Homework2

Exercise 1:

a. $T(n) = n \log n + 3n + 2$

$\Rightarrow O(n \log n)$

b. $T(n) = n \log(n!) + 5n^2 + 7$

Proved that: $n! \leq \frac{n^{n+1}}{e^{n-1}}$

$\Rightarrow \log(n!) \leq log(n^{n+1}) - log(e^{n-1})$

WLOG, consider that $\log \approx \ln$

$\Rightarrow log(n!) \leq (n+1) \log n - n + 1$

$\Rightarrow n \log(n!) + 5n^2 + 7 \leq n(n+1) \log n - n^2 + n + 5n^2 + 7$

$= n^2 \log n + 4n^2 + n \log n + n + 7$

$\Rightarrow O(n^2 \log n)$

c. $T(n) = 1000n + 0.01n^2$

Regardless 0.01 is an small const, $T(n) = O(n^2)$

d. $T(n) = 100n \log n + n^3 + 100n$

Because $n^3 \geq 100n \log n$ when $n \geq N_0$ with (N_0 big enough)

$\Rightarrow O(n^3)$

e. $T(n) = 0.01n \log n + n(\log n)^2$

$\Rightarrow O(n \log^2 n)$

Exercise 2:

a. Because of only 1 for loop $\Rightarrow$ O(n)

b. There are $n/2$ operator in this loop, but time complexity is still $O(n)$

c. The size of 2 for loop is $n \times (n-1)/2 \Rightarrow O(n^2)$

d. O(n)

e. 3 for loop, with $n \times n \times (n-1)/2 \Rightarrow O(n^3)$