robust outputs.

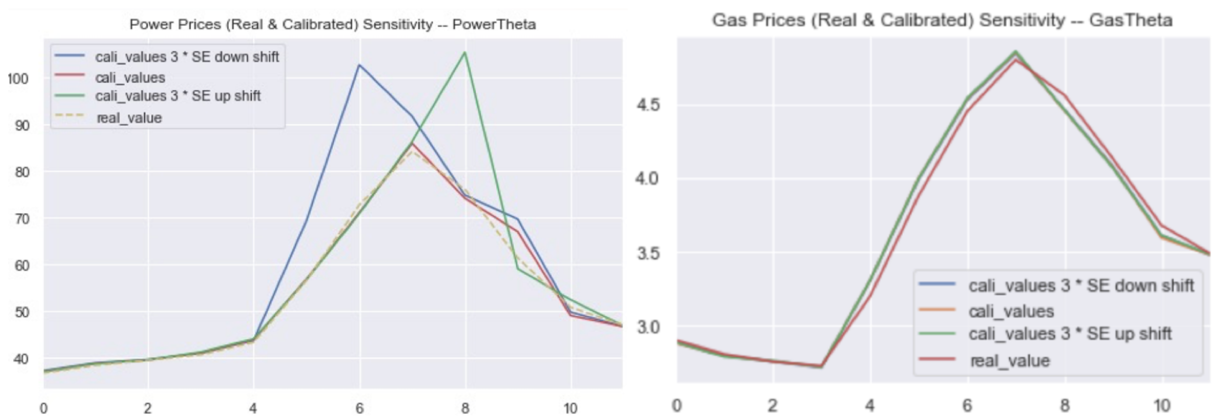


Figure 3: Sensitivity Illustration Parameters

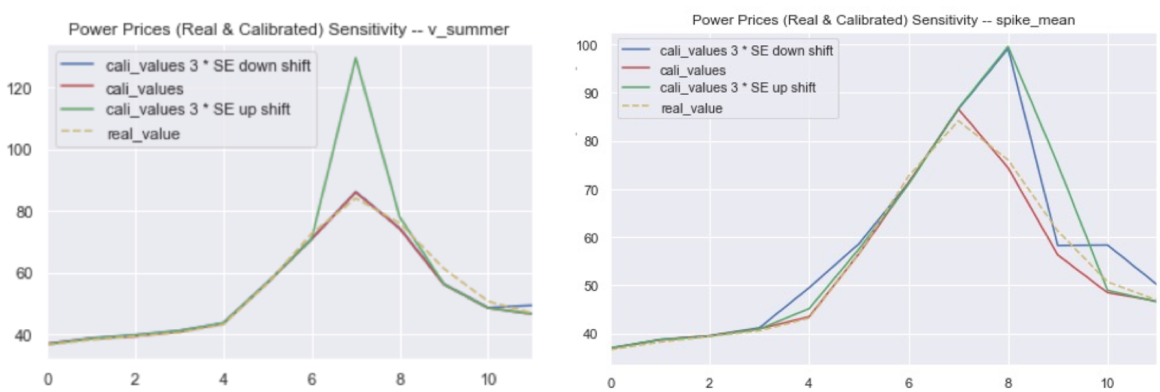


Figure 4: Other Selected Sensitivity Illustration Parameters

We tried out different parameters input into our calibration model to test whether the random disturbance on the the estimation of model parameters of will make a huge difference on the calibration model's output.

As we can see from the Figure 3&4, the calibrated values and model's real values are close to each other, but when we add some random disturbance onto it, we can see some large result shift on the model's output. The Gas process's calibration is relative stable than the Power process.

To evaluate the parameter's sensitivity with respect to the final valuation estimation, we get the follow **Percentage Mean Absolute Deviation**(and its standard error of this deviation) for all the parameters:

$$v_1, v_2, ...v_n \in [v - 3SE, v + 3SE]$$

$$PMAE = \frac{1}{n}\sum_{i=1}^{n}|\frac{P_i - P_0}{P_0}|$$

where P_i is final profits valuation

	v_{summer}	v_{other}	v_{gas}	m	ρ	q_t	θ_{power}	θ_{gas}
PMAE	0.14	0.17	0.17	0.16	0.13	0.16	0.18	0.16

For the final result, we also conducted another sets of sensitivity analysis. Through this, we want show the influence of parameters estimation error on the final model output. As we can see from the table, the mean percent deviation is always within the acceptable range ($\pm 1\%$)

Conclusion

In conclusion, we apply bootstrap methodology to calibration parameters and use dynamic threshold operating policy to calculate final valuation results. For long-term strategy, net operating earning for Muddy River Power plant is 93.98 Million, which is largely above 24 Million. This result is robust after we apply Model and Valuation Sensitivity Analysis.

Calibration Questionnaire

1. What are the values of the parameters of your calibrated gas and power processes?

Theta parameter for Power as follows:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}
40	41.5	41	40.5	37.5	75	55	95	72	44	35.5	28.5

We can get theta parameter for Gas as follows:

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}
2.55	2.8	2.65	2.15	5.65	5.2	6.25	7.3	2.55	2.55	2.4	2.05

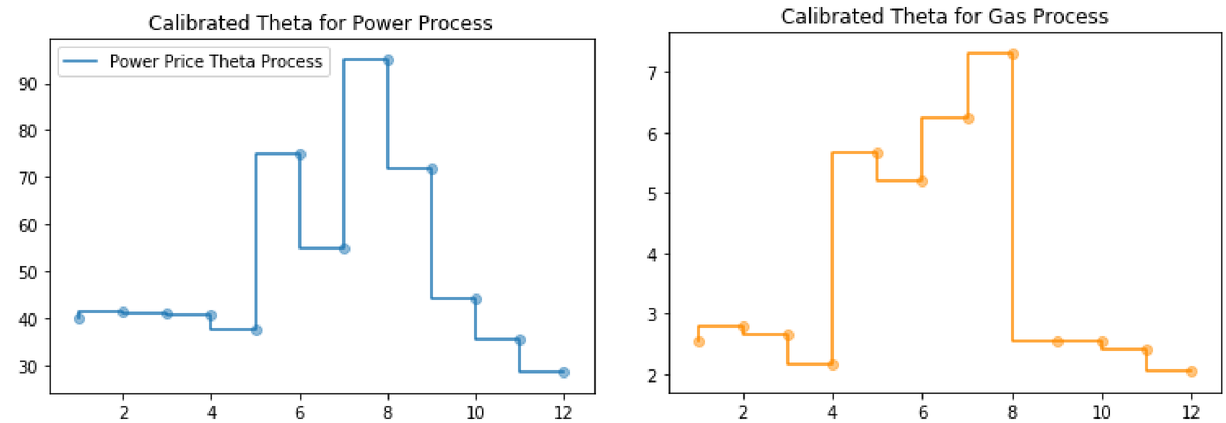


Figure 5: Power and Gas Calibrated Theta Processes

2. What are the cross-path means and standard deviations for weekly gas, power prices, and spark spreads over the deal?

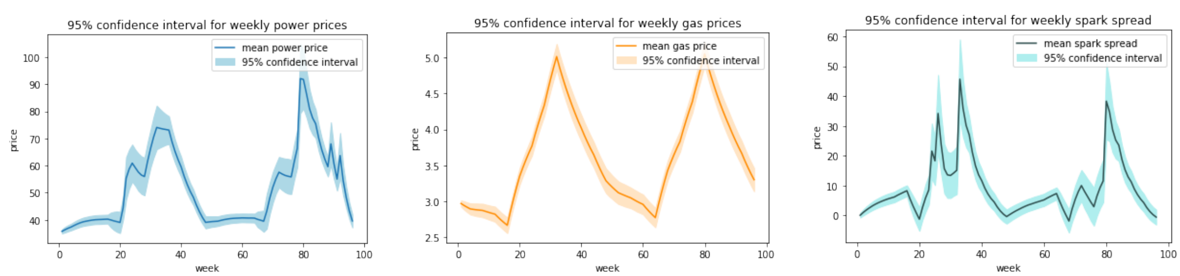


Figure 6: Power, Gas, Spark Spread Weekly Price Cross-Path mean and Standard Deviation

3. What is your operating policy?

Muddy River operates based on two thresholds – On and Off and detailed decision rule is shown below:

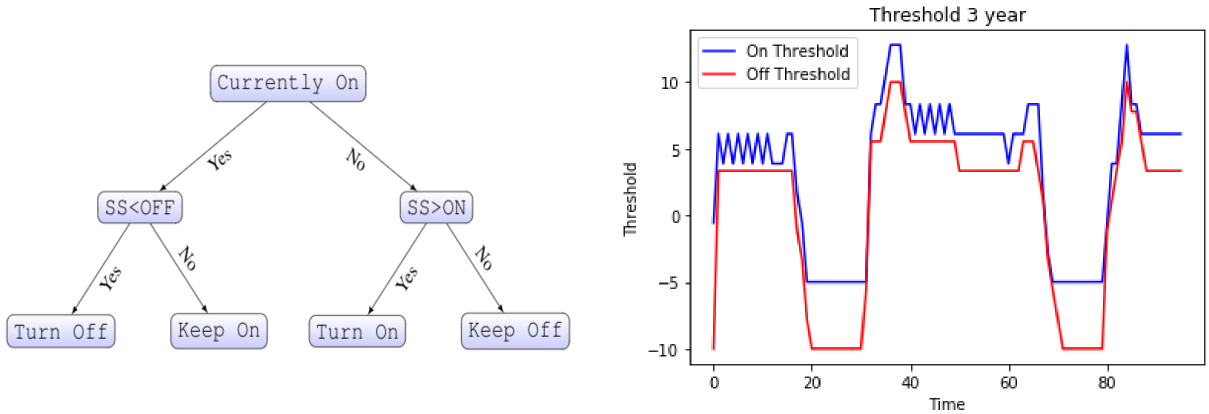


Figure 7: Decision rule and thresholds

References and Notes

[1] Göran Svensson Longstaff and Schwartz models for American Options Master Thesis, Department of mathematics Royal Institute of Technology, 06/02/2004.